Title: Researcher Presentations

Speakers: Astrid Eichhorn, Colleen Delaney, Roberta Iseppi, Lisa Glaser, Mahumm Ghaffar, Eilind Karlsson, Evelyn Yoczira Lira Torres, Sharmila Gunasekaran

Collection: Women at the Intersection of Mathematics and Theoretical Physics

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Women at the Intersection of Mathematics and Theoretical Physics
Perimeter Institute

Quantum Gravity and Riemannian Geometry on The Fuzzy Sphere

Joint work with Prof. Shahn Majid

Evelyn Yoczira Lira Torres
Queen Mary University of London

23 February 2021

Arxiv 2004.14363

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Quantum spacetime hypothesis:

Spacetime is not a continuum → More effectively described by a noncommutative coordinate algebra where {x,y,z,t} do not fully commute

Quantum Gravity → Quantum Geometry → Classical GR

Geometry	Algebra
Topological space	Coordinate algebra *(not required commutative)*
Differentiable manifold	Differencial graded algebra
Riemannian structure …	Noncommutative metric, connections, …

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Classical Riemannian Geometry

- Spaces as commutative algebras

$$C^\infty(M) = \Omega^0(M) \subset \Omega(M) = \bigoplus_i \Omega^i(M)$$

- Ω^1 space of 1-forms ('differentials')
  - dx commute

$$f dg = (dg) f \in \Omega^1$$

  - Leibniz rule

$$d(fg) = (df)g + f dg$$

- Exterior product with graded Leibniz rule

$$d(\omega \wedge \eta) = (d\omega) \wedge \eta (-1)^{|\omega|} \omega \wedge d\eta$$

$$d^2 = 0$$

Quantum Riemannian Geometry

- Unital algebra over a field.
  - No commutativity needed.
  - Exterior product noncommutative.
- Ω^1 as bimodule

$$a((db)c) = (a(db))c$$

  - Leibniz rule

$$d(ab) = (da)b + a(db)$$

- 'Surjectivity' condition

$$\Omega^1 = span\{a(db)|a, b \in A\}$$

$$d^2 = 0$$

- This has to extend to a DGA
- A first order differential calculus (A, d, Ω^1)

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(A, d, Ω^1)

QRG

- Quantum metric $g \in \Omega^1 \otimes_A \Omega^1$ ⟶ g is central
  - Inverse, bimodule inner product $(\ ,\ ) : \Omega^1 \otimes_A \Omega^1 \to A$
- Connection (left & right)

$$\nabla(a.\omega) = da \otimes \omega + a.\nabla\omega, \qquad \nabla(\eta a) = (\nabla\eta)a + \sigma(\eta \otimes da)$$

  - Where generalised braiding

$$\sigma : \Omega^1 \otimes_A \Omega^1 \to \Omega^1 \otimes_A \Omega^1$$

  - Extended to tensor products

$$\nabla(\omega \otimes \eta) = \nabla\omega \otimes \eta + (\sigma(\omega \otimes ()) \otimes \mathrm{id})\Delta\eta$$

- Torsion, $T_\nabla : \Omega^1 \to \Omega^2, \quad T_\nabla = \wedge\nabla - d$
- Metric compatibility: $\nabla(g) = 0$ and Torsion free: $T_\nabla = 0$ ⟶ QLC
- Curvature: $R_\nabla : \Omega^1 \to \Omega^2 \otimes_A \Omega^1, R_\nabla = (d \otimes \mathrm{id} - \mathrm{id} \wedge \nabla)\nabla$
- Ricci: $\mathrm{Ricci} \in \Omega^1 \otimes_A \Omega^1$ ⟶ $i : \Omega^2 \to \Omega^1 \otimes_A \Omega^1$ Lifting map
  - Scalar curvature $S = (\ ,\ )\ \mathrm{Ricci} \in A$

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QRG

- Real quantum metric, $g = g_{ij} s^i \otimes s^j$
  - With inverse, $(s^i, s^j) = g^{ij}$
- Connection ,

$$\nabla s^i = -\tfrac{1}{2} \Gamma^i{}_{jk} s^j \otimes s^k$$

  - Where generalised braiding,

$$\sigma(s^i \otimes s^j) = s^j \otimes s^i + \dots$$

- Curvature,

$$R_\nabla(s^i) = \rho^i{}_{jn} \epsilon_{jim} s^m \otimes s^n$$

$$R_{mn} = \rho^i{}_{jn} \epsilon_{jim}, \quad S = \rho^i{}_{jn} \epsilon_{jim} g^{mn}$$

- Scalar curvature $S = \tfrac{1}{2}(\mathrm{Tr}(g^2) - \tfrac{1}{2}\mathrm{Tr}(g)^2)/\det(g)$

- QLC unique, $\Gamma_{ijk} = 2\epsilon_{ikm} g_{mj} + Tr(g)\epsilon_{ijk}$ any metric

Constant

Fuzzy Sphere

- Algebra

$$[x_i, x_j] = 2\imath \lambda_p \epsilon_{ijk} x_k$$

λ_p Plank scale

$$\mathbb{C}_\lambda[S^2] = U(\mathfrak{su}_2)/\langle \textstyle\sum_j x_j^2 - (1 - \lambda_p^2) \rangle$$

- Calculus $\Omega^1 = \{s^1, s^2, s^3\}$,
  - $[s^i, x_j] = 0$
  - $s^i \wedge s^j + s^j \wedge s^i = 0$
  - $dx_i = \epsilon_{ijk} x_j s^k$
  - $ds^i = -\tfrac{1}{2} \epsilon_{ijk} s^j \wedge s^k$

Inner calculus, $\theta = \frac{1}{2\imath\lambda_p} x_i s^i$

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QG $\sum_i x_i^2 = 1 - \lambda_p^2$ ⟹ Euclideanised QG (+)

- Scalar curvature

$$S[g] = \tfrac{1}{2}(\mathrm{Tr}(g^2) - \tfrac{1}{2}\mathrm{Tr}(g)^2)/\det(g) \quad \Longrightarrow \quad \int 1 = |\det(g)|$$

- Functional integral:

$$Z = \int \mathcal{D}g\, e^{-\frac{2}{G}\int S[g]}$$

  - The metric in the 3x3 positive definite symmetric matrices $\mathcal{P}_3 \longrightarrow GL_3(\mathbb{R})/O_3(\mathbb{R})$ ↙ Riemannian manifold w/ invariant metric $\mathfrak{g}_{\mathcal{P}_3}$

And, ⟵ Lebesgue measure ⟵ Line element

$$\sqrt{|\det(\mathfrak{g}_{\mathcal{P}_3})|} = |\det(g)|^{-2} \qquad \sqrt{|\det(\mathfrak{g}_{\mathcal{P}_3})|} \qquad ds^2 = \mathrm{Tr}((g^{-1}dg)^2)$$

$$\Longrightarrow Z = \int_{\mathcal{P}_3} \prod_{i\leq j} dg_{ij}\, |\det(g)|^{-2}\, e^{-\frac{1}{G}(\mathrm{Tr}(g^2) - \frac{1}{2}\mathrm{Tr}(g)^2)}$$

Spectral descomposition for g

$$g = E(\theta,\phi,\psi)^t \mathrm{diag}(\lambda_1,\lambda_2,\lambda_3) E(\theta,\phi,\psi)$$

The Jacobean,

$$\prod_{i\leq j} dg_{ij} = d\theta\, d\phi\, d\psi\, |\sin(\phi)| \prod_i d\lambda_i\, |(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)(\lambda_2-\lambda_3)|$$

