Title: State sum models with defects

Speakers: Catherine Meusburger

Collection: Women at the Intersection of Mathematics and Theoretical Physics

Date: February 22, 2021 - 10:30 AM

URL: http://pirsa.org/21020028

Abstract: "We explain how to construct a Turaev-Viro state sum model with defect planes, defect lines and defect points. This is work in progress with John Barrett."

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Talk PI

State sum models with defects

Catherine Meusburger

Friedrich-Alexander-Universität Erlangen-Nürnberg

Work in progress with John Barrett

Women at the intersection of Mathematics and Theoretical Physics

Perimeter Institute

February 22, 2021

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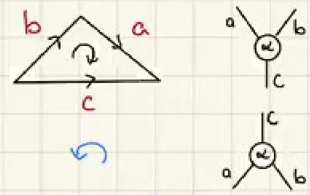
Turaev-Viro Barrett-Westbury invariants

• ingredients: • triangulated 3-mfold M
• spherical fusion category $\mathcal{C}$ over $\mathbb{C}$

• Construction:

• labeling ℓ: • edge $e \xrightarrow{\ell}$ simple object $\overset{a \in I}{\longrightarrow}$ $\underset{a^* \in I}{\longleftarrow}$

• triangle

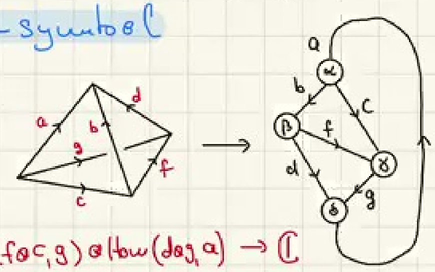


$\alpha \in \mathrm{Hom}(a \otimes b, c)$

$\alpha \in \mathrm{Hom}(c, a \otimes b)$

• tetrahedron $t \longrightarrow$ 6j-symbol

~ associator in terms of simple objects



$\in \mathbb{C}$

$\mathrm{Hom}(a, b \otimes c) \otimes \mathrm{Hom}(b, d \otimes f) \otimes \mathrm{Hom}(f \otimes c, g) \otimes \mathrm{Hom}(d \otimes g, a) \to \mathbb{C}$

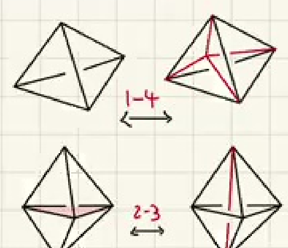
~> State Sum

$$\boxed{Z(M,T) = \frac{1}{\dim \mathcal{C}^{v}} \sum_{\ell: E \to I} \left(\prod_{t \in T} 6j(\ell, t) \right) \left(\prod_{e \in E} \dim \ell(e) \right)}$$

• topological invariant: Pachner move invariance

2-3: pentagon for associator
1-4: invertibility of associator

• extends to 3-mfolds with boundary



Turaev-Viro Barrett-Westbury invariants

• ingredients: • triangulated 3-mfold M

• spherical fusion category $\mathcal{C}$ over $\mathbb{C}$

$\otimes$ $\oplus$ $*$

• Construction:

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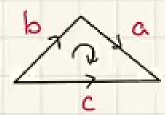

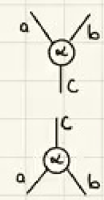

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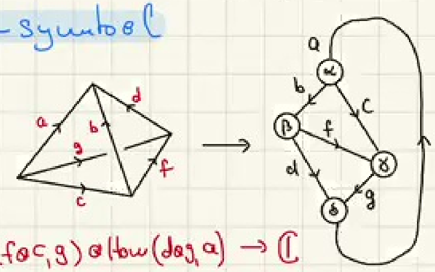

$\in \mathbb{C}$

$\mathrm{Hom}(a, b \otimes c) \otimes \mathrm{Hom}(b, d \otimes f) \otimes \mathrm{Hom}(f \otimes c, g) \otimes \mathrm{Hom}(d \otimes g, a) \to \mathbb{C}$

⇝ State sum $\dim \mathcal{C} = \sum_{a \in I} \dim(a)^2$

$\dim a = tr(1_a)$

a

$$\boxed{Z(M,T) = \frac{1}{\dim \mathcal{C}^v} \sum_{\ell: E \to I} \left(\prod_{t \in T} 6j(\ell,t) \right) \left(\prod_{e \in E} \dim \ell(e) \right)}$$

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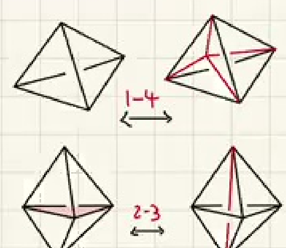


