Title: Researcher Presentations

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Collection: Women at the Intersection of Mathematics and Theoretical Physics

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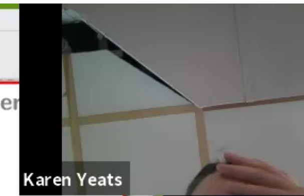

An alternative to Feynman diagram expansions

Karen Yeats, University of Waterloo

An alternative expansion for solutions to many Dyson-Schwinger equations. (With Marie, Hihn, and in progress with Nabergall)

- Shape:

$$G(x, L) = 1 - \sum_{C} x^{|C|} f_{t_k - t_{k-1}} \cdots f_{t_2 - t_1} f_0^{|C|-k} \sum_{i \leq t_1} f_{t_1 - i} \frac{(-L)^i}{i!}$$

  in the simplest case and similarly.
- **NOT** from Wick's theorem.
- Applicable to many theories.
- Factorial growth remains, but each chord diagram contributes something simple.

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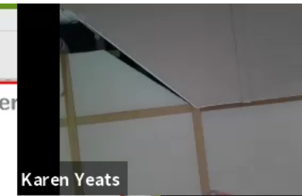

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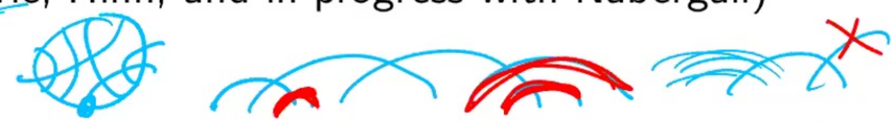

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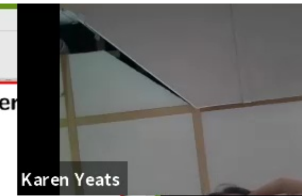

Using chord diagram expansions

Chord diagram expansions are mathematically and physically useful. (With Courtiel, Courtiel and Zeilberger, Mahmoud, and in progress with Nabergall.)

- Well suited to answering asyptotic questions, especially in next-to^k leading logs.

$$[z^n]H_k(z)$$

$$\sim \frac{(-1)^n}{\Gamma(1-\frac{1}{s})k!} a_{1,0}^{n-k}(a_{2,0}+(s-1)a_{1,1}a_{1,0})^k \log(n)^k n^{k-\frac{1}{s}-1} s^{n-1} \quad \text{for } s \geq 2$$

$$\sim \frac{(-1)^n}{(k-1)!} a_{1,0}^{n-k} a_{2,0}^k \log(n)^{k-1} n^{k-2} \quad \text{for } s = 1.$$

- Rejuvenating pure combinatorics of chord diagrams.

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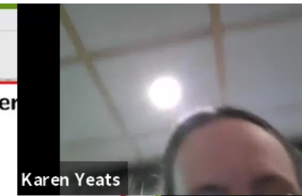
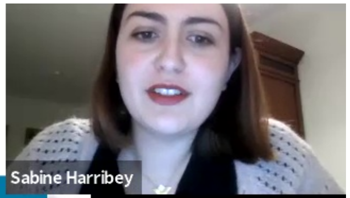


- Bootstrap combinatorics of the instanton expansions?

	$F_0(\rho)$	$F_1(\rho)$	$F_2(\rho)$	$F_3(\rho)$	$\cdots$
$C^{(0)}(x)\,\frac{1}{x}$	1	1	4	27	$\cdots$
$C^{(1)}(x)\,\frac{e\sqrt{2\pi}}{\xi}$	1	$-\frac{5}{2}$	$-\frac{43}{8}$	$-\frac{579}{16}$	$\cdots$
$C^{(2)}(x)\,\frac{xe^2 2\pi}{\xi^2}$	-1	5	$\frac{11}{2}$	$\frac{97}{2}$	$\cdots$
$C^{(3)}(x)\,\frac{x^2e^3(2\pi)^{3/2}}{\xi^2}$	$\frac{3}{2}$	$-\frac{47}{4}$	$\frac{67}{16}$	$-\frac{2157}{32}$	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

- Should apply more broadly to all physically interesting Dyson-Schwinger equations.

Now, listen to all these other people doing awesome things!



Renormalization in tensor field theories and the melonic fixed point.

Sabine Harribey

Joint work with Dario Benedetti, Razvan Gurau and Kenta Suzuki

February 2021, WIMTH 2021

CPHT Centre de Physique Théorique

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DEPARTMENT OF PHYSICS AND ASTRONOMY

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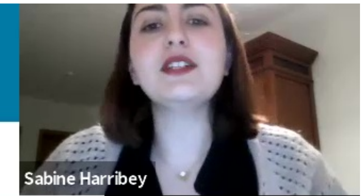
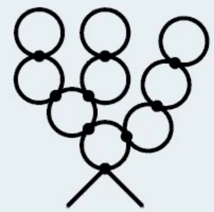
From vector to tensor models



Vector ϕ_a

$\frac{\lambda}{N}(\phi_a\phi_a)^2$

Cactus diagrams

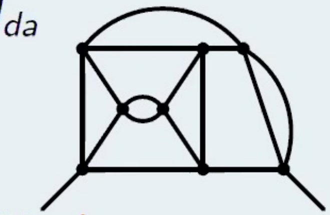


$\rightarrow$ Easy

Matrix M_{ab}

$\frac{\lambda}{N}M_{ab}M_{bc}M_{cd}M_{da}$

Planar diagrams

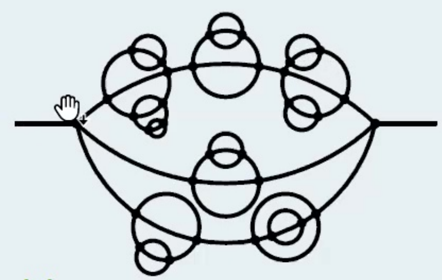


$\rightarrow$ Hard

Tensor T_{abc}

$\frac{\lambda}{N^{3/2}}T_{aeb}T_{cfb}T_{ced}T_{afd}$

Melon diagrams



$\rightarrow$ Tractable

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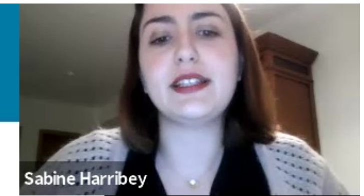
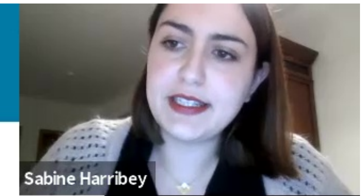
Melonic quantum field theory



