Title: Mathematical Puzzles from Causal Set Quantum Gravity

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Collection: Women at the Intersection of Mathematics and Theoretical Physics

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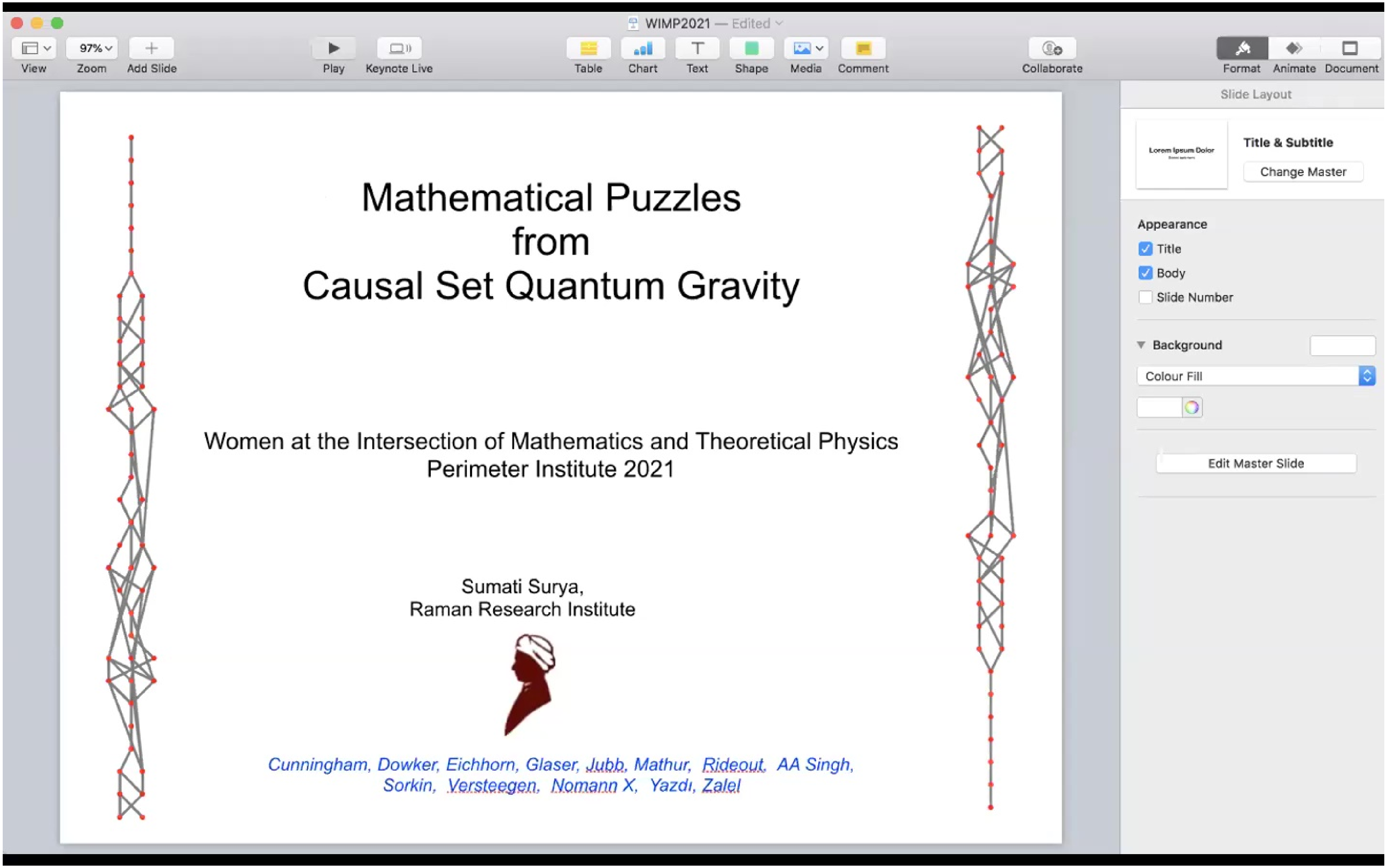
Abstract: I will discuss some of the mathematical puzzles that arise from the causal set approach to quantum gravity. In this approach, any causal continuum spacetime is said to be emergent from an underlying ensemble of locally finite posets which represents a discretisation of the causal structure. If the discrete substructure is to capture continuum geometry to sufficient accuracy, then it must be "approximately" close to it. How can we quantify this closeness? This discreteness, while also preserving local Lorentz invariance, leads to a fundamental non-locality. This is not only an obstacle to a â€œtraditionalâ€• initial value formulation, but also to the geometric interpretation of entanglement entropy. Is there an analytic way to quantify the remanent Planckian non-locality? These questions, as well as others arising from the quantum dynamics of causal, may be of potential interest to mathematicians, in particular Geometers and Combinatorists.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Mathematical Puzzles
from
Causal Set Quantum Gravity
Women at the Intersection of Mathematics and Theoretical Physics
Perimeter Institute 2021
Sumati Surya,
Raman Research Institute
Cunningham, Dowker, Eichhorn, Glaser, Jubb, Mathur, Rideout, AA Singh,
Sorkin, Versteegen, Nomann X, Yazdi, Zalel
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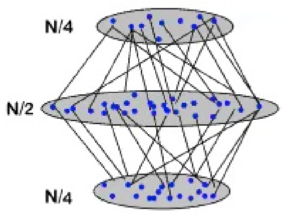
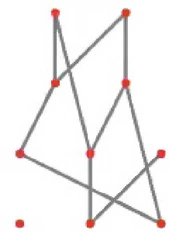
Causal Set Theory

