Title: Conformal correlators and AdS2/CFT1

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Conformal correlators and AdS_2/CFT_1

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(2020 & work soon to appear)

Women at the Intersection of Mathematics and Theoretical physics

Perimeter Institute @ Zoom, February 24 2021

Motivation: Wilson loops as 1-dimensional defects

Why CFT_1 (1-dimensional conformal field theories) are interesting?

- A simpler but still constraining setup to test ideas about higher-d CFTs (in fact, every higher-d CFT is a 1d CFT!)
- Emerge naturally as of defect CFT, a simple laboratory to study defects

In a CFT, for instance $\mathcal{N} = 4$ SYM in $d = 4$ or ABJM in $d = 3$, a Wilson line can be viewed as a conformal defect.

[Giombi Roiban Tseytlin 17] [Giombi Beccaria Tseytlin 18]

$$\langle W \rangle = P \exp \left(- i \int_{t_1}^{t_2} dt\, \mathcal{L}(t) \right)$$

A straight line breaks the original conformal symmetry to

a) rotations around the line

b) dilatations, translations and special conformal transformations

Thus the Wilson loop implicitly defines a defect CFT_1.

Can we study this “simpler” CFT?

A defect CFT$_1$: the 1/2 BPS Wilson line in ABJM theory

The set of correlators of operator insertions along the line

$$\langle \mathcal{O}(t_1)\mathcal{O}(t_2)\ldots\mathcal{O}(t_n)\rangle_W = \frac{\langle \mathrm{Tr}\mathcal{O}_1(t_1)W\mathcal{O}_2(t_2)\ldots\mathcal{O}_{n-1}(t_{n-1})W\mathcal{O}_n(t_n)\rangle}{\langle W\rangle}$$

where

$$\langle W\rangle = P\exp\left(-i\int_{t_1}^{t_2} dt\,\mathcal{L}(t)\right)$$

can be interpreted as characterizing a defect CFT$_1$.

It should be fully determined by its spectrum of dimensions and OPE coefficients.

Consider the $\mathcal{N}=6$ superconformal Chern-Simons-matter theory in $d=3$ (ABJM).

Its original symmetry, $OSp(6|4)$, is broken by the 1/2 BPS Wilson line to $SU(1,1|3)$, the $\mathcal{N}=6$ superconformal group in $d=1$.

Its bosonic subgroup is $SO(2,1)\times U(1)_M\times SU(3)_R$.

Operator insertions along the Wilson line are labelled by $[\Delta; m; j_1, j_2]$.

The displacement supermultiplet

Among the possible operator insertions (defect operators), a special role is played by a set of "elementary excitations" with protected scaling dimension.
They fall into a short representation of the $SU(1,1|3)$ subalgebra
It is a chiral multiplet, the displacement supermultiplet

$$[\tfrac{1}{2},\tfrac{3}{2},0,0] \qquad \mathbb{F},\ \bar{\mathbb{F}}$$
$$\searrow$$
$$[1,2,1,0] \qquad \mathbb{O}^a,\ \bar{\mathbb{O}}_a \quad a=1,2,3$$
$$\searrow$$
$$[\tfrac{3}{2},\tfrac{5}{2},0,1] \qquad \Lambda^a,\ \bar{\Lambda}_a \quad a=1,2,3$$
$$\searrow$$
$$[2,3,0,0] \qquad \mathbb{D},\ \bar{\mathbb{D}} \qquad \partial_\mu T^{\mu m} = \delta^2(x^\perp)\,\mathbb{D}^m(t)$$

Translational invariance is broken, the stress tensor is no longer conserved and the usual conservation law needs to be modified by some additional terms localized on the defect.

The displacement supermultiplet

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$\searrow$ $[2,3,0,0]$	$\mathbb{D},\ \bar{\mathbb{D}}$

8F+8B
like the DOF of transverse string fluctuations

All operators in the supermultiplet can be related to broken symmetry generators.

