Title: Division algebraic symmetry breaking

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Abstract: Can the 32C-dimensional algebra RCHO offer anything new for particle physics? Indeed it can. Here we identify a sequence of complex structures within RCHO which sets in motion a cascade of breaking symmetries: Spin(10) -> Pati-Salam -> Left-Right symmetric -> Standard model + B-L (both pre- and post-Higgs-mechanism). These complex structures derive from the octonions, then from the quaternions, then from the complex numbers. It should be noted that this pattern would not have been obvious within the standard formalism.

Division algebraic symmetry breaking

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R, C, H, O

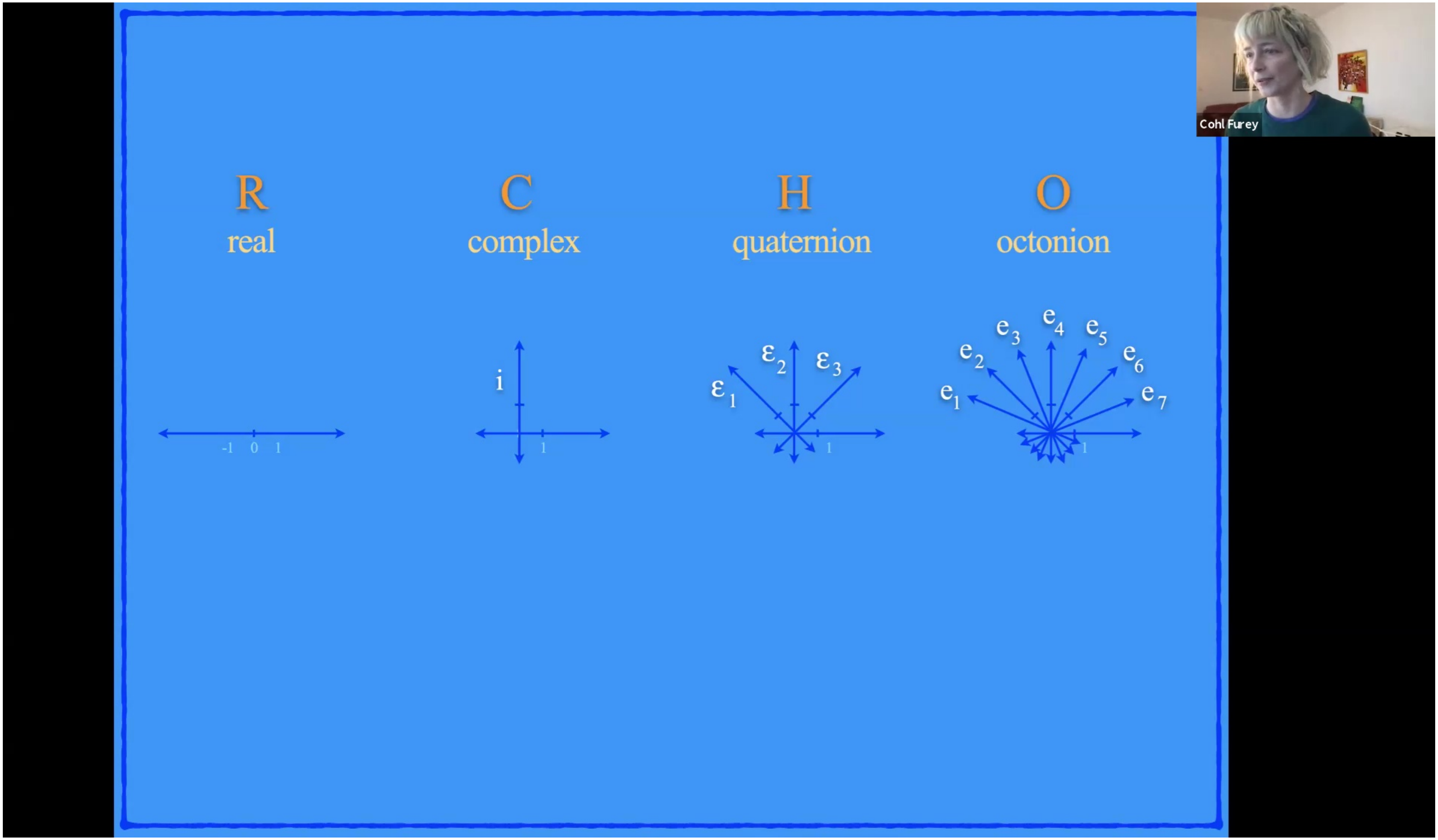
special, and yet not special

R, C, H, O

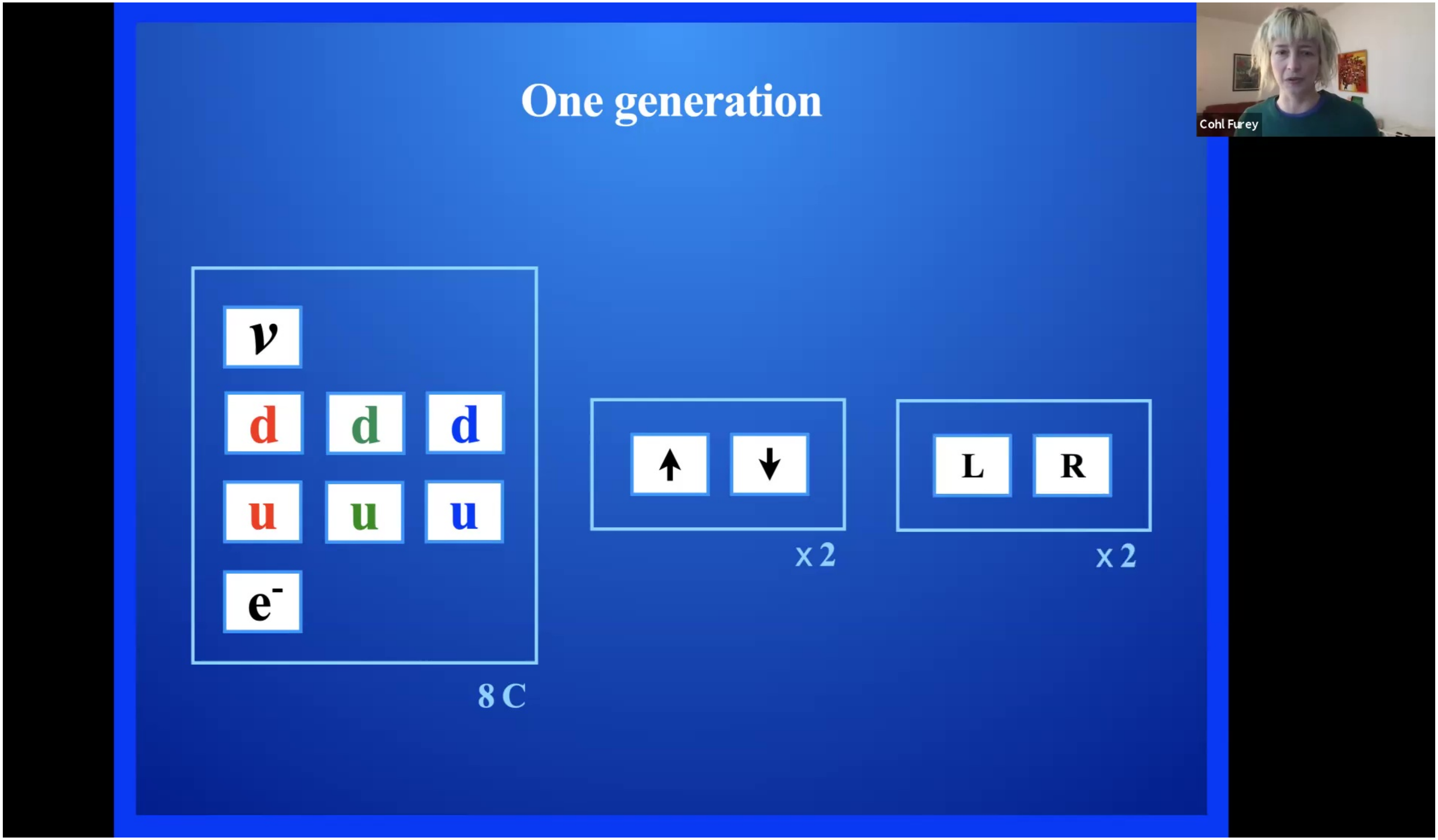
R	C	H	O
↓	↓	↓	↓
everywhere	quantum	special relativity	strong nuclear