— Bombelli, Lee, Meyer and Sorkin, 1987

The fine grained structure of spacetime is that of a causal set



- **Acyclic:** $x \prec y, y \prec x \Rightarrow x = y$
- **Transitive:** $x \prec y, y \prec z \Rightarrow x \prec z$
- **Local Finiteness:** $|\mathrm{Fut}(x) \cap \mathrm{Past}(y)| < \infty$

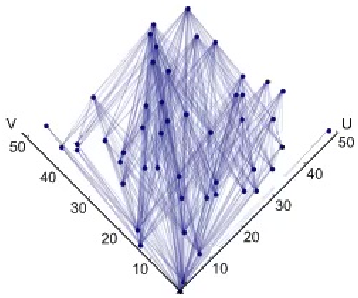


Causal Structure Poset — *Alexandrov, Seifert, Zeeman,* ***Penrose****, Geroch, Kronheimer, Finkelstein, Myrheim, Hemion..*

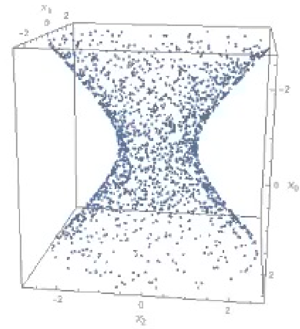
$$(M, \prec) + \text{vol element} = (M, g)$$

— Hawking, King, McCarthy, 1976
— Malament, 1977

$$\boxed{\text{Order + Number} \sim \text{Spacetime}}$$



The Continuum Approximation



Order + Number ~ Spacetime

$C \sim (M, g)$

* Order ↔ Causal Structure
* Number ↔ Volume

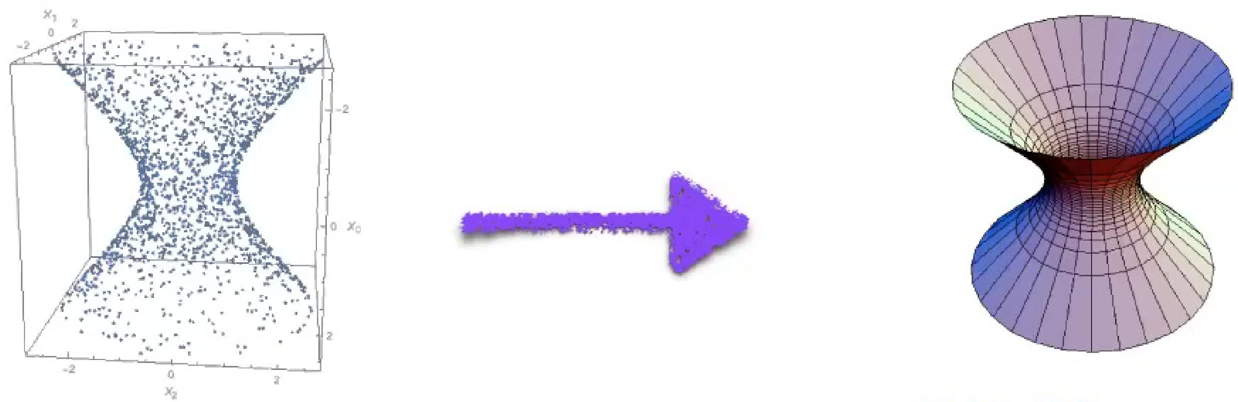
$$P_V(n) = \frac{(\rho V)^n}{n!} e^{-\rho V}$$

$$\langle N \rangle = \rho V, \qquad \Delta N = \sqrt{\rho V}$$

Random Discreteness => Lorentz invariance — *Bombelli, Henson and Sorkin, 2006*

Q) How unique/special is Poisson? —*Aslanbeigi and Saravani, 2014*

The Continuum is Emergent



Geometrical Reconstruction:

