Title: The quantum sine-Gordon model with quantum circuits

Speakers: Ananda Roy

Series: Quantum Fields and Strings

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Abstract: Analog quantum simulation has the potential to be an indispensable technique in the investigation of complex quantum systems. In this work, we numerically investigate a one-dimensional, faithful, analog, quantum electronic circuit simulator built out of Josephson junctions for one of the paradigmatic models of an integrable quantum field theory: the quantum sine-Gordon (qSG) model in 1+1 space-time dimensions. We analyze the lattice model using the density matrix renormalization group technique and benchmark our numerical results with existing Bethe ansatz computations. Furthermore, we perform analytical form-factor calculations for the two-point correlation function of vertex operators, which closely agree with our numerical computations. Finally, we compute the entanglement spectrum of the qSG model. We compare our results with those obtained using the integrable lattice-regularization based on the quantum XYZ chain and show that the quantum circuit model is less susceptible to corrections to scaling compared to the XYZ chain. We provide numerical evidence that the parameters required to realize the qSG model are accessible with modern-day superconducting circuit technology, thus providing additional credence towards the viability of the latter platform for simulating strongly interacting quantum field theories.

The Quantum sine-Gordon Model with Quantum Circuits

Ananda Roy

Technical University Munich

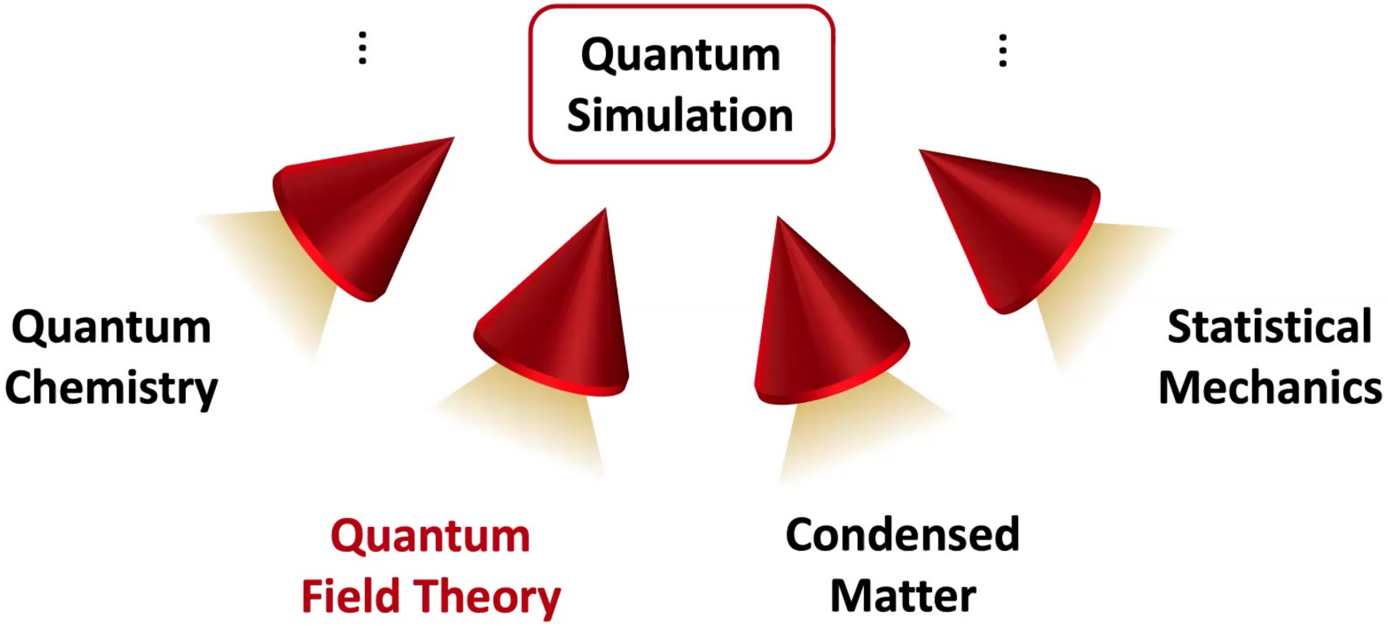
Perimeter Institute, Feb 23, 2021


Image: BBC UK

What kind of computer are we going to use to simulate physics?

… you can simulate this with a quantum system, with quantum computer elements. It's not a Turing machine, but a machine of a different kind.

Feynman (1982)



⋮

Quantum Simulation

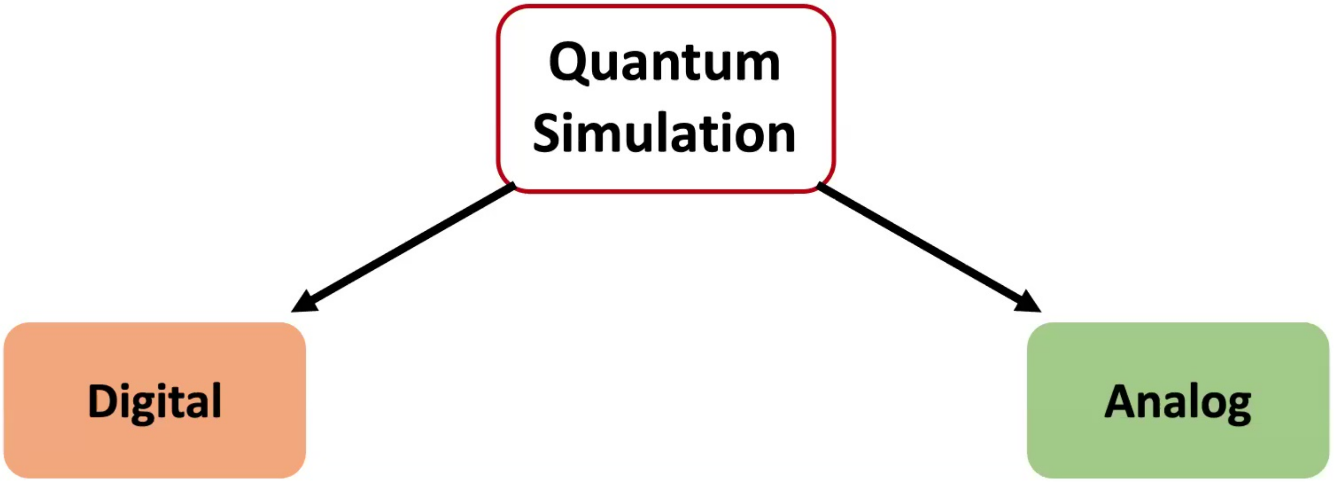
⋮

Quantum Chemistry

Quantum Field Theory

Condensed Matter

Statistical Mechanics



Quantum Simulation

Digital

Analog

A universal quantum computer is also a universal digital quantum simulator

Lloyd (1996)

<u>Needs Quantum Error Correction</u>

Shor (1995), Steane (1996)

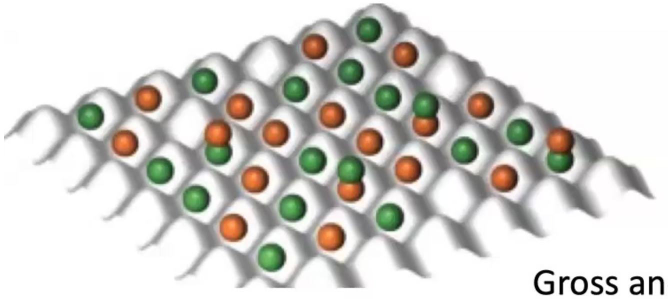
Trade-off universality for near-term achievability

<u>Does not need Quantum Error Correction</u>

Wide range of platforms available

What kind of analog simulator are we going to use to simulate physics?

Cold atoms

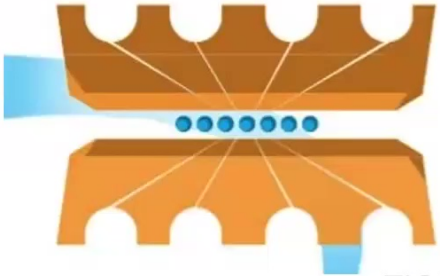


Gross and Bloch (2017)

strongly correlated systems, topological phases, gauge theories, …

Munich, Harvard, Boulder, Rutgers, …

Trapped ions

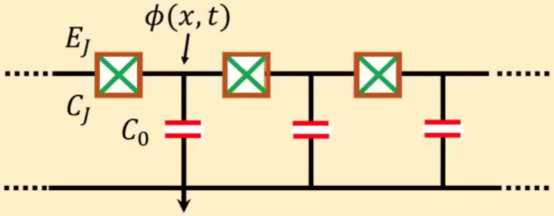


TIQI, UMD

Quantum magnetism, open quantum systems, …

Maryland, Innsbruck, …

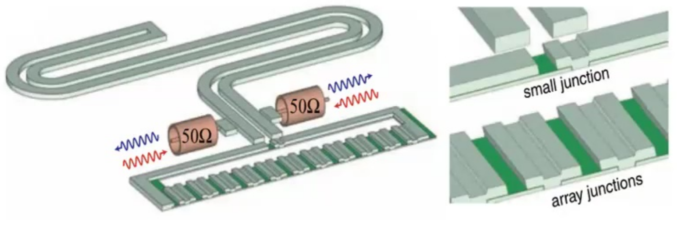
Quantum circuits



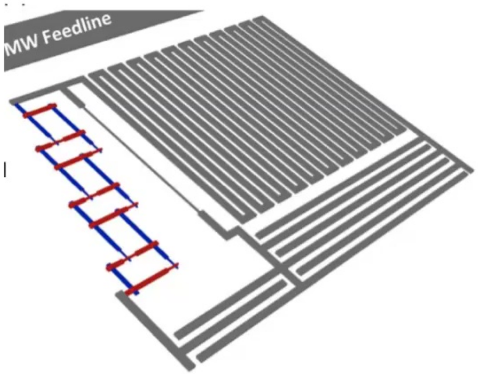
Impurity systems, bosonic many-body systems, …

Yale, Rutgers, Maryland, Princeton, Grenoble, …

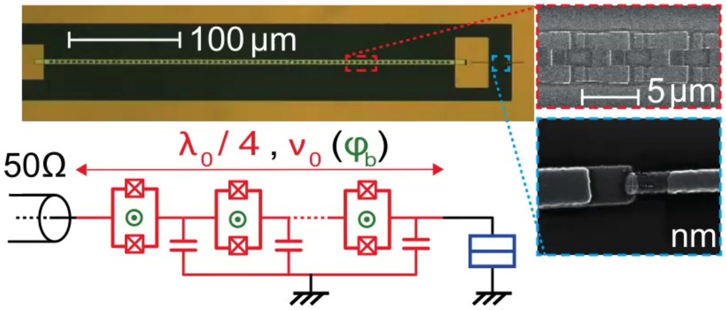
Quantum circuits as analog quantum simulators



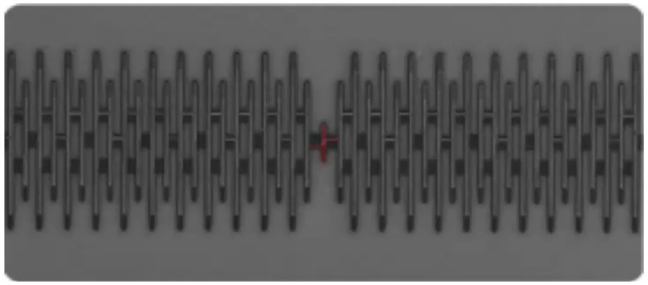
Devoret group (Yale, 2009): $N = 43$



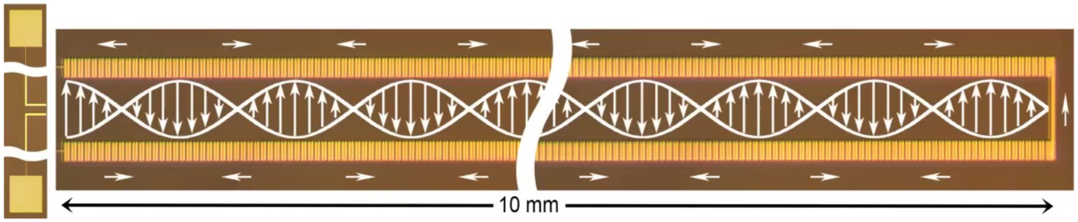
Gershenson group (Rutgers, 2012): $N = 6,\ 24$



Esteve group (Saclay, 2013): $N \sim 100$



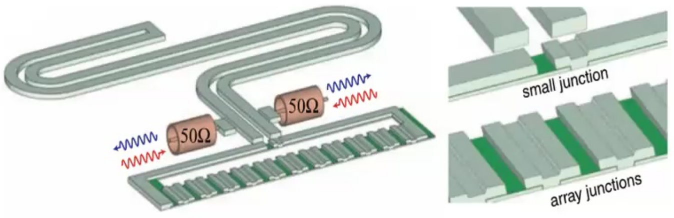
Roch group (Grenoble , 2019): $N \sim 1500$



Manucharyan group (Maryland, 2018, 2019): $N \sim 33000$

Early experiments: Delft (1990-s)

Quantum circuits as analog quantum simulators



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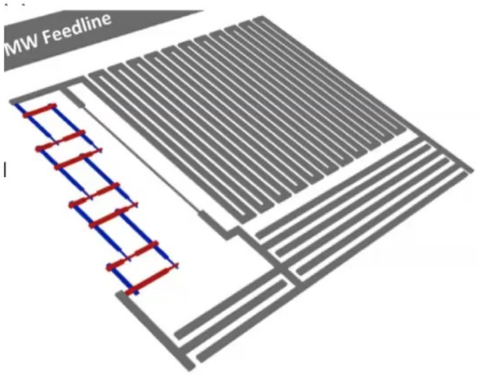

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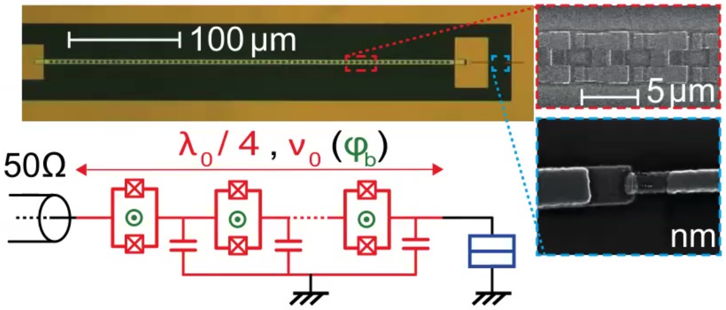

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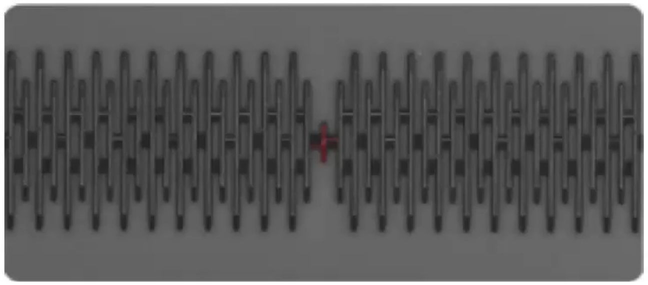

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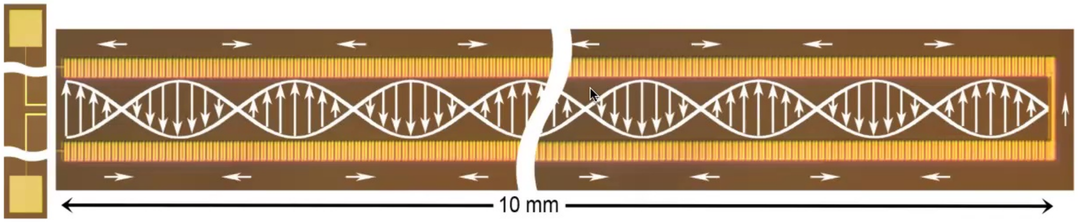