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Riemannian manifold w/ invariant metric $\mathfrak{g}_{\mathcal{P}_3}$ → Line element $ds^2 = \mathrm{Tr}((g^{-1}dg)^2)$ → Lebesgue measure $\sqrt{|\det(\mathfrak{g}_{\mathcal{P}_3})|}$ → And, $\sqrt{|\det(\mathfrak{g}_{\mathcal{P}_3})|} = |\det(g)|^{-2}$

$$\Longrightarrow \quad Z = \int_{\mathcal{P}_3} \prod_{i\leq j} dg_{ij}\; |\det(g)|^{-2}\, e^{-\frac{1}{G}(\mathrm{Tr}(g^2) - \frac{1}{2}\mathrm{Tr}(g)^2)}$$

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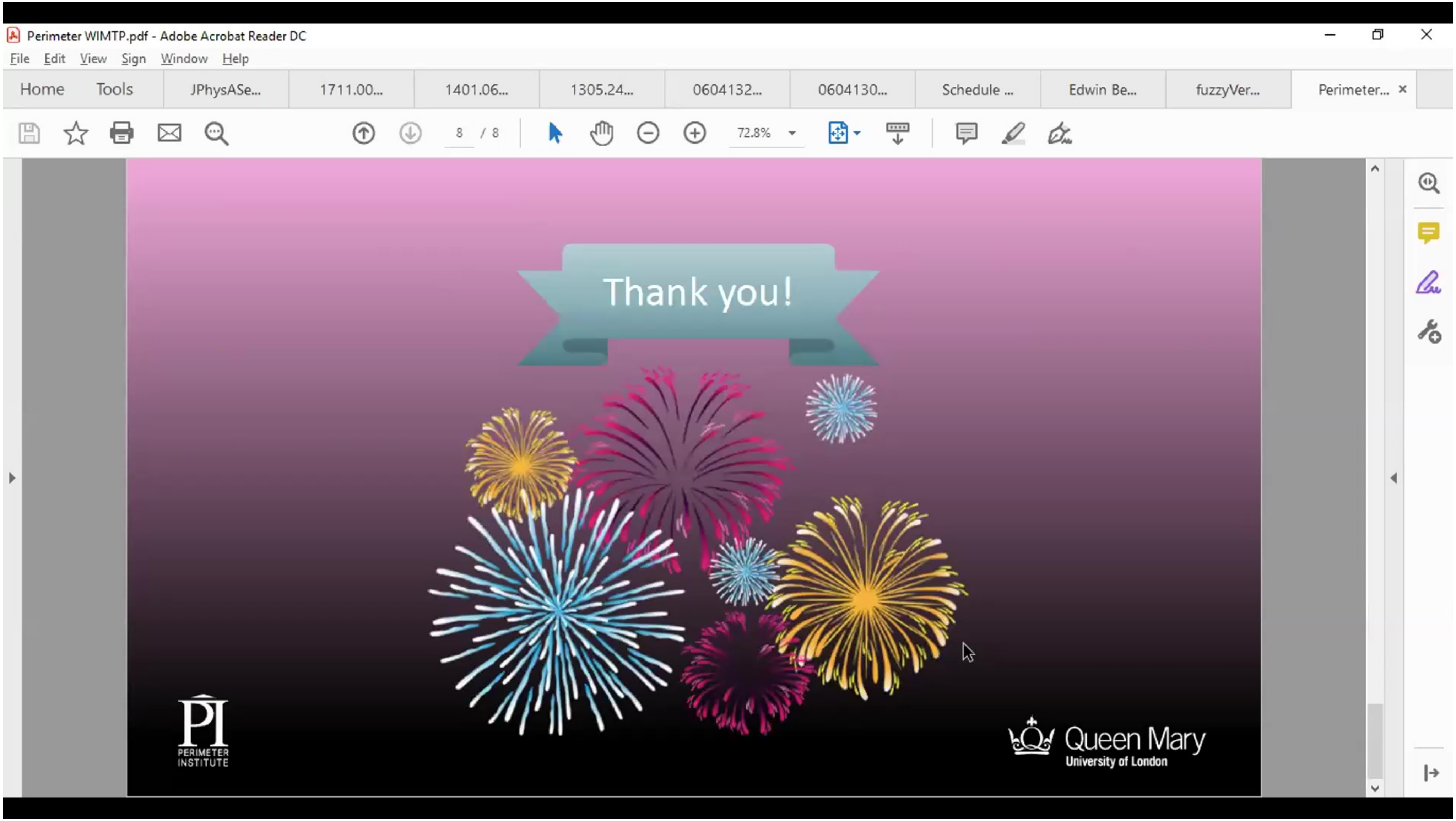
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19:29 Tue 23. Feb

Eilind Karlsson, TUM

Fully Extended Topological Field Theories

↳ Eilind Karlsson, TUM

$$Z : \text{Bord} \longrightarrow \text{Vect}$$

Fully Extended Topological Field Theories

↳ Eilind Karlsson, TUM

$$Z : \mathrm{Bord} \longrightarrow \mathrm{Vect}$$

e.g. $S = \mathrm{Vect}$, Chain, CAT

$$Z : \mathrm{Bord}_d^{fr} \longrightarrow \mathrm{Alg}_n(S)$$

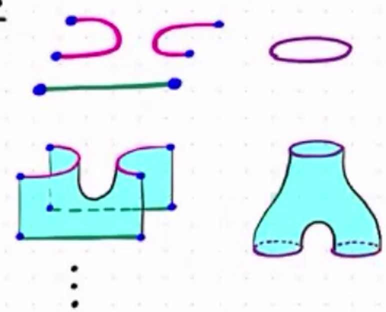
- Z: symmetric monoidal functor
- Bord_d^{fr}: category of (framed) bordisms
- $\mathrm{Alg}_n(S)$: Higher Morita category

informal

obj • • (framed) 0-mflds

1-morp

2-morp



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Eilind Karlsson, TUM

informal

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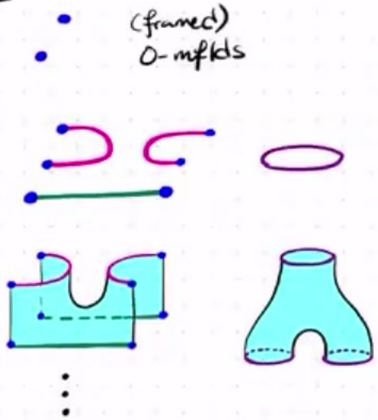
1-morp

2-morp



informal

obj E_n-algebras (in S)

1-morp Bimodule of E_{n-1}-algs

2-morp Bimod. of bimod. of E_{n-2}-algs

⋮

n-morp Bimodule of ... of bimod

(n+1)-morp bimodule map

⋮

Examples of E_n-algs.

S=Vect { n=1 assoc. algebras; n=2 commutative algs.

S=CAT { n=1 tensor category; n=2 braided tensor cat.

"local observables for a TFT"

Examples of E_n-algs.

$S = \text{Vect}$:
- $n=1$ assoc. algebras
- $n=2$ commutative algs.

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- $n=1$ tensor category
- $n=2$ braided tensor cat.