- Dimension
- Time-like, Space-like and Induced Spatial Distance
- Spatial Topology
- Locality
- D'Alembertian
- Einstein-Hilbert Action
- Gibbons Hawking York Boundary Term
- Scalar fields..

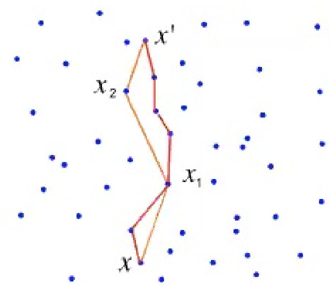
— Myrrheim, 1978
— Meyer, 1988
— David Reid 2002
— Brightwell & Gregory, 1991
— Rideout & Wallden, 2009
— Roy, Sinha & Surya 2012
— Glaser & Surya, 2013
— Eichhorn, Surya & Versteegen, 2018
— Major, Rideout & Surya, 2006,2008,
— Benincasa & Dowker, 2010
— Dowker & Glaser, 2013
—Glaser, 2014
— Benincasa, Dowker & Schnitzer, 2011
— Buck, Dowker, Jubb & Surya, 2015
— Cunningham, 2018
—Dable-Heath, Fewster, Rejzner, Woods, 2019
—Dowker 2020
—Machet & Wang 2020

Q) Are there more geometrically interesting order invariants?!

Examples

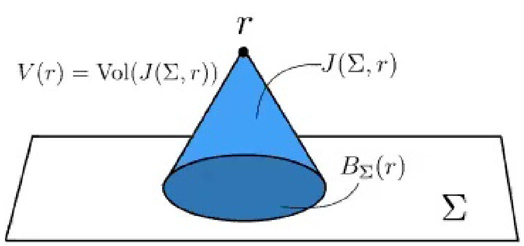
Time-like distance

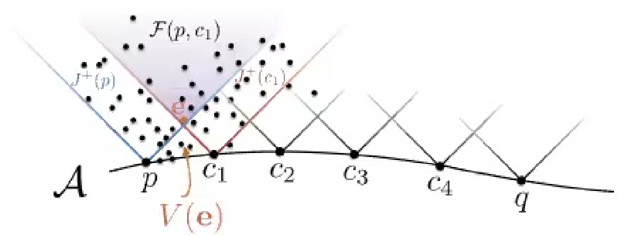
$\tau(p,q) \propto$ **length of longest chain**



— *Brightwell & Gregory, 1991*
— *Myrrheim. 1978*
— *Meyer, 1988*
—*Bachmat*
— *Roy, Sinha & Surya, 2013*
— *Nomaan & Kambor, 2020*

Spatial distance





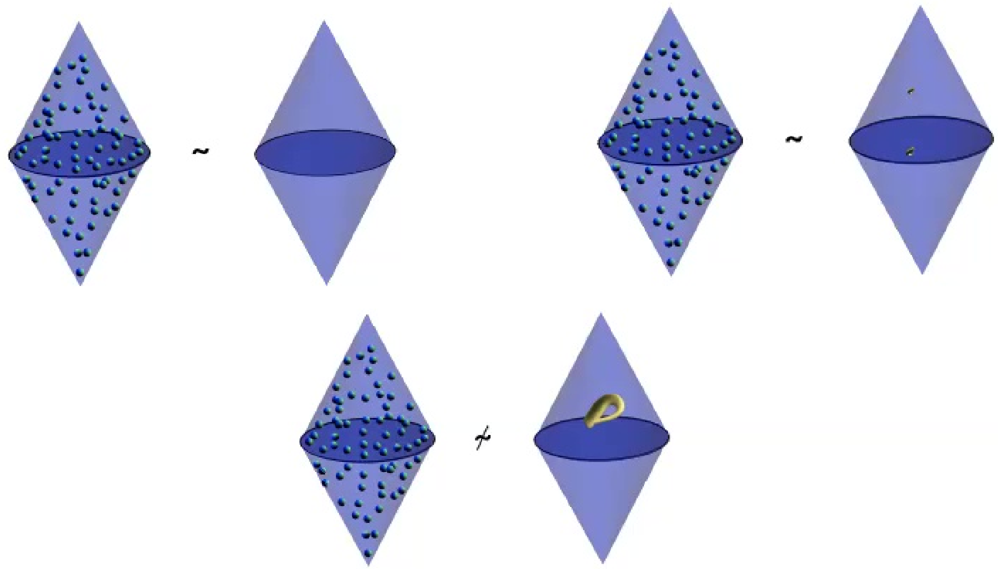
$$d(p,q) = 2\left(\frac{V(e)}{\zeta_{\mathrm{dim}}}\right)^{\frac{1}{\mathrm{dim}}}$$