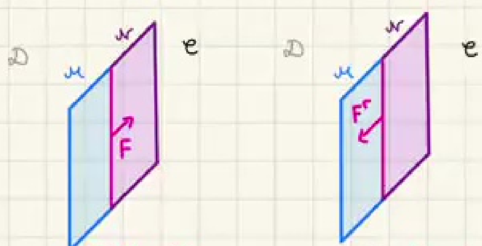
• extends to 3-mfolds with boundary

defect data

- 3d regions — spherical fusion categories $\mathcal{C}, \mathcal{D}, \dots$
- defect planes — finite semisimple bimodule categories with bimodule traces

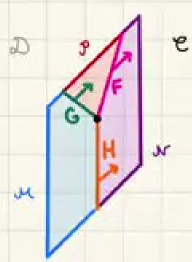
with $d \triangleright^{op} m \triangleleft^{op} c = c^* \triangleright m \triangleleft d^*$

- defect lines — bimodule functors compatible with bimodule traces



$F: \mathcal{M} \to \mathcal{N}$ — right adjoint $F^r: \mathcal{N} \to \mathcal{M}$

- defect vertices — bimodule natural transformations



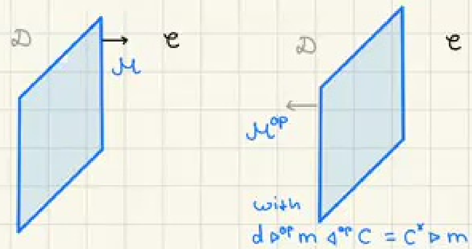
$\nu: FG \to H$

Ex: $\mathcal{M} = \mathcal{N} = \mathcal{C} = \mathcal{D}$ spherical category as bimodule category over itself

defect data

- 3d regions — spherical fusion categories $\mathcal{C}, \mathcal{D}, \dots$
- defect planes — finite semisimple bimodule categories with bimodule traces

$\triangleright : \mathcal{C} \times \mathcal{M} \to \mathcal{M}$
$\triangleleft : \mathcal{D} \times \mathcal{M} \to \mathcal{M}$



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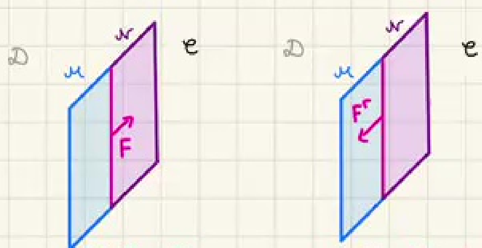
- defect lines — bimodule functors compatible with bimodule traces



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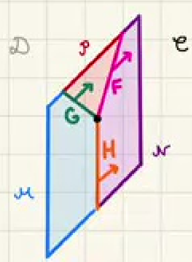


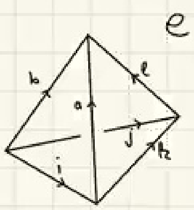
$\nu : FG \to H$

Ex: $\mathcal{M} = \mathcal{N} = \mathcal{C} = \mathcal{D}$ spherical category as bimodule category over itself

State sum model via generalised 6j symbols [J.B., C.M.]

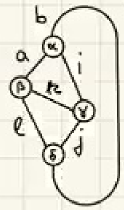
tetrahedra

6j symbols $\in \mathbb{C}$



associator for $\mathcal{C}$

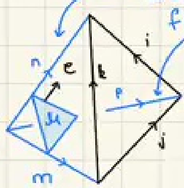
$a: \otimes(\otimes \times \mathrm{id}) \xrightarrow{\sim} \otimes(\mathrm{id} \times \otimes)$



trace in $\mathcal{C}$

$\alpha: b \to a \otimes i$
$\beta: a \to \ell \otimes k$
$\gamma: k \otimes i \to j$
$\delta: \ell \otimes j \to b$

orientation fixed by plane

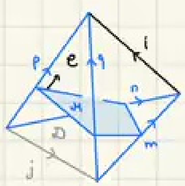


coherence data for action functor $\triangleright: \mathcal{C} \times \mathcal{M} \to \mathcal{M}$

$c: \triangleright(\otimes \times \mathrm{id}) \xrightarrow{\sim} \triangleright(\mathrm{id} \times \triangleright)$

bimodule trace in $\mathcal{M}$

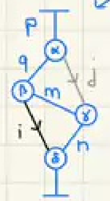
$\alpha: n \to k \triangleright m$
$\beta: k \to i \otimes j$
$\gamma: m \triangleleft j \to p$
$\delta: i \triangleright p \to n$