"local observables for a TFT"

Cobordism hypothesis

= Locality statement

$$d\,\text{TFT} = \text{Fun}^{\otimes}(\text{Bord}_d^{fr}, \mathcal{C}) \simeq \mathcal{C}^{f.d}$$

$\mathcal{C}^{f.d}$: d-dualizable objects in $\mathcal{C}$

19:34 Tue 23. Feb

Eilind Karlsson, TUM

[Lurie] [Brochier - Jordan - Snyder for S=CAT]

My thesis project

Thm (for $n=1,2$) / Conjecture ($n \geq 3$)

An E_n-algebra $A \in Alg_n(S)$ is $(n+1)$-dualizable iff it is dualizable over the factorization homologies

$$\int_{S^{k-1} \times \mathbb{R}^{n-k+1}} A \qquad \text{for } k=0,\dots,n.$$

19:35 Tue 23. Feb — Eilind Karlsson, TUM — 98 %

[Lurie] [Brochier-Jordan-Snyder for S=CAT]

Thm (for n=1,2) / Conjecture (n ≥ 3) ← My thesis project

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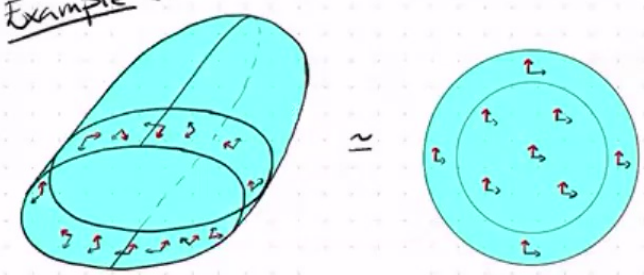
Example (n=2, k=2)



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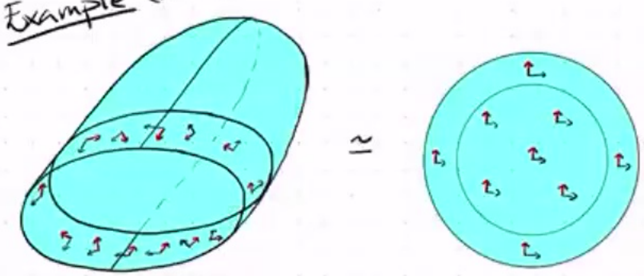


* Why care?
  - Gives higher-dimensional TFT's (TV, RT, CY)
  - Needed for invertibility ⇒ invertible TFT's

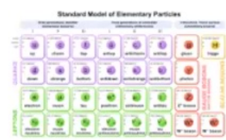
Why noncommutative geometry?

The concept of *quantization* brought two new ideas into the mathematical formalization of physics laws: discreteness & noncommutativity

Gravity:

- curvature of the spacetime $\rightsquigarrow$ continuous nature
- framework: Riemannian diff. geometry $\rightsquigarrow$ commutative

Fundamental interactions



- mediated by particle $\rightsquigarrow$ discrete nature
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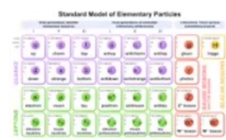

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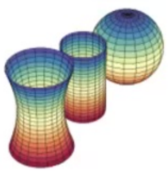
University of Southern Denmark

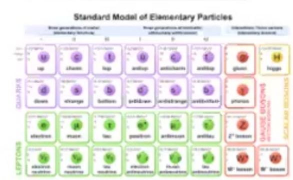
Noncommutative geometry and the BV formalism: the case of finite dimensional noncommutative manifolds

Roberta A. Iseppi

Women at the intersection of mathematics and theoretical physics

February 2021





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Goal: provide a unifying background $\rightsquigarrow$ *Idea:* describe geometric objects in algebraic terms

A. Connes

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A. Connes

Topology:

loc. compact Hausdorff sp $\leftrightsquigarrow$ (Gelfand Neimark Th. ['43]) comm. $\mathcal{C}^*$-alg.

noncomm. topological sp. $\leftrightsquigarrow$ noncomm. $\mathcal{C}^*$-alg.

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A. Connes

Goal: provide a unifying background $\rightsquigarrow$ *Idea:* describe <u>geometric</u> objects in <u>algebraic</u> terms

<u>Topology:</u>

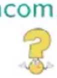
loc. compact Hausdorff sp $\leftrightsquigarrow$ (Gelfand Neimark Th. ['43]) comm. C^*-alg.

noncomm. topological sp. $\leftrightsquigarrow$ noncomm. C^*-alg.

<u>Metric:</u>

compact Riem. spin manifold $\leftrightsquigarrow$ (Reconstr. Th. Connes [2008]) canonical spectral triple $(C^\infty(M), L^2(M,S), D, J, \gamma)$

noncom. mfd $\leftrightsquigarrow$ noncom. spectral tr. $(\mathcal{A}, \mathcal{H}, D)$



Roberta A. Iseppi

Noncommutative geometry and the BV formalism: the case of finite dimensional noncommutative manifolds 2

Why NCG can be interesting to study QFT?

spectral triple		gauge theory
$(\mathcal{A}, \mathcal{H}, D)$	➡	$(X_0, S_0, \mathcal{G})$

Each spectral triples encodes a gauge theory

- $\mathcal{A}$ = unital *-alg., $\mathcal{A} \cong \mathcal{B}(\mathcal{H})$
- $\mathcal{H}$ = Hilbert sp.
- $D : \mathcal{H} \to \mathcal{H}$ = self-adj. op.

- $X_0 = \left\{ \varphi = \sum_j a_j[D, b_j] : \varphi^* = \varphi \right\} \rightsquigarrow$ conf. sp = inner fluctuations
- $S_0[D + \varphi] = Tr(f(D + \varphi)), \quad f \in \mathbb{R}[x] \rightsquigarrow$ action func. = spectral action
- $\mathcal{G} = \mathcal{U}(\mathcal{A}) \rightsquigarrow$ gauge group = unitary elements in $\mathcal{A}$

Does all of this describe any physically relevant model?
$\rightsquigarrow$ the Standard Model as an almost-commut. spectral triple:

[A.H. Chamseddine, A. Connes, M. Marcolli, '07]

$$M \times F$$

M = compact Riem. spin manifold with canonical spectral triple

$(\mathcal{C}^\infty(M), L^2(M, S), D_M, J_M, \gamma_M)$

F= finite noncomm. space with finite real spectral triple

$(\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C}), \mathbb{C}^{96}, D_{SM}, J_{SM}, \gamma_{SM})$

The BV construction and the BRST cohomology (Batalin-Vilkovisky, [1983]

Context: quantization of a gauge theory (X_0, S_0) via a path integral approach

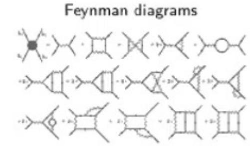
$$\langle g \rangle = \frac{1}{Z}\int_{X_0} g e^{\frac{i}{\hbar}S_0}[d\mu], \qquad \text{where} \qquad Z := \int_{X_0} e^{\frac{i}{\hbar}S_0}[d\mu].$$

$\langle g \rangle$ ↓ expectation value of a reg. funct. g on X_0

Z ↓ partition function

path integral

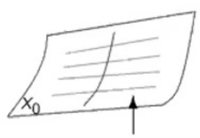
Feynman diagrams



Problem 1: the measure is not well-defined ⇝ perturbative approach with Feynman diagram

$$\int_{X_0} e^{\frac{i}{\hbar}S_0}[d\mu] \underset{\hbar\to 0}{\backsim} \sum_{x_0 \in \{\text{crit. pts } S_0\}} e^{\frac{i}{\hbar}S_0(x_0)} \,|\det S_0''(x_0)|^{-\frac{1}{2}}\, e^{\frac{\pi i}{4} sign(S_0''(x_0))} (2\pi\hbar)^{\frac{\dim X_0}{2}} \sum_\Gamma \frac{\hbar^{-\chi(\Gamma)}}{|Aut(\Gamma)|}\Phi_\Gamma.$$

Problem 2: the presence of orbits of critical points for gauge invariant action functional ⇝ ghost fields



orbits of $\mathcal{G}$

$$(X_0, S_0) \xrightarrow{\text{BV construction}} (\widetilde{X}, \widetilde{S})$$