- First introduced in zero dimension: random geometry and quantum gravity [Gurau, Bonzom, Rivasseau, . . .]
- Strongly coupled QFTs and holography : SYK model without disorder [Witten, Klebanov, Tarnopolsky, . . .]
- Tensor models in higher dimension: new class of conformal field theories

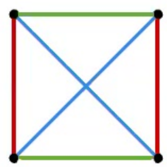
- Problem: divergences in the computation of the two and four-point functions
- Renormalization group: computation of beta functions, existence of an attractive IR fixed point

Model and renormalization

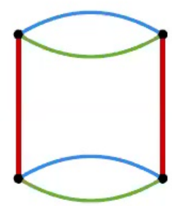


$$S[\varphi] = \frac{1}{2} \int d^d x \, \varphi_{abc}(x)(-\Delta)^{\zeta} \varphi_{abc}(x) + S^{\text{int}}[\varphi] \, ,$$

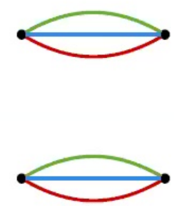
- $O(N)^3$ tensor model with quartic interactions [Carrozza,Tanasa,...]



Tetrahedron



Pillow



Double-trace

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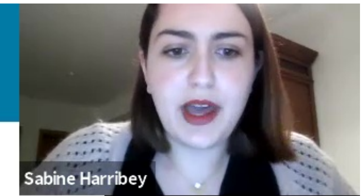

Model and renormalization

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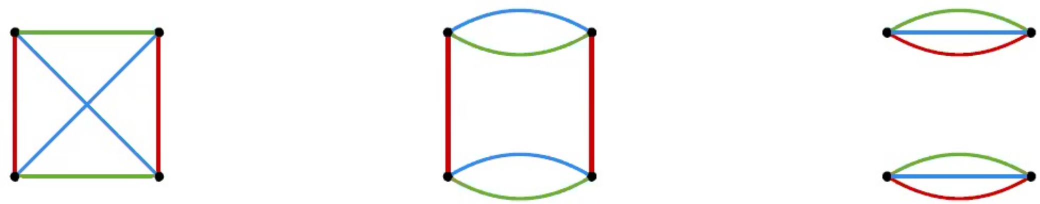

Tetrahedron   Pillow   Double-trace

- $0 < \zeta < 1$: long-range model with $d < 4$ fixed
- Weakly relevant case $\zeta = \frac{d+\epsilon}{4}$
- IR regularization: mass regulator $\mu > 0$
- BPHZ subtraction scheme at zero momentum

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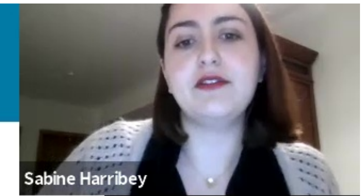
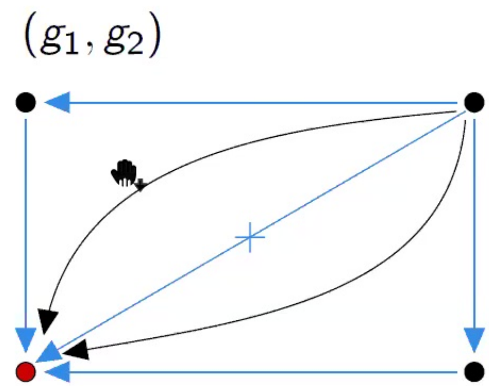


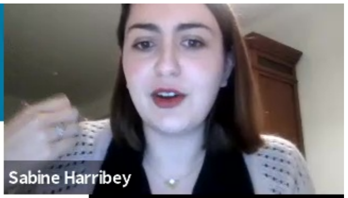
RG trajectories

At large N: four lines of fixed point paramatrized by the tetrahedral coupling.

- One infrared attractive fixed point, stable and strongly interacting
- Explicit renormalization group trajectory from UV to IR fixed point



Further work



- Detailed study of this new type of CFT: *Melonic CFT*
- Computation of dimensions of bilinears and OPE coefficients See poster session
- Renormalization for other tensor models : sextic interactions, bipartite models, fermionic fields, other symmetry groups, . . .
- What is the holographic dual of these melonic CFTs ?

Conformal_Defects_and_Emergent_SUSY (1).pdf - Foxit Reader

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Conformal Defects and Emergent SUSY