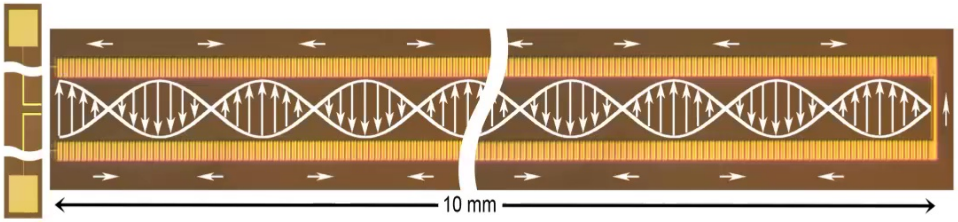
Manucharyan group (Maryland, 2018, 2019):
$N \sim 33000$

Probing strongly-interacting quantum many-body systems

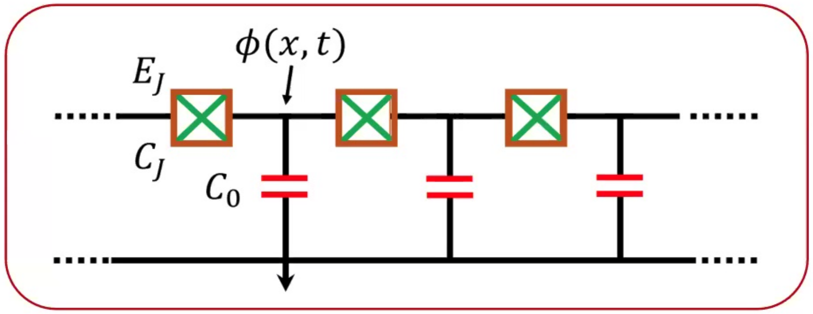
Early experiments: Delft (1990-s)

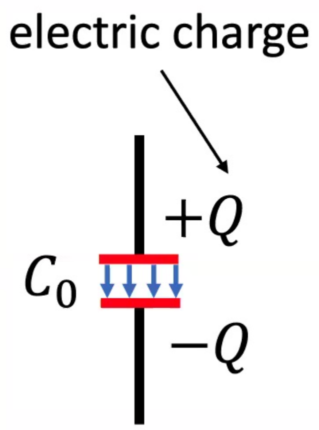
The Bose-Hubbard model with quantum circuits

Experimental Setup: Maryland group (2018)



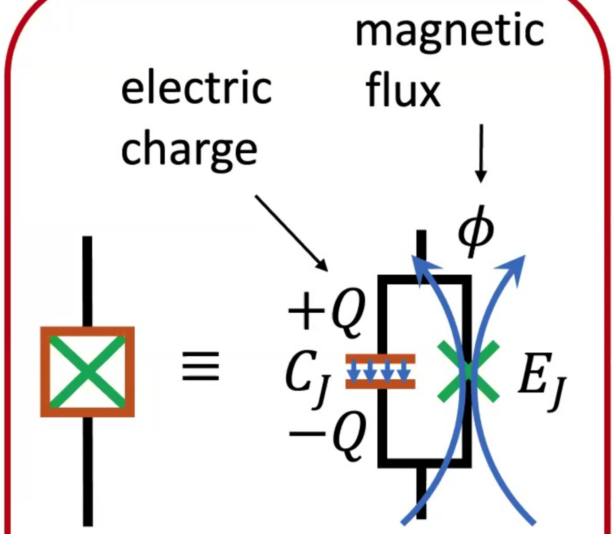
Equivalent quantum circuit





Capacitor

$$H = \frac{Q^2}{2C_0}$$



Josephson junction

$$H = \frac{Q^2}{2C_J} - E_J \cos\frac{\phi}{\varphi_0}$$

$$[\phi, Q] = i\hbar$$

$\varphi_0 = \hbar/2e$

The Bose-Hubbard model with quantum circuits

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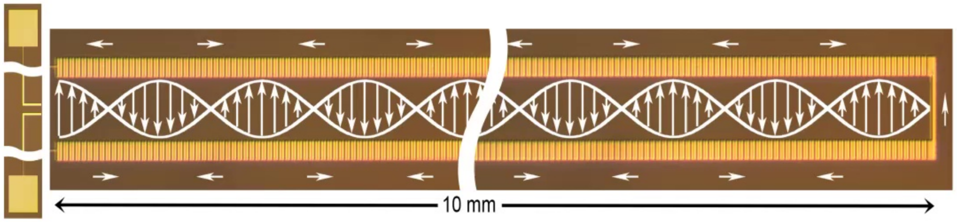
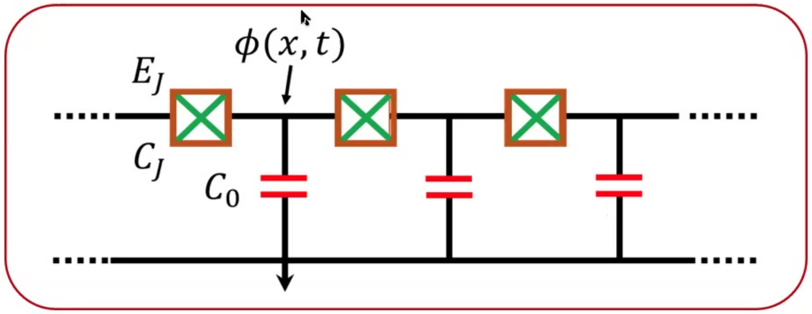


Quantum circuit, first step: without disorder



$$E_{C_0} = \frac{2e^2}{C_0}$$

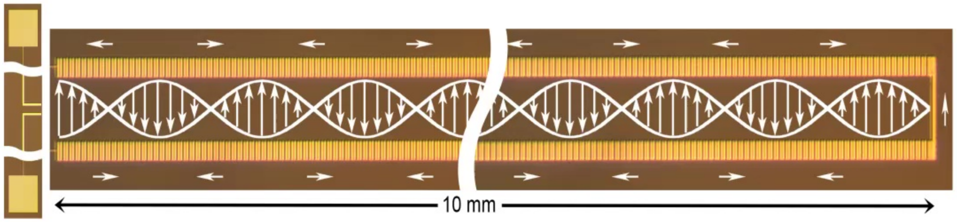
$$\epsilon < 1$$

$$H_{\text{circuit}} = E_{C_0} \sum_i n_i^2 + \epsilon\, E_{C_0} \sum_i n_i n_{i+1} - E_g \sum_i n_i - E_J \sum_i \cos(\phi_i - \phi_{i+1})$$

onsite repulsion nearest-neighbor repulsion `chemical potential’

The Bose-Hubbard model with quantum circuits

Experimental Setup: Maryland group (2018)



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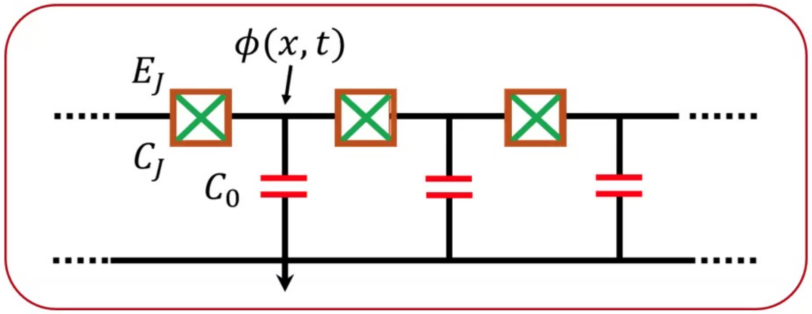

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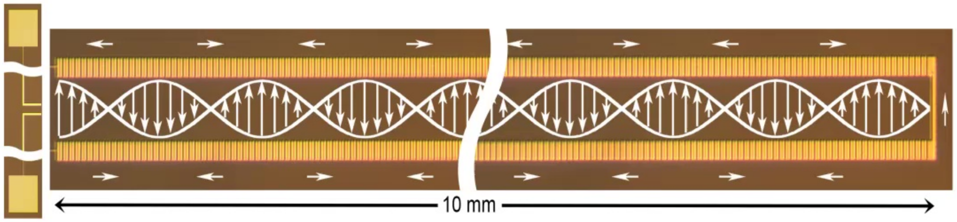
$$[n_i, e^{\pm i\phi_j}] = \pm e^{\pm i\phi_j}\, \delta_{ij}$$

Bradley and Doniach (1984), Korshunov (1989), Bruder *et al* (1993), Glazman and Larkin (1997)

The Bose-Hubbard model with quantum circuits

Experimental Setup: Maryland group (2018)



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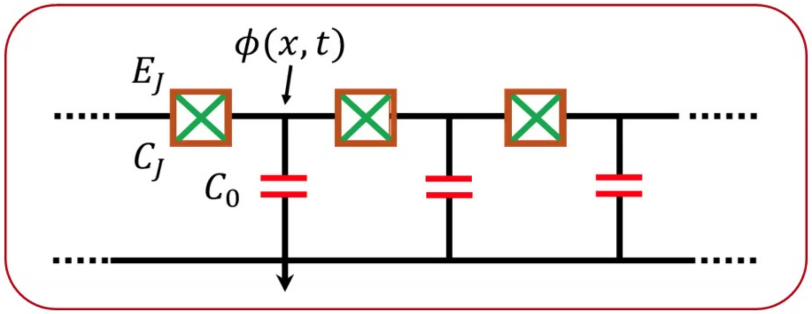

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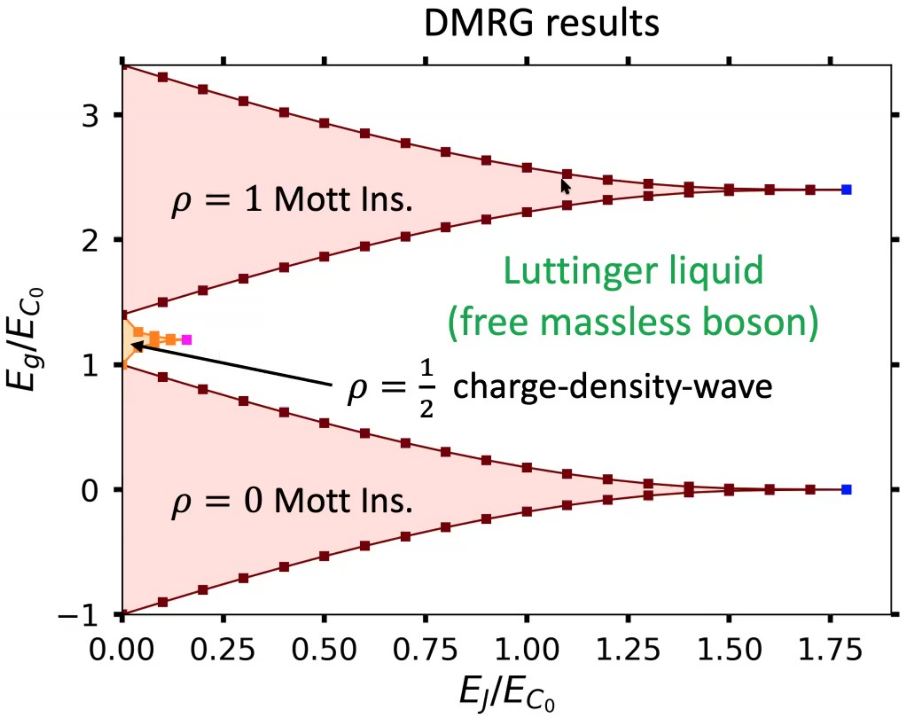

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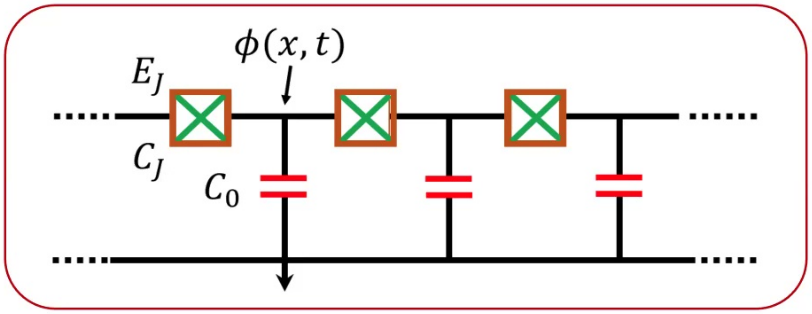
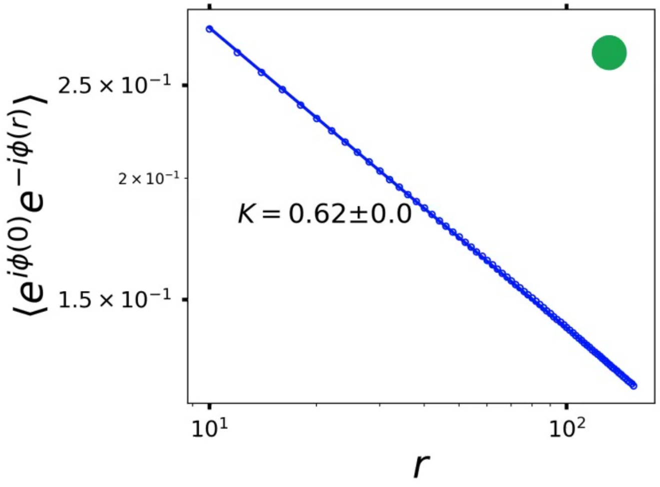
AR *et al*, J. Stat. Mech. (2020)

The Bose-Hubbard model with quantum circuits

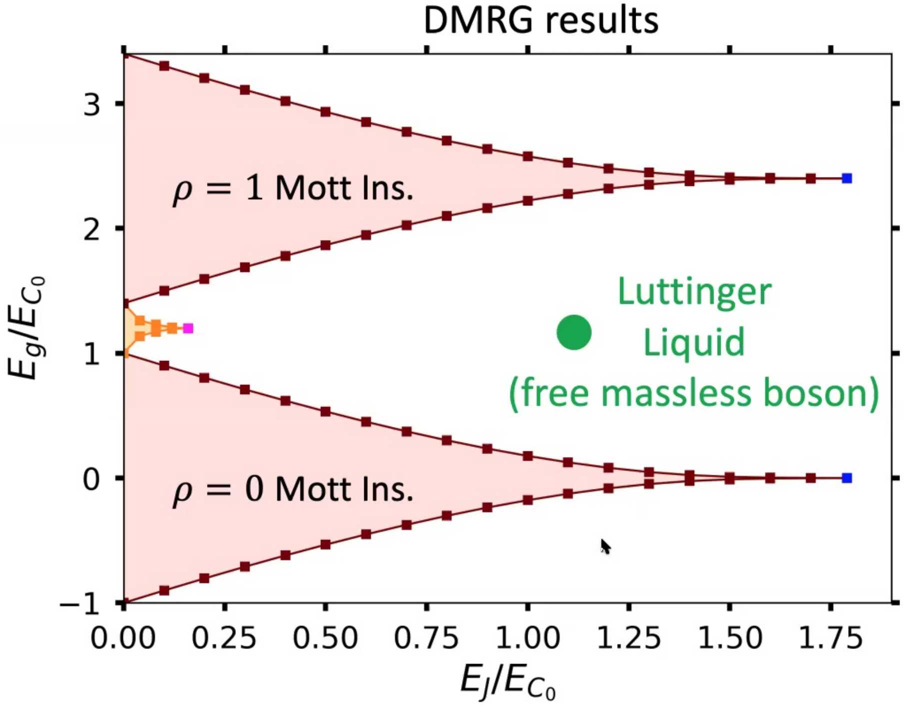
Quantum circuit



Luttinger liquid behavior: $\langle e^{i\phi(0)} e^{-i\phi(r)} \rangle \sim \frac{1}{r^{K/2}}$



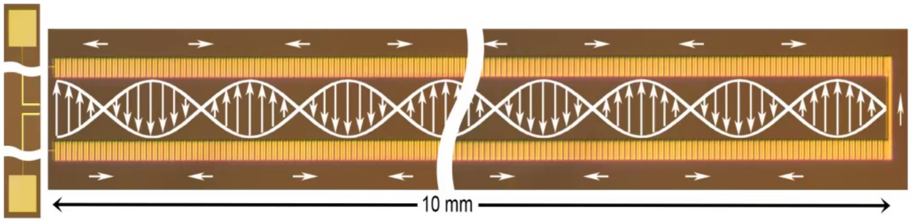
DMRG results



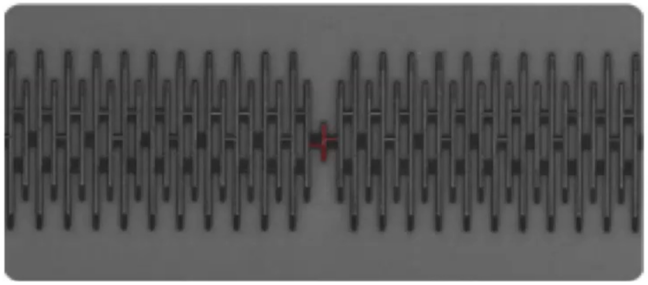
Most experiments have disorder, the lobes are replaced by Bose-glass phase, but the Luttinger liquid (free boson) phase persists