$$D(p,q) = \inf_{\gamma} d_\gamma(p,q)$$

—*Eichhorn, Mizera & Surya, 2016*
—*Eichhorn, Surya & Versteegen, 2018, 2019*

The Causal Set Hauptvermutung

$$C \approx (M_1, g_1),\ C \approx (M_2, g_2) \ \Rightarrow\ (M_1, g_1) \sim (M_2, g_2)$$



Q) When are two Lorentzian manifolds "close"?
When are two causal sets/structures "close" ?

Lorentzian Gromov Hausdorff Distance — *Bombelli & Noldus 2004*

Looking for PF invariants —*Sorkin and Zwane*

Scalar Fields on Causal Sets

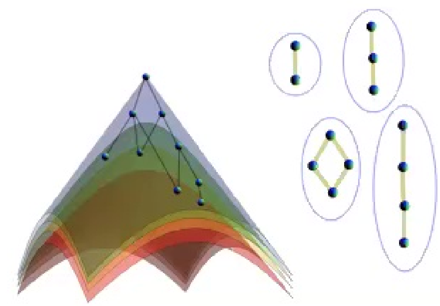
— *Sorkin 2007*
— *Benincasa & Dowker, 2010*
— *Benincasa 2012*
— *Dowker & Glaser, 2013*

D'Alembertian

$$B\phi(x) = \frac{4}{\sqrt{6}l^2}\left(-\phi(x) - \sum_{y\in L_1(x)}\phi(y) - 9\sum_{y\in L_1(x)}\phi(y) + 16\sum_{y\in L_1(x)}\phi(y) - 8\sum_{y\in L_1(x)}\phi(y)\right)$$

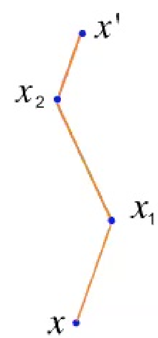
The Benincasa-Dowker Action

$$\frac{1}{\hbar}S_{BD} = \frac{4l^2}{\sqrt{6}l_p^2}\left(N - N_0 + 9N_1 - 16N_2 + 8N_3\right)$$



Green Functions

$$K(x,x') = \sum_k a^{k+1}b^k\langle P_k\rangle$$



— *Johnston, 2008*
— *Dowker, Surya & Nomaan, 2015*

Q) Is there a purely order theoretic dimension-independent Green Function? If not, can one find it for arbitrary d?

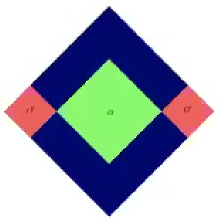
Quantum Scalar Fields : the SJ vacuum and Sorkin's Spacetime EE

$$[\phi(x), \phi(x')] = i\Delta(x, x'), \qquad (i\Delta \circ f)(x) \equiv \int_R dV_{x'} i\Delta(x, x')f(x'), \quad W_{SJ} = \mathrm{Pos}(\mathrm{i}\Delta)$$

— Johnston, 2010
—Sorkin, 2011

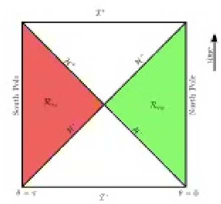
$$S = \sum_\mu \mu \ln|\mu|, \quad W_O(x, x') \circ v = i\mu\Delta_O(x, x') \circ v, \quad \Delta_O v \neq 0$$