Philine van Vliet

DESY Hamburg

February 22, 2021

Based on ArXiV: 2012.00018 with Pedro Liendo and Aleix Gimenez-Grau

Conformal_Defects_and_Emergent_SUSY (1).pdf - Foxit Reader

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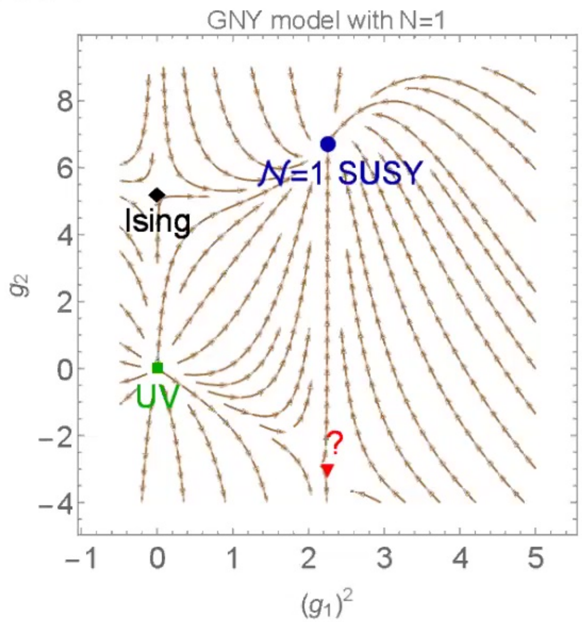
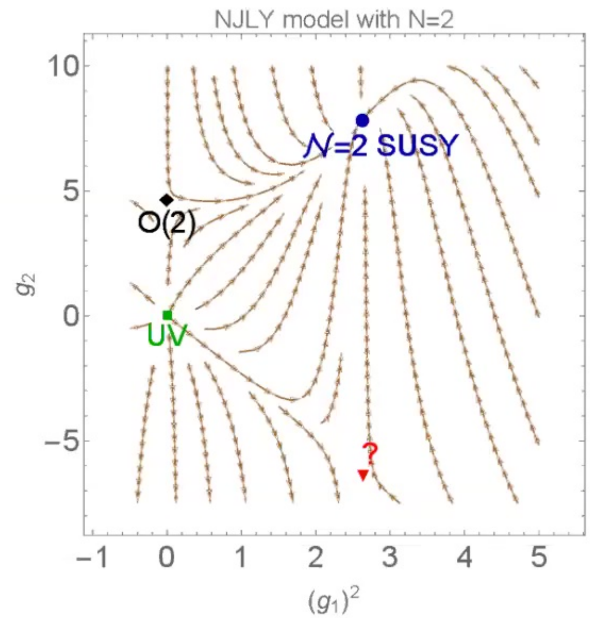
Our setup

- Three-dimensional CFT
- Low amount of SUSY: $\mathcal{N} = 2$
- Included a boundary
- Studied using the defect bootstrap in $d = 4 - \epsilon$ dimensions
- Checked with perturbative calculations for WZ model

Emergent Supersymmetry

- Add SUSY for more control: protected dimensions, extra symmetries.
- SUSY can arise at fixed points as enhanced symmetry: emergent SUSY





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Conformal Defects

- We can generalize a CFT by adding extended objects: Wilson lines, boundaries, surfaces

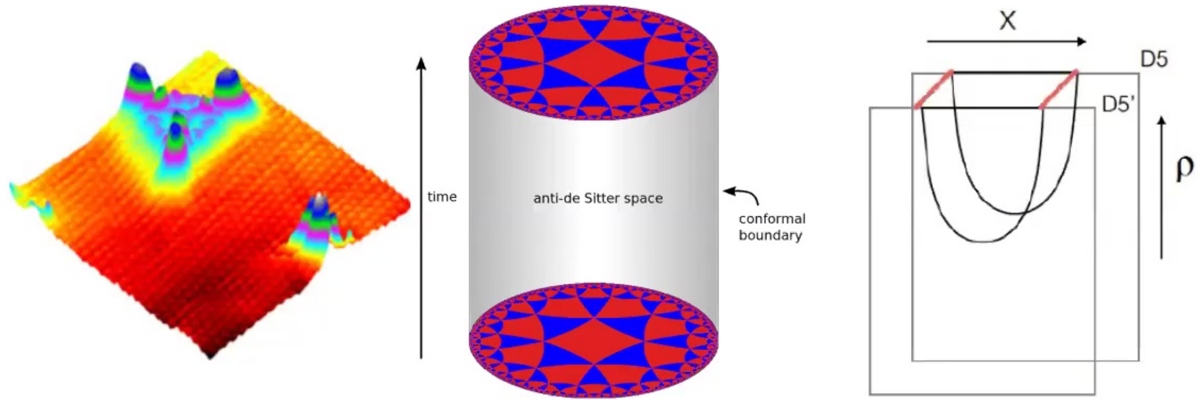

Figure: Examples of extended objects

- Extended objects in the vacuum break part of the conformal symmetry of your theory.

Conformal Defects

- *Conformal* defect preserves a conformal subgroup of the original symmetry

$$SO(d+1,1) \to SO(p+1,1) \times SO(q), \quad d = p + q\,. \tag{1}$$

- The defect preserves conformal symmetry on the defect and rotations around the defect.
- There is no conserved stress-energy tensor on the defect. Instead, they have a *displacement operator* D

$$\partial_\mu T^{\mu d}(x) = -D(x^a)\delta(x^d), \tag{2}$$

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Conformal Defects and Supersymmetry

- Some defects preserve part of the original SUSY
- The breaking of the bulk supergroup into a defect supergroup depends on the defect, dimension of the bulk, amount of SUSY, ...
- A defect can break the bulk supergroup in multiple ways.

Philine van Vliet

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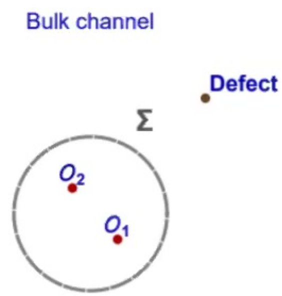
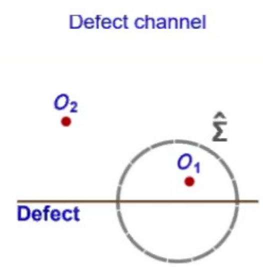
The Defect Bootstrap

- 2-pt bulk correlators not fixed, but depend on cross-ratios

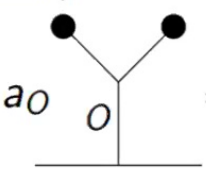
$$\langle \mathcal{O}_1 \mathcal{O}_2 \rangle = \frac{G(\xi, \phi)}{|2x_1^{\perp}|^{\Delta_1} |2x_2^{\perp}|^{\Delta_2}} \qquad (3)$$

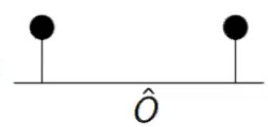
- Use *bulk* or *boundary* OPE to expand in conformal blocks





- Impose *crossing symmetry*

$$\sum_{\Delta,\ell} C_{\phi\phi O}\, a_O$$


$$= \sum_{\widehat{\Delta},s} b^2_{\phi\hat{O}}$$


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References

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[3] E. Lauria, P. Liendo, B. C. Van Rees and X. Zhao, [arXiv:2005.02413 [hep-th]].