AR *et al*, J Stat. Mech. (2020)

Quantum circuits as analog free boson QFT simulators

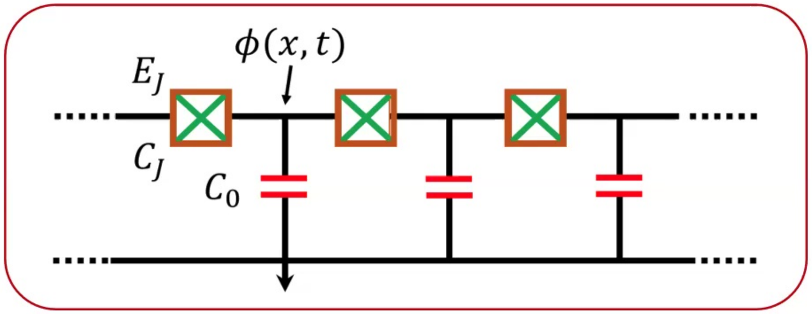


Maryland group (2018, 2019): $N \sim 33000$

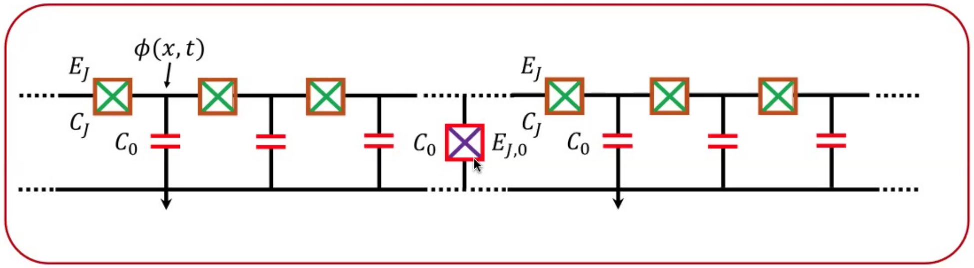


Grenoble group (2019): $N \sim 1500$

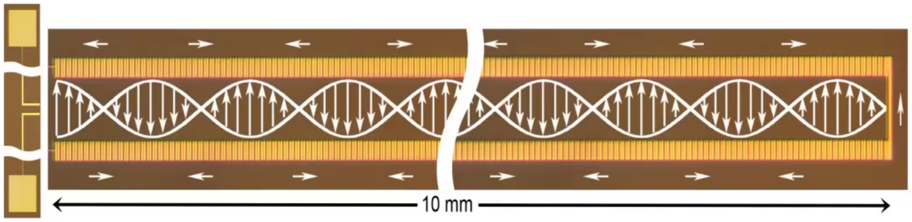
Quantum circuit



Quantum circuit



Quantum circuits as analog free boson QFT simulators



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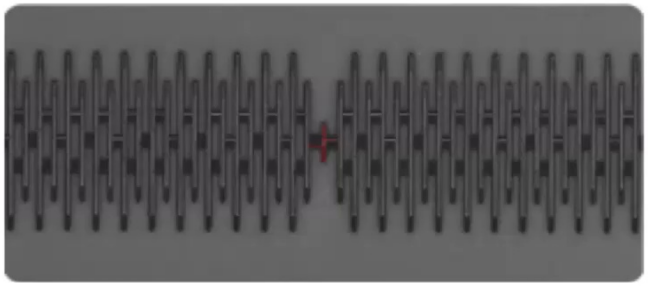

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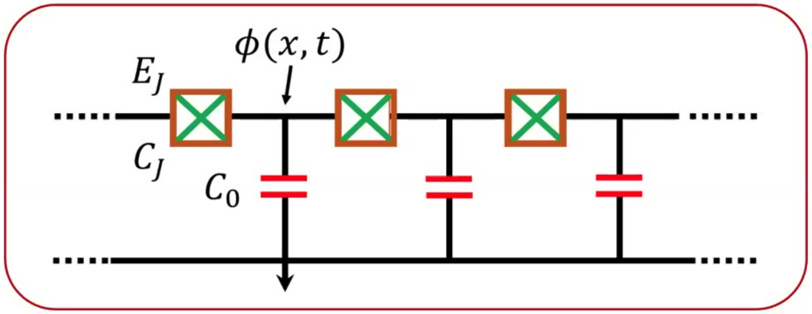
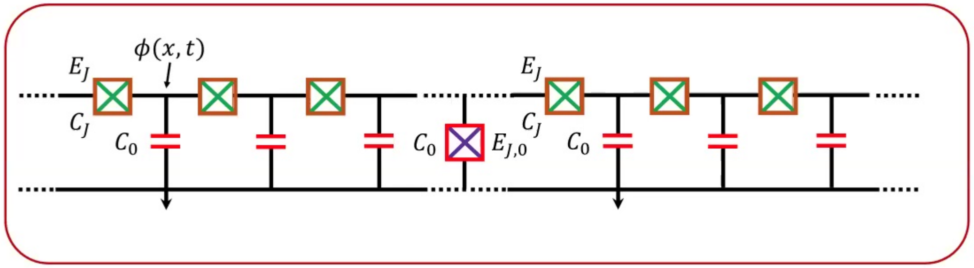
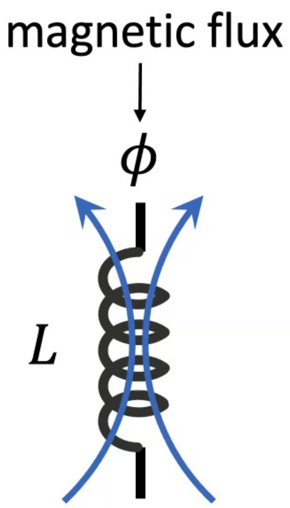


Quantum circuit



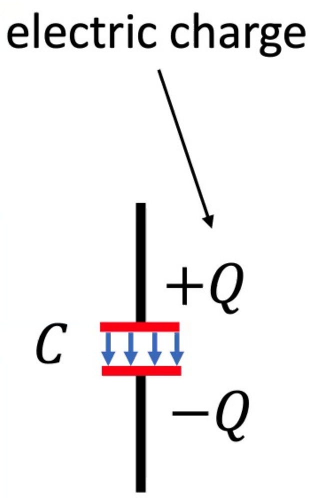
It is the tip of the iceberg…

Essential quantum circuit elements



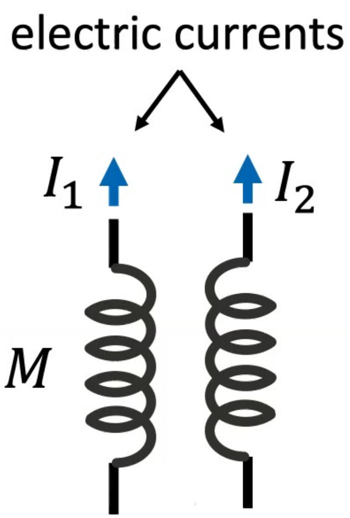
Inductor

$$H = \frac{\phi^2}{2L}$$



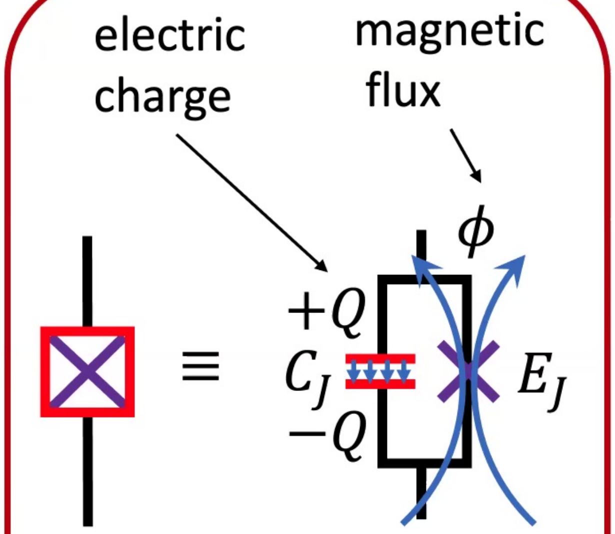
Capacitor

$$H = \frac{Q^2}{2C}$$



Mutual Inductor

$$H = -M I_1 I_2$$



Josephson junction

$$H = \frac{Q^2}{2C_J} - E_J \cos\frac{\phi}{\varphi_0}$$

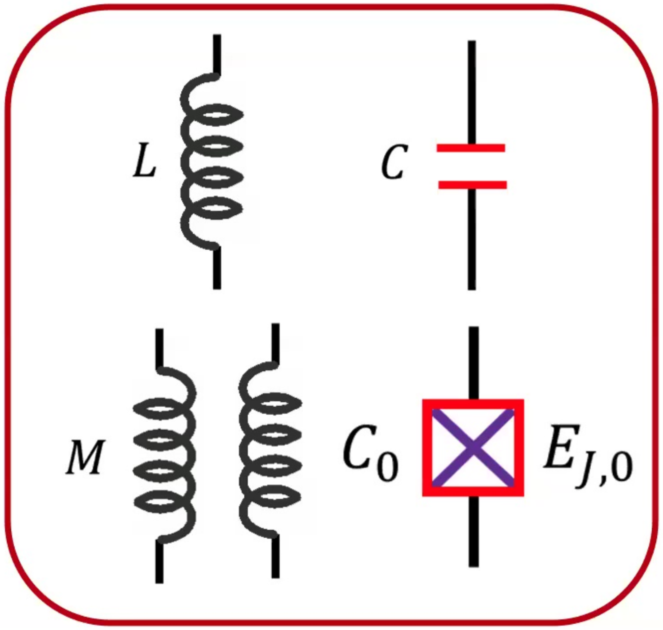
$$[\phi, Q] = i\hbar$$

$\varphi_0 = \hbar/2e$

Pozar (1976), Caldeira and Leggett (1983), Devoret (1997)

Building blocks of Quantum Electronic Circuit Lattices

1. The magnetic flux at a point in space-time
   → the bosonic field ϕ
2. Magnetic energy stored in an inductor
   → potential energy $(\partial_x \phi)^2$ or ϕ^2
3. Charging energy of a capacitor
   → kinetic energy $(\partial_t \phi)^2$ or interaction $\partial_t \phi_1 \partial_t \phi_2$
4. Energy stored in a mutual inductor
   → interaction $\partial_x \phi_1 \partial_x \phi_2$
5. Nonlinearity of a Josephson junction
   → source of nonlinear interaction: $\cos \phi$



Fermions may be included using Majorana zero modes

AR and H. Saleur, Phys. Rev. B (2019)

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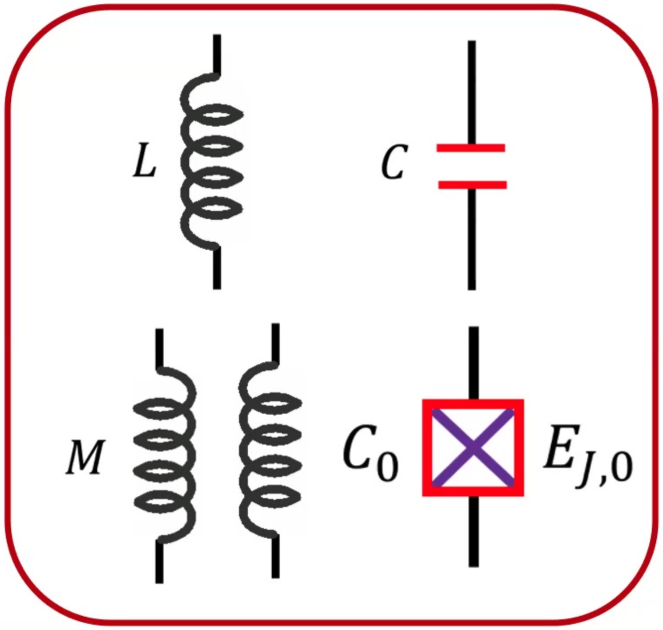
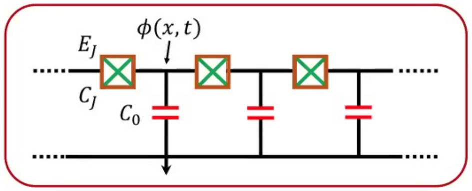


This talk: bosonic theories, with circuit elements that *currently* exist

AR and H. Saleur, Phys. Rev. B (2019)

Free QFT

Massless boson



Integrable QFT

Quantum sine-Gordon model

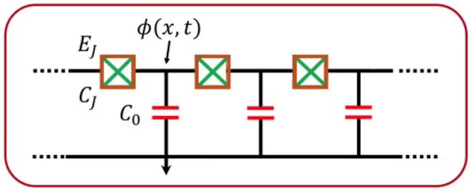
classical + quantum integrable

Quantum double sine-Gordon model

can be purely quantum integrable!

Free QFT

Massless boson



↓

Integrable QFT

Quantum sine-Gordon model

classical + quantum integrable

Quantum double sine-Gordon model

can be purely quantum integrable!

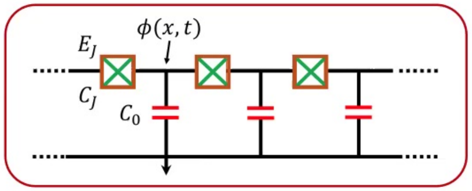
↓

Perturbed integrable QFT

Perturbed quantum sine-Gordon model

Free QFT

Massless boson



Integrable QFT

Quantum sine-Gordon model

classical + quantum integrable

Quantum double sine-Gordon model

can be purely quantum integrable!

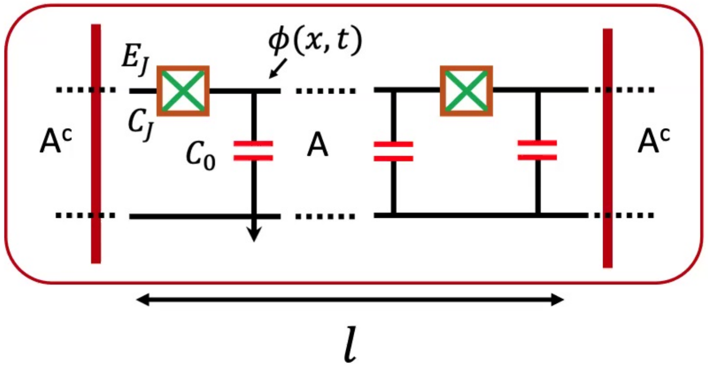
Perturbed integrable QFT

Perturbed quantum sine-Gordon model

Entanglement in the free, massless boson QFT

Entanglement entropy:

$$S = -\mathrm{Tr}\rho_A \ln \rho_A\,, \quad \rho_A = \mathrm{Tr}_{\mathrm{A^c}}(\rho)$$



Logarithmic scaling in the ground state:

$$S = \frac{c}{3} \ln l + S_0$$

central charge (= 1)

subsystem size

non-universal

Holzhey *et al* (1994),
Calabrese and Cardy (2004)

Questions:

1) Signature of the Luttinger parameter in entanglement?

2) Entanglement in the presence of boundaries?