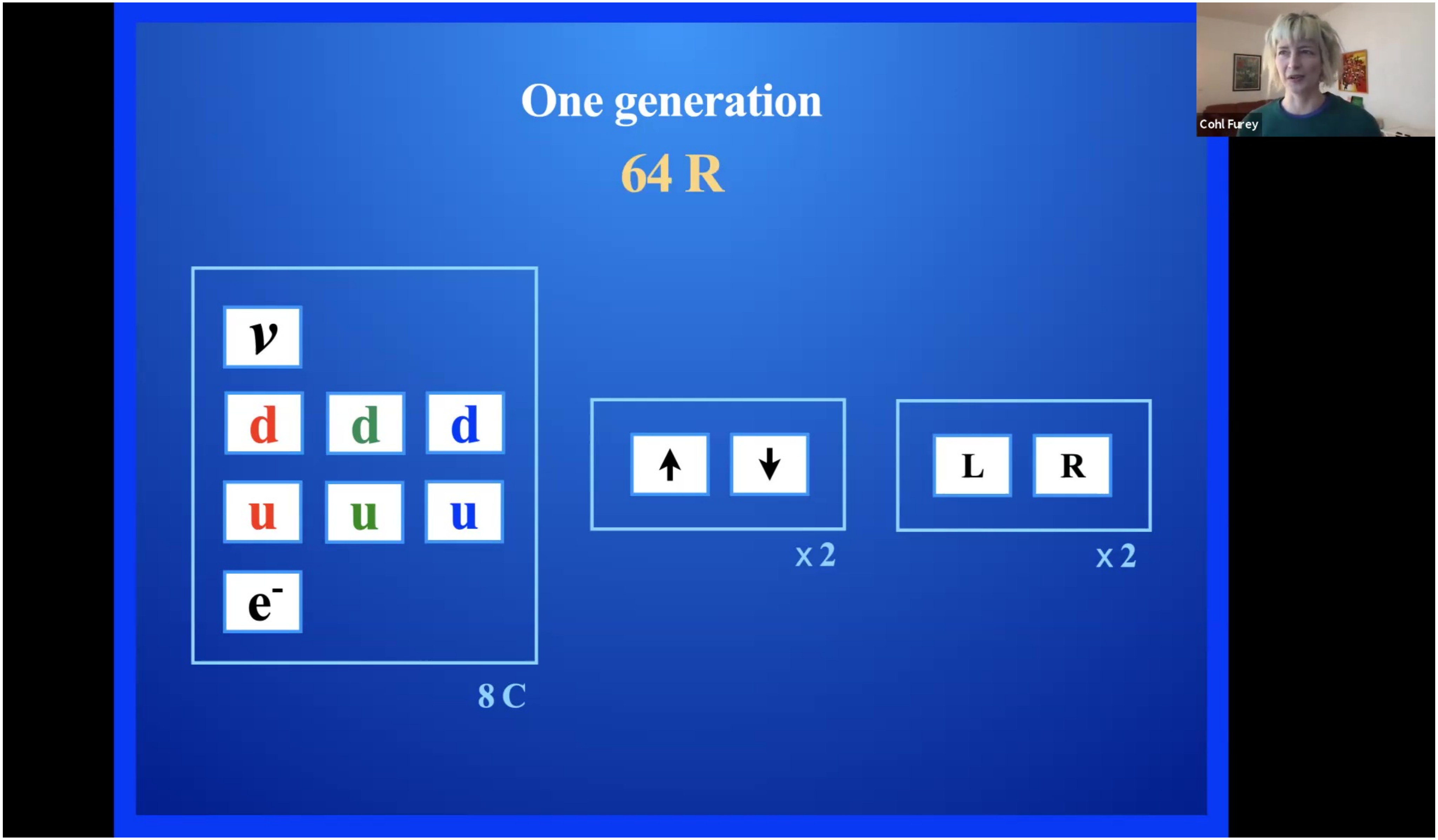
electromagnetism

isospin



Cohl Furey
R
real
C
complex
H
quaternion
O
octonion
-1 0 1
i
1
ε1 ε2 ε3
1
e1 e2 e3 e4 e5 e6 e7
1


One generation
ν
d
d
d
u
u
u
e-
8 C
↑
↓
x 2
L
R
x 2
Cohl Furey


One generation
64 R
ν
d d d
u u u
e⁻
↑ ↓
x 2
L R
x 2
8 C
Cohl Furey