[4] L. Fei, S. Giombi, I. Klebanov and G. Tarnopolsky, PTEP **2016** (2016), 12C105 [arXiv:1607.05316 [hep-th]].

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xQCD amplitudes and in-medium factorization

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February 22, 2021

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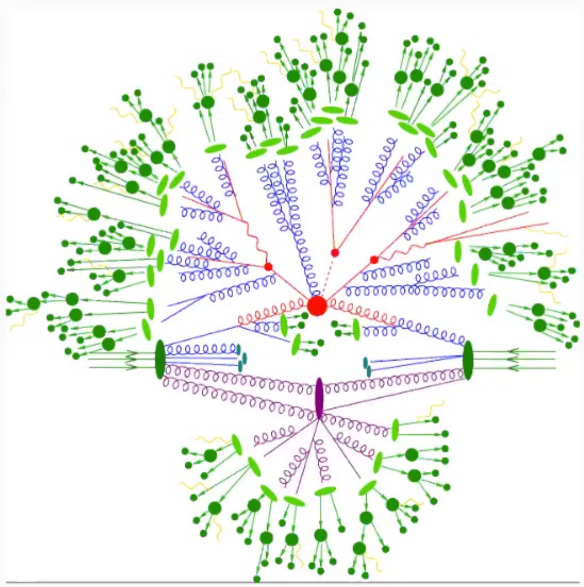
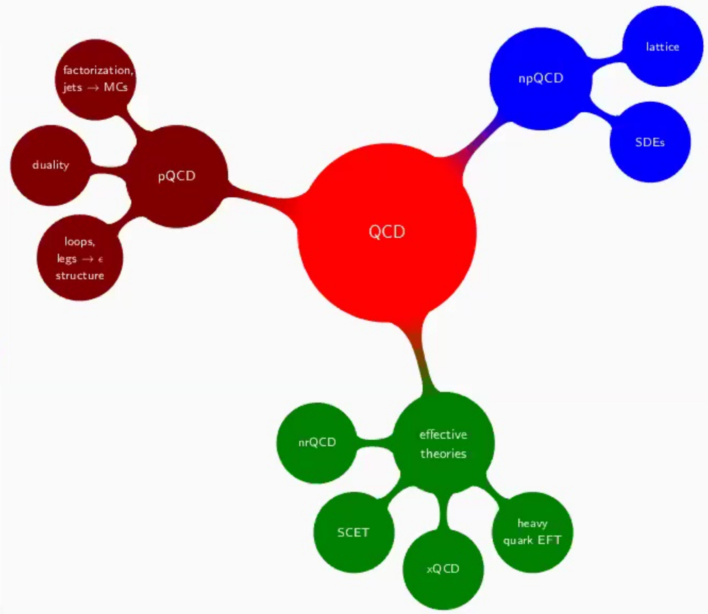
Women at the Intersection of Mathematics and Theoretical Physics
Perimeter Institute, 22-25 Feb. 2021

QCD at the heart of hadron/nuclear matter





QCD: factorization theorem, evolution and resummation

In hadron-hadron collisions it is an approximation, corrections suppressed by inverse powers of Q:

$$Q^2\,\sigma_{phys}(Q,m) = S(Q/\mu, \alpha_s(\mu)) \otimes L(\mu, m) + \mathcal{O}(1/Q^n)$$

- Short distance physics - new physics, μ factorization scale
- Long distance physics - universal (PDFs or FFs), m IR scale
- inverse power of momentum transfer Q in hard scattering

factorization $\longrightarrow$ evolution

$$\mu\frac{d}{d\mu}\ln\sigma_{phys}(Q,m) = 0 \quad\Longrightarrow\quad \mu\frac{d\ln L}{d\mu} = -P(\alpha_s(Q)) = -\mu\frac{d\ln S}{d\mu}$$

evolution $\longrightarrow$ resummation

$$\ln\sigma_{phys}(Q,m) = \exp\left\{\int_q^Q \frac{d\mu'}{\mu'} P(\alpha_s(\mu'))\right\}$$

boost invariance & scale invariance!

Factorization of QCD amplitudes

[Sen (1983); Kidonakis, Oderda and Sterman (1998); Sterman and METY (2002)]

The amplitude $\mathcal{M}^{[\mathrm{f}]}$ can be factorized into functions that organize momentum regions relevant to ϵ poles in the scattering amplitude

$$\mathcal{M}_L^{[\mathrm{f}]} = S_{LI}^{[\mathrm{f}]}\left(\wp_i, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) \otimes h_I^{[\mathrm{f}]}\left(\wp_i, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) \quad J^{[\mathrm{f}]}\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right)$$

- (blue) process-dependent functions that describe the short-distance dynamics of the hard scattering
- (orange) matrix of functions that describes the coherent soft radiation arising from the overall color flow
- (pink) independent of the color flow, which describes the long-distance perturbative evolution of partons

Factorized QCD amplitudes at hadron colliders

$$\mathcal{M}_L^{[f]} = S_{LI}^{[f]}\left(\wp_i, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) \otimes h_I^{[f]}\left(\wp_i, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) \quad J^{[f]}\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right)$$

factorization $\longrightarrow$ evolution: variations of the jet functions and the soft matrix with the scale Q are compensated by variations of the hard function