Coherence data for $\triangleright: \mathcal{C} \times \mathcal{M} \to \mathcal{M}$, $\triangleleft: \mathcal{M} \times \mathcal{D} \to \mathcal{M}$

$b: \triangleleft(\triangleright \times \mathrm{id}) \xrightarrow{\sim} \triangleright(\mathrm{id} \times \triangleleft)$

bimodule trace in $\mathcal{M}$



$\alpha: p \to q \triangleleft j$
$\beta: q \to i \triangleright m$
$\gamma: m \triangleleft j \to n$
$\delta: i \triangleright n \to p$

Coherence data for $\mathcal{D}$-module functor $F: \mathcal{M} \to \mathcal{N}$

$t: \triangleleft(F \times \mathrm{id}) \xrightarrow{\sim} F \triangleleft$

orientation fixed by functor

bimodule trace in $\mathcal{N}$

$\alpha: n \to F(m)$
$\beta: m \to p \triangleleft i$
$\gamma: F(p) \to q$
$\delta: q \triangleleft i \to n$

bimodule natural transformation

$\nu: H \to FG$

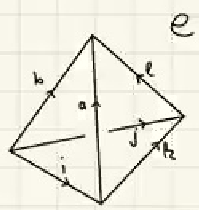
bimodule trace in $\mathcal{N}$

$\alpha: n \to H(m)$
$\nu_m: H(m) \to FG(m)$
$\gamma: G(m) \to p$
$\delta: F(p) \to n$

State sum model via generalised 6j symbols [J.B., C.M.]

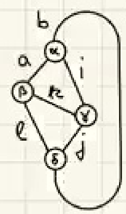
tetrahedra

6j symbols $\in \mathbb{C}$



associator for $\mathcal{C}$

$a: \otimes(\otimes\times id) \xrightarrow{\sim} \otimes(id\times\otimes)$



← trace in $\mathcal{C}$

$\alpha: b \to a\otimes i$
$\beta: a \to \ell\otimes k$
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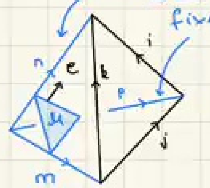
orientation fixed by plane



coherence data for action functor $\triangleright: \mathcal{C}\times\mathcal{M}\to\mathcal{M}$

$c: \triangleright(\otimes\times id) \xrightarrow{\sim} \triangleright(id\times\triangleright)$

← bimodule trace in $\mathcal{M}$

$\alpha: n \to k\triangleright m$
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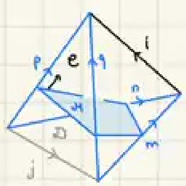

coherence data for $\triangleright: \mathcal{C}\times\mathcal{M}\to\mathcal{M}$, $\triangleleft: \mathcal{M}\times\mathcal{D}\to\mathcal{M}$

$b: \triangleleft(\triangleright\times id) \xrightarrow{\sim} \triangleright(id\times\triangleleft)$

← bimodule trace in $\mathcal{M}$

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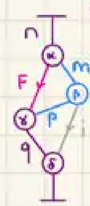
coherence data for $\mathcal{D}$-module functor $F: \mathcal{M}\to\mathcal{N}$

$t: \triangleleft(F\times id) \xrightarrow{\sim} F\triangleleft$

orientation fixed by functor



← bimodule trace in $\mathcal{N}$

$\alpha: n \to F(m)$
$\beta: m \to p\triangleleft i$
$\gamma: F(p) \to q$
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bimodule natural transformation

$\nu: H \Rightarrow FG$

← bimodule trace in $\mathcal{N}$

$\alpha: n \to H(m)$
$\nu_m: H(m) \to FG(m)$
$\gamma: G(m) \to p$
$\delta: F(p) \to n$

State Sum model [J.B., C.M.]

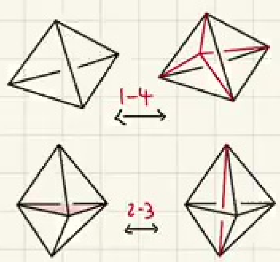
$$\mathcal{Z}(\mathcal{M},T) = \prod_{e_x} \dim(e_x) \sum_{\ell:\, E_X \to I_X} \left(\prod_{t\in T} Gj(\ell,t) \right) \left(\prod_{e\in E_X} \dim(\ell(e)) \right)$$

spherical fusion categories and bimodule categories

- Summation: Simple objects in spherical fusion categories and bimodule categories
- Gj symbols: independent of choices (orientation etc) encode coherence data for defect data

topological invariance ~ Pachner move invariance

- 2-3 move: pentagon relations for coherence data
- 1-4 move: invertibility of coherence data