The displacement supermultiplet

Their 2-point functions are particularly simple, e.g.

$$\langle \mathbb{D}(t_1)\bar{\mathbb{D}}(t_2)\rangle = \frac{C_{\mathbb{D}}}{t_{12}^4}$$

where the normalization constant $C_{\mathbb{D}} = 12\, B_{1/2}(\lambda)$ has a physical meaning:
it coincides with the Bremsstrahlung function, one of the few unprotected observables known to each order in AdS/CFT. [Correa Henn Maldacena Sever 12] [Bianchi Griguolo Preti Seminara17] [Bianchi Preti Vescovi 18]

Their 3-point functions vanish by symmetry.

The displacement supermultiplet

Their 4-point functions, on the other hand, have a less costrained form

$$\langle \mathcal{O}_\Delta(t_1)\mathcal{O}_\Delta(t_2)\mathcal{O}_\Delta(t_3)\mathcal{O}_\Delta(t_4)\rangle = \frac{1}{(t_{12}t_{34})^{2\Delta}}G(\chi).$$

G has non-trivial dependence on the coupling and conformal cross ratio $\chi = \frac{t_{12}t_{34}}{t_{13}t_{24}}$

They encode in particular scaling dimensions and structure constants of unprotected operators appearing in the OPE.

G has a decomposition in blocks, corresponding to the exchange of a given conformal multiplet

$$G(\chi) = \sum_h c_h\, \chi^h\, {}_2F_1(h,h,2h;\chi)$$

with h is the dimension of the exchanged operators

$c_h = \frac{c^2_{\mathcal{O}_\Delta\mathcal{O}_\Delta\mathcal{O}_h}}{c^2_{\mathcal{O}_\Delta\mathcal{O}_\Delta}\, c_{\mathcal{O}_h\mathcal{O}_h}}$ are normalized OPE coefficients

and ${}_2F_1(h,h,2h;\chi)$ is the exact conformal block in $d=1$.

The displacement supermultiplet

Their 4-point functions, on the other hand, have a less costrained form

$$\langle \mathcal{O}_\Delta(t_1)\mathcal{O}_\Delta(t_2)\mathcal{O}_\Delta(t_3)\mathcal{O}_\Delta(t_4)\rangle = \frac{1}{(t_{12}t_{34})^{2\Delta}} G(\chi)\,.$$

G has non-trivial dependence on the coupling and conformal cross ratio $\chi = \frac{t_{12}t_{34}}{t_{13}t_{24}}$

They encode in particular scaling dimensions and structure constants of unprotected operators appearing in the OPE.

Little is known about their structure!

Our study **[Bianchi Bliard Forini Griguolo Seminara 20]**

- superspace analysis
- analytic bootstrap
- direct (Witten) diagrammatics at strong coupling via AdS/CFT

Four-point functions for the defect operators (from superspace analysis)

$$\langle \mathbb{F}(t_1)\bar{\mathbb{F}}(t_2)\mathbb{F}(t_3)\bar{\mathbb{F}}(t_4)\rangle = \frac{f(z)}{t_{12}t_{34}}$$

$$\langle \mathbb{O}^{a_1}(t_1)\bar{\mathbb{O}}_{a_2}(t_2)\mathbb{O}^{a_3}(t_3)\bar{\mathbb{O}}_{a_4}(t_4)\rangle = \frac{4}{t_{12}^2 t_{34}^2}\Big[\delta^{a_1}_{a_2}\delta^{a_3}_{a_4}\left(f(z)+zf'(z)+z^2f''(z)\right) + \delta^{a_1}_{a_4}\delta^{a_3}_{a_2}\left(z^2f'(z)-z^3f''(z)\right)\Big]$$

$$\langle \mathbb{D}(t_1)\bar{\mathbb{D}}(t_2)\mathbb{D}(t_3)\bar{\mathbb{D}}(t_4)\rangle = \frac{64}{t_{12}^4 t_{34}^4}\Big[z^6(1-z)^3f^{(6)}(z) - 3f^{(5)}(z)z^5(1-z)^2(7z+1) + 3f^{(4)}(z)z^4(-46z^3+63z^2-18z+1) + 6f^{(3)}(z)z^3(55z^3-39z^2+3z+1) + 18f''(z)(-14z^5+3z^4+z^2) - 36f'(z)z(1-z^3) + 36f(z)\Big]$$

$$\langle \mathbb{D}(t_1)\bar{\mathbb{D}}(t_2)\mathbb{O}^{a_3}(t_3)\bar{\mathbb{O}}_{a_4}(t_4)\rangle = -\frac{16\,\delta^{a_3}_{a_4}}{t_{12}^4 t_{34}^4}\Big[(1-z)z^4f^{(4)}(z) + (3z+1)z^3f^{(3)}(z) + 3z^2f''(z) + 6zf'(z) + 6f(z)\Big]$$

The correlation function $f(z)$ of the superconformal primary completely determines that of its superdescendants.