$$\left\{ \begin{array}{c} \text{One generation} \\ 64\ \mathbb{R} \end{array} \right\}$$

single package?

One generation

$$\boxed{64\,\mathbb{R}}$$

$$\updownarrow$$

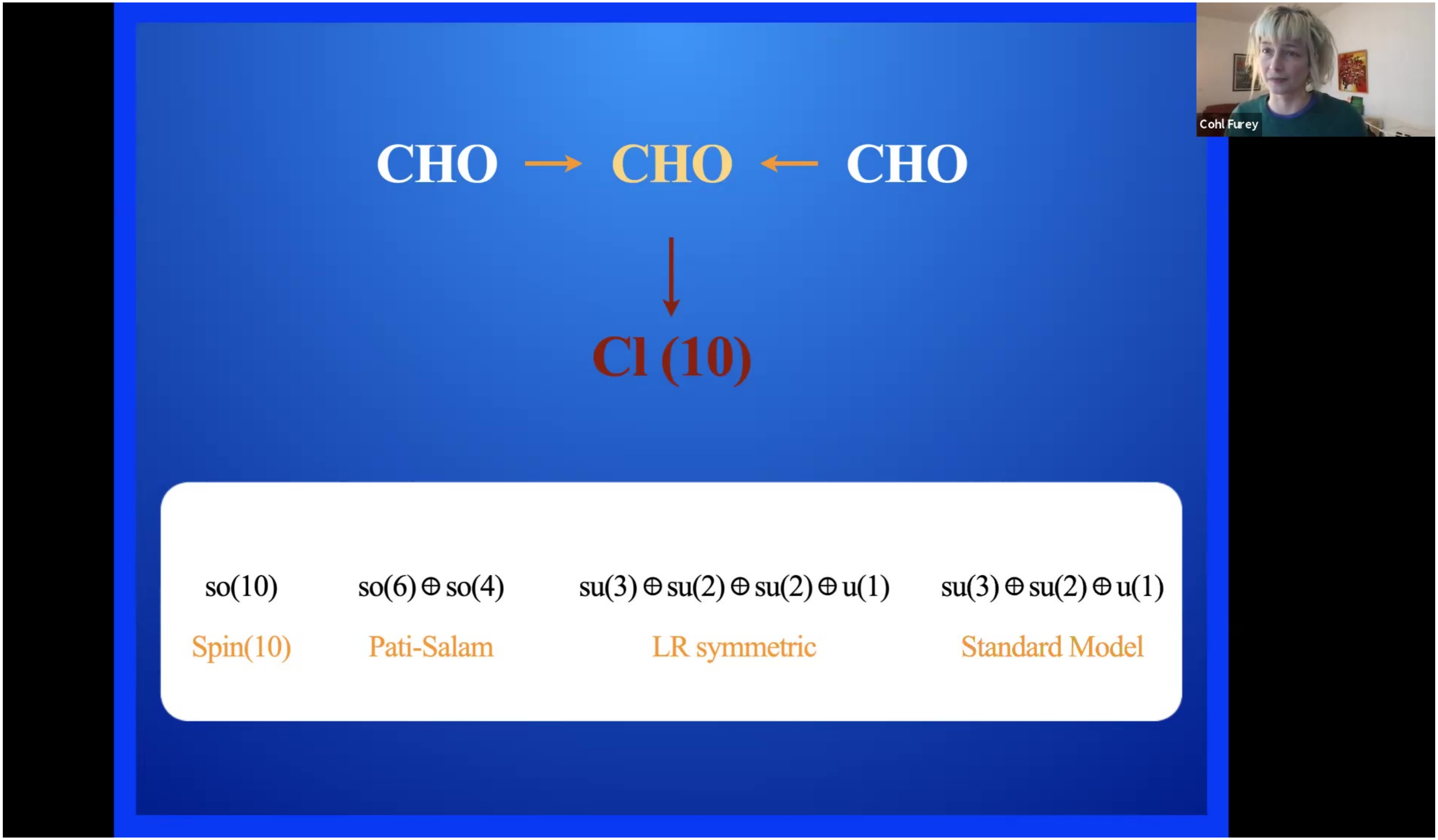
$$\underset{1}{\mathbb{R}} \otimes \underset{2}{\mathbb{C}} \otimes \underset{4}{\mathbb{H}} \otimes \underset{8}{\mathbb{O}}$$