3) Complete spectrum from an entanglement measure?

conformal invariance allows exact analytical computations

Entanglement in the free, massless boson QFT

Entanglement entropy:

$$S = -\mathrm{Tr}\rho_A \ln \rho_A\,, \quad \rho_A = \mathrm{Tr}_{A^c}(\rho)$$

Logarithmic scaling in the ground state:

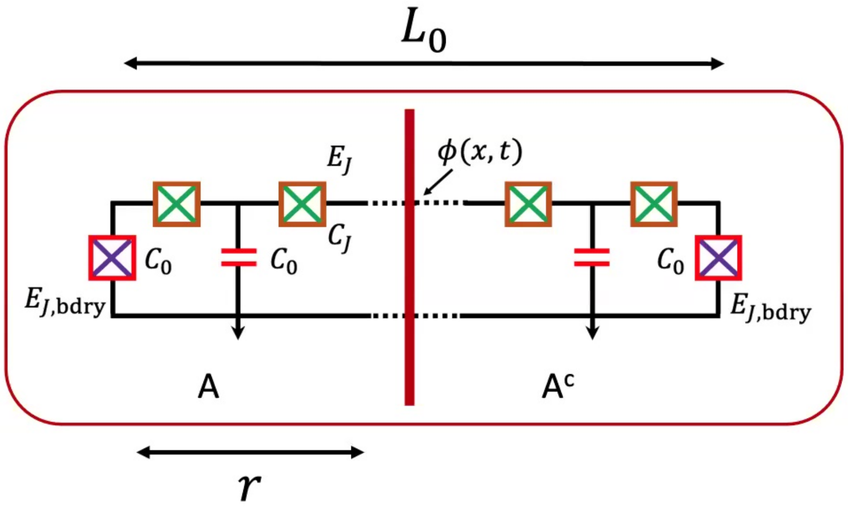
$$S = \frac{c}{6}\ln L_{\mathrm{eff}} + S_b + S_0$$

central charge (= 1)

effective subsystem size

boundary entropy

non-universal



Boundary conditions:

1. Dirichlet ($\phi =$ constant) -- $E_{J,\mathrm{bdry}} \to \infty$
2. Neumann ($\partial_x \phi = 0$) -- $E_{J,\mathrm{bdry}} \to 0$

Entanglement Hamiltonian: $\mathcal{H}_A = -\frac{1}{2\pi}\ln \rho_A$

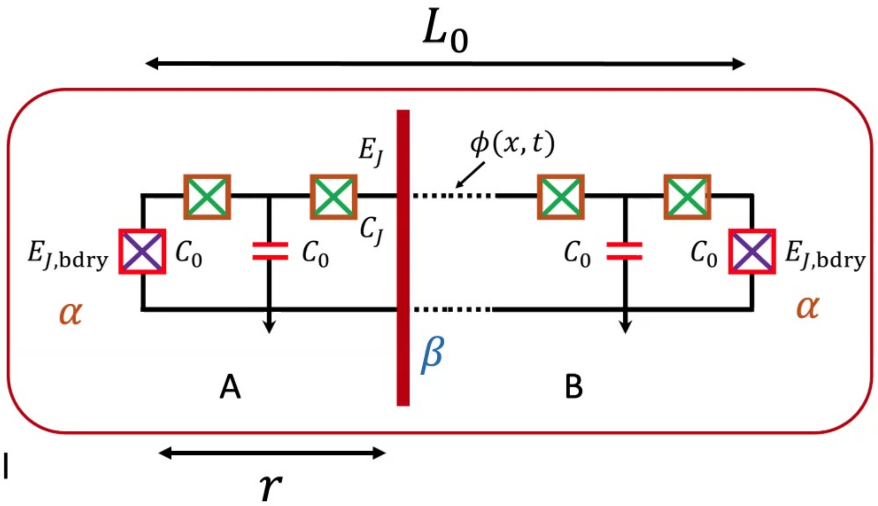
spectrum gives the complete spectrum of the QFT

Accessible with DMRG, complements exact diagonalization/truncated conformal space approaches

Haag (1992), Li and Haldane (2008), Cardy and Tonni (2016)

Entanglement spectrum of the compactified massless field theory: exact results

Problem is the same as computing partition function of boundary CFTs



Entanglement energy ($\alpha = \beta =$ Neumann):

$$\varepsilon_N(k,l) = \varepsilon_N(0,0) + \frac{\pi}{L}\left(\frac{K}{2}k^2 + l\right),$$

exact form available   dim. of primary fields   descendant level

degeneracy $p(l) =$ # of integer partitioning of l

$$\mathrm{L} = \ln\left[\frac{2L_0}{\pi a}\sin\frac{\pi r}{L_0}\right],$$

K = Luttinger parameter

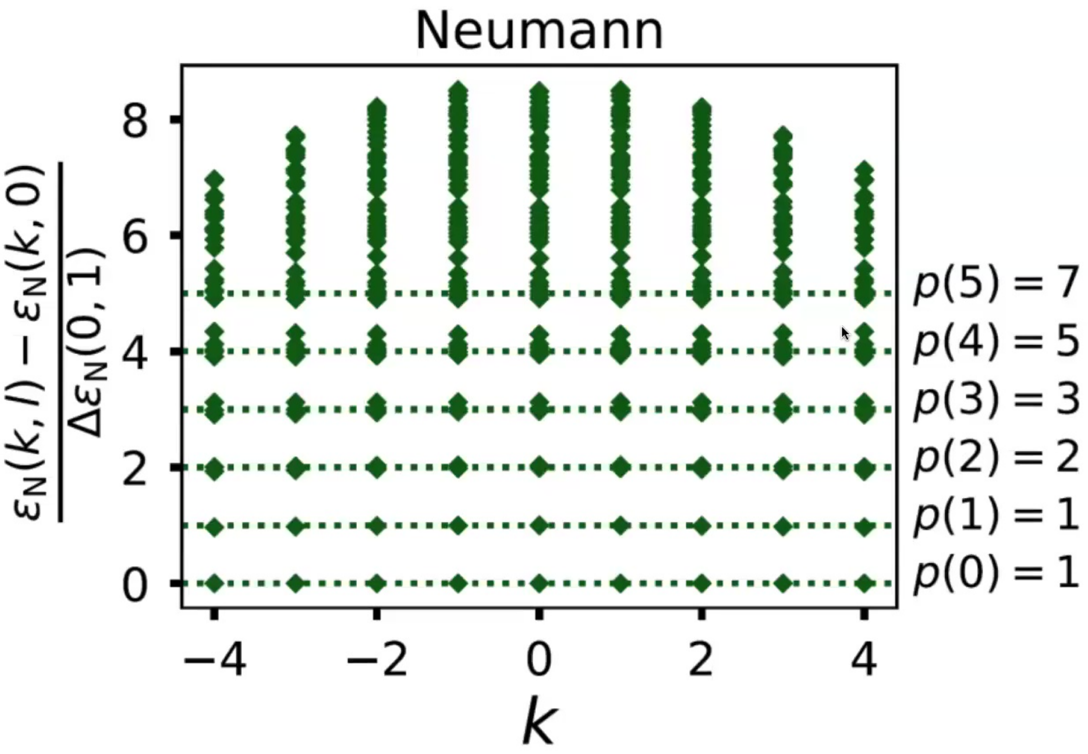
Change in boundary entropy (Neumann → Dirichlet):

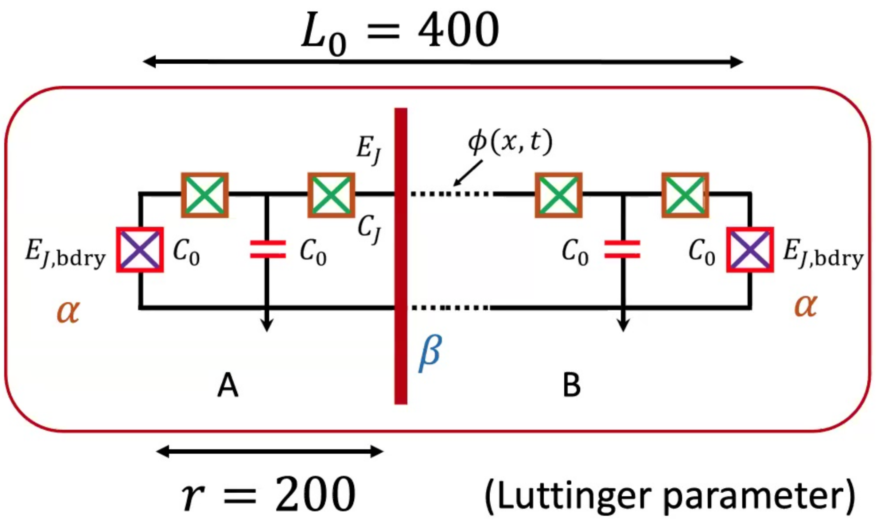
$$\Delta S_{N\to D} = \frac{1}{2}\ln\frac{2}{K}$$

A.R. *et al* (2020)

Related: DiGiulio and Tonni (2020), Giudici *et al* (2018)

Entanglement spectrum of the free, compactified boson QFT: DMRG results





(Luttinger parameter)
$K = 0.192$

Entanglement energy (Neumann):

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AR *et al,* J. Stat. Mech (2020)

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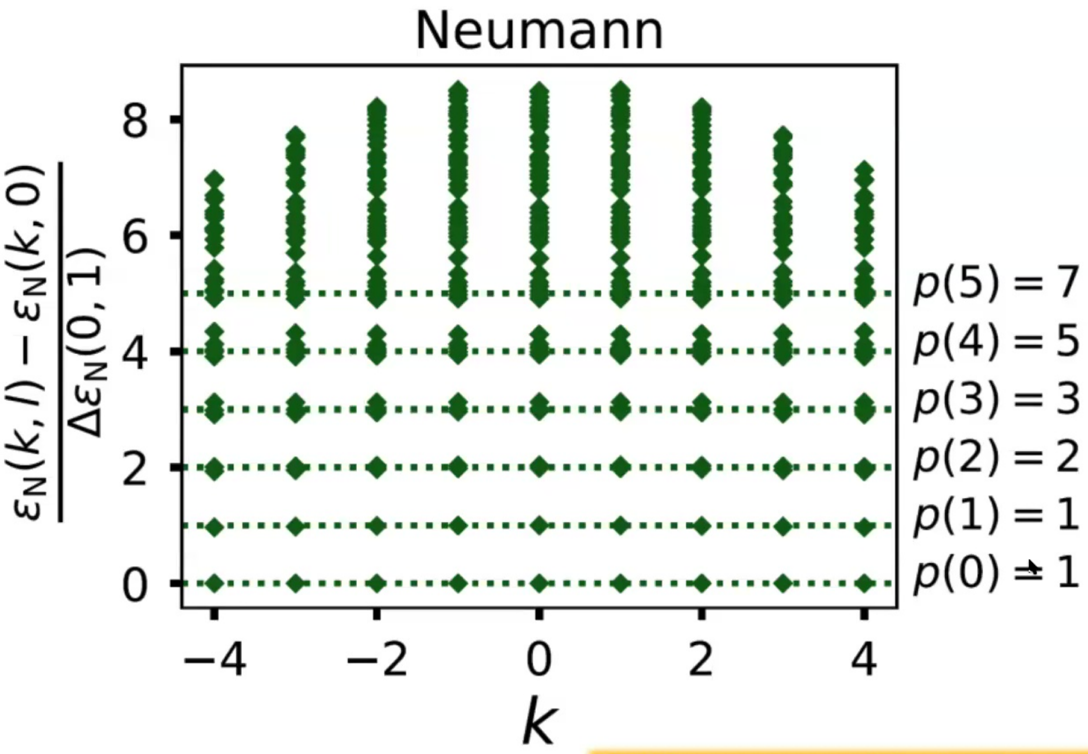

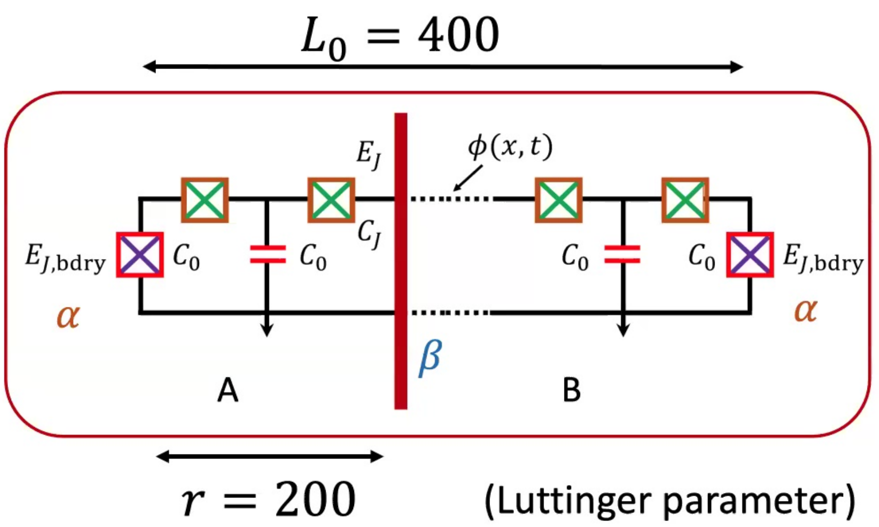

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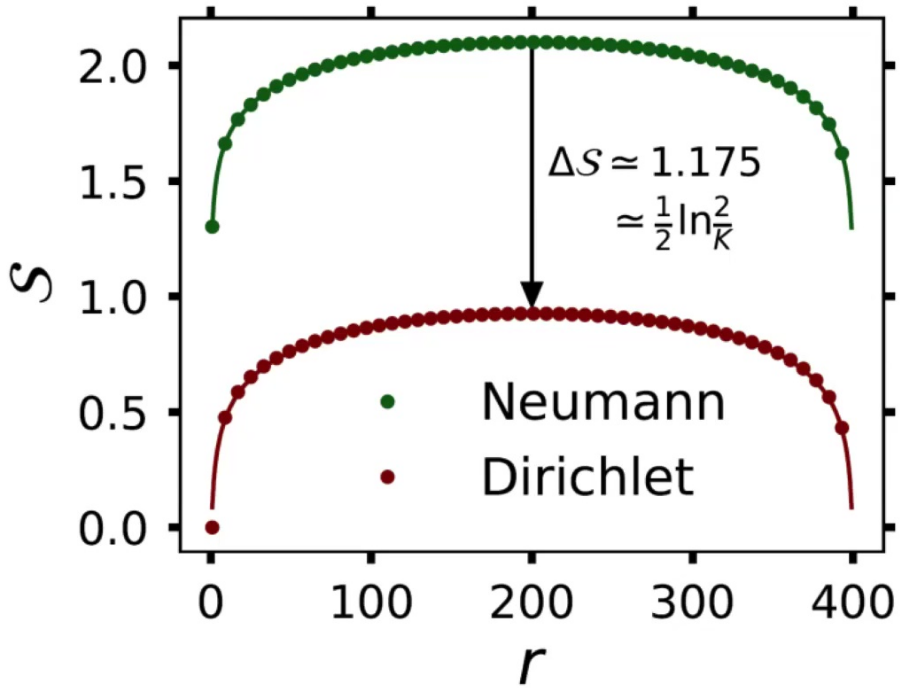
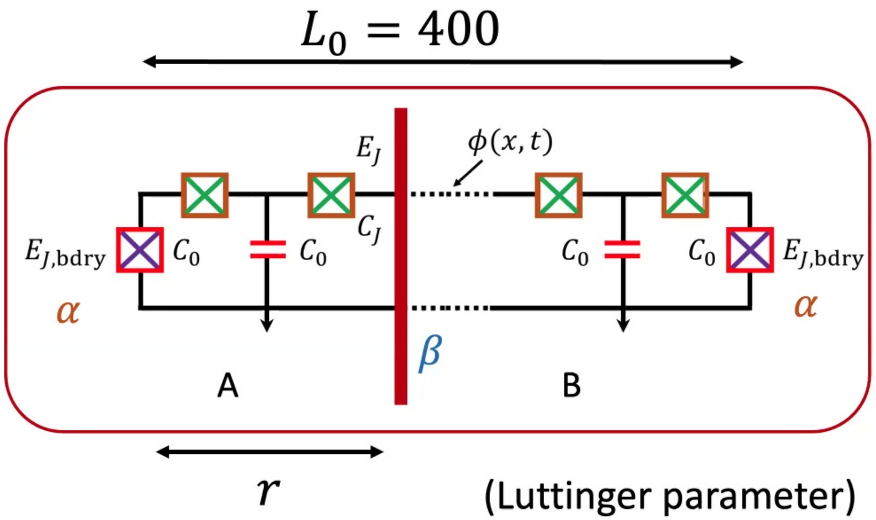
AR *et al,* J. Stat. Mech (2020)

Highly non-generic behavior, characteristic of an integrable model

Entanglement spectrum of the free, compactified boson QFT: DMRG results





(Luttinger parameter)
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Change in boundary entropy (Neumann → Dirichlet):