$\widetilde{X} = X_0 \cup \{\text{ghost/anti-ghost fields}\}$

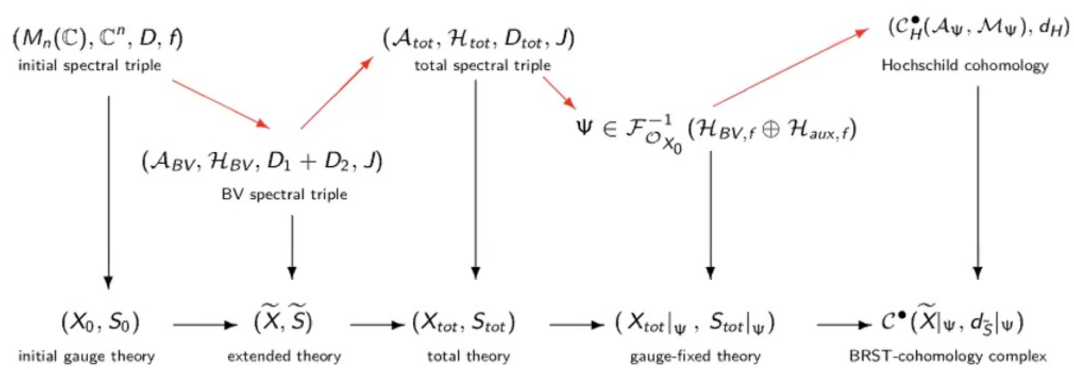
$\widetilde{S} = S_0 +$ terms depending on gh./anti-gh.

ghost fields ↓

- non-existing fields bosonic/fermionic,
- auxiliary variables

Where we are:

Noncommutative geometry



BV construction & BRST cohomology

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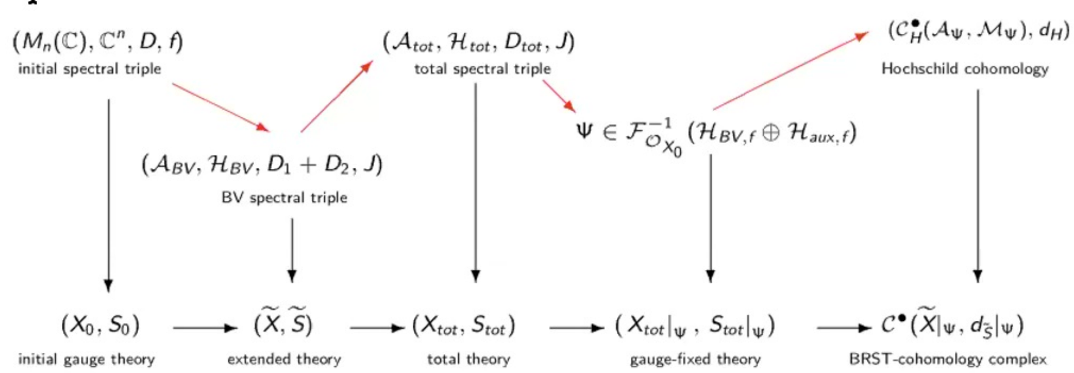
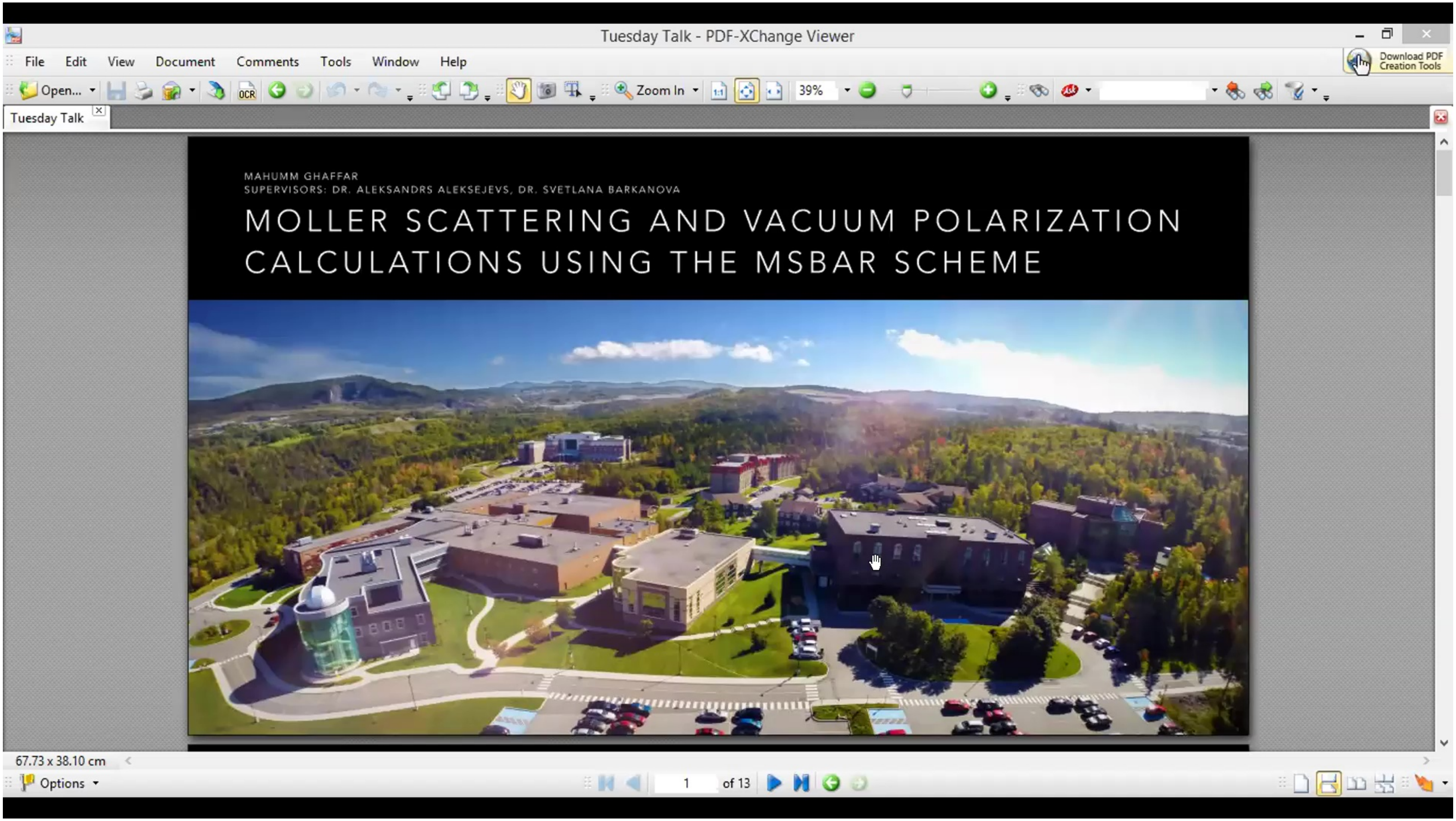


BV construction & BRST cohomology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MAHUMM GHAFFAR
SUPERVISORS: DR. ALEKSANDRS ALEKSEJEVS, DR. SVETLANA BARKANOVA
MOLLER SCATTERING AND VACUUM POLARIZATION CALCULATIONS USING THE MSBAR SCHEME
67.73 x 38.10 cm
Options
1
of 13

OVERVIEW

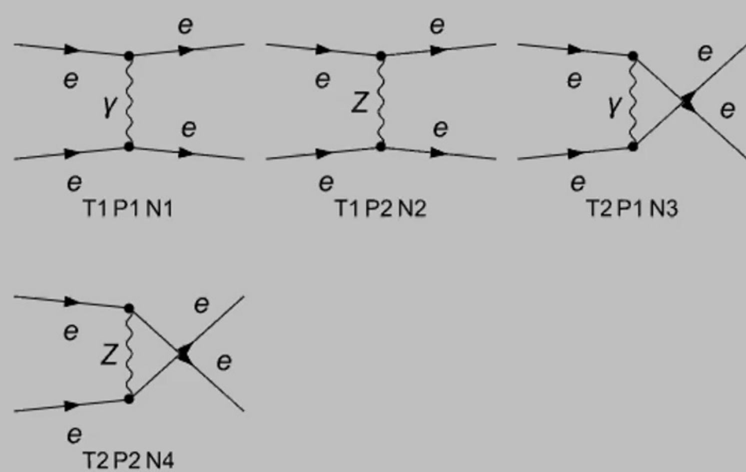
- Moller tree level Parity Violating Asymmetry calculations.
- Kinematical dependence of Parity Violating Asymmetry.
- Vacuum Polarization calculations using PV decomposition.
- Renormalization of vacuum polarization using MSbar scheme.
- Our future goals.