$$\boxed{64\,\mathbb{R}}$$

$$\mathbf{R} \otimes \mathbf{C} \otimes \mathbf{H} \otimes \mathbf{O}$$

$$= \mathbf{C} \otimes \mathbf{H} \otimes \mathbf{O}$$

$$\longrightarrow \mathbf{CHO}$$


CHO → CHO ← CHO
Cl (10)
so(10)
Spin(10)
so(6) ⊕ so(4)
Pati-Salam
su(3) ⊕ su(2) ⊕ su(2) ⊕ u(1)
LR symmetric
su(3) ⊕ su(2) ⊕ u(1)
Standard Model
Cohl Furey

$\boxed{1}$

One generation

$$\boxed{64\ \mathbb{R}}$$

$$\updownarrow$$

$$\underset{1}{\mathbb{R}} \otimes \underset{2}{\mathbb{C}} \otimes \underset{4}{\mathbb{H}} \otimes \underset{8}{\mathbb{O}}$$

$$\boxed{64\ \mathbb{R}}$$

2

CHO → CHO ← CHO

↓

Cl(10)

$so(10)$	$so(6) \oplus so(4)$	$su(3) \oplus su(2) \oplus su(2) \oplus u(1)$	$su(3) \oplus su(2) \oplus u(1)$
Spin(10)	Pati-Salam	LR symmetric	Standard Model

Anything new beyond Cl(10)?

Spin(10)

$$r_{ij}\ \gamma_i\ \gamma_j$$

old way

Spin(10)

$$r''_{ij}\, e_i(e_j \cdot) \;+\; r_n\, P_L \varepsilon_n \cdot \;+\; r'_n\, P_R \varepsilon_n \cdot$$
$$+\; r_{ni}\;\, i\,\varepsilon_n e_i(e_7 \cdot) \;+\; r''_i (e_i \cdot)$$

real

$n = 1, 2, 3 \qquad i,j = 1 \ldots 6$

new way

Spin(10)

$$r''_{ij}\, e_i(\, e_j \cdot) \;+\; r_n\, P_L \varepsilon_n \cdot \;+\; r'_n\, P_R \varepsilon_n \cdot$$

$$+\; r_{ni}\;\, i\, \varepsilon_n e_i(\, e_7 \cdot) \;+\; r''_i (e_i \cdot)$$

quaternionic

$n = 1, 2, 3 \qquad i,j = 1 \ldots 6$

new way

Spin(10)

$$r''_{ij}\, e_i(\, e_j \cdot) \quad + \quad r_n\, P_L \varepsilon_n \cdot \quad + \quad r'_n\, P_R \varepsilon_n \cdot$$

$$+ \quad r_{ni} \;\; i\, \varepsilon_n e_i (\, e_7 \cdot) \quad + \quad r''_i (e_i \cdot)$$

octonionic

$n = 1, 2, 3 \qquad i,j = 1 \ldots 6$

new way

Spin(10)

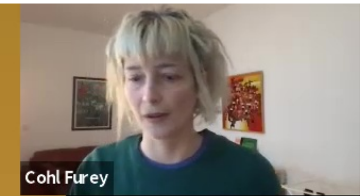
$$P_L = (\,1 + ie_7\,)/2$$

$$r''_{ij}\, e_i(\,e_j \cdot\,) \;+\; r_n P_L \varepsilon_n \cdot \;+\; r'_n P_R \varepsilon_n \cdot$$

$$+\; r_{ni}\;\, i\,\varepsilon_n e_i(\,e_7 \cdot\,) \;+\; r''_i(e_i \cdot)$$

$n = 1, 2, 3 \qquad i,j = 1 \ldots 6$

new way



Spin(10)

$$P_R = (1 - ie_7)/2$$

$$r''_{ij}\, e_i(e_j \cdot) \;+\; r_n P_L \varepsilon_n \cdot \;+\; r'_n P_R \varepsilon_n \cdot$$

$$+\; r_{ni}\; i\,\varepsilon_n e_i (e_7 \cdot) \;+\; r''_i (e_i \cdot)$$

$n = 1, 2, 3 \qquad i,j = 1 \ldots 6$

new way

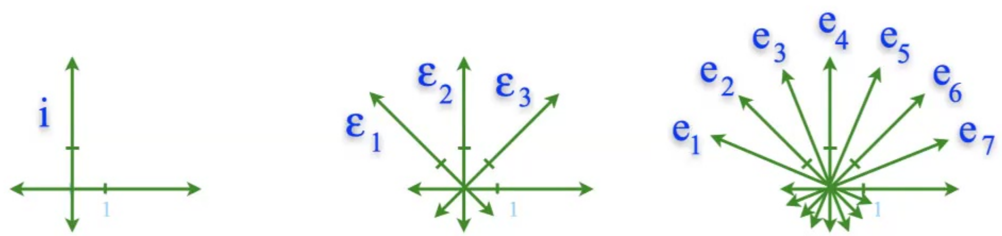
Complex structure

A complex structure is a linear map J
on a R vector space
for which $J^2 = -1$.