Four-point functions for the defect operators

Expanding in Graßmann variables we get

$$\langle \mathbb{F}(t_1)\bar{\mathbb{F}}(t_2)\mathbb{F}(t_3)\bar{\mathbb{F}}(t_4)\rangle = \frac{f(z)}{t_{12}t_{34}}$$

$$\langle \mathbb{O}^{a_1}(t_1)\bar{\mathbb{O}}_{a_2}(t_2)\mathbb{O}^{a_3}(t_3)\bar{\mathbb{O}}_{a_4}(t_4)\rangle = \frac{4}{t_{12}^2 t_{34}^2}\Big[\delta^{a_1}_{a_2}\delta^{a_3}_{a_4}\left(f(z) + zf'(z) + z^2 f''(z)\right) + \delta^{a_1}_{a_4}\delta^{a_3}_{a_2}\left(z^2 f'(z) - z^3 f''(z)\right)\Big]$$

$$\begin{aligned}\langle \mathbb{D}(t_1)\bar{\mathbb{D}}(t_2)\mathbb{D}(t_3)\bar{\mathbb{D}}(t_4)\rangle = \frac{64}{t_{12}^4 t_{34}^4}\Big[& z^6(1-z)^3 f^{(6)}(z) - 3 f^{(5)}(z) z^5 (1-z)^2(7z+1) \\ & + 3 f^{(4)}(z) z^4(-46z^3 + 63z^2 - 18z + 1) \\ & + 6 f^{(3)}(z) z^3(55z^3 - 39z^2 + 3z + 1) \\ & + 18 f''(z)(-14z^5 + 3z^4 + z^2) - 36 f'(z)\, z(1-z^3) + 36 f(z)\Big]\end{aligned}$$

$$\langle \mathbb{D}(t_1)\bar{\mathbb{D}}(t_2)\mathbb{O}^{a_3}(t_3)\bar{\mathbb{O}}_{a_4}(t_4)\rangle = -\frac{16\,\delta^{a_3}_{a_4}}{t_{12}^4 t_{34}^4}\Big[(1-z)\, z^4 f^{(4)}(z) + (3z+1)\, z^3 f^{(3)}(z) + 3z^2 f''(z) + 6z f'(z) + 6 f(z)\Big]$$

The correlation function $f(z)$ of the superconformal primary completely determines that of its superdescendants. → non-trivial function of the coupling!

We can evaluate $f(z)$, and thus ALL the correlators, at strong coupling using string worldsheet worldsheet perturbation theory: via Witten diagrams.

String dual - AdS_2 minimal surface

In AdS/CFT dictionary, the Wilson loop operator is dual to a minimal string surface ending on the contour defining the operator at the boundary.
For ABJM, the dual is a fundamental Type IIA string in a $\mathrm{AdS}_4 \times \mathbb{C}\mathrm{P}^3$ background.
The bosonic part of the Nambu-Goto string action reads

$$S_B = T \int d^2\sigma \sqrt{\det \frac{1}{z^2}\left(\partial_\mu x^r \partial_\nu x^r + \partial_\mu z \partial_\nu z\right) + 4\Big(\frac{\partial_\mu \bar{w}_a \partial_\nu w^a}{1+|w|^2} - \frac{\partial_\mu \bar{w}_a w^a \bar{w}_b \partial_\nu w^b}{(1+|w|^2)^2}\Big)}$$

where T is the effective string tension

$$T = \frac{R^2}{2\pi\alpha'} = 2\sqrt{2\lambda}\,, \qquad \lambda = \frac{N}{k}\,.$$