$$\frac{d}{d \ln\, Q} S_{LI} = -\Gamma_{LJ}^{[f]}\, S_{JI}$$

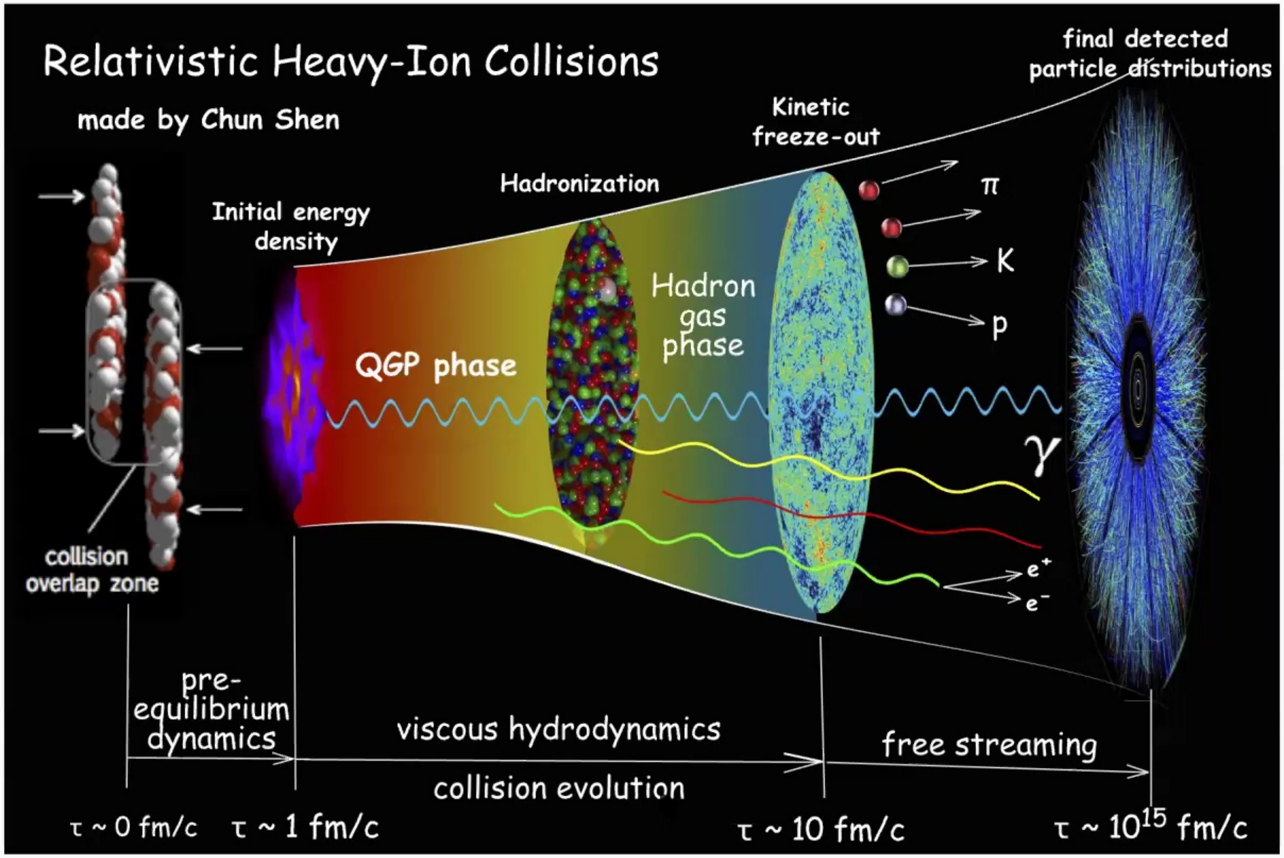
evolution $\longrightarrow$ resummation: mixing of color structures due to soft parton exchange is contained within anomalous dimension matrix $\Gamma^{[f]}$

$$\mathbf{S}^{[f]} = \mathrm{P}\,\exp\left[-\frac{1}{2}\int_0^{-Q^2} \frac{d\tilde{\mu}^2}{\tilde{\mu}^2}\Gamma^{[f]}\left(\bar{\alpha}_s\left(\frac{\mu^2}{\tilde{\mu}^2}, \alpha_s(\mu^2), \epsilon\right)\right)\right]$$

Jet and soft functions can be defined with specific QCD matrix elements, e.g. $0 \to 2$ singlet, EM Sudakov FF

$$J^{[i\bar{i}]} = \left[\mathcal{M}^{[i\bar{i}\to 1]}\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right)\right]^{\frac{1}{2}}$$