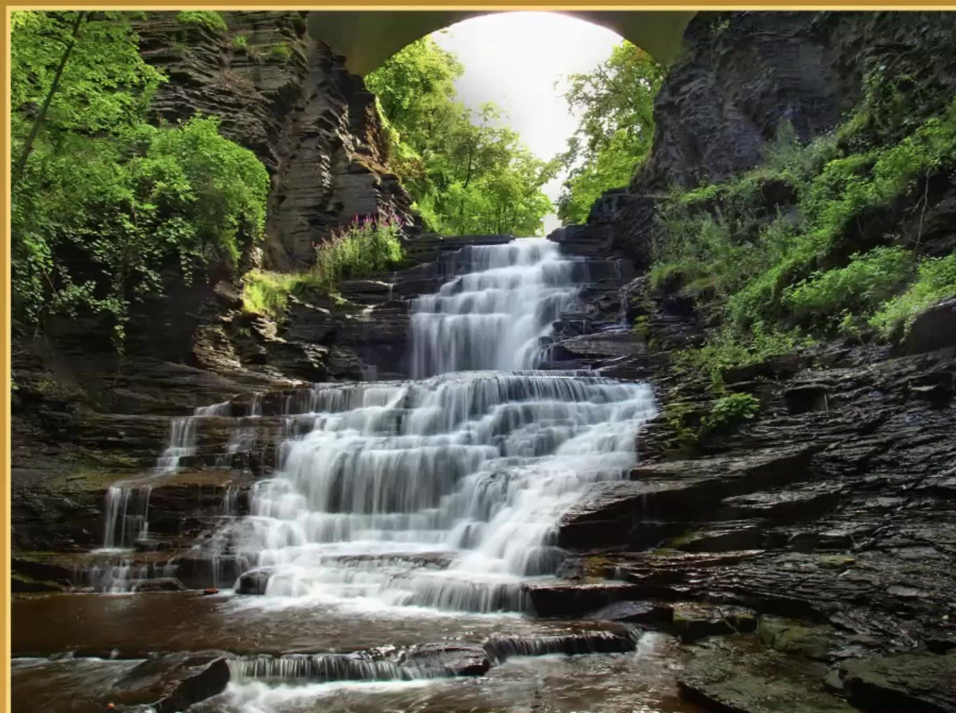
Generalizes notion of multiplying by complex i.

Complex structure

C	H	O
complex	quaternion	octonion



Cascade of complex structures



$e_j \rightarrow$

$\leftarrow e_j$

$\varepsilon_n \rightarrow$

$\leftarrow \varepsilon_n$

$\leftarrow i \rightarrow$

Spin(10)

$$r''_{ij}\, e_i(\, e_j \cdot) \;+\; r_n P_L \varepsilon_n \cdot \;+\; r'_n P_R \varepsilon_n \cdot$$

$$+\; r_{ni}\; i\,\varepsilon_n e_i(\, e_7 \cdot) \;+\; r''_i (e_i \cdot)$$

$n = 1, 2, 3$ $\quad$ $i,j = 1 \dots 6$

$e_7 \Psi$

Spin(10)

$$r''_{ij}\, e_i(e_j\,\cdot) \;+\; r_n\, P_L\, \varepsilon_n\cdot \;+\; r'_n\, P_R\, \varepsilon_n\cdot$$

$$+\; r_{ni}\;\, i\,\varepsilon_n e_i(e_7\,\cdot) \;+\; r''_i(e_i\,\cdot)$$

$n = 1, 2, 3$ $\quad$ $i,j = 1 \ldots 6$

$e_7\Psi$

Spin(10)

$$r''_{ij}\, e_i(e_j \cdot) \;+\; r_n P_L \varepsilon_n \cdot \;+\; r'_n P_R \varepsilon_n \cdot$$

$$+\; r_{ni}\; i\varepsilon_n e_i(e_7 \cdot) \;+\; r''_i(e_i \cdot)$$

$n = 1, 2, 3 \qquad i,j = 1 \ldots 6$

$e_7\Psi$

Spin(10)

$$r''_{ij}\, e_i(\,e_j\,\cdot\,) \;+\; r_n\, P_L\, \varepsilon_n \cdot \;+\; r'_n\, P_R\, \varepsilon_n \cdot$$

$$+\; r_{ni}\;\, i\,\varepsilon_n e_i(\,e_7\,\cdot\,) \;+\; r''_i(e_i\,\cdot\,)$$

$n = 1, 2, 3 \quad i,j = 1 \ldots 6$

$e_7\Psi \quad \Downarrow$

$$\underbrace{r''_{ij}\, e_i(\,e_j\,\cdot\,)}^{so(6)} \;+\; r_n\, P_L\, \varepsilon_n \cdot \;+\; r'_n\, P_R\, \varepsilon_n \cdot$$

$n = 1, 2, 3 \quad i,j = 1 \ldots 6$

Spin(10)

$$r''_{ij}\, e_i(\,e_j\cdot) \;+\; r_n\, P_L \varepsilon_n\cdot \;+\; r'_n\, P_R \varepsilon_n\cdot$$

$$+\; r_{ni}\;\, i\,\varepsilon_n e_i(\,e_7\cdot) \;+\; r''_i(e_i\cdot)$$

$n = 1, 2, 3 \qquad i,j = 1 \dots 6$

$e_7 \Psi \quad \Downarrow$

$$\overset{so(6)}{r''_{ij}\, e_i(\,e_j\cdot)} \;+\; \overset{su(2)}{r_n\, P_L \varepsilon_n\cdot} \;+\; \overset{su(2)}{r'_n\, P_R \varepsilon_n\cdot}$$