The minimal surface dual to the 1/2-BPS Wilson line is given by

$$z = s\,, \qquad x^0 = t\,, \qquad x^i = 0\,, \qquad w^a = 0$$

The induced metric is just that of AdS_2

$$ds^2 = \frac{1}{s^2}(dt^2 + ds^2)$$

String dual - AdS_2 minimal surface

This setup preserves same superconformal symmetry $SU(1,1|3)$ of our defect CFT_1!
In particular, the isometry of AdS_2 is the conformal group in $d=1$,

Fluctuation modes over the minimal surface are scalar fields over AdS_2 and their dynamics is governed by the fluctuation Lagrangian

$$S_B \equiv T\int d^2\sigma\sqrt{g}\,L_B\,,\qquad L_B = L_2 + L_{4X} + L_{2X,2w} + L_{4w} + \dots\,,$$

$$L_2 = g^{\mu\nu}\partial_\mu X\partial_\nu\bar{X} + 2|X|^2 + g^{\mu\nu}\partial_\mu w^a\partial_\nu\bar{w}_a\,,$$

$$L_{4X} = 2|X|^4 + |X|^2\left(g^{\mu\nu}\partial_\mu X\partial_\nu\bar{X}\right) - \tfrac{1}{2}\left(g^{\mu\nu}\partial_\mu X\partial_\nu X\right)\left(g^{\rho\kappa}\partial_\rho\bar{X}\partial_\kappa\bar{X}\right),$$

$$L_{2X,2w} = \left(g^{\mu\nu}\partial_\mu X\partial_\nu\bar{X}\right)\left(g^{\rho\kappa}\partial_\rho w^a\partial_\kappa\bar{w}_a\right) - 2\left(g^{\mu\nu}\partial_\mu X\partial_\nu w^a\right)\left(g^{\rho\kappa}\partial_\rho\bar{X}\partial_\kappa\bar{w}_a\right)\,,$$

$$L_{4w} = -\tfrac{1}{2}(w^a\bar{w}_a)(g^{\mu\nu}\partial_\mu w^b\partial_\nu\bar{w}_b) - \tfrac{1}{2}(w^a\bar{w}_b)(g^{\mu\nu}\partial_\mu w^b\partial_\nu\bar{w}_a) + \tfrac{1}{2}\left(g^{\mu\nu}\partial_\mu w^a\partial_\nu\bar{w}_a\right)^2$$
$$- \tfrac{1}{2}(g^{\mu\nu}\partial_\mu w^a\partial_\nu\bar{w}_b)\left(g^{\rho\kappa}\partial_\rho\bar{w}_a\partial_\kappa w^b\right) - \tfrac{1}{2}(g^{\mu\nu}\partial_\mu w^a\partial_\nu w^b)\left(g^{\rho\kappa}\partial_\rho\bar{w}_a\partial_\kappa\bar{w}_b\right)\,.$$

String dual - AdS_2 minimal surface

This setup preserves same superconformal symmetry $SU(1,1|3)$ of our defect CFT_1!
In particular, the isometry of AdS_2 is the conformal group in $d = 1$,

Fluctuation modes over the minimal surface are scalar fields over AdS_2

Then AdS_2/CFT_1 states that they should be dual to operators inserted at the $d = 1$ boundary with dimensions

$$\Delta(\Delta - 1) = m^2 \quad \text{bosons} \qquad \Delta = \frac{1}{2} + |m| \quad \text{spinors}$$

Hence, we recover the eight bosonic operators in the super-displacement multiplet!

$\Delta = \frac{1}{2}$	$\mathbb{F}, \bar{\mathbb{F}}$	$\Longleftrightarrow$	$\psi, \bar{\psi}$	$m^2 = 0$
$\Delta = 1$	$\mathbb{O}^a, \bar{\mathbb{O}}_a \quad a = 1,2,3$	$\Longleftrightarrow$	$w^a, \bar{w}_a$	$m^2 = 0$
$\Delta = \frac{3}{2}$	$\mathbb{\Lambda}^a, \bar{\mathbb{\Lambda}}_a \quad a = 1,2,3$	$\Longleftrightarrow$	$\psi^a, \bar{\psi}_a$	$m_F = \pm 1$
$\Delta = 2$	$\mathbb{D}, \bar{\mathbb{D}}$	$\Longleftrightarrow$	$X, \bar{X}$	$m^2 = 2$

Witten diagrams in AdS_2

The four-point functions of the dual operators at strong coupling can then be obtained from familiar AdS/CFT techniques by computing Witten diagrams in AdS_2.