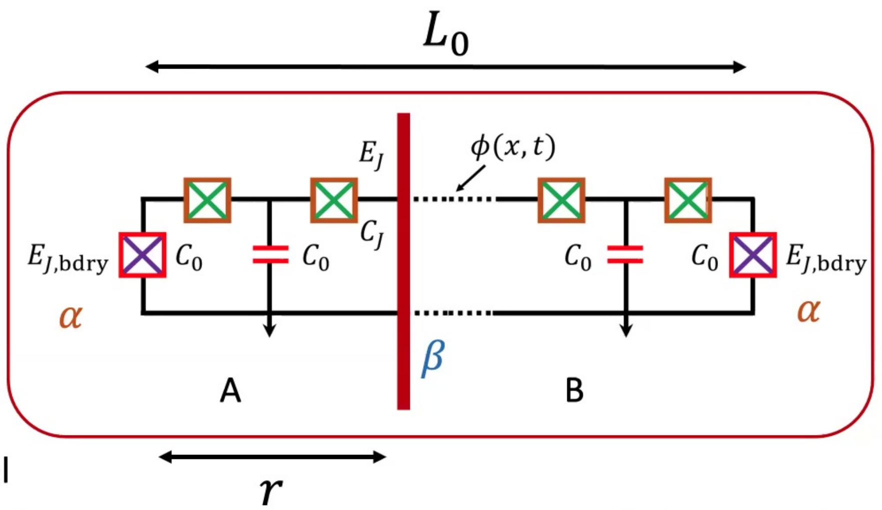
$$\Delta S_{N\to D} = \frac{1}{2}\ln\frac{2}{K}$$

AR *et al,* J. Stat. Mech (2020)

Related: Affleck *et al* (2009)

Entanglement spectrum of the compactified massless field theory: exact results

Problem is the same as computing partition function of boundary CFTs



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exact form available   dim. of primary fields   descendant level

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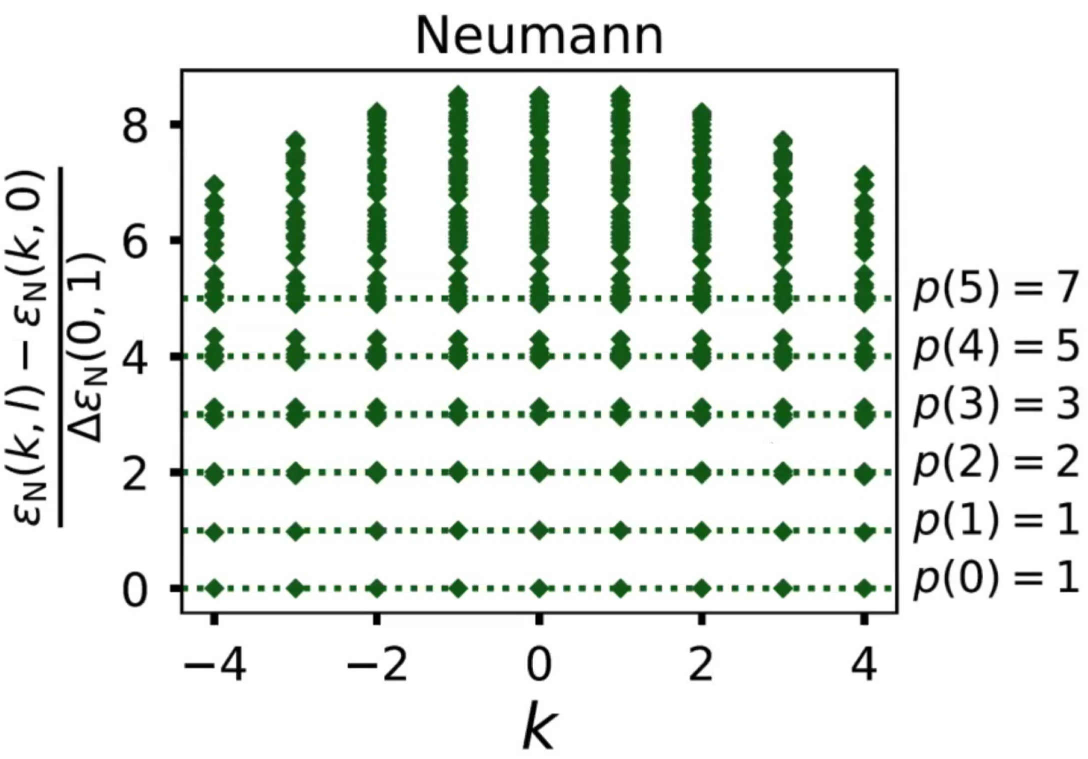

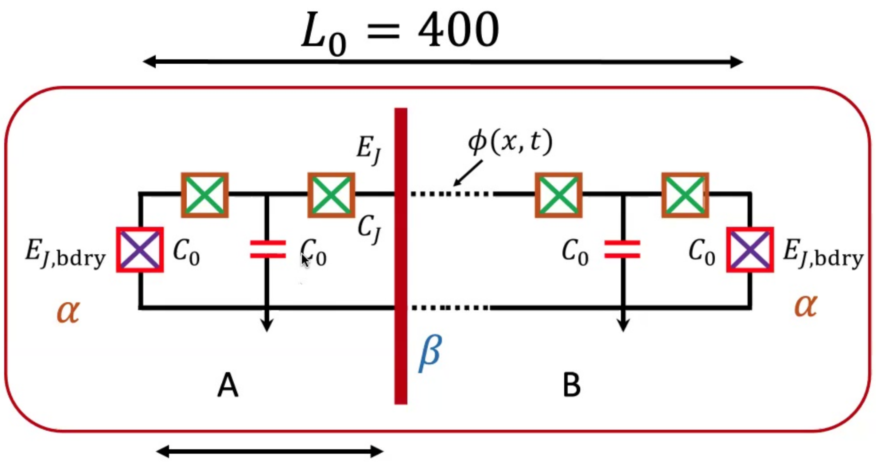

$r = 200$

(Luttinger parameter)
$K = 0.192$

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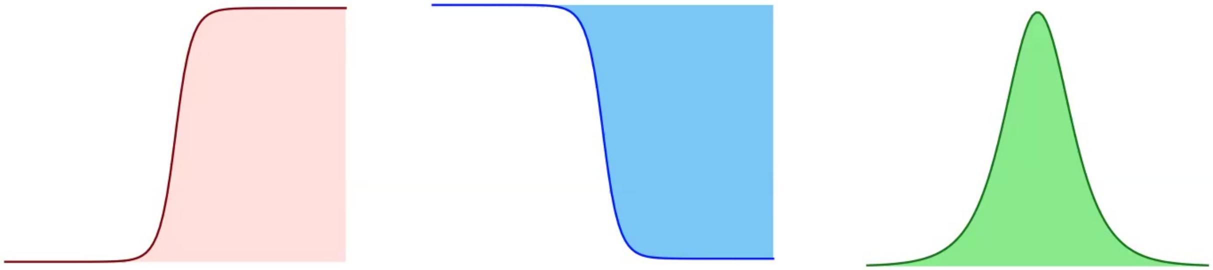
AR *et al,* J. Stat. Mech (2020)

The quantum sine-Gordon model

Hamiltonian: $H_{\mathrm{sG}} = H_{\mathrm{free}} - M_0 \int dx \cos \beta\phi$

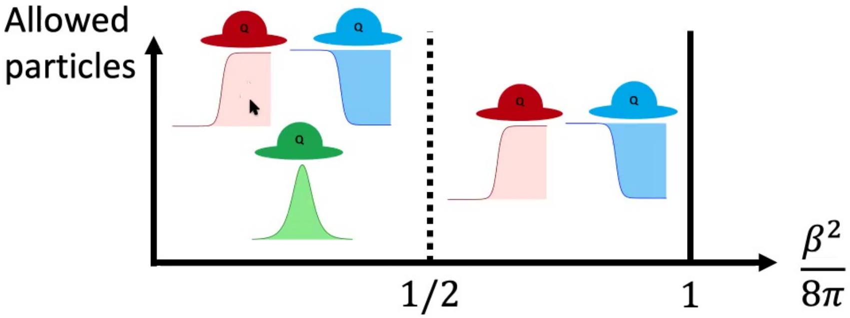
Classically integrable equations of motion, supports:

$(\beta \to 0)$



soliton antisoliton breather

Quantum mechanically conserved currents, hence integrable, spectrum depends on β:

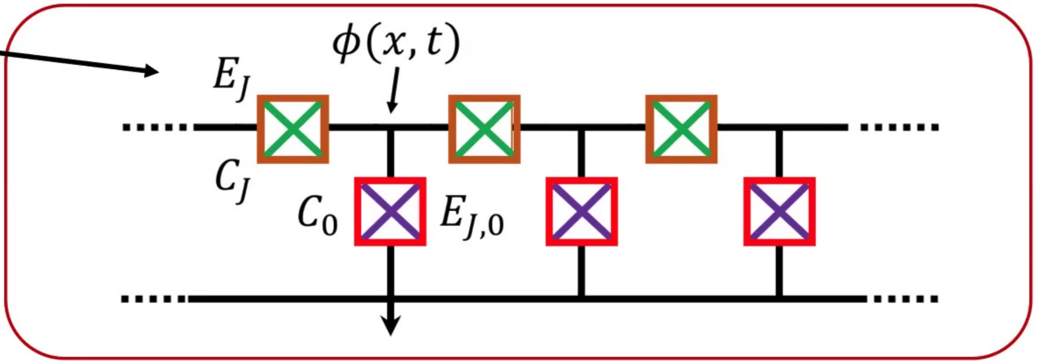


Dashen *et al* (1975), Al. and A. Zamolodchikov (1979), Dennis and Bernard (1991)

From the free boson to the sine-Gordon model

The quantum sine-Gordon model: $H_{\mathrm{sG}} = H_{\mathrm{free}} - M_0 \int dx \cos \beta\phi$

Quantum regime: $\beta \sim 1$



$$\beta = \sqrt{\pi K}$$

sine-Gordon coupling

Luttinger parameter

$E_J, E_{C_0} \gg E_{C_J}$,
$E_{J,0} \sim E_{C_0}$
$E_{C_{0,J}} = 2e^2/C_{0,J}$

AR *et al,* arXiv:2007.06874

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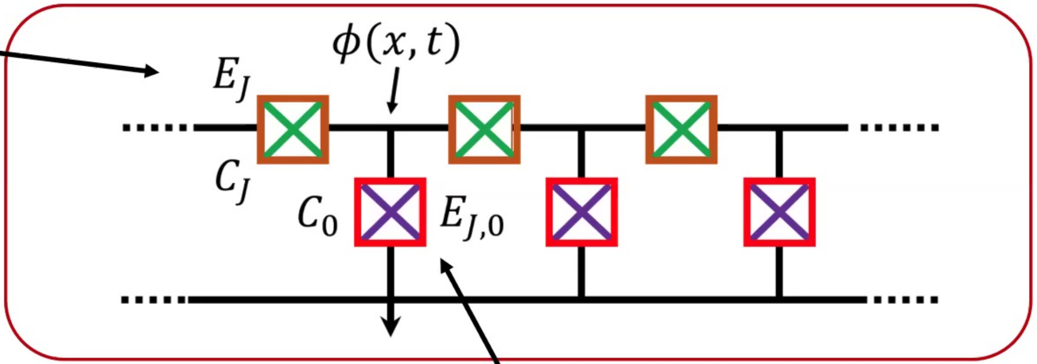

sine-Gordon nonlinearity

$$\beta = \sqrt{\pi K}$$

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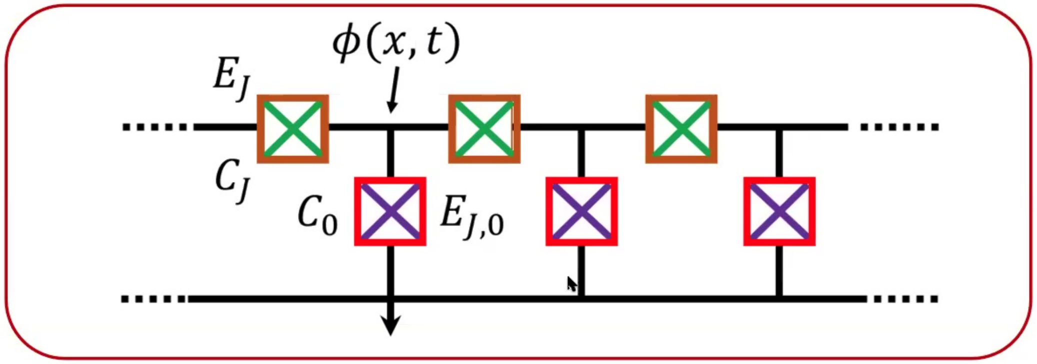
The quantum sine-Gordon model with quantum circuits

Quantum circuit:



$$[n_i, e^{\pm i\phi_j}] = \pm e^{\pm i\phi_j}\,\delta_{ij}$$

Hamiltonian:

$$H_{\text{circuit}} = E_{C_0}\sum_i n_i^2 + \epsilon\, E_{C_0}\sum_i n_i n_{i+1} - E_J\sum_i \cos(\phi_i - \phi_{i+1}) - E_{J,0}\sum_i \cos\phi_i$$

onsite repulsion

nearest-neighbor repulsion

nearest-neighbor hopping

sine-Gordon nonlinearity

Related work: Solitons in long Josephson junctions, Ustinov (1988), Walraff *et al* (2003)

AR *et al,* arXiv:2007.06874

The quantum sine-Gordon model with quantum circuits: DMRG results

Expectation value of the lattice vertex operator:

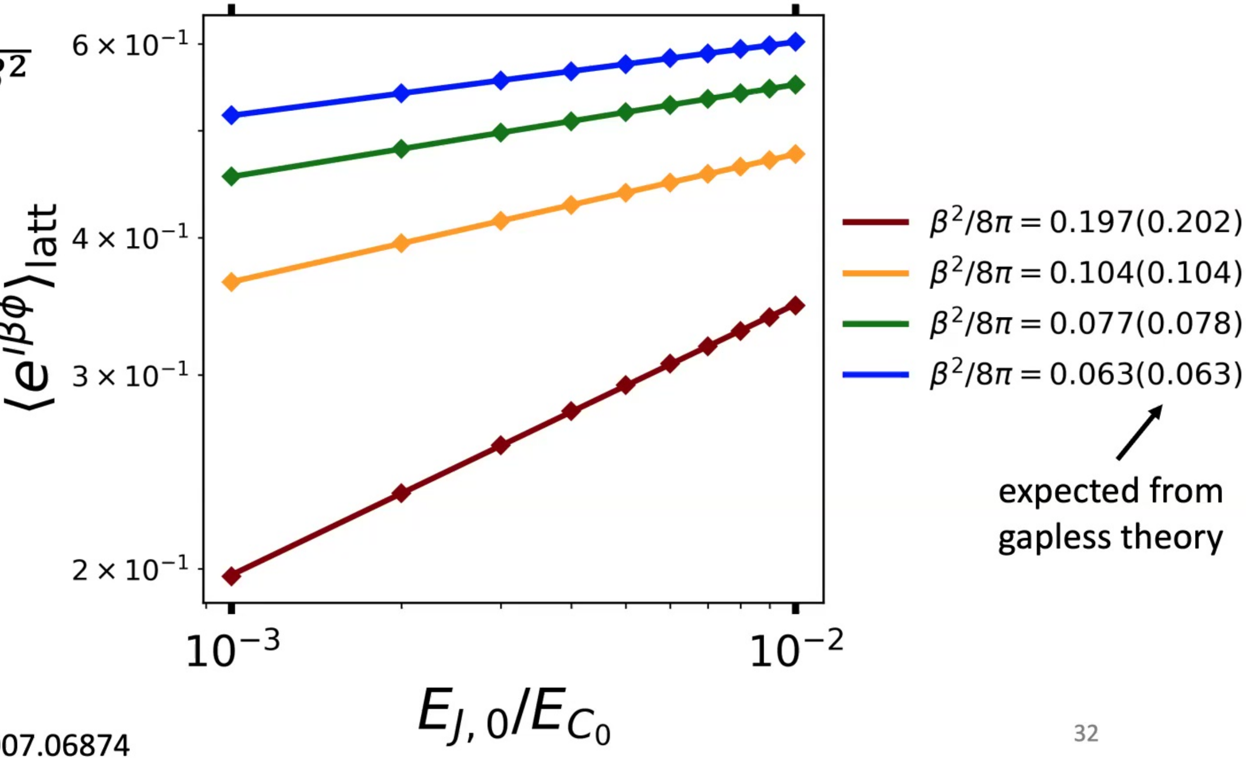
$$\langle e^{i\beta\phi}\rangle_{\text{latt}} \propto f(\beta)\left(\frac{E_{J,0}}{E_{C_0}}\right)^{\frac{\beta^2}{8\pi-\beta^2}}$$

