

Title: Einstein's Equivalence principle for superpositions of gravitational fields

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Series: Quantum Gravity

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Abstract: The Principle of Equivalence, stating that all laws of physics take their special-relativistic form in any local inertial frame, lies at the core of General Relativity. Because of its fundamental status, this principle could be a very powerful guide in formulating physical laws at regimes where both gravitational and quantum effects are relevant. However, its formulation implicitly presupposes that reference frames are abstracted from classical systems (rods and clocks) and that the spacetime background is well defined. Here, we generalise the Einstein Equivalence Principle to quantum reference frames (QRFs) and to superpositions of spacetimes. We build a unitary transformation to the QRF of a quantum system in curved spacetime, and in a superposition thereof. In both cases, a QRF can be found such that the metric looks locally flat. Hence, one cannot distinguish, with a local measurement, if the spacetime is flat or curved, or in a superposition of such spacetimes. This transformation identifies a Quantum Local Inertial Frame. These results extend the Principle of Equivalence to QRFs in a superposition of gravitational fields. Verifying this principle may pave a fruitful path to establishing solid conceptual grounds for a future theory of quantum gravity.

# Einstein's Equivalence Principle for superpositions of gravitational fields

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F. Giacomini, Č. Brukner, arXiv:2012.13754 (2020)

# Einstein Equivalence Principle

Fundamental principle  
of the theory

Experimental tool

*Metric structure of GR*

?

*Alternative theories*

*Gravitating quantum systems?\**



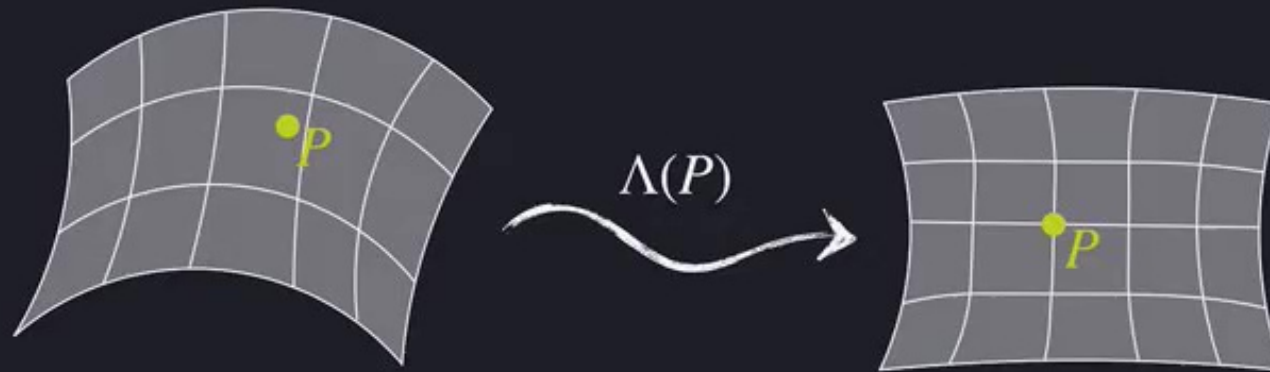
Lack of classical spacetime

**Validity**  
*Elements to retain*

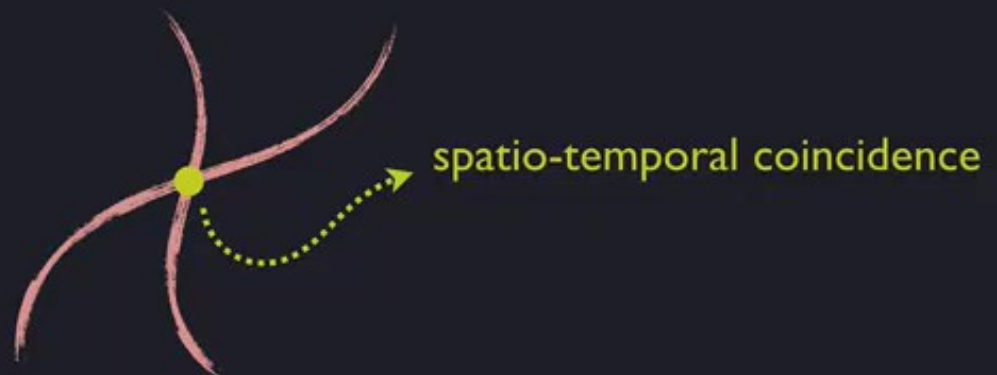
*Does the EEP hold?*

**Violations**  
*Elements to modify*

# Einstein Equivalence Principle



Principle of Relativity  
(implies General Covariance)