Extreme QCD

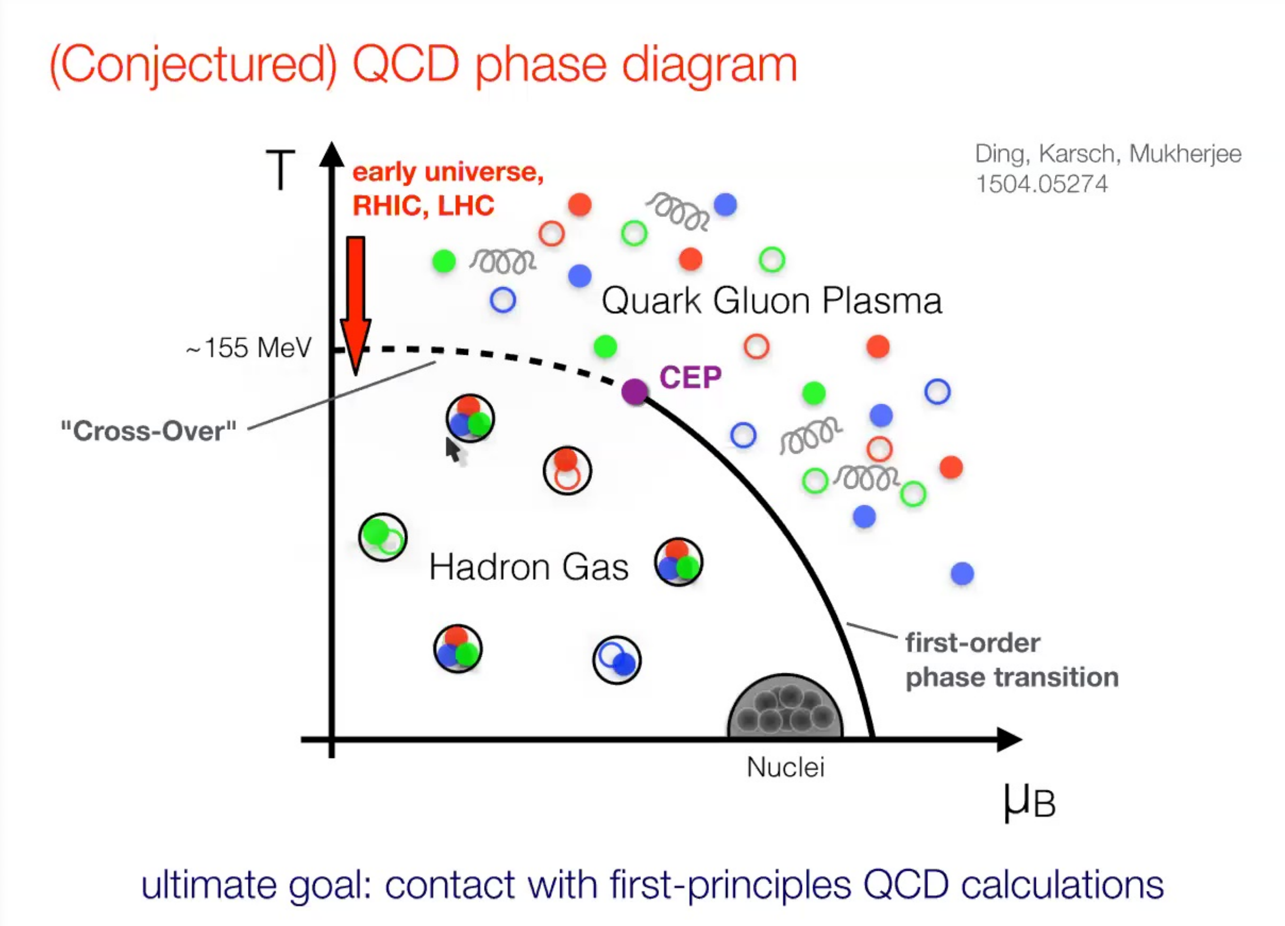

Relativistic Heavy-Ion Collisions
made by Chun Shen
final detected particle distributions
Kinetic freeze-out
Hadronization
Initial energy density
Hadron gas phase
QGP phase
π
K
p
γ
e+
e−
collision overlap zone
pre-equilibrium dynamics
viscous hydrodynamics
free streaming
collision evolution
τ ~ 0 fm/c
τ ~ 1 fm/c
τ ~ 10 fm/c
τ ~ 10^15 fm/c


Maria Elena Tejeda Y...

Extreme QCD

- important parameters for QCD in equilibrium: **temperature** T and **baryon number density** n_B (or chemical potential μ_B).
- intrinsic scale $\Lambda_{QCD} \sim 200$ MeV: expect a transition around $T \simeq \Lambda_{\rm QCD} \sim 200$ MeV, $n_B \sim \Lambda^3_{QCD} \sim 1\ \text{fm}^{-3}$.
- α_{QCD} runs to smaller values with increasing energy scale: **anticipate confined and chiral symmetry broken QCD matter undergoes a phase transition at high energy densities**
- in heavy-ion collisions we can create the conditions to study the QGP and phase transitions

The phase diagram: QCD at finite T and μ_B.



QCD factorization in-medium: what is known

- proof of factorization theorems at the leading power of the large momentum transfer is independent of the details of the identified hadrons
- the corrections to the factorized formalism are very much sensitive to which hadrons are colliding or observed in the final-state
- only the first subleading power contributions to hadronic observables can be factorized to all orders in a similar way to the leading power contributions.
- subleading power contributions to the hadronic observables are very sensitive to QCD multiple scattering and, therefore, depend on where the collision is taking place... $p+p$ (in vaccum?) or $A+A$ (in medium?)

$\rightarrow$ the kinematic regime where the leading power formalism is applicable could be very different for $p+p$, $p+A$ or $A+A$ collisions.

QCD factorization in-medium: what is done

Qiua, Ringer, Satoa, Zurita, *QCD factorization and universality of jet cross sections in heavy-ion collisions.* Nucl. Phys. A (2020)

At leading power, factorized xsec for inclusive jets

$$\frac{\mathrm{d}\sigma^{pp\to \mathrm{jet}+X}}{\mathrm{d}p_T\,\mathrm{d}\eta} = \sum_{abc} f_{a/p} \otimes f_{b/p} \otimes H^c_{ab} \otimes J_c\,,$$

The jet functions satisfy the usual timelike DGLAP (since the photon yield in heavy-ion collisions is consistent with no modification):

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} J_c = \sum_d P_{dc} \otimes J_d\,.$$

Ansatz for the heavy-ion cross section where we replace the vacuum jet function as

$$J_c(z, p_T R, \mu) \to J_c^{\mathrm{med}}(z, p_T R, \mu) = W_c(z) \otimes J_c(z, p_T R, \mu)\,.$$

QCD factorization in-medium: what is underlying

Gross, Pisarski, Yaffe (1981). Weldon (1982). Kajantie, Kapusta (1985). Landsman, Van Weert (1987). Braaten, Pisarski (1990-1992).