— Sorkin 2014

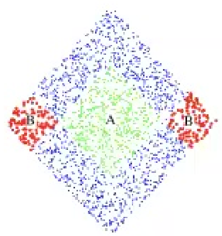


Area Law: $S \approx \frac{1}{3} \ln \frac{\sqrt{N_l}}{4\pi}$

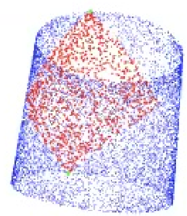
—Saravani Sorkin & Yazdi, 2014



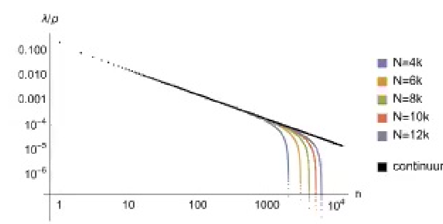
—Mathur, Surya, Nomaan X



— Sorkin & Yazdi, 2018



— Surya, Yazdi, Nomaan X, 2020



$S \propto V$: only after double truncation $S \propto A$!

Q) How does non-locality affect Entanglement Entropy and Black Hole Thermodynamics?

Causal Set Path Sum

$$Z_n = \sum_{c \in \Omega_n} e^{i\frac{S(c)}{\hbar}}$$

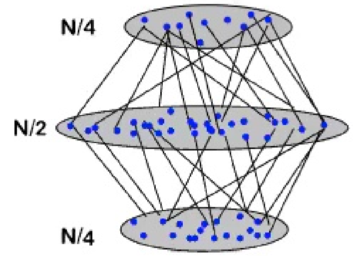
Which Action to use? Does it contain higher derivative terms?

What does a typical causal set look like?

The Sample Space Ω_n

$|\Omega_n| \sim 2^{\frac{n^2}{4}+\frac{3n}{2}+o(n))}$ **is dominated by KR posets**

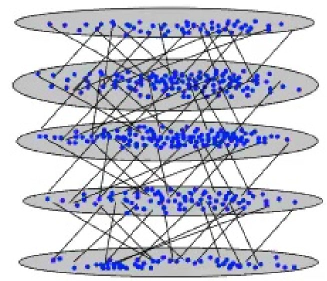
— *Kleitmann and Rothschild, 1975*

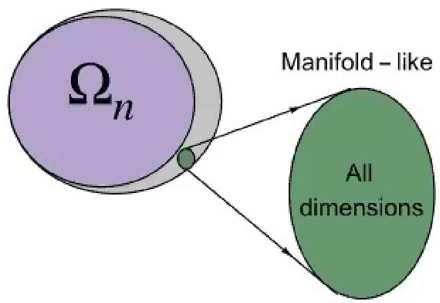


Non manifold-like

Layered posets are subdominant

— *Dhar, 1978, 1980*





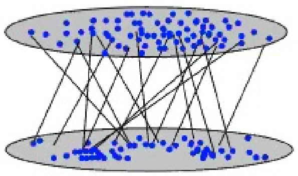
Suppressions in the Lorentzian Path Integral

—*Loomis & Carlip, 2017*

$$Z = \sum_{c\in\Omega} e^{i\frac{S(c)}{\hbar}} = \sum_{c\in A} e^{i\frac{S(c)}{\hbar}} + \sum_{c\in B} e^{i\frac{S(c)}{\hbar}}$$

Is this suppressed?

Bilayer Posets



$$\frac{1}{\hbar}S_{BD}(c) = \mu\Big(N + \sum_{j=0}^{j_{max}} \lambda_j N_j\Big) = \mu(N + \lambda_0 N_0)$$

—*Benincasa & Dowker, 2010*

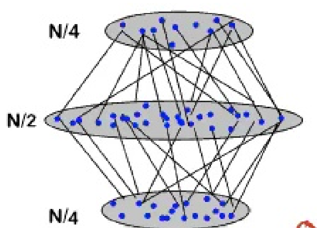
$$Z_{bilayer}[\mu, \lambda_0] \sim \int_0^{1/2} \mathrm{d}p\, |\mathscr{C}_{p,N}| \exp(iS_l(p))$$