# Locally Inertial Frames

$$x^\mu = x_P^\mu + \left. \frac{\partial x^\mu}{\partial \xi^\alpha} \right|_{x_P} \xi^\alpha + \dots \quad \text{Linear term}$$

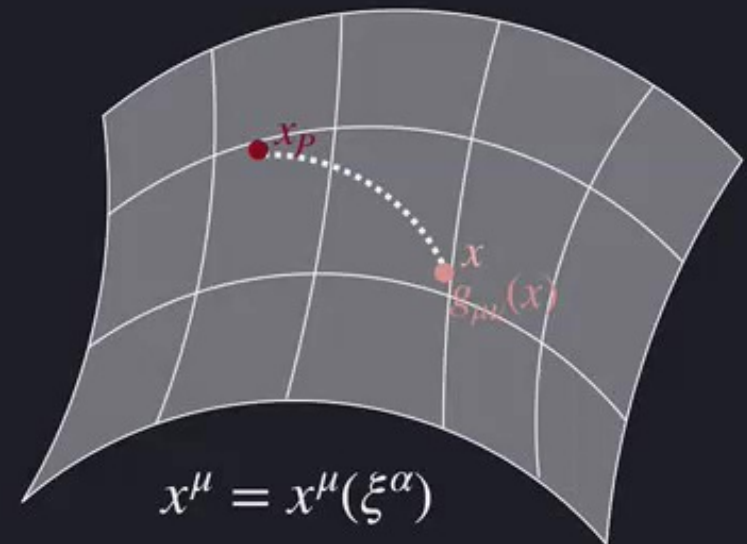
$$f_\alpha^\mu = \left. \frac{\partial x^\mu}{\partial \xi^\alpha} \right|_{x_P} \quad \text{16 free parameters}$$

$$\tilde{g}_{\alpha\beta}(\xi) = \frac{\partial x^\mu}{\partial \xi^\alpha} \frac{\partial x^\nu}{\partial \xi^\beta} g_{\mu\nu}(x_P + f\xi)$$

$$\tilde{g}_{\alpha\beta}(0) = f_\alpha^\mu f_\beta^\nu g_{\mu\nu}(x_P) \quad \text{10 quantities to fix}$$

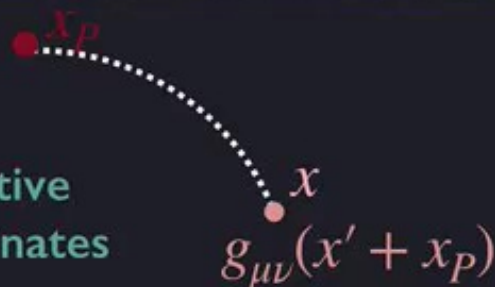
$$\tilde{g}_{\alpha\beta}(\xi) = \eta_{\alpha\beta} + O(\xi^2)$$

Locally Minkowski!