For the 4-point function of fields e.g. in AdS

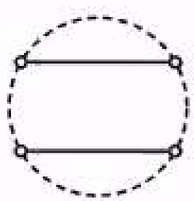
$$\langle X(t_1)\,\bar{X}(t_2)\,X(t_3)\,\bar{X}(t_4)\rangle = \frac{1}{t_{12}^2 t_{34}^2} G(z)\,,$$

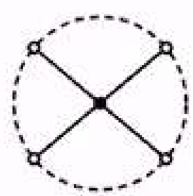
where $G(z)$ has the strong coupling expansion

$$G(z) = G^{(0)}(z) + \frac{1}{T} G^{(1)}(z) + \ldots$$

disconnected contribution (diagrams with 2 "boundary-to-boundary" propagators)

tree-level contact diagrams (4-vertices with 4 bulk-to-boundary propagators attached)





Summary of 4-point function results

The correlators of string worldsheet excitations read

The superspace analysis of correlators for defect operators gives

$$\langle X(t_1)\,\bar{X}(t_2)\,X(t_3)\,\bar{X}(t_4)\rangle = \frac{1}{t_{12}^4 t_{34}^4}\Big[1 + z^4 + \tfrac{1}{T}\big[-8z^4 - (3-8z)z^4(\ln z - \ln(1-z)) - z^3 - \tfrac{7}{6}z^2 - z - (8-3z)\tfrac{\ln(1-z)}{z} - 8\big]\Big]$$

$$\Updownarrow$$

$$\langle \mathbb{D}(t_1)\bar{\mathbb{D}}(t_2)\mathbb{D}(t_3)\bar{\mathbb{D}}(t_4)\rangle = \frac{64}{t_{12}^4 t_{34}^4}\Big[z^6(1-z)^3 f^{(6)}(z) - 3\,f^{(5)}(z)z^5(1-z)^2(7z+1) + 3\,f^{(4)}(z)z^4(-46z^3+63z^2-18z+1) + 6\,f^{(3)}(z)z^3(55z^3-39z^2+3z+1) + 18\,f''(z)\,(-14z^5+3z^4+z^2) - 36\,f'(z)\,z(1-z^3) + 36\,f(z)\Big]$$

$$\langle w^{a_1}(t_1)\,\bar{w}_{a_2}(t_2)\,w^{a_3}(t_3)\,\bar{w}_{a_4}(t_4)\rangle = \frac{1}{t_{12}^2 t_{34}^2}\Big[\delta^{a_1}_{a_2}\delta^{a_3}_{a_4}\big[1 + \tfrac{1}{2T}(z^2\ln z - (z^2 - \tfrac{4}{z} + 3)\ln(1-z) - z + 4)\big] + \delta^{a_1}_{a_4}\delta^{a_3}_{a_2}\big[z^2 + \tfrac{1}{2T}((3-4z)z^2\ln z + (4z^3-3z^2-1)\ln(1-z) + (4z-1)z)\big]\Big]$$

$$\Updownarrow$$

$$\langle \mathbb{O}^{a_1}(t_1)\bar{\mathbb{O}}_{a_2}(t_2)\mathbb{O}^{a_3}(t_3)\bar{\mathbb{O}}_{a_4}(t_4)\rangle = \frac{4}{t_{12}^2 t_{34}^2}\Big[\delta^{a_1}_{a_2}\delta^{a_3}_{a_4}\,(f(z) + zf'(z) + z^2 f''(z)) + \delta^{a_1}_{a_4}\delta^{a_3}_{a_2}\,(z^2 f'(z) - z^3 f''(z))\Big]$$

$$\langle X(t_1)\,\bar{X}(t_2)\,w^{a_3}(t_3)\,\bar{w}_{a_4}(t_4)\rangle = \frac{1}{t_{12}^4 t_{34}^2}\delta^{a_3}_{a_4}\big[1 + \tfrac{1}{T}(2(z-2)\tfrac{\ln(1-z)}{z} - 4)\big]$$

$$\Updownarrow$$

$$\langle \mathbb{D}(t_1)\bar{\mathbb{D}}(t_2)\mathbb{O}^{a_3}(t_3)\bar{\mathbb{O}}_{a_4}(t_4)\rangle = -\frac{16\,\delta^{a_3}_{a_4}}{t_{12}^4 t_{34}^4}\Big[(1-z)\,z^4 f^{(4)}(z) + (3z+1)\,z^3 f^{(3)}(z) + 3z^2 f''(z) + 6zf'(z) + 6f(z)\Big]$$