Hot gauge theories $\rightarrow$ HTL

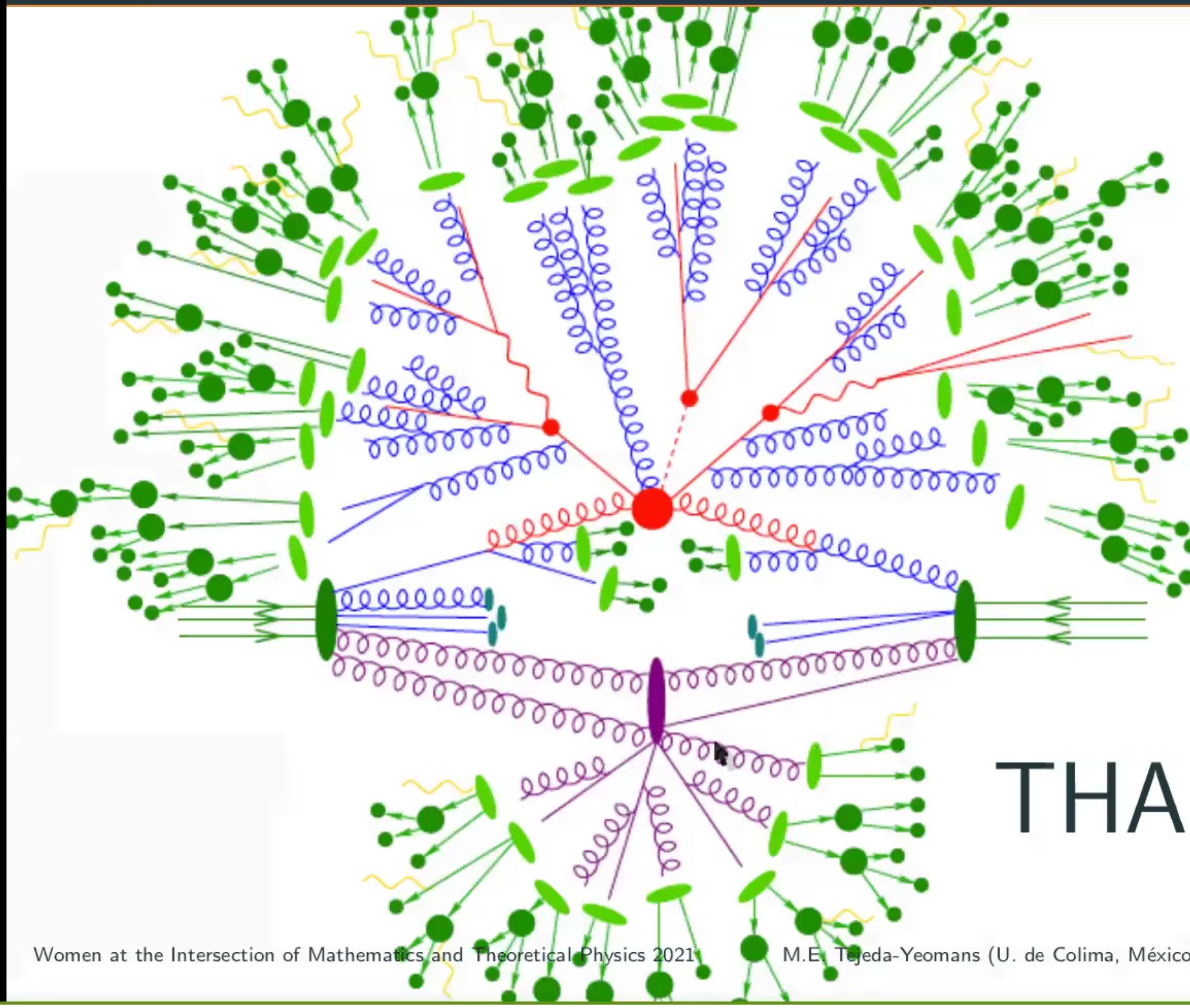
- many studies of the high-temperature behavior of Green functions in thermal QCD
- mainly Hard Thermal Loop approximation (HTL)
  - $\rightarrow$ systematic method for the calculation of amplitudes in hot gauge theories
  - $\rightarrow$ distinguish between hard momenta $\mathcal{O}(T)$ and soft momenta $\mathcal{O}(gT)$
  - $\rightarrow$ use PT over hard momenta, use EFT over soft momenta (resummation)
  - $\rightarrow$ hard thermal loops arise solely from subdiagrams at one loop order: UV finite, gauge independent, and satisfy simple Ward Id

Amplitudes of hot - and dense? - gauge theories (xQCD)

$$\mathcal{M} = S\left(\frac{T^2}{\mu^2}, \alpha_s(T^2, \mu^2), \epsilon, T\right) \otimes h\left(\frac{T^2}{\mu^2}, \alpha_s(T^2, \mu^2)\right) \quad J\left(\frac{T^2}{\mu^2}, \alpha_s(\mu^2), \epsilon, T\right)$$

Ansatz for xQCD amplitudes singular structure using only causality, unitarity and symmetry, based on several ideas out there

- HTL amplitudes as maps of classical ($\hbar \to 0$) limits of off-shell currents at $T = 0$ (see for example, L. de la Cruz 2012.07714 [hep-th] (2021))
- Classical limits of scattering amplitudes: distinguish between the momentum p of a particle and its wavenumber $\bar{p}$: $p \equiv \hbar\bar{p} \leftrightarrow$ distinction between soft and hard in HTL currents
- Thermal amplitudes can be constructed using the Schwinger-Keldysh formalism + possibility of extension to other regimes - no need for Lorentz-invariant states (see for example, S. Caron-Huot JHEP 05 (2011))



THANKS

Elliptic curves and Thompson's group

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Women at the Intersection of Mathematics and Theoretical Physics
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Main Idea

Can moonshine help answer number theoretic questions?

Elliptic Curves

Let E be an elliptic curve defined over $\mathbb{Q}$, and let $E(\mathbb{Q})$ denote the set of $\mathbb{Q}$-rational points of E.

Theorem 1 (Mordell).

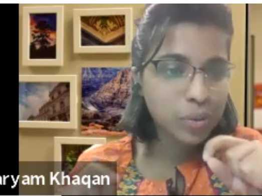
$$E(\mathbb{Q}) = \mathbb{Z}^r \oplus E(\mathbb{Q})_{\mathrm{tor}}.$$

Computing the rank r of a general elliptic curve is considered a hard problem in number theory.

Conjecture (Birch and Swinnerton–Dyer).

The rank of an elliptic curve equals the order of vanishing of its L-function $L_E(s)$ at $s = 1$.

An Elliptic Curve

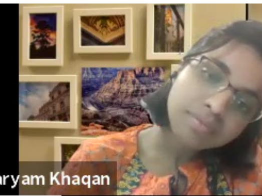


Let E be the following elliptic curve over $\mathbb{Q}$,

$$y^2 = x^3 + 864x - 432$$

For $d < 0$ a fundamental discriminant, let E^d be the quadratic twist,

$$E^d: \ y^2 = x^3 + 864d^2x - 432d^3$$



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Question: How does rank(E^d) vary with d?

Some Data

We will restrict to discriminants such that $\left(\frac{d}{19}\right) = -1$.

$\lvert d \rvert$	rank(E^d)
4	0
7	0
11	0
20	0
23	2
24	0
$\vdots$	$\vdots$
83	2
87	2
104	2
111	0

Modular Form

Let $F(\tau)$ denote the unique (weakly holomorphic) modular form in $M^{+,!}_{\frac{3}{2}}(\Gamma_0(4))$ such that

$$F(q) = q^{-5} + O(q)$$

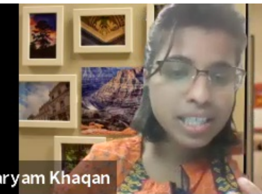
Let $c(d)$ denote the coefficient of q^{-d} in the q-expansion for F.