$$\tan(-\frac{\mu\lambda_0}{2}) > \sqrt{3}$$

$$d = 4: \quad l \geq 1.452 l_p$$

$$d = 2: \text{no} \quad \text{suppression}$$

KR Posets



$$\frac{1}{\hbar}S_L(c) = \mu(N + \lambda_0 N_0)$$

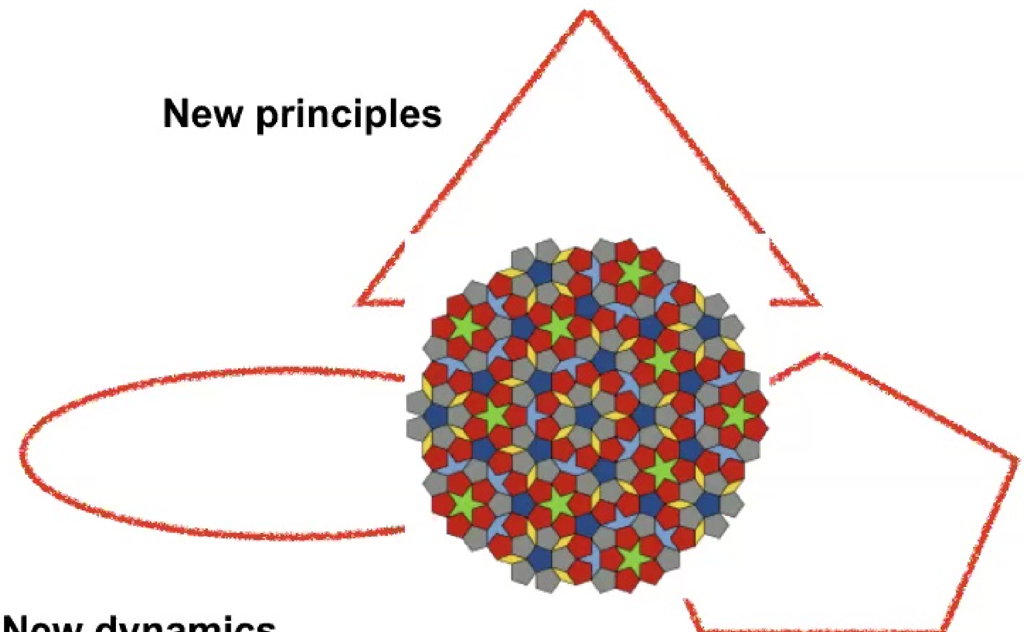
KR posets are also suppressed for same parameter range!

—*Mathur, Singh & Surya, 2020*

Q) Is there an order theoretic characterisation of manifold-like causal sets? What action would make 4+D manifold-like causal sets dominate?