Is there a single f(z) solving simultaneously these non-trivial ODEs?

Summary of 4-point function results

The correlators of string worldsheet excitations read

The superspace analysis of correlators for defect operators gives

$$\langle X(t_1)\bar{X}(t_2)X(t_3)\bar{X}(t_4)\rangle = \frac{1}{t_{12}^4 t_{34}^4}\Big[1+z^4+\frac{1}{T}\big[-8z^4-(3-8z)z^4(\ln z-\ln(1-z)) - z^3-\frac{8}{6}z^2-z-(8-3z)\frac{\ln(1-z)}{z}-8\big]\Big]$$

$$\langle \mathbb{D}(t_1)\bar{\mathbb{D}}(t_2)\mathbb{D}(t_3)\bar{\mathbb{D}}(t_4)\rangle = \frac{64}{t_{12}^4 t_{34}^4}\Big[z^6(1-z)^2 f^{(6)}(z)-3f^{(5)}(z)z^5(1-z)^2(7z+1) +3f^{(4)}(z)z^4(-46z^3+63z^2-18z+1) +6f^{(3)}(z)z^3(55z^3-39z^2+3z+1) +18f''(z)(-14z^5+3z^4+z^3)-36f'(z)z(1-z^3)+36f(z)\Big]$$

$$\langle w^{a_1}(t_1)\bar{w}_{a_2}(t_2)w^{a_3}(t_3)\bar{w}_{a_4}(t_4)\rangle = \frac{1}{t_{12}^2 t_{34}^2}\Big[\delta^{a_1}_{a_2}\delta^{a_3}_{a_4}\big[1+\frac{1}{2T}\big(z^2\ln z-(z^2-\frac{4}{z}+3)\ln(1-z)-z+4\big)\big] +\delta^{a_1}_{a_4}\delta^{a_3}_{a_2}\big[z^2+\frac{1}{2T}\big((3-4z)z^2\ln z+(4z^3-3z^2-1)\ln(1-z)+(4z-1)z\big)\big]\Big]$$

$$\langle \mathbb{O}^{a_1}(t_1)\bar{\mathbb{O}}_{a_2}(t_2)\mathbb{O}^{a_3}(t_3)\bar{\mathbb{O}}_{a_4}(t_4)\rangle = \frac{4}{t_{12}^2 t_{34}^2}\Big[\delta^{a_1}_{a_2}\delta^{a_3}_{a_4}\big(f(z)+zf'(z)+z^2f''(z)\big) +\delta^{a_1}_{a_4}\delta^{a_3}_{a_2}\big(z^2f'(z)-z^3f''(z)\big)\Big]$$

$$\langle X(t_1)\bar{X}(t_2)w^{a_3}(t_3)\bar{w}_{a_4}(t_4)\rangle = \frac{1}{t_{12}^4 t_{34}^2}\delta^{a_3}_{a_4}\Big[1+\frac{1}{T}\big(2(z-2)\frac{\ln(1-z)}{z}-4\big)\Big]$$

$$\langle \mathbb{D}(t_1)\bar{\mathbb{D}}(t_2)\mathbb{O}^{a_3}(t_3)\bar{\mathbb{O}}_{a_4}(t_4)\rangle = -\frac{16\,\delta^{a_3}_{a_4}}{t_{12}^4 t_{34}^4}\Big[(1-z)z^4f^{(4)}(z)+(3z+1)z^3f^{(3)}(z)+3z^2f''(z)+6zf'(z)+6f(z)\Big]$$

These differential equations are all solved by the simple function

$$f(z) = 1 - z + \frac{1}{T}\Big(1 - z - (3-z)z\ln z + \frac{(1-z)^3}{z}\ln(1-z)\Big) + \mathcal{O}\Big(\frac{1}{T^2}\Big)$$

This is the strong coupling expansion of the function governing all correlation functions of operators in the displacement supermultiplet.