MOLLER SCATTERING

- $e^- + e^- -> e^- + e^-$
- Very clean process with extremely suppressed backgrounds.
- Longitudinally polarized e^- beam strikes on un-polarized e^-'s of liquid Hydrogen target.
- Weinberg mixing angle 'θ_W' is determined which becomes energy dependent at higher loop order.

TREE LEVEL MOLLER SCATTERING

$$e \;\; e \rightarrow e \;\; e$$



PARITY VIOLATING ASYMMETRY

- Formula: $A_{PV} = \dfrac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$, where: $\sigma_R \propto |M_R|^2$ and $\sigma_L \propto |M_L|^2$

- For QED, $|M_{\gamma R}| = |M_{\gamma L}|$, numerator contains just weak+electroweak cross terms.

- Denominator contains just QED terms as m_Z (90 GeV)> m_{e^-}(0.5 MeV)

- $A_{PV} = \dfrac{|M_{ZZ}|_R^2 - |M_{ZZ}|_L^2 + |M_{\gamma Z}|_R^2 - |M_{\gamma Z}|_L^2}{|M_{\gamma\gamma}|_R^2 + |M_{\gamma\gamma}|_L^2}$

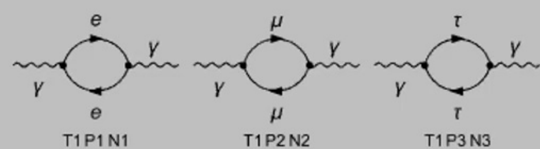
- Combining all ingredients and using: $a_v = \dfrac{I^3 - 2\sin^2\theta_W Q_f}{2\sin\theta_W\cos\theta_W}$, $a_p = \dfrac{I^3}{2\sin\theta_W\cos\theta_W}$, where $Q_f = -1(e^-)$ and $I_3 = -\dfrac{1}{2}$.
- θ_W is the Weinberg mixing angle and electroweak charge: $Q_W^e = 1 - 4\sin^2\theta_W$.
- $A_{PV} = \dfrac{16E^2\sin\theta^2\left(\frac{1}{4} - \sin^2\theta_W\right)}{m_Z^2\sin^2\theta_W\cos^2\theta_W(3+\cos^2\theta)^2}$
- In terms of Fermi constant: $\dfrac{\sqrt{2}G_F}{\pi\alpha} = \dfrac{1}{m_Z^2\sin^2\theta_W\cos^2\theta_W}$
- **FINAL RESULT:** $A_{PV} = \dfrac{4E^2G_F\sin^2\theta Q_W^e}{\sqrt{2}\pi\alpha(3+\cos^2\theta)^2}$

NEED TO INCLUDE HIGHER ORDER TERMS

- Electroweak charge at tree level is : $Q_W^e = (1 - 4\sin^2\theta_W)$
- At 1-loop level this is modified and becomes energy dependent at which the measurement is carried out i.e, $sin^2\theta_W$ runs.
- Results become more precise at higher orders.
- 1-loop level running of Q^e is studied by considering **vacuum polarization** diagram.

PHOTON VACUUM POLARIZATION

$\gamma \rightarrow \gamma$



- $$\Pi_{\mu\nu} = Tr\left[\int d^D q (\iota e \gamma_\nu)\frac{\not{q}+m}{q^2-m^2}(\iota e \gamma_\mu)\frac{\not{k}+\not{q}+m}{(k+q)^2-m^2}\right] + \ldots.$$

- $$B_\nu = \int d^D q \frac{q_\nu}{D_1 D_2},\ B_{\mu\nu} = \int d^D q \frac{q_\mu q_\nu}{D_1 D_2},\ B_0 = \int d^D q \frac{1}{D_1 D_2}$$ and $$A_0 = \int \frac{d^D q}{D_1} or \int \frac{d^D q}{D_2}$$

- $$\Pi_{\mu\nu} = -(16\pi\alpha)[k_\mu B_\nu + k_\nu B_\mu + 2B_{\mu\nu} - g_{\mu\nu}K^\alpha . B_\alpha + g_{\mu\nu} m_1 m_2 B_0 - (A_0(m_1^2) g_{\mu\nu} + m_1^2 B_0) g_{\mu\nu}]$$

- **Using Mathematica X-package**

$$A_0(m^2) = m^2\left(1 + \frac{1}{\epsilon} + \ln\frac{\mu^2}{m^2}\right),\ B_0(0, m^2, m^2) = \frac{1}{\epsilon} + \ln\frac{\mu^2}{m^2},$$

$$B_0(k^2, m^2, m^2) = 2 + \frac{1}{\epsilon} + \frac{\sqrt{k^2(k^2-4m^2)}\,\log\left[\frac{-k^2+2m^2+\sqrt{k^2(k^2-4m^2)}}{2m^2}\right]}{k^2} + \ln\frac{\mu^2}{m^2}$$

take $\epsilon -> 0$ (except $\frac{1}{\epsilon}$)

- $$\Pi(k^2) = -\frac{\alpha k^2}{3\pi}\left[k^2\frac{m_e^2 m_\tau^2 + m_e^2 m_\mu^2 + m_\mu^2 m_\tau^2}{5 m_e^2 m_\mu^2 m_\tau^2} + \log\left[\frac{m_e^2}{m_\tau^2}\right] + \log\left[\frac{m_e^2}{m_\mu^2}\right] + \frac{3}{\epsilon} + k' + 3\log\frac{\mu^2}{m_e^2}\right]$$

MSBAR SCHEME AND RUNNING OF CHARGE

- $$\Pi(\mu^2) = -\frac{\alpha}{3\pi}\left[5 + \log\left[\frac{m_e^2}{m_\tau^2}\right] + \log\left[\frac{m_e^2}{m_\mu^2}\right] + \frac{3}{\epsilon} + 3\log\left(\frac{\mu^2}{m_e^2}\right)\right], \text{ (large k)}$$