exact form known

$$\beta = \sqrt{\pi K}$$

sine-Gordon coupling

Luttinger parameter



expected from gapless theory

Lukyanov and Zamolodchikov (1996), A.R. *et al,* arXiv:2007.06874

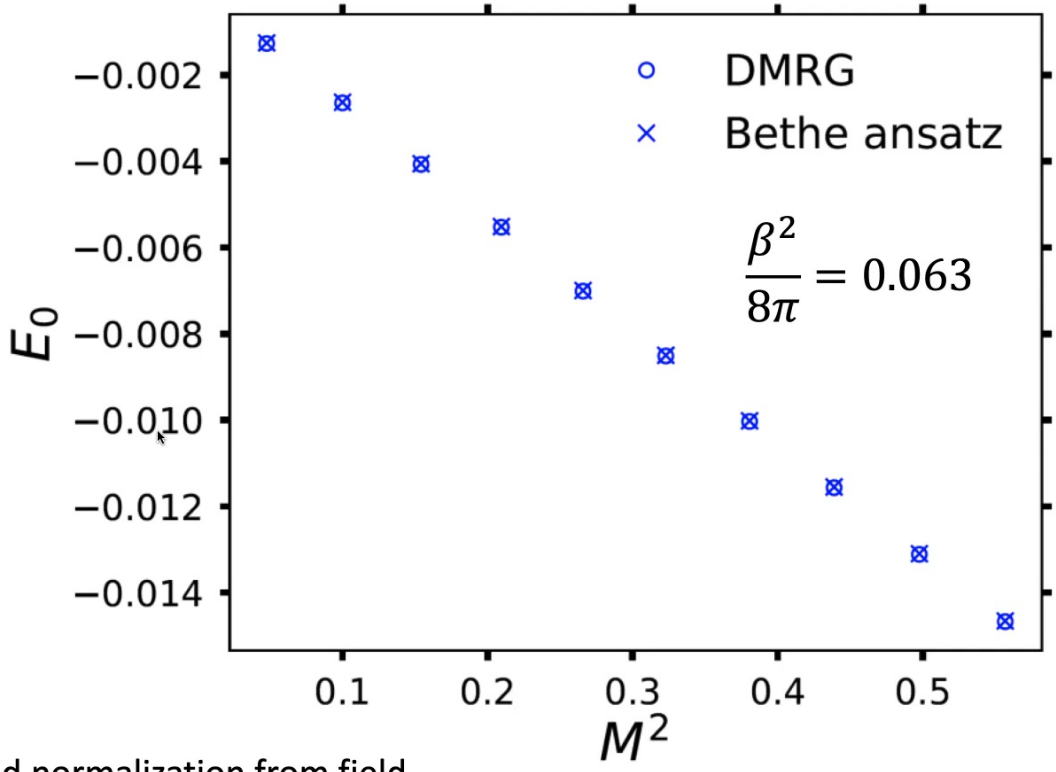
The quantum sine-Gordon model with quantum circuits: DMRG results

Ground state energy:

$$E_0 = -\frac{M^2}{4}\tan\frac{\pi\xi}{2},$$

$$\xi = \frac{\beta^2}{8\pi - \beta^2}$$

M = mass of soliton

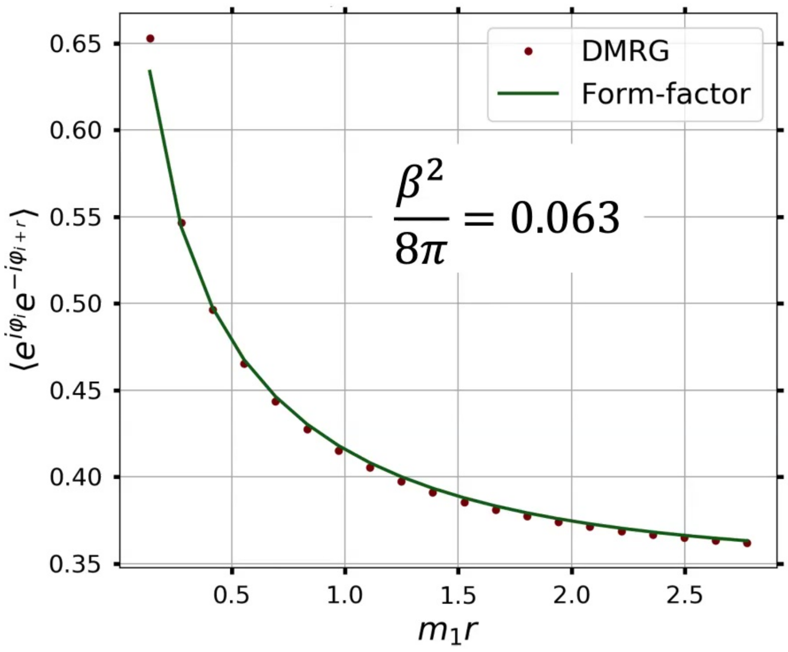


--- field normalization from field expectation value

Zamolodchikov (1995), A.R. *et al,* arXiv:2007.06874

The quantum sine-Gordon model with quantum circuits: DMRG results

Correlation functions using form-factors:



$$\frac{\beta^2}{8\pi} = 0.063$$

--- contributions up to second breather mass included

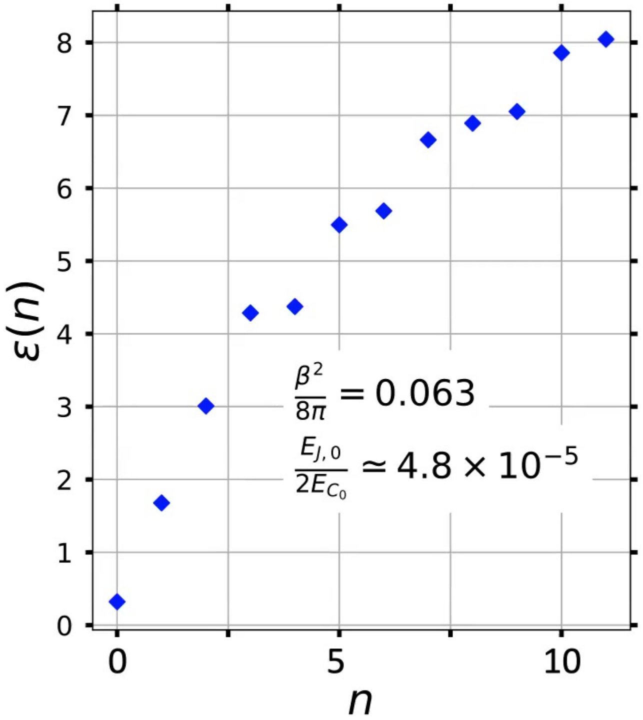
--- field normalization from field expectation value

A.R. *et al,* arXiv:2007.06874

m_1 = first breather mass

Entanglement spectrum of the quantum sine-Gordon model : DMRG results

Entanglement spectrum of the sine-Gordon model = Hamiltonian spectrum of the boundary sine-Gordon model



Cho, Ryu and Ludwig (2017),
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AR *et al,* J. Stat. Mech (2020),
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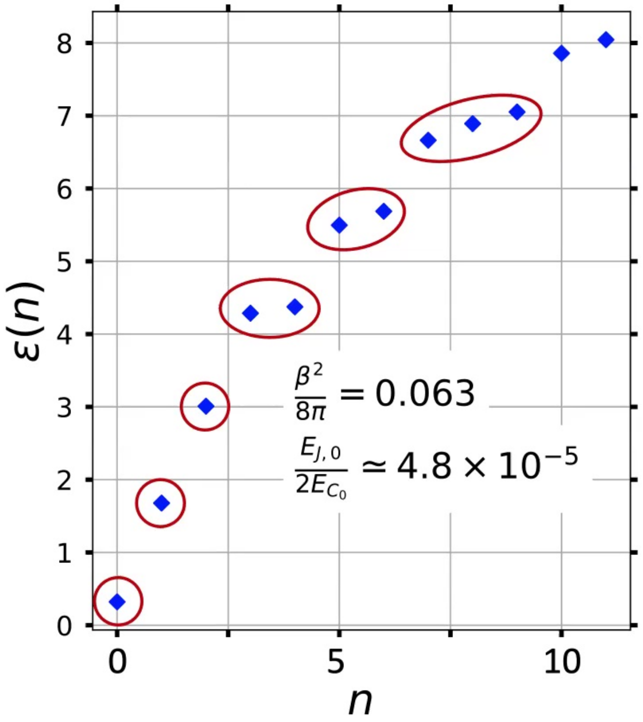

Predicted degeneracies: 1,1,1,2,2,3,4,…

Sources of discrepancies:

1. Finite-entanglement truncation in DMRG
2. Non-integrable lattice model

Cho, Ryu and Ludwig (2017),
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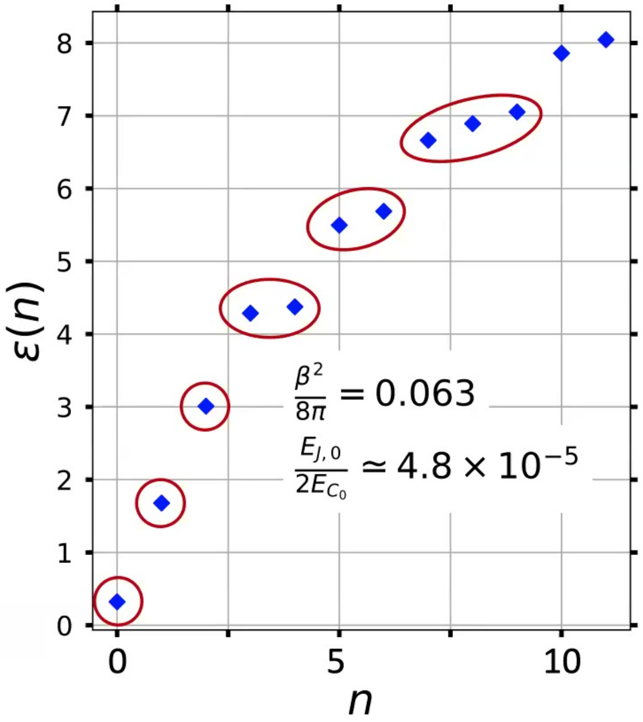

Exact solution for entanglement spectrum from XYZ spin-chain

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Cho, Ryu and Ludwig (2017),
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AR *et al,* J. Stat. Mech (2020),
AR *et al,* arXiv:2007.06874

The sine-Gordon model with XYZ spin-chain

Baxter’s XYZ spin-chain

$$H_{\mathrm{XYZ}} = -\frac{1}{2}\sum_{i=1}^{L}\left[J_x\sigma_i^x\sigma_{i+1}^x + J_y\sigma_i^y\sigma_{i+1}^y + J_z\sigma_i^z\sigma_{i+1}^z\right], J_x > J_y \geq |J_z|$$

XYZ to sine-Gordon operator mapping: $\sigma^+ \sim e^{\frac{i\beta\phi}{2}}$

The QFT predictions apply in the regime: $\underset{\substack{\uparrow \\ \text{lattice} \\ \text{spacing}}}{a} \ll \underset{\substack{\uparrow \\ \text{correlation} \\ \text{length}}}{\xi} \ll \underset{\substack{\uparrow \\ \text{system} \\ \text{size}}}{L}$

Cold atomic simulators of XYZ spin chains exist!

Murmann *et al* (2015),…

Baxter (1982), Luther (1975), Lukyanov (1997, 2003)

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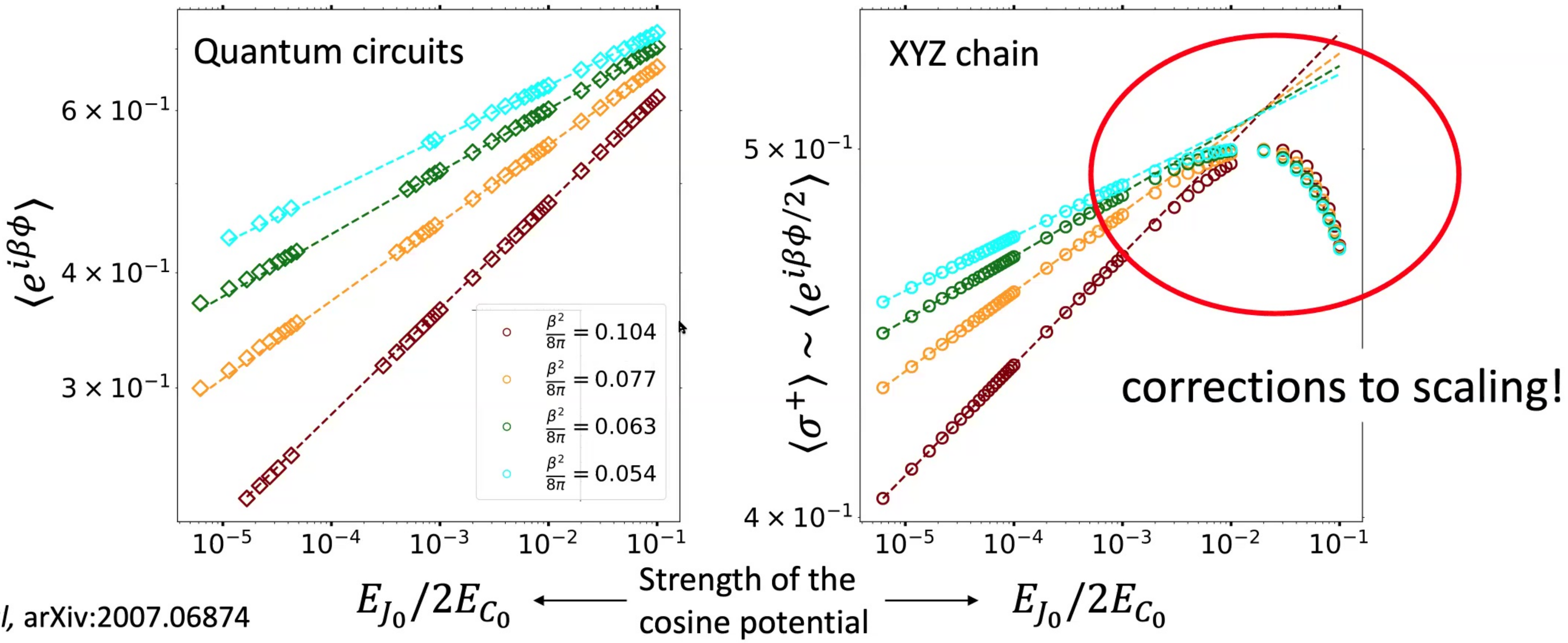
Quantum circuits start from compact, bosonic lattice degrees of freedom



corrections to scaling!