$a:\ \otimes(\otimes\times id) \xrightarrow{\sim} \otimes(id\times\otimes)$ spherical categories

$c:\ \triangleright(\otimes\times id) \xrightarrow{\sim} \triangleright(id\times\triangleright)$
$d:\ \triangleleft(\triangleleft\times id) \xrightarrow{\sim} \triangleleft(id\times\otimes)$
$b:\ \triangleleft(\triangleright\times id) \xrightarrow{\sim} \triangleright(id\times\triangleleft)$
bimodule categories

$s:\ F\triangleright \xrightarrow{\sim} \triangleright(id\times F)$
$t:\ \triangleleft(F\times id) \xrightarrow{\sim} F\triangleleft$
bimodule functors

+ bimodule compatibility of bimodule nat. transformation

- additional move from hexagon for bimodule functors

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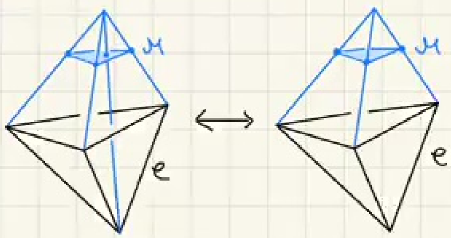
CALENDAR Tomorrow

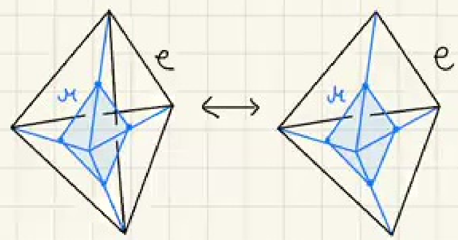
Geburtstag Simone

Tomorrow

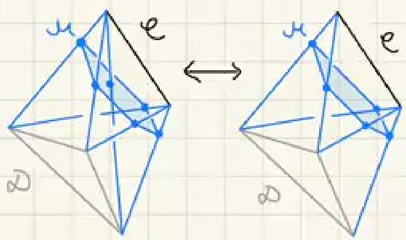
2-3 moves [J.B, C.M.]



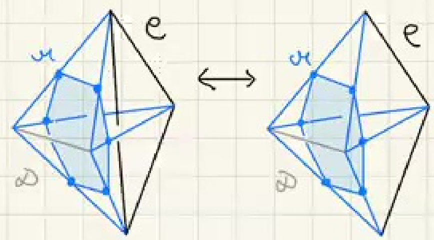
$\mathcal{C}$ - module category



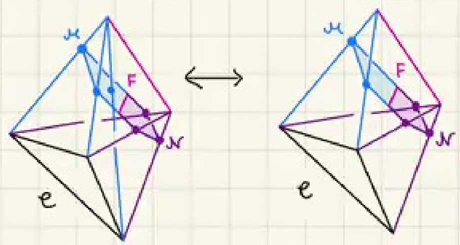
$c:\ \triangleright(\otimes\times \mathrm{id}) \xrightarrow{\sim} \triangleright(\mathrm{id}\times\triangleright)$



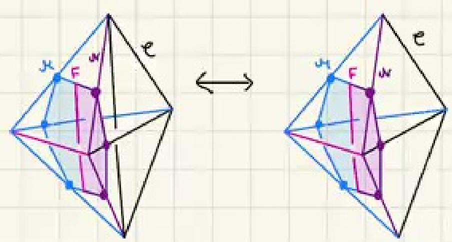
$(\mathcal{C},\mathcal{D})$ - bimodule category



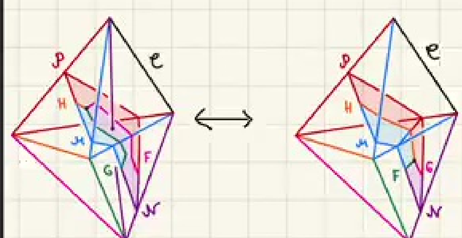
$b:\ \triangleleft(\triangleright\times \mathrm{id}) \xrightarrow{\sim} \triangleright(\mathrm{id}\times\triangleleft)$