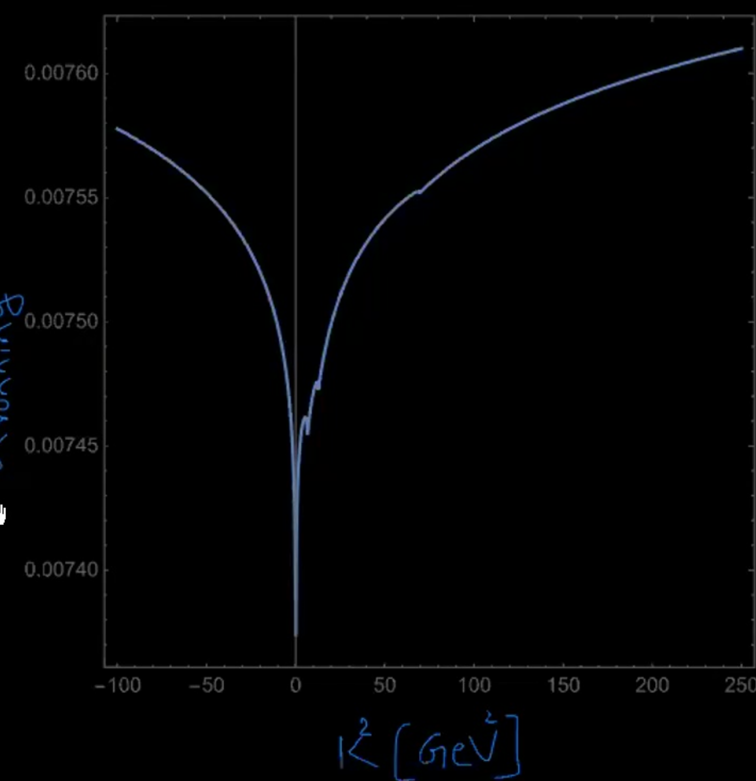
  $$\Pi(\mu^2) = -\frac{\alpha}{3\pi}\left[5 + \log\left[\frac{m_e^2}{m_\tau^2}\right] + \log\left[\frac{m_e^2}{m_\mu^2}\right] + 3\log\left(\frac{\mu^2}{m_e^2}\right)\right] \text{(MSBar scheme)}$$

- Running of charge ($\alpha = \frac{e^2}{4\pi}$):

$$\alpha_{eff}(\mu^2) = \frac{\alpha}{1 - \Pi(\mu^2)} \text{ (MSBar scheme)}$$

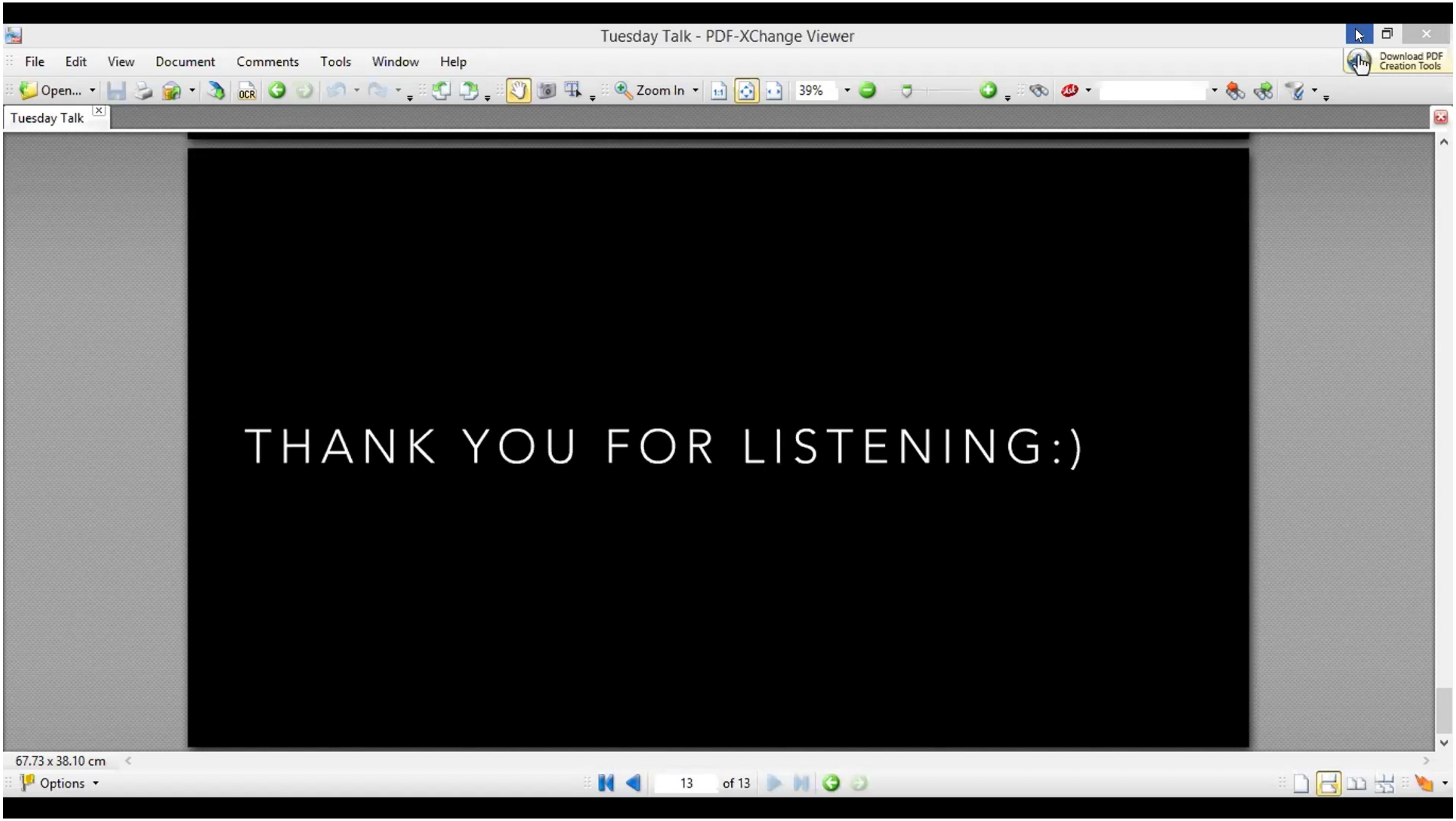
RUNNNING OF ALPHA WITH k^2

- Large momentum transfer->α increases (screening effect decreases)
- Production resonance e.g. at k^2= 70 GeV^2, b-quark is produced (≈4GeV)
- Includes all SM particles (fermion+quark families) in 1-loop.



WHAT WE WANT TO DO NEXT?

- Our next goal is to perform complete 1-Loop level calculations for Moller A_{PV}, using MSBar renormalization scheme.
- By determining the running of electroweak charge (Q_W^e), will try to get more precise value of θ_W.
- The consistency of these results could enable to search for the signals of physics beyond the Standard Model.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Slow decay of waves in gravitational solitons

Women at the Intersection of Mathematics and Theoretical Physics

Sharmila Gunasekaran

(joint work with Hari Kunduri)

Department of Mathematical Sciences
University of Alberta

[arXiv:2007.04283 : to appear in Annales Henri Poincaré]

Feb 23, 2021

Outline

Problem :

Quantitative behavior of solutions Φ of $\Box_g \Phi = 0$ in the background of (M, g) (gravitational solitons)

1. *"The what" : spacetime of interest and wave equation*

2. *"The why" : microstate interpretation and the stability question*

3. *"The how" : quasimode construction to prove slow (logarithmic as opposed to inverse polynomial) decay*

"The what" : (M, g)

- Gravitational solitons : smooth, globally stationary, AF spacetimes with positive energy
- $d = 4$ EM theories do not admit solitons *[Lichnerowicsz]*
- solution to $5d$ supergravity theory :

$$S = \frac{1}{16}\int_M \left(\star R - 2F \wedge \star F - \frac{8}{3\sqrt{3}} \boldsymbol{F} \wedge \boldsymbol{F} \wedge \boldsymbol{A}\right) \qquad (1)$$

- sourced by a Chern-Simons term :

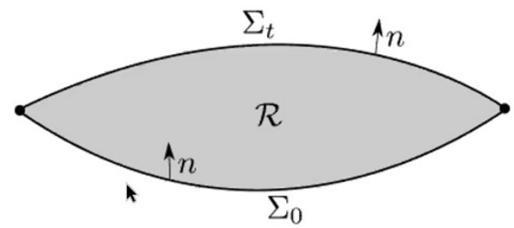
$$\mathrm{d} \star F = -\frac{2}{\sqrt{3}} F \wedge F \qquad (2)$$

- $\mathbb{R} \times SU(2) \times U(1)$ isometry
- $M \cong \Sigma \times \mathbb{R}$; $\Sigma \cong \mathbb{CP}^2 \setminus \mathrm{pt.}$

"The what" : wave equation

1. Φ - perturbation obeying $\Box_g \Phi = 0$

2. In the context of the initial value problem in GR :



- $\Phi(0)$ and $\Phi_t(0)$ given
- understand the evolution of Φ i.e., $\Phi(t)$

3. From a PDE perspective, we ask
   - boundedness : $E[\Phi(t)] \leq CE[\Phi(0)]$?
   - decay : $E_\Omega[\Phi(t)] \leq C\delta(t)E[\Phi(0)]$ where $\delta(t) \to 0$?