Strength of the cosine potential

AR *et al*, arXiv:2007.06874

Perturbed integrable QFTs with quantum circuits

A perturbed sine-Gordon model

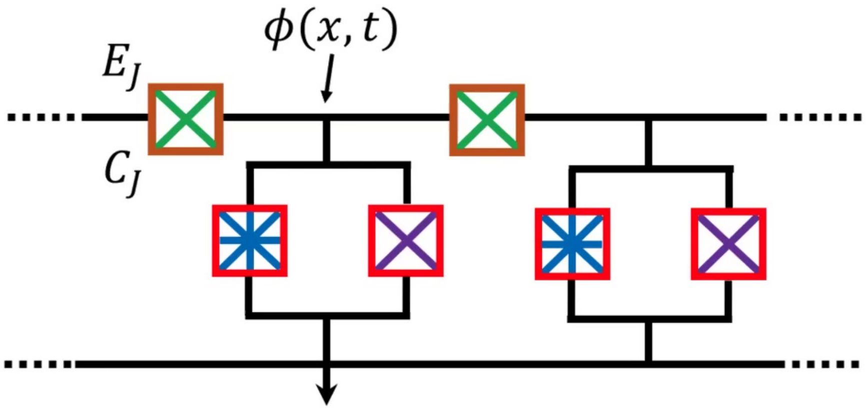
$$H_{\mathrm{psG}} = H_{\mathrm{free}} - M_0 \int dx \cos\beta\phi - M_0' \int dx \cos(2\beta\phi + \delta)$$

$2e$ Josephson junction

An Ising phase-transition occurs as M_0/M_0' increases

$4e$ Josephson junction

Quantum circuit:



Mussardo *et al* (2004), Bajnok *et al* (2018)

AR (unpublished)

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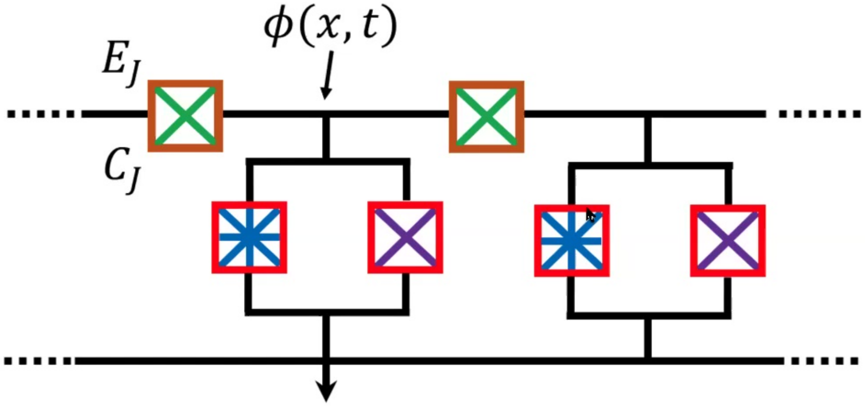

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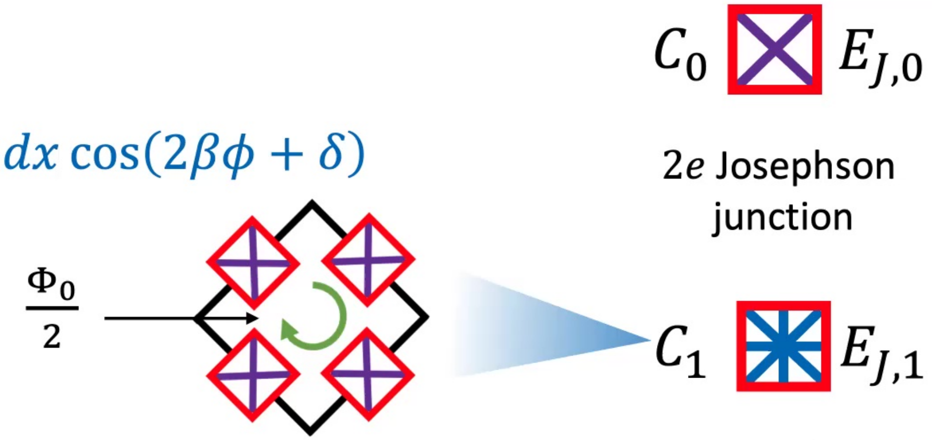
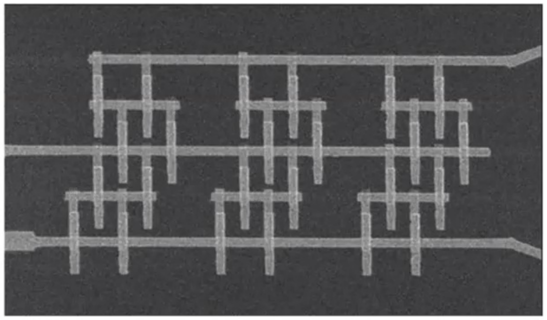


2e Josephson junction

4e Josephson junction



Gershenson group, Rutgers (2008)

Doucot, Ioffe *et al* (2001),
Brooks *et al* (2013),
Bell *et al* (2018),
Larsen *et al* (2020)

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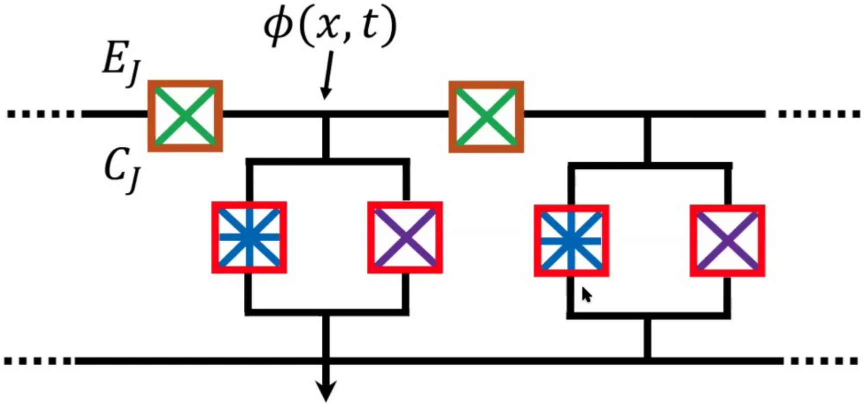
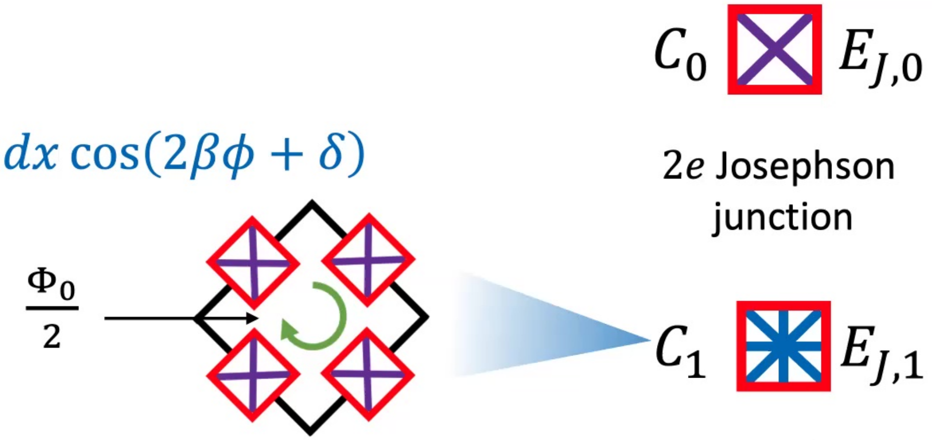
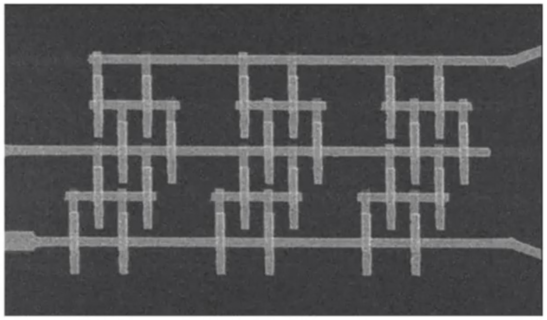


AR (unpublished)

Ongoing: Entanglement dynamics under time-evolution



4e Josephson junction



Gershenson group, Rutgers (2008)

Doucot, Ioffe *et al* (2001),
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Bell *et al* (2018),
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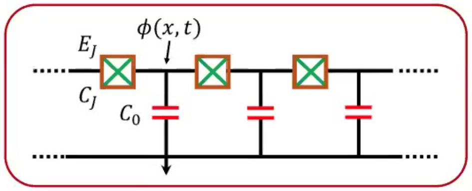
Potentially generalizable to arbitrary interactions

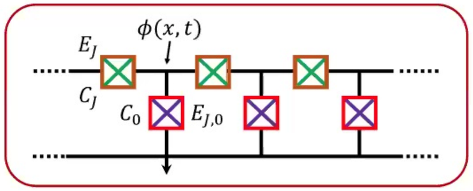
Free QFT

Massless boson



Integrable QFT

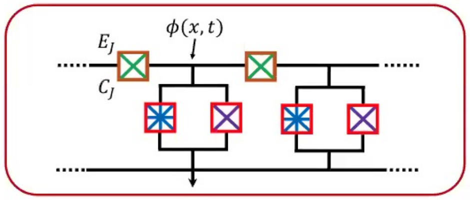
Quantum sine-Gordon model



Quantum double sine-Gordon model

Perturbed integrable QFT

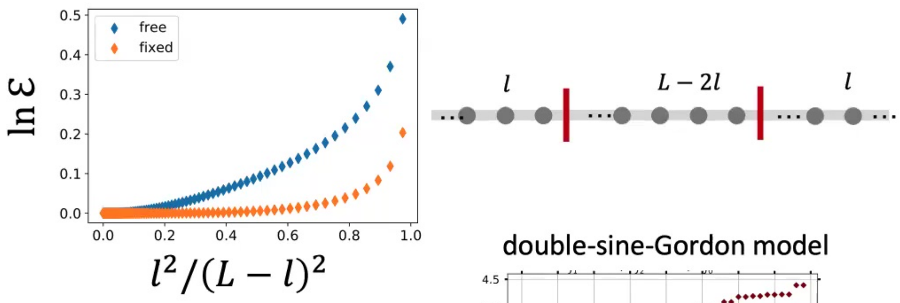
Perturbed Quantum sine-Gordon model



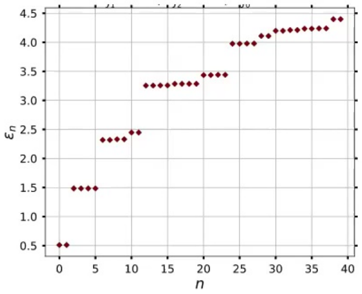
Outlook

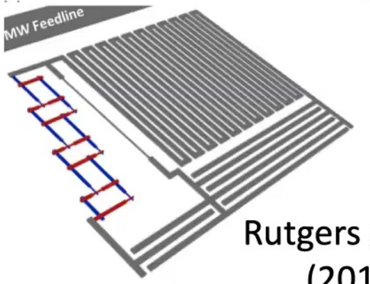
1. Quantifying entanglement in mixed states using entanglement negativity
2. Absence of thermalization in a pure quantum-integrable model – entanglement signatures
3. Experimental signatures of quantum integrability
4. Opening quantum field theories to dissipation

Boundary RG flow in Ising CFT



double-sine-Gordon model





Rutgers group (2012)

Vidal and Werner (2002), Calabrese *et al* (2012), G. Brandino *et al* (2010), Yang *et al* (2017), AR *et al* (in progress)

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Thank You!

Johannes Hauschild (UC Berkeley)
Frank Pollmann (TU Munich)
Hubert Saleur (CEA Saclay)
Dirk Schuricht (Utrecht University)

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European
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The sine-Gordon model with XYZ spin-chain

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XYZ to sine-Gordon operator mapping: $\sigma^+ \sim e^{\frac{i\beta\phi}{2}} + \cdots$

corrections to scaling important for finite systems!

The QFT predictions apply in the regime: $a \ll \xi \ll L$

lattice spacing

correlation length

system size

Cold atomic simulators of XYZ spin chains exist!

Murmann *et al* (2015),…

Baxter (1982), Luther (1975), Lukyanov (1997, 2003)

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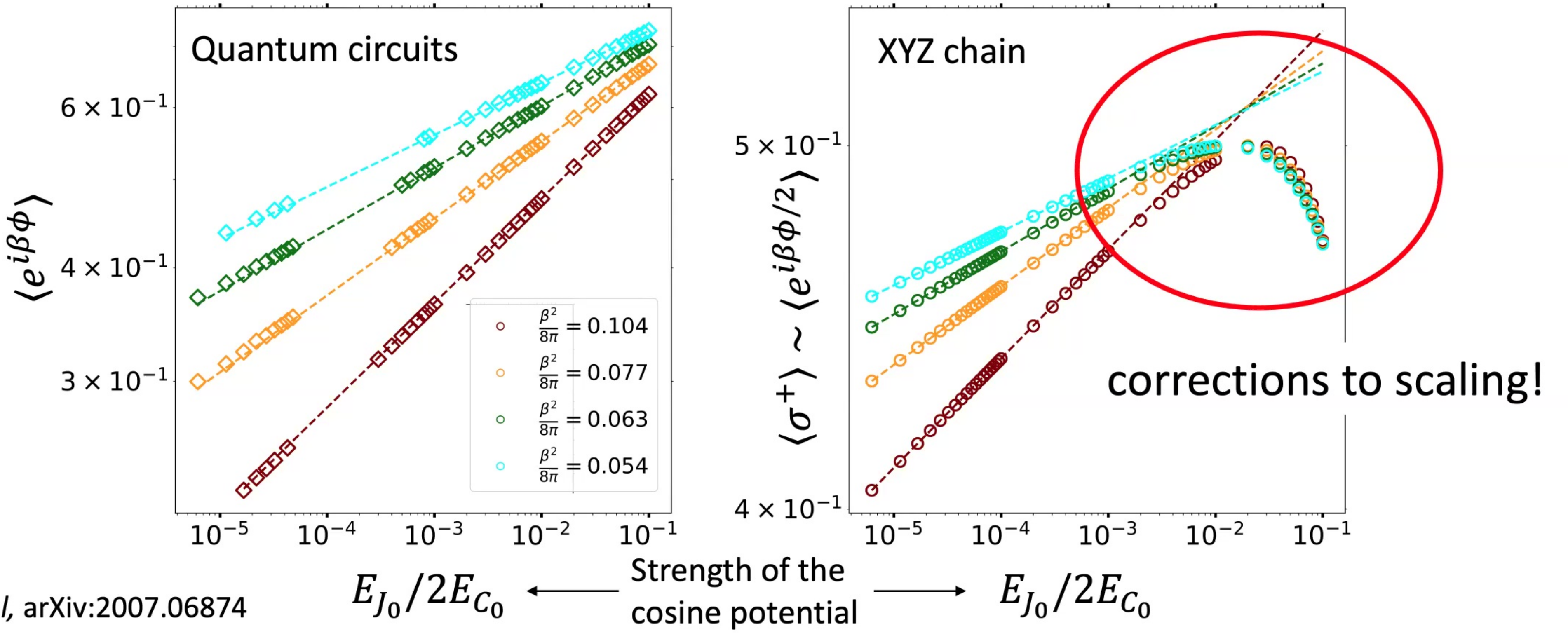

corrections to scaling!

Strength of the cosine potential

AR *et al*, arXiv:2007.06874

The quantum double sine-Gordon model

Euclidean action:

$$\mathcal{A}_{\mathrm{dSG}} = \int d^2x \left[\frac{1}{2}\left\{\left(\partial_\mu \phi_1\right)^2 + \left(\partial_\mu \phi_2\right)^2\right\} + \frac{2M_0}{\pi} \cos \alpha_1 \phi_1 \cos \alpha_2 \phi_2\right]$$

Exists conserved currents leading to factorized scattering

Two cases when the model is quantum integrable:

1. Symmetric case: $\alpha_1 = \alpha_2$ --- also classically integrable
2. Relevant case: $\alpha_1^2 + \alpha_2^2 = 4\pi$ --- purely quantum integrable!