Old building blocks aren’t enough

New principles

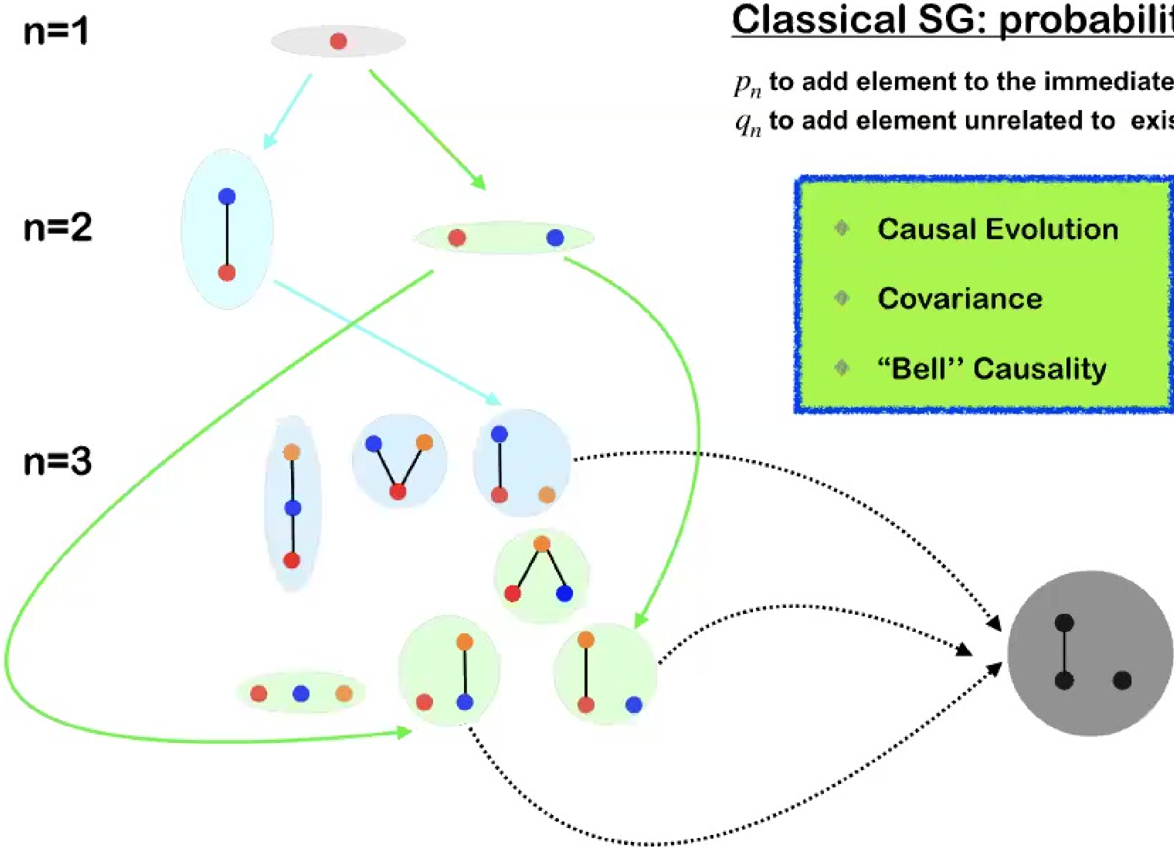


New symmetries

New dynamics

Sequential Growth Models

—Rideout & Sorkin,2000



Classical SG: probabilities

p_n to add element to the immediate future of an element
q_n to add element unrelated to existing element

- Causal Evolution
- Covariance
- "Bell" Causality

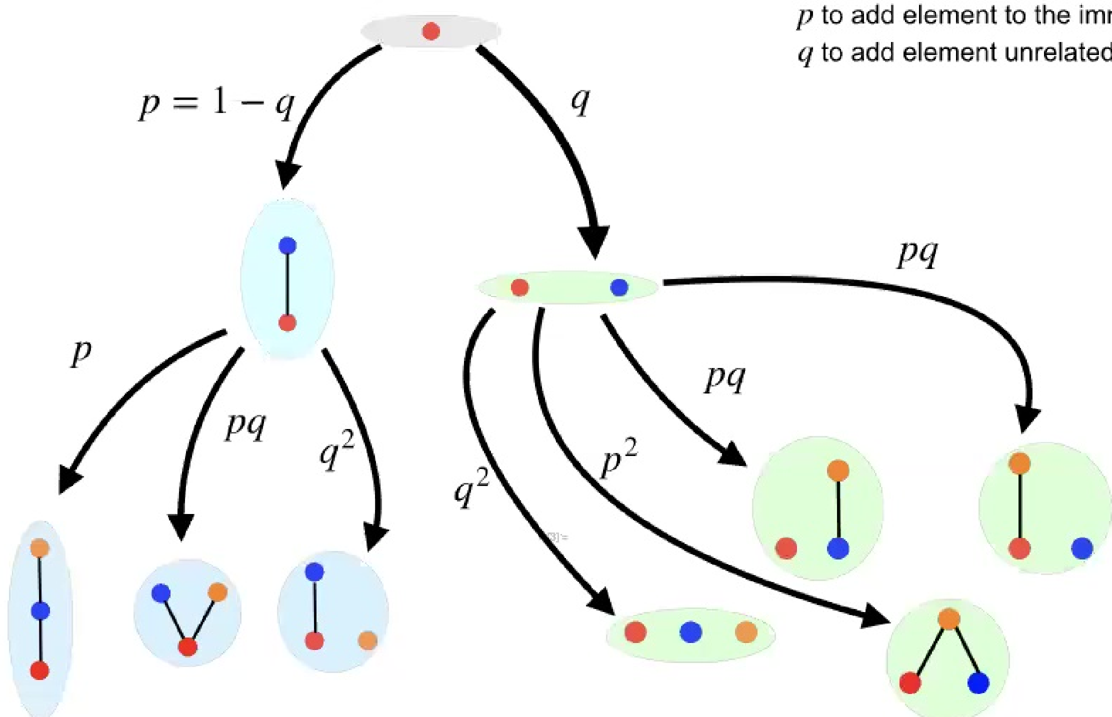
$$\Rightarrow \{t_1, t_2, \ldots\}$$

$$\frac{1}{q_n} = \sum_j \binom{n}{j} t_j$$

Transitive Percolation

—*Rideout & Sorkin, 2000*

p to add element to the immediate future of an element
q to add element unrelated to existing element

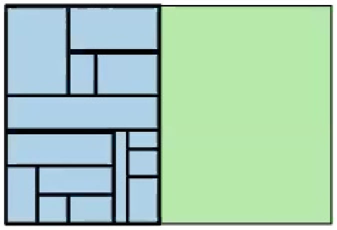


Transitive Percolation $p \in [0,1]$ overcomes KR entropy!