"The why" : microstate interpretation and the stability question

1. gravitational solitons : microstate interpretation in string theory
2. Einstein equations :

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} = 8\pi T_{\mu\nu} \quad (3)$$

$$R_{\mu\nu} \sim \mathcal{A}_{\mu\nu}(\partial^2 g, (g^{-1}\partial g)^2) \quad (4)$$

3. In wave gauge (say vacuum),

$$\Box_g g_{\mu\nu} = Q_{\mu\nu}(g, \partial g) \quad (5)$$

$\rightarrow$ coupled PDE with quadratic nonlinearities in ∂g

4. Strategy :
   1) study an appropriate linearization of EFEs
   2) introduce nonlinearities

Hence, the first step is understanding $\Box_g \Phi = 0$

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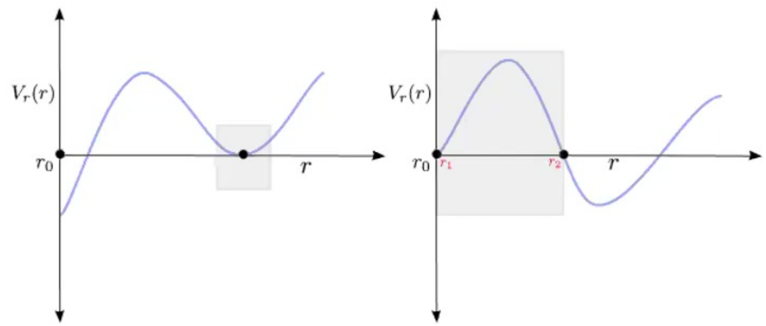
"The how" : trapping

Trapping of null geodesics : $\dot{r}^2 = V_r(r)$



(a) Unstable (Schwarzschild) *polynomial decay*

(b) Stable (solitons) *logarithmic decay*

Stable trapping slows down decay of waves! *[Holzegel–Smulevici, Keir, Benomio]*

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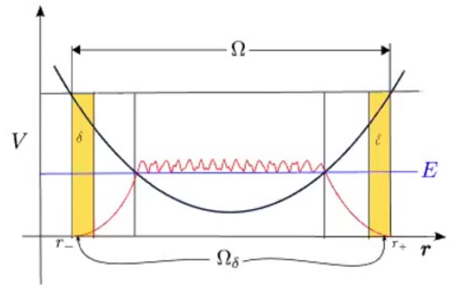
"The how" : sketch of proof for slow decay

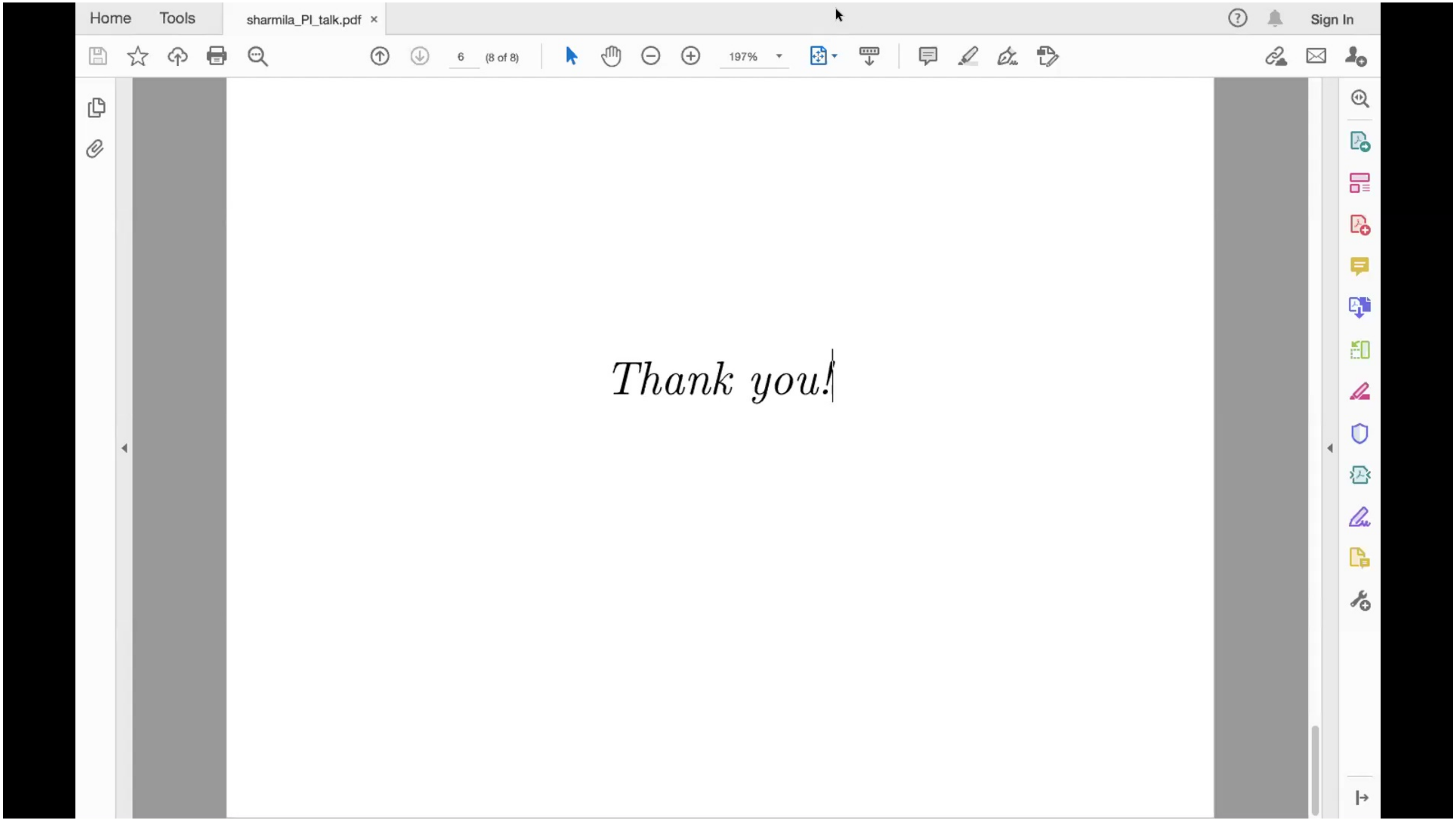
1. Null geodesics and waves



2. Quasimodes :
   - there are time periodic bound state solutions
   - can extended to the whole spacetime with a small error induced due to the extension
   - existence of such approximate solutions proves logarithmic decay

$$\text{Result (sharp)}: \; E_\Omega[\Phi(t)] \leq \frac{C}{\log(2+t)^2} E_2[\Phi(0)] \tag{6}$$

Open : BH-soliton configurations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Thank you!