Step 1

Relative  
coordinates



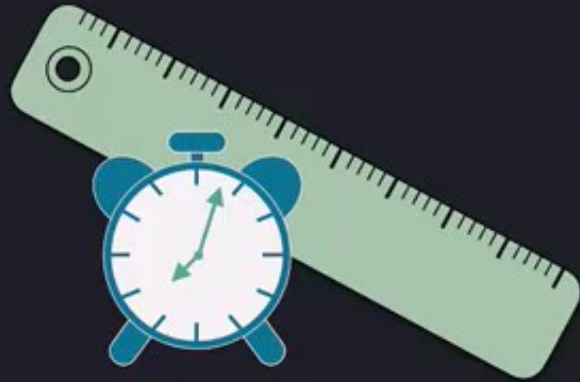
Step 2

“Straighten” the  
metric at one point





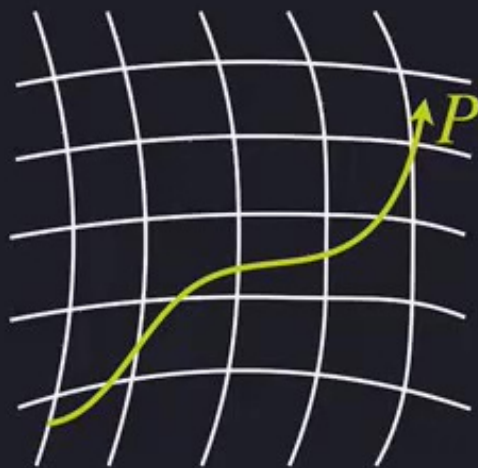
# What are reference frames?



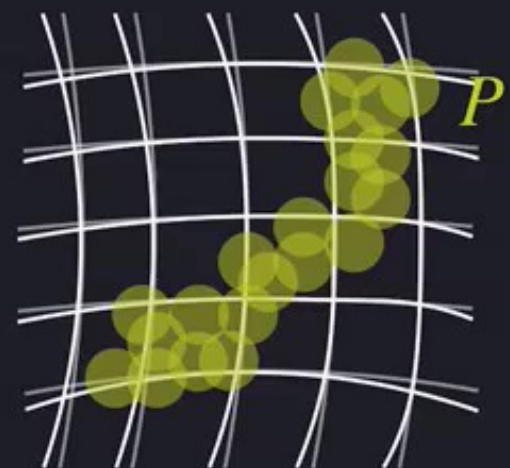
*Operational view of reference frames*

*Rods and clocks*

*Quantum systems: superposition, entanglement*



*Classical system on  
classical spacetime*



*Quantum system on  
superposition of spacetimes*

# What are reference frames?

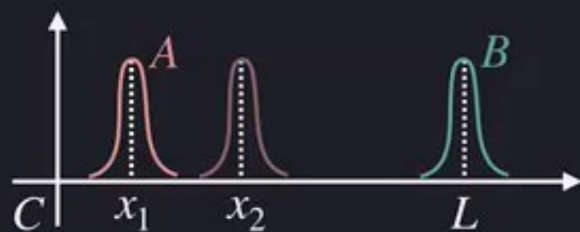


*Operational view of reference frames*

*Rods and clocks*

*Quantum systems: superposition, entanglement*

## Quantum Reference Frames



*Giacomini, Castro Ruiz, Brukner, Nat. Commun. (2019)*

*classical spacetime*

*superposition of spacetimes*

*Adapted from Misner, Thorne, Wheeler "Gravitation"*

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*In any and every **Quantum** Locally Inertial Frame (QLIF), anywhere and anytime in the universe, all the (nongravitational) laws of physics must take on their familiar non-relativistic form.*

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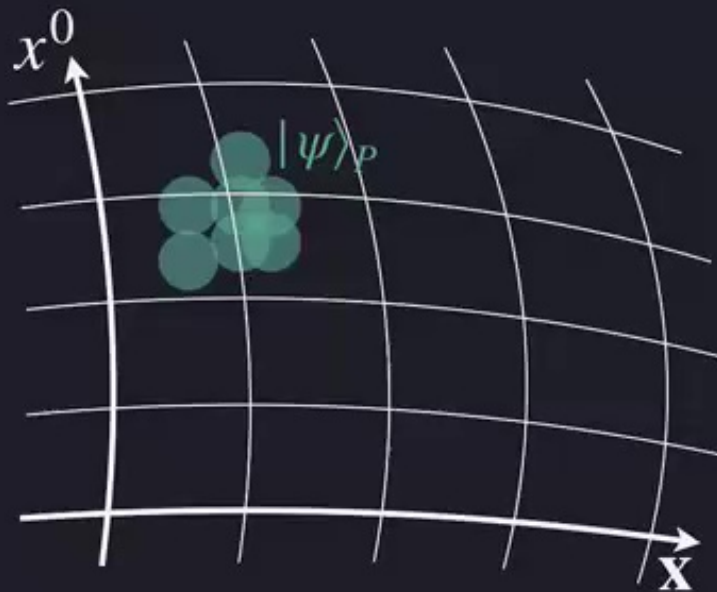
*Gravitational field  $G$*