Bukhvostov and Lipatov (1980), Fateev (1996), Lesage *et al* (1997, 1998), A.R. and H. Saleur (2019)

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Occurs in spinful Luttinger liquids and in quantum circuits

Bukhvostov and Lipatov (1980), Fateev (1996), Lesage *et al* (1997, 1998), A.R. and H. Saleur (2019)

The quantum double sine-Gordon model

$\alpha_1^2 + \alpha_2^2 = 4\pi$

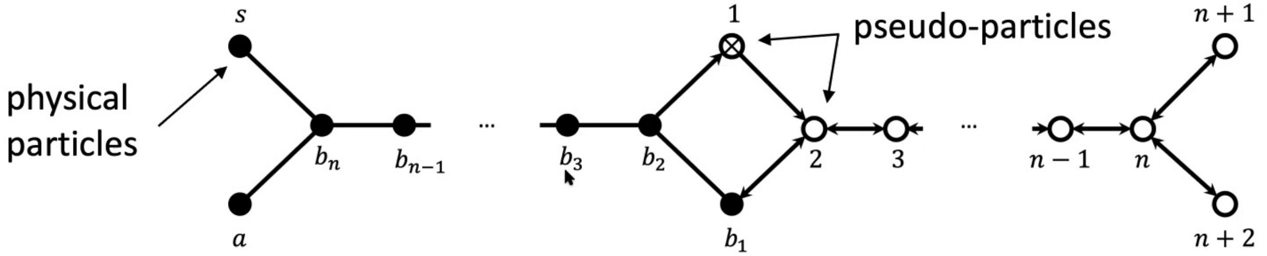
Euclidean action:

$$\mathcal{A}_{\mathrm{dsG}} = \int d^2x \left[\frac{1}{2}\left\{\left(\partial_\mu \phi_1\right)^2 + \left(\partial_\mu \phi_2\right)^2\right\} + \frac{2M_0}{\pi} \cos\alpha_1\phi_1 \cos\alpha_2\phi_2\right]$$

Solitons exist of fields $\varphi_{1,2} = \frac{\alpha_1\phi_1 \pm \alpha_2\phi_2}{2\sqrt{\pi}}$, have a pair of quantum numbers which scatter independently

Factorized scattering matrix: $S = S_{p_1} \otimes S_{p_2}$

Bethe ansatz calculation at zero and nonzero temperature

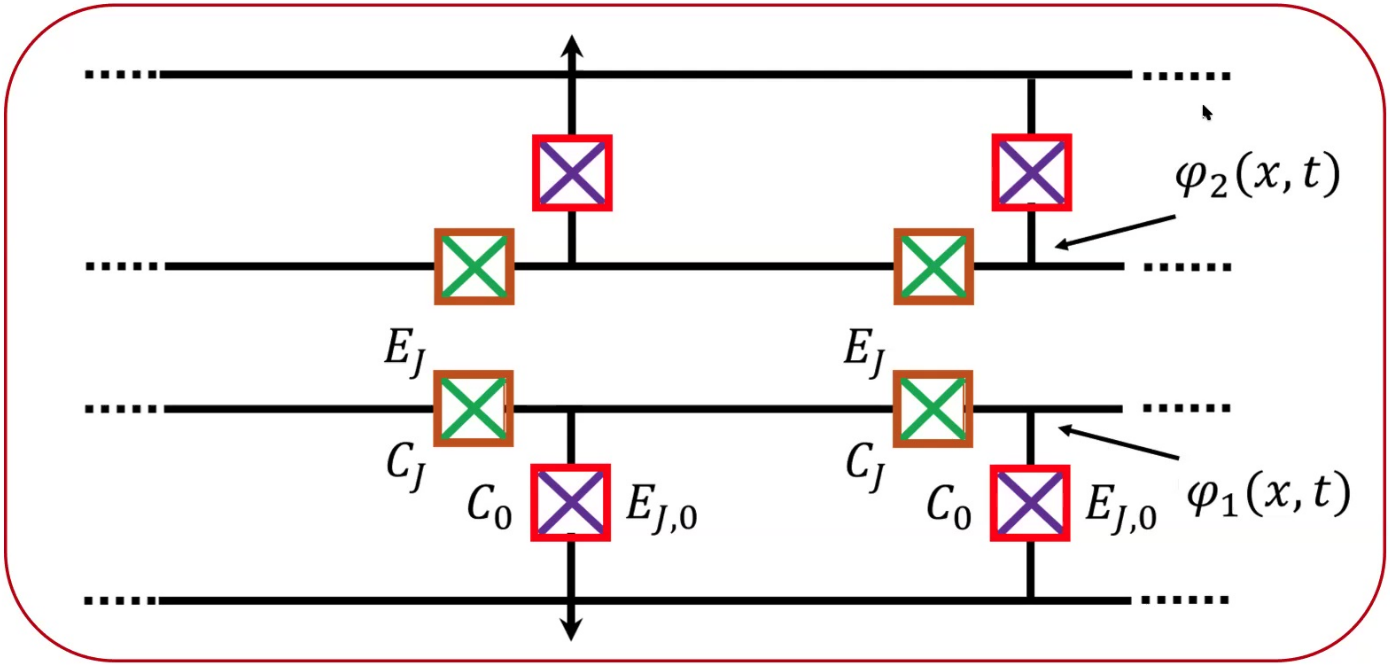


Thermodynamic Bethe ansatz equations

A. R. and H. Saleur (2019)

The double sine-Gordon model with quantum circuits

Two coupled sine-Gordon models, coupled by $\partial_x\varphi_1\partial_x\varphi_2$ and $\partial_t\varphi_1\partial_t\varphi_2$



Rotated fields: $\varphi_{1,2} = \frac{\alpha_1\phi_1 \pm \alpha_2\phi_2}{2\sqrt{\pi}}$

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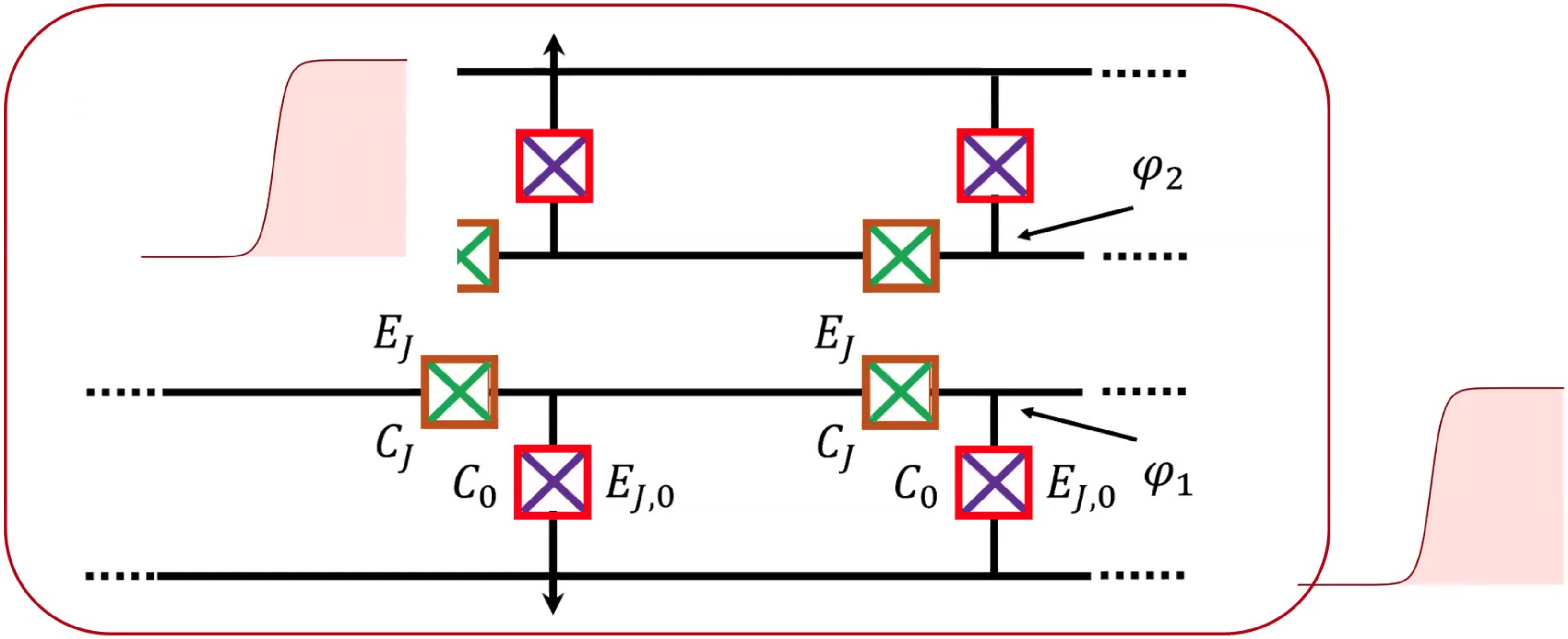
The double sine-Gordon model with quantum circuits

classical integrable manifold: two decoupled sine-Gordon models



Rotated fields: $\varphi_{1,2} = \frac{\alpha_1\phi_1 \pm \alpha_2\phi_2}{2\sqrt{\pi}}$

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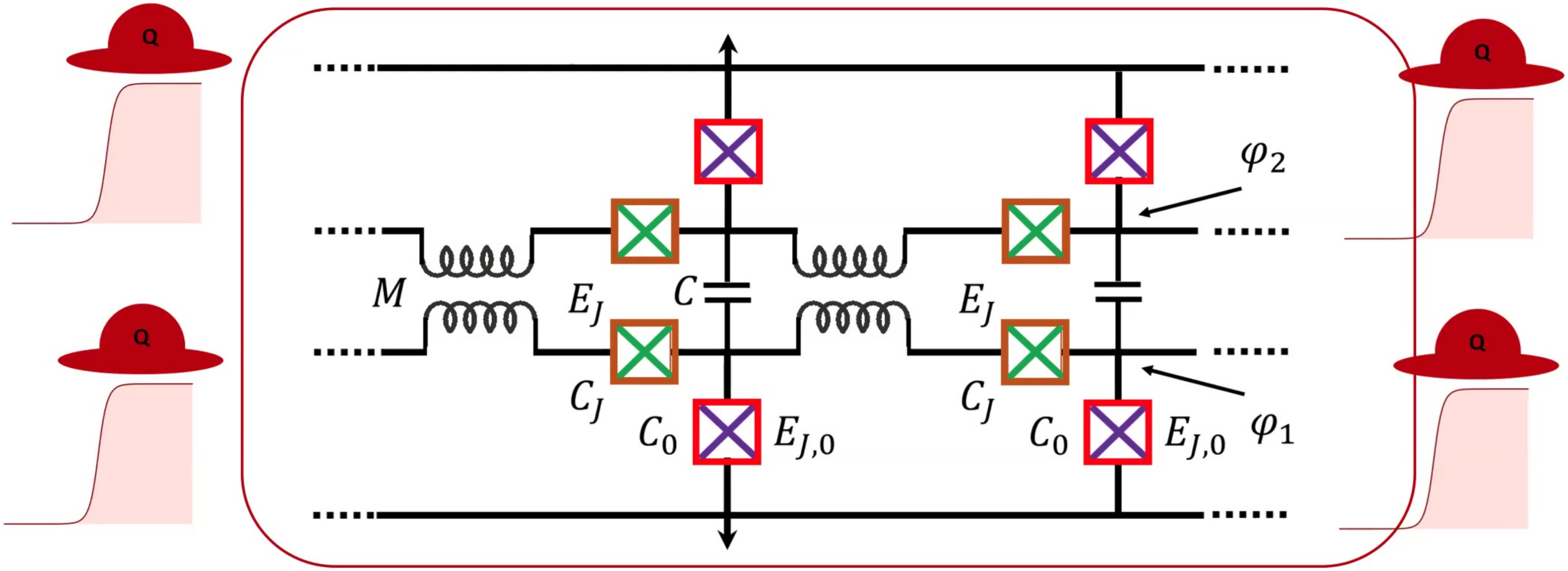
The double sine-Gordon model with quantum circuits

quantum integrable manifold: two coupled sine-Gordon models



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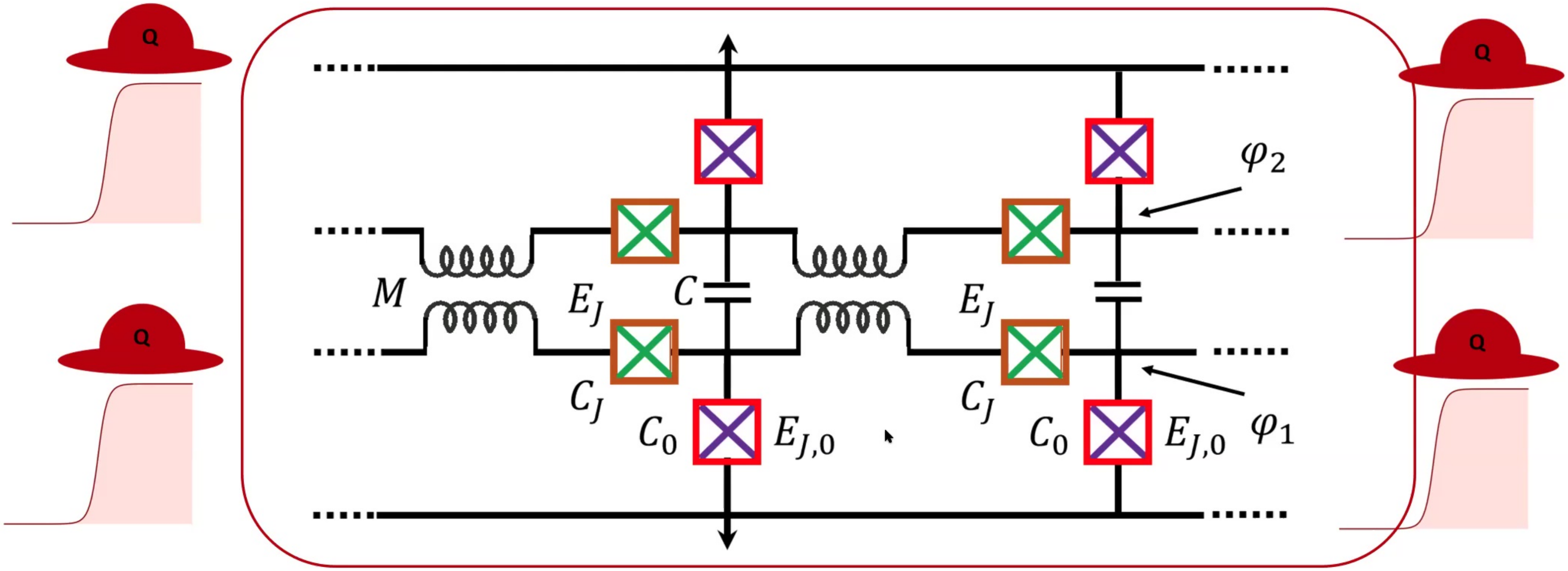
The double sine-Gordon model with quantum circuits

quantum integrable manifold: two coupled sine-Gordon models



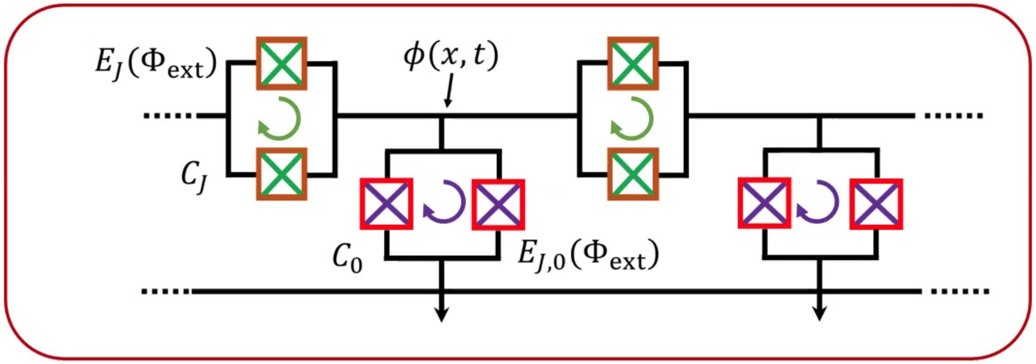
A testbed for classical vs quantum integrability
--- signatures in transport and specific heat measurements

A.R. and H. Saleur (2019)

Outlook: quantum quenches with quantum circuits

The sine-Gordon model: $H_{\text{free}} - M_0 \int dx \cos \beta\phi$

Quantum circuit:



$$\Phi_0 = \frac{h}{2e}$$

Two possible quench scenarios by applying magnetic flux:

1. Quench in sine-Gordon coupling $\beta^2 \simeq \frac{1}{2}\sqrt{\frac{2E_{C_0}}{E_J}}$
2. Quench in sine-Gordon mass parameter $M_0 \simeq E_{J,0}/E_{C_0}$

Tunable Josephson energy:

$$E_J(\Phi_{\text{ ext}}) = E_J(0)\cos\frac{\pi\Phi_{\text{ext}}}{\Phi_0}$$

A.R. (unpublished)

Related analytical works: Bertini *et al* (2014), Rylands and Andrei (2019)