Simulating spectral triples

universität wien

Lisa Glaser

2nd February 2021

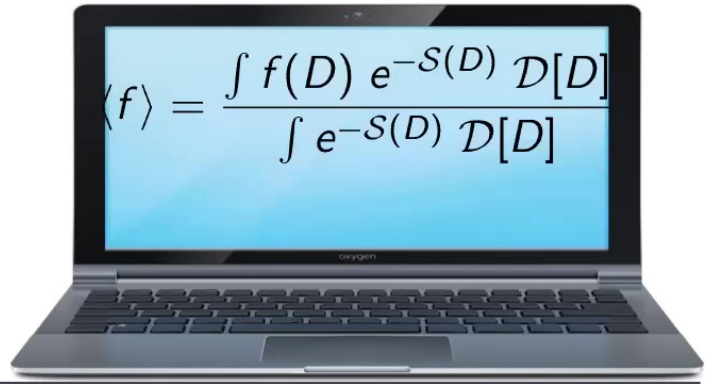
Introduction:

- Postdoc in Vienna
- Work on causal set theory and non-commutative geometry (have a past in CDT)
- Co-organizer of the Quantum Gravity Across approaches seminar sites.google.com/view/qg-aa

Quantum Gravity on the computer

Recipe:

- Define your theory
- Pick your observables
- Choose your algorithm



Ingredients:

- Geometry, here the Dirac operator D and measure $\mathcal{D}[D]$
- Functions of the geometry f
- Action $\mathcal{S}$ (in the physicists sense, an energy, part of the measure)

(Barrett, LG J.Phys. A49, 245001 (2016))

(LG J.Phys.A50 275201 (2017))

(Barrett, Druce, LG J.Phys. A52, 275203 (2019))

Geometry as a spectral triple

$(\mathcal{A}, \mathcal{H}, D)$

- an Algebra $\mathcal{A}$ with action on $\mathcal{H}$
- a Hilbert space $\mathcal{H}$
- a Dirac operator D acting on $\mathcal{H}$

The 'sound' is not enough

- $\{\lambda_n\} = spec(D)$ is not enough, we need to know $\mathcal{A}$, algebra of functions on the space (drum)
- For manifolds $\mathcal{A}$ is commutative, can generalize to non-commutative

(A. Connes, Int.J.Geom.Meth.Mod.Phys. 5, 1215-1242 (2008))

(more detail e.g. A. Connes, Commun.Math.Phys. 182, 155-176 (1996))

Geometry as a spectral triple

$(\mathcal{A}, \mathcal{H}, D)$

- an Algebra $\mathcal{A}$ with action on $\mathcal{H}$
- a Hilbert space $\mathcal{H}$
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Axioms of non-commutative geometry [a]

- $\exists$ a faithfull action $\mathcal{A}$ in $\mathcal{H}$
- $\mathcal{H}$ is a bimodule over $\mathcal{A}$ (there is a left and a right action of $\mathcal{A}$ in $\mathcal{H}$)
- First order condition $[[D, a\triangleright], \triangleleft b] = 0$ for $a, b \in \mathcal{A}$

[a] Abridged version

(A. Connes, Int.J.Geom.Meth.Mod.Phys. 5, 1215-1242 (2008))

(more detail e.g. A. Connes, Commun.Math.Phys. 182, 155-176 (1996))

Non-commutative distance

Distance measure in non-commutative geometry

(A. Connes, Noncommutative Geometry. (Academic Press, 1994))

$$d(\omega_1, \omega_2) = \sup_{a \in \mathcal{A}} \{ |\omega_1(a) - \omega_2(a)| : ||[D, a]|| \leq 1 \}$$

Example:

Calculate distance between points x, y from function f

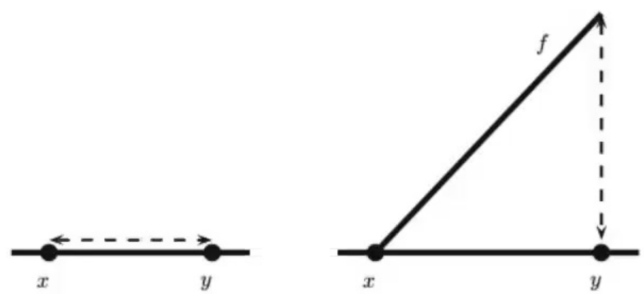

figure from
(W.D. van Suijlekom "Noncommutative Geometry and Particle Physics" Springer (2015))

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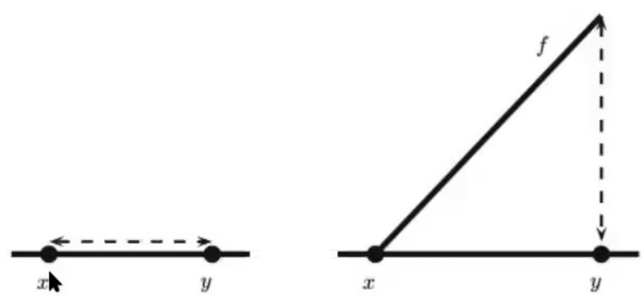

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Idea:

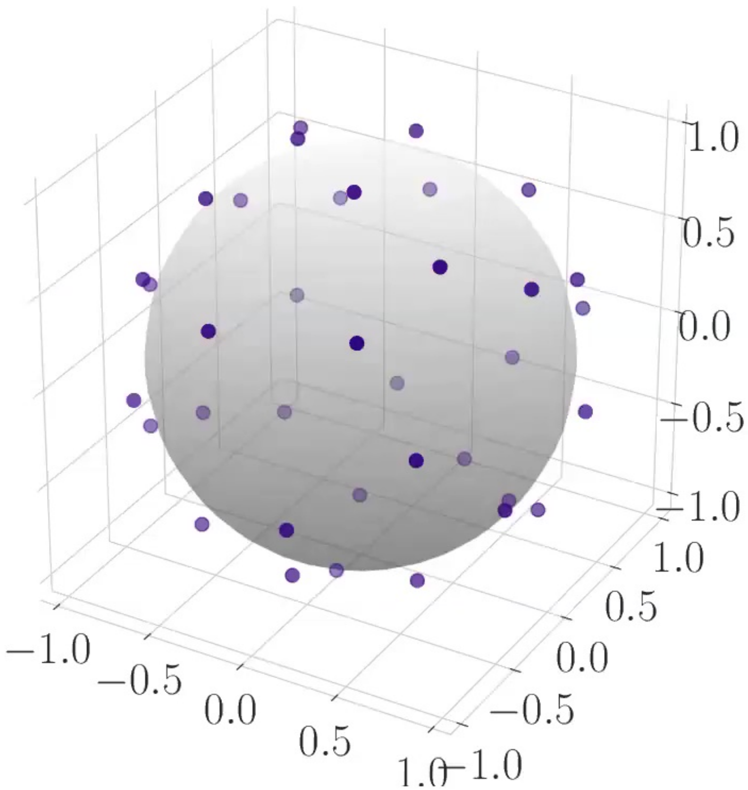
If we can calculate this numerically we can plot our geometry!
Maybe we can see a difference between the two Dirac operators?

Aim:

Find states of small dispersion, then use the distance between states of small dispersion to build a picture of M.

A picture of geometry

The truncated sphere at $\Lambda = 5$



- run the algorithm to generate a set of states and their distance matrix
- use graph embedding algorithm to find a locally isometric embedding
- be sad that pdf presentations don't deal well with gif images

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Future plans

What do I want to do?

- Explore more geometries
- More visualisations:
  - try using wider range of a
  - use algorithm on other spectral triples
- spectral triples as matter in other QG approaches? (e.g. causal sets)