$n = 1, 2, 3 \qquad i,j = 1 \dots 6$

$e_7\Psi$ $\Downarrow$

Pati-Salam

$$\overset{so(6)}{r''_{ij}\, e_i\,(\,e_j\,\cdot\,)} \;+\; \overset{su(2)}{r_n\, P_L\, \varepsilon_n \cdot} \;+\; \overset{su(2)}{r'_n\, P_R\, \varepsilon_n \cdot}$$

$n = 1, 2, 3 \qquad i,j = 1 \ldots 6$

Pati-Salam

$e_7\Psi$ $\Downarrow$

so(6) su(2) su(2)

$$r''_{ij}\, e_i(e_j\,\cdot) \;+\; r_n P_L \varepsilon_n \cdot \;+\; r'_n P_R \varepsilon_n \cdot$$

$n = 1, 2, 3$ $i,j = 1 \ldots 6$

Ψe_7 $\Downarrow$

u(3) su(2) su(2)

$$r'''_{ij}\, e_i(e_j\,\cdot) \;+\; r_n P_L \varepsilon_n \cdot \;+\; r'_n P_R \varepsilon_n \cdot$$

$n = 1, 2, 3$ $i,j = 1 \ldots 6$

Ψe_7 $\Downarrow$

LR symmetric

$$\overset{u(3)}{r'''_{ij}\, e_i\,(e_j\,\cdot)} \;+\; \overset{su(2)}{r_n\, P_L\, \varepsilon_n\,\cdot} \;+\; \overset{su(2)}{r'_n\, P_R\, \varepsilon_n\,\cdot}$$

$n = 1, 2, 3 \qquad i,j = 1 \ldots 6$

Ψe_7

LR symmetric

$$\overset{u(3)}{r_{ij}''' e_i (e_j \cdot)} \quad + \quad \overset{su(2)}{r_n P_L \varepsilon_n \cdot} \quad + \quad \overset{su(2)}{r_n' P_R \varepsilon_n \cdot}$$

$n = 1, 2, 3 \qquad i,j = 1 \ldots 6$

$\varepsilon_3 \Psi_R$

$\Psi e_7 \quad \Downarrow$

LR symmetric

$$\overset{u(3)}{r'''_{ij}\, e_i(\, e_j \cdot)} \;+\; \overset{su(2)}{r_n\, P_L\, \varepsilon_n \cdot} \;+\; \overset{su(2)}{r'_n\, P_R\, \varepsilon_n \cdot}$$

$n = 1, 2, 3 \qquad i,j = 1 \ldots 6$

$\varepsilon_3 \Psi_R \quad \Downarrow$

$$\overset{u(3)}{r'''_{ij}\, e_i(\, e_j \cdot)} \;+\; \overset{su(2)}{r_n\, P_L\, \varepsilon_n \cdot} \;+\; \overset{u(1)}{u(1)_Y \cdot}$$

$n = 1, 2, 3 \qquad i,j = 1 \ldots 6$

$$\varepsilon_3 \Psi_R \quad \Downarrow$$

Standard model
+ B-L

$$\overset{u(3)}{r_{ij}^{\prime\prime\prime} e_i (e_j \cdot)} \;+\; \overset{su(2)}{r_n P_L \varepsilon_n \cdot} \;+\; \overset{u(1)}{u(1)_Y \cdot}$$