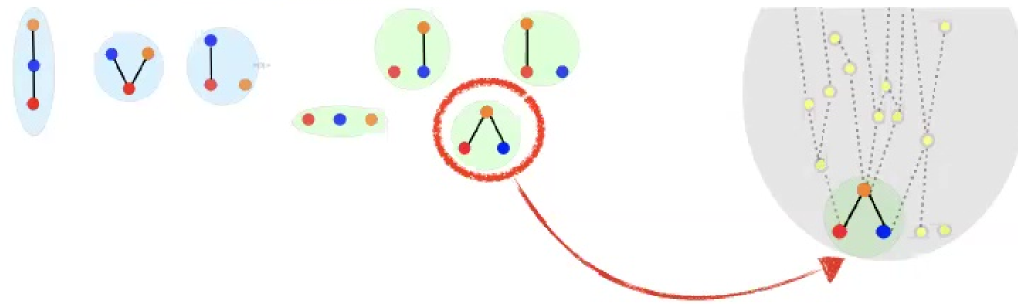
—*Brightwell & Luczak, 2015*

Measure Space: $(\Omega, \mathfrak{A}, \mu)$

—Brightwell, Dowker, Garcia, Henson & Sorkin, 2003



Ω : past finite labelled causal sets
$\mathfrak{A}$: event algebra generated by cylinder sets
$\mu : \mathfrak{A} \to [0,1]$: probability measure



Event :

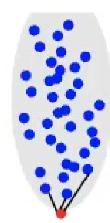
$$\alpha(c_n) \in \mathfrak{A}, \quad \alpha(c_n) = \{c \in \Omega \mid c|_n = c_n\}$$

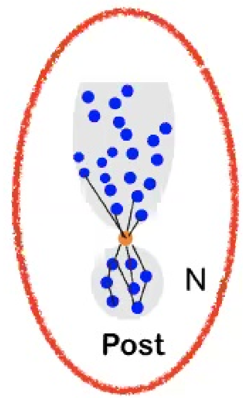
Caratheodary-Hahn extension theorem: μ extends to $\mathfrak{S}(\mathfrak{A}) \Rightarrow$ Covariance

Covariant event algebra $\widetilde{\mathfrak{S}}(\mathfrak{A}) = \mathfrak{S}(\mathfrak{A})/\sim$

Covariant Observables are Covariant events

Covariant Event Algebras using Past Sets
— Dowker & Zalel, 2019





Quantum Sequential Growth $(\Omega, \mathfrak{A}, \mu_{\mathrm{v}})$

— *Sorkin, 1996*

$$\mu_{\mathrm{v}} : \mathfrak{A} \to \mathscr{H}$$

Vector Measure

Histories Hilbert Space

— *Dowker, Johnston & Sorkin, 2010*

- $\mu_{\mathrm{v}}(\alpha) \equiv [\chi_\alpha] \in \mathscr{H}$
- $\langle \mu_{\mathrm{v}}(\alpha), \mu_{\mathrm{v}}(\beta) \rangle = D(\alpha, \beta)$

- **Free vector space V of functions :** $f : \mathfrak{A} \to \mathbb{C}$
- $\langle u, v \rangle \equiv \sum_{\alpha, \beta \in \mathfrak{A}} u^*(\alpha) v(\beta) D(\alpha, \beta)$
- $\lim_{i \to \infty} || u_i - v_i ||_V = 0 \Rightarrow \{u_i\} \sim \{v_i\}$
- $\mathscr{H} \equiv V/\sim, \quad [\{u_i\}] \in \mathscr{H}$

Is QSG Covariant?

— *Dowker, Johnston & Surya, 2010*

— *Surya & Zalel, 2020*

Caratheodary-Hahn-Kluvnek theorem:

μ_{v} extends to $\mathfrak{S}(\mathfrak{A})$ only if a set of convergence conditions are satisfied by μ_{v}

Q) Can we Classify Covariant Quantum Measures?

Q) How unique/special is Poisson?

Q) Are there more geometrically interesting order invariants?!

Q) When are two Lorentzian manifolds "close"?
When are two causal sets/structures "close" ?

Q) Is there a purely order theoretic dimension independent Green Function? If not can one find it for arbitrary d?

Q) What of Gauge Fields and Spinors?

Q) How does non-locality affect Entanglement Entropy and Black Hole Thermodynamics?

Q) Can we Classify Covariant Quantum Measures?