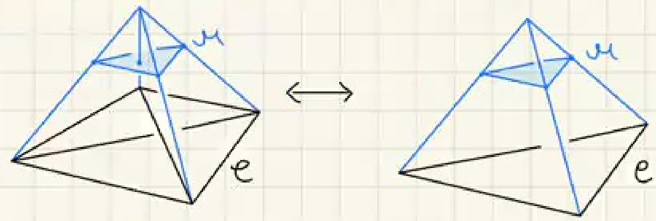
$\mathcal{C}$ - module functor



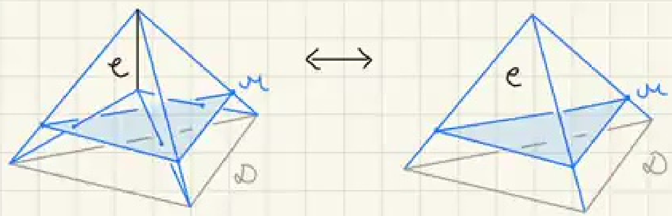
$s:\ \triangleright(\mathrm{id}\times F) \xrightarrow{\sim} F\triangleright$



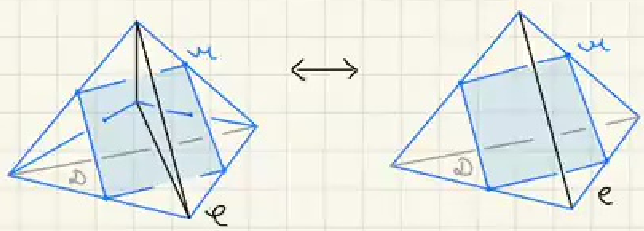
1-4 moves [J.B., C.M.]

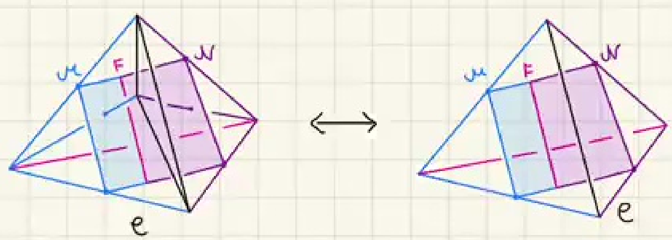


$\mathcal{C}$-module category

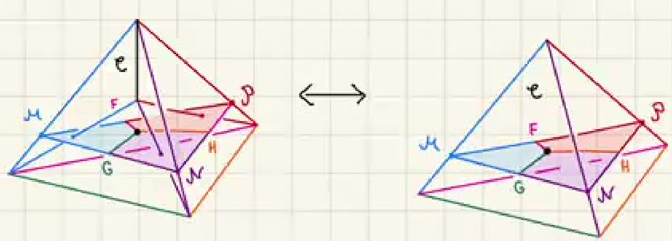


$(\mathcal{C},\mathcal{D})$-bimodule category



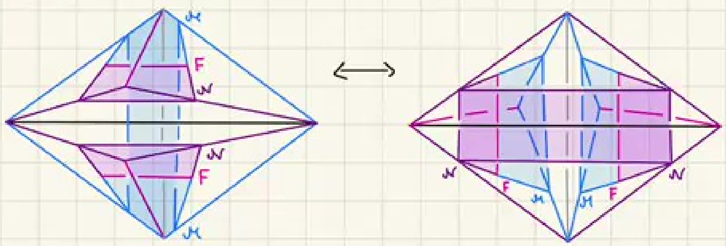


$\mathcal{C}$-module functor



$\mathcal{C}$-module natural transformation

additional move



hexagon relation for bimodule functor

Summary

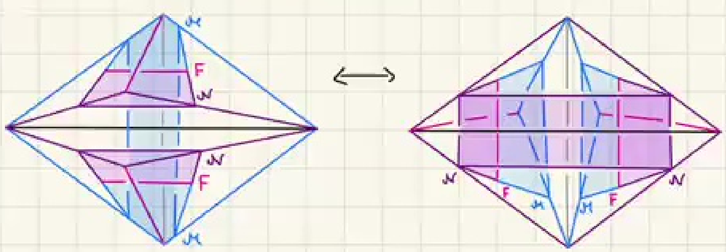
- Simple and explicit Turaev-Viro model with defects
- generalised 6j symbols $\longleftrightarrow$ coherence data for defect data
- triangulation independence via Pachner moves
  $\longleftrightarrow$ pentagon axioms & invertibility for defect data

To Dos

- use semisimplicity of $\mathrm{Fun}(\mathcal{M},\mathcal{N})$ for topological invariance of defect data
- Concrete examples

Thank you for your attention!

additional move



hexagon relation for bimodule functor

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To Dos

- use semisimplicity of $\mathrm{Fun}(\mathcal{M},\mathcal{N})$ for topological invariance of defect data
- Concrete examples
  $Z(\mathcal{C})$ ~ transparent defect planes
  from group representations, ...

Thank you for your attention!