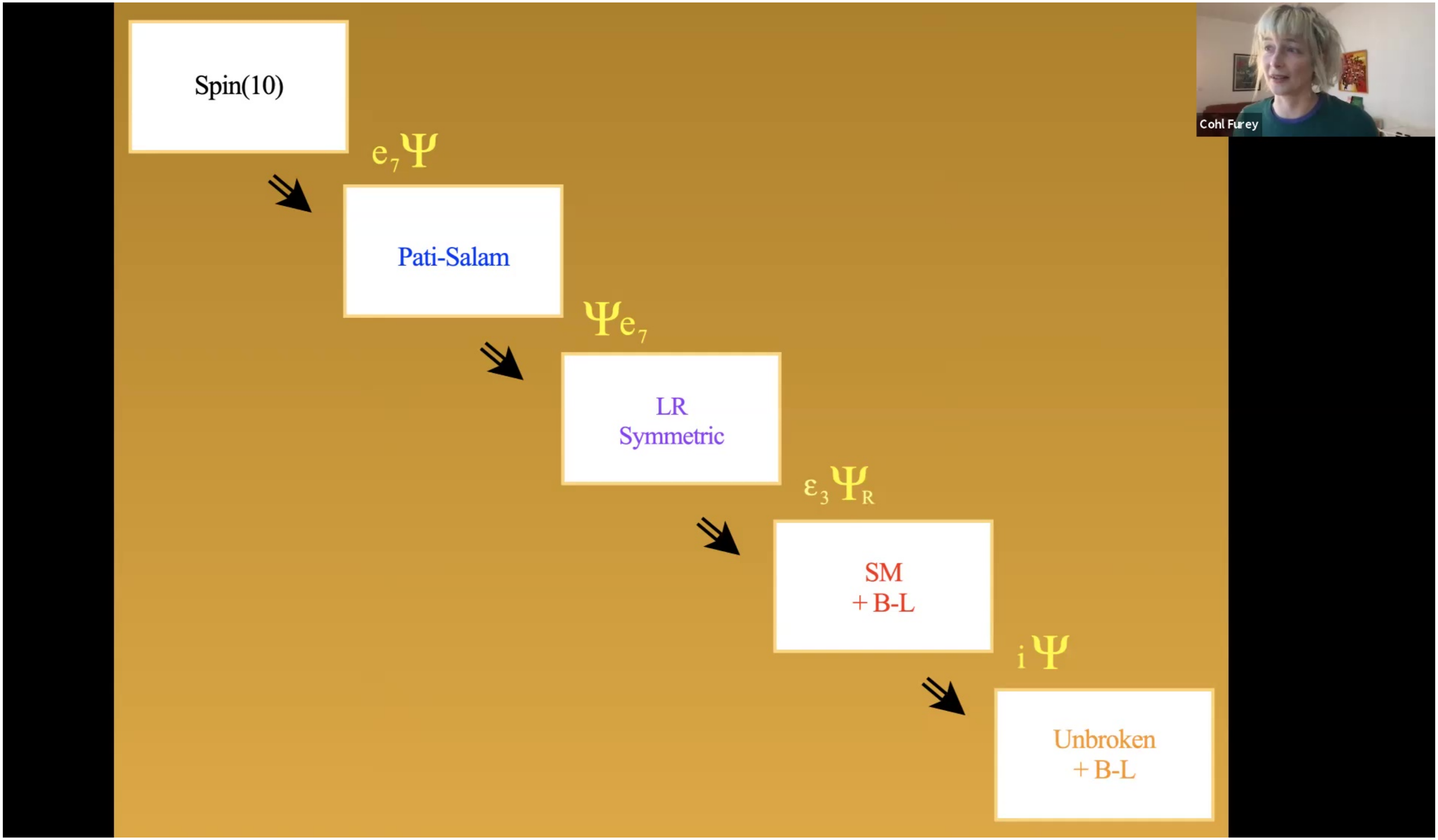
$n = 1, 2, 3 \qquad i,j = 1 \ldots 6$

$$i\,\Psi \quad \Downarrow$$

Unbroken
+ B-L

$$\overset{u(3)}{r_{ij}^{\prime\prime\prime} e_i (e_j \cdot)} \;+\; \overset{u(1)}{u(1)_{EM} \cdot}$$

$i,j = 1 \ldots 6$


Spin(10)
$e_7\Psi$
Pati-Salam
Ψe_7
LR
Symmetric
$\varepsilon_3\Psi_R$
SM
+ B-L
$i\Psi$
Unbroken
+ B-L
Cohl Furey