This correction at NLO was derived also using analytic bootstrap!

CFT data at strong coupling

The four-point function has an OPE expansion in superblocks

$$\langle \mathbb{F}(t_1)\bar{\mathbb{F}}(t_2)\mathbb{F}(t_3)\bar{\mathbb{F}}(t_4)\rangle = \frac{1}{t_{12}t_{34}} f(z) = \frac{1}{t_{12}t_{34}} \sum_h c_h (-z)^h \, {}_2F_1(h, h, 2h+3, z)\,.$$

At this order, operators exchanged in the OPE are just the identity and the tower of operators $\mathcal{O}\partial^n\mathcal{O}$, built out of the elementary excitations. Therefore,

$$h = 1 + n + \frac{1}{T}\gamma_n^{(1)} \qquad c_n = c_n^{(0)} + \frac{1}{T} c_n^{(1)}$$

$$\gamma_n^{(1)} = 3 + 4n + n^2$$

$$c_n = \sqrt{\pi}2^{-2n-3}(n+3)\frac{\Gamma(n+1)}{\Gamma\left(n+\frac{5}{2}\right)}\Big[(n+2) + \frac{1}{T}\big[4n^2 - 2\left(n^3+6n^2+11n+6\right)\ln 2 + 15n$$
$$+(n+1)(n+2)(n+3)\psi^{(0)}(n+1) - (n+1)(n+2)(n+3)\psi^{(0)}\left(n+\tfrac{5}{2}\right) + 13\big]\Big]$$

"inverting" for the coefficients in the sum, namely using orthogonality relations for the hypergeometric functions.

16:57 Wed 24. Feb

Perimeter_February_2021

1D Mellin formalism for 4 point correlators of identical scalars (1)

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Intermediate conclusions and questions

- We have considered a class of four-point correlators in the CFT_1 defined on the 1/2-BPS Wilson line in the 3d superconformal ABJM theory.
- Superconformal symmetry determines four-point correlators of the displacement supermultiplet in terms of a single function, that we evaluate at strong coupling using holography and Witten diagrams and the analytic boostrap. We can extract CFT data.

What is the relation to the (believed) integrability of the string sigma model?
Is there an AdS_2 analog of e.g. S-matrix factorization of integrable theories?
What happens beyond tree-level? Witten diagrams with loops in AdS should be well-defined, since the 2d theory is supposed to be UV finite.
However, issues of regularization seem to appear.

Is there a representation ("momentum space") in which these computations simplify?

Conformal correlators and Mellin space

Yes. In Mellin space the scattering nature of the correlator becomes more transparent.

Higher dimensions [Mack 2009] [Penedones 2010]

$$\langle\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{1}{x_{13}^{2\Delta}x_{24}^{2\Delta}} F(u,v)\,, \qquad u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}\,, \qquad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

$$F(u,v) = \int_{\mathcal{C}} d\gamma_{12} d\gamma_{14}\, M(\gamma_{12},\gamma_{14})\, \Gamma^2(\gamma_{12})\, \Gamma^2(\gamma_{14}) \Gamma^2(\Delta-\gamma_{12}-\gamma_{14})\, u^{-\gamma_{12}} v^{-\gamma_{14}}$$

$M(\gamma_{12},\gamma_{14})$ has the properties of a scattering amplitude:

- Crossing symmetry
- Poles corresponding to (the twist of) operators exchanged in the OPE
- Asymptotic behavior compatible with the Regge limit
- Simple expression for Witten diagrams

Conformal correlators and Mellin space

Yes. In Mellin space the scattering nature of the correlator becomes more transparent.