More Data

$\|d\|$	$c(d)$	rank(E^d)
4	- 565760	0
7	52756480	0
11	5874905295	0
20	- 19691491018752	0
23	19134687117312 0	2
24	- 394919975761920	0
⋮	⋮	⋮
83	27859572924157397484965799 00	2
87	1278910078579392904191246336 0	2
104	-579539154122485522172914516992 0	2
111	62099872645859114904016024043520	0

More Data

$\lvert d \rvert$	$c(d) \mod 19$	$\mathrm{rank}(E^d)$
4	3	0
7	16	0
11	16	0
20	3	0
23	0	2
24	13	0
$\vdots$	$\vdots$	$\vdots$
83	0	2
87	0	2
104	0	2
111	13	0



Theorem

Let Th denote *Thompson's group*, the sporadic simple group of order $2^{15} \cdot 3^{10} \cdot 5^3 \cdot 7^2 \cdot 13 \cdot 19 \cdot 31$

Theorem 2.

There exists an infinite-dimensional graded Th-module $W = \oplus_{n \in \mathbb{Z}} W_n$ such that if

$$\dim(W_{|d|}) \not\equiv 0 \pmod{19},$$

then the Mordell–Weil group $E^d(\mathbb{Q})$ is finite for each elliptic curve E of conductor 19, and each $d < 0$ as above.

A note on the theorem

Each $c(d) = \dim(W_d)$ is given by the finite sum

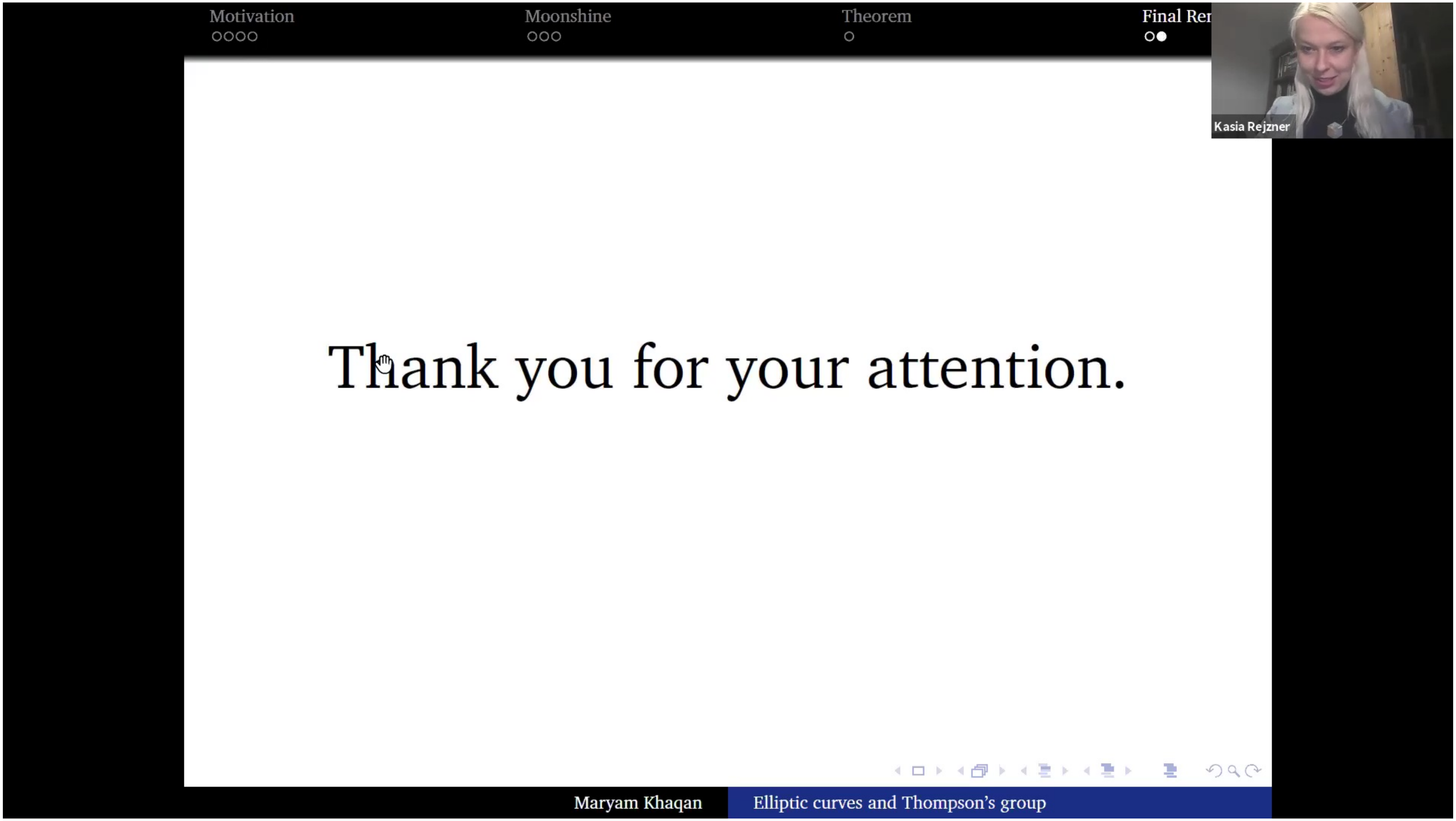
$$c(d) = \frac{-1}{\sqrt{5}} \sum_{Q \in \mathcal{Q}_{5d}^{(1)}} \chi(Q) j(\tau_Q)$$

where

$\mathcal{Q}_{5d}^{(1)} :=$ set of positive definite quadratic forms with discriminant $5d$,

$\tau_Q :=$ the unique root of Q in $\mathbb{H}$,

and $j(\tau)$ is the usual elliptic modular invariant.


Motivation
Moonshine
Theorem
Final Rer
Kasia Rejzner
Thank you for your attention.
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Elliptic curves and Thompson's group