Summary

One generation

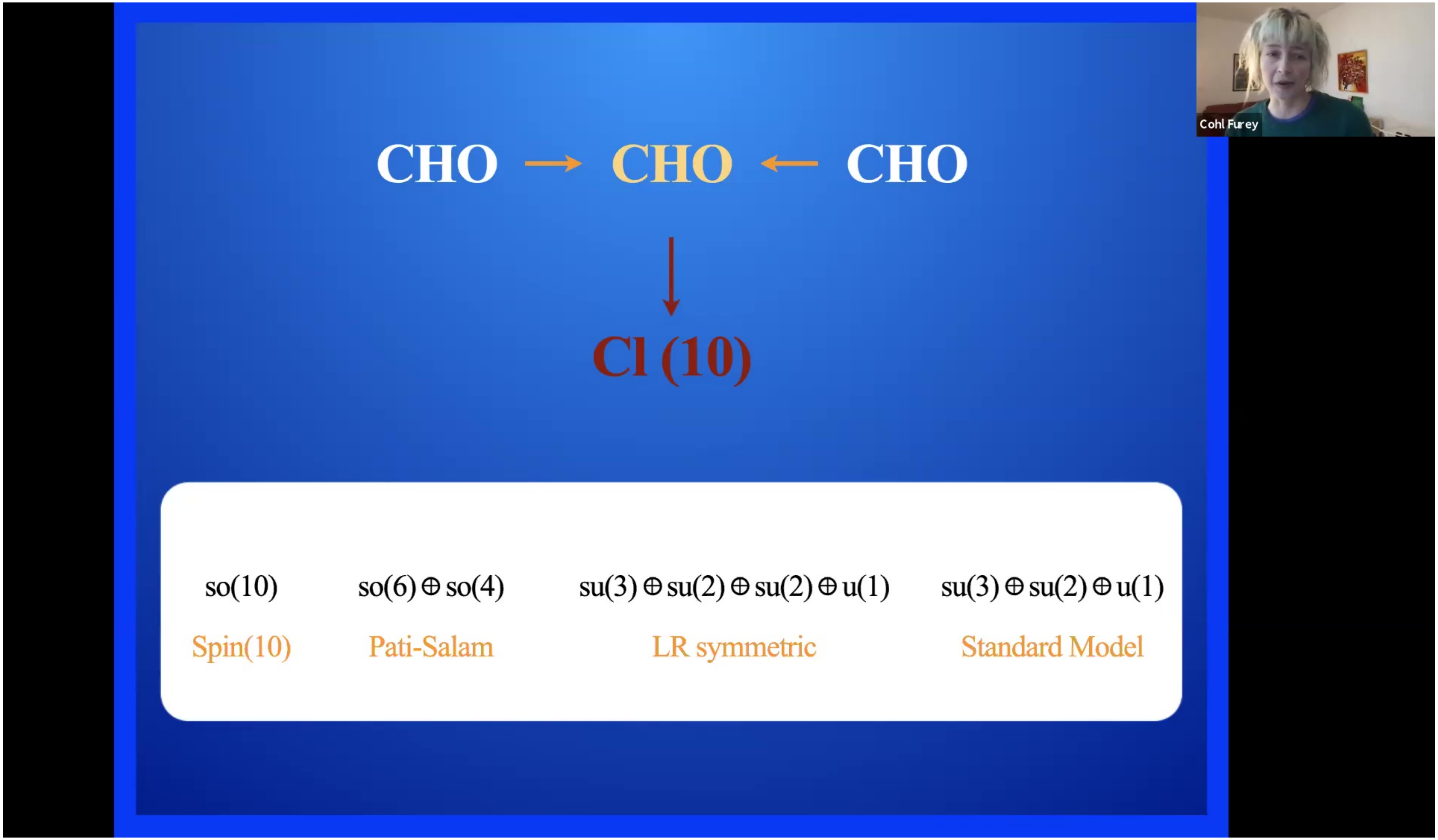
$$\boxed{64\,\mathbb{R}}$$

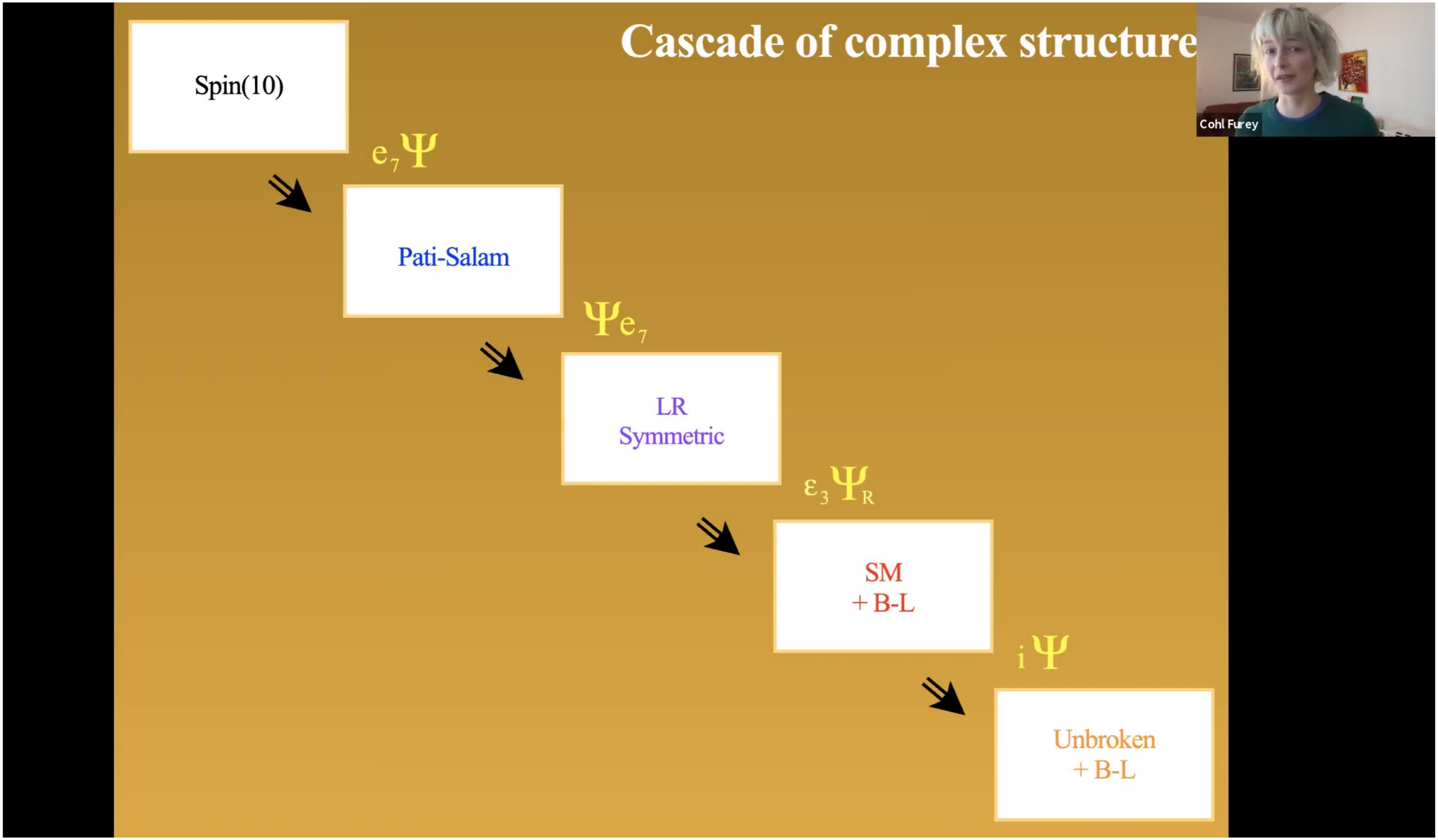
$$\updownarrow$$

$$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$$

1 2 4 8

$$\boxed{64\,\mathbb{R}}$$


Cohl Furey
CHO → CHO ← CHO
Cl (10)
so(10)
Spin(10)
so(6) ⊕ so(4)
Pati-Salam
su(3) ⊕ su(2) ⊕ su(2) ⊕ u(1)
LR symmetric
su(3) ⊕ su(2) ⊕ u(1)
Standard Model


Cascade of complex structure
Spin(10)
$e_7 \Psi$
Pati-Salam
Ψe_7
LR
Symmetric
$\varepsilon_3 \Psi_R$
SM
+ B-L
$i \Psi$
Unbroken
+ B-L
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