Higher dimensions

[Mack 2009] [Penedones 2010]

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{1}{x_{13}^{2\Delta}x_{24}^{2\Delta}} F(u,v)\,, \qquad u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}\,, \qquad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

$$F(u,v) = \int_{\mathcal{C}} d\gamma_{12} d\gamma_{14}\, M(\gamma_{12},\gamma_{14})\, \Gamma^2(\gamma_{12})\, \Gamma^2(\gamma_{14}) \Gamma^2(\Delta - \gamma_{12} - \gamma_{14})\, u^{-\gamma_{12}} v^{-\gamma_{14}}$$

$M(\gamma_{12},\gamma_{14})$ has the properties of a scattering amplitude:

- Crossing symmetry
- Poles corresponding to (the twist of) operators exchanged in the OPE
- Asymptotic behavior compatible with the Regge limit
- Simple expression for Witten diagrams

Conformal correlators and Mellin space in $d = 1$

In $d = 1$, just one independent cross ratio and thus one independent Mellin variable (correspondingly, scattering in $d = 2$ described by a single Mandelstam variable)

- Reduce the higher-dimensional case: possible, technically involved
- Define an inherently $d = 1$ Mellin amplitude inspired by same guiding principles

$$M(s) = \frac{1}{\Gamma(s)\Gamma(2\Delta - s)} \int_0^\infty dz\, z^{-1-s} f(z)\,, \qquad z = \frac{x_{12}\, x_{34}}{x_{14} x_{23}} > 0$$

with inverse

$$f(z) = \int_{\mathcal{C}} \frac{ds}{2\pi i}\, \Gamma(s)\, \Gamma(2\Delta - s)\, M(s)\, z^s\,,$$

where

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{1}{x_{12}^{2\Delta}\, x_{34}^{2\Delta}}\, f(z)\,,$$

Crossing $f(z) = z^{2\Delta}\, f(1/z)$ translates to

$$M(s) = M(2\Delta - s).$$

reminiscent of the crossing $S(s) = S(4m^2 - s)$ in two (flat) dimensions.

17:00 Wed 24. Feb

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1D Mellin formalism for 4 point correlators of identical scalars (1)

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Nonperturbative Mellin amplitude in $d = 1$

$$M(s) = \frac{1}{\Gamma(s)\Gamma(2\Delta - s)} \int_0^{\infty} dz\, z^{-1-s} f(z)\,,$$

- $M(s)$ is crossing-invariant
- $M(s)$ is regular in $2\Delta - h_0 <$ Re$(s) < h_0$
- Asymptotic behavior: $M(s) \sim \frac{1}{s^a}\,, a > 1$

- $M(s)$ has poles for physical exchanged operators at $s = h + k$ k=0,1,2,..with residue Res$[M(s)]|_{s=h+k} = \frac{(-1)^k \Gamma(2h)\Gamma(h+k)}{k!\,\Gamma(h)^2\Gamma(2h+k)\Gamma(2\Delta-h-k)}$
- $M(s)$ has zeros (generically) at $s = 2\Delta + k\,,\ k = 0, 1, 2, ..$ (canceling unwanted OPE contributions)

Using this properties, we can derive sum rules constraining the CFT data

$$\sum_h c_h\, F(h) = 0$$

Sum rules

Define a family of functionals

$$\omega_p = \oint_{\mathbb{C}|_\infty} \frac{ds}{2\pi i} \frac{M(s)}{s - 2\Delta - p} = 0$$

Then

$$\omega_p = \sum_{s^*} \frac{\text{Res}_{s=s^*}(M(s))}{s^* - 2\Delta - p} + M(2\Delta - p)$$

and the sum rules read

$$\omega_p = \sum_{h,k} c_h \frac{(-1)^{k+1}\Gamma(2h)\Gamma(h+k)\, 2(h+k-\Delta)}{\Gamma(h)^2\Gamma(2h+k)\Gamma(2\Delta-h-k)\Gamma(k+1)\,(h+k+p)(h+k-2\Delta-p)} = 0$$

Tested on generalised free field theory and perturbatively on ϕ^4 model.

Conclusions

The one-dimensional conformal defect theories of which I sketched an example are very interesting models to study defect theories with a variety of techniques.

I have sketched a proposal of a consistent "Mellin amplitude" in $d = 1$, which in a general CFT$_1$ provides

- simple, closed formulas for four-point correlators corresponding to quartic interactions (not shown today)
- an infinite set of sum rules, of which we have tested the consistency and that would be great to use in a predictive way.