*Test particle  $M$  "surveying" the gravitational field*

*Particle  $P$  serving as a Quantum Reference Frame*

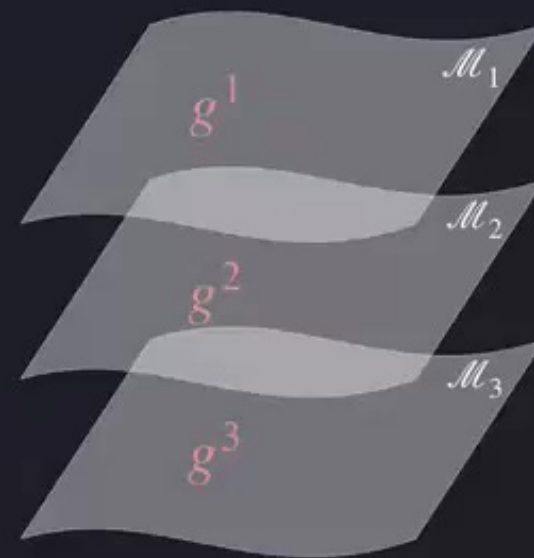


# Generalisation of the EEP



Quantum system in a superposition in spacetime

$$|\psi\rangle_P = \int d^4x \sqrt{-g(x)} \psi(x) |x\rangle_P$$



Quantum superposition of classical spacetimes

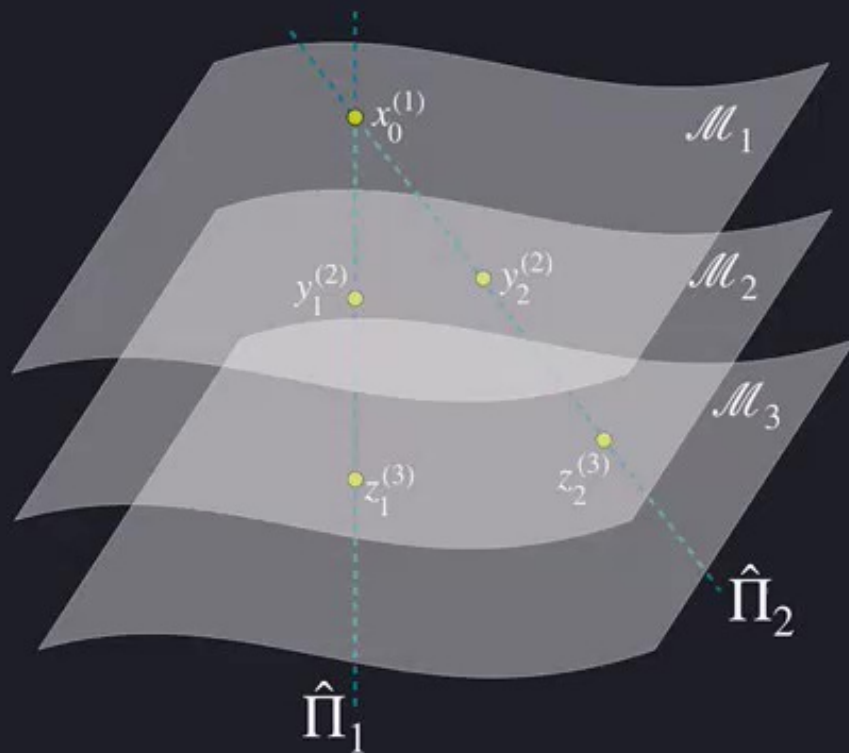
$$|g^1\rangle \quad |g^2\rangle \quad |g^3\rangle$$

Macroscopically distinguishable

Superposition principle holds

# Identification across spacetimes

$$|\phi \triangleright g\rangle = \sum_i |\phi_i \triangleright g_i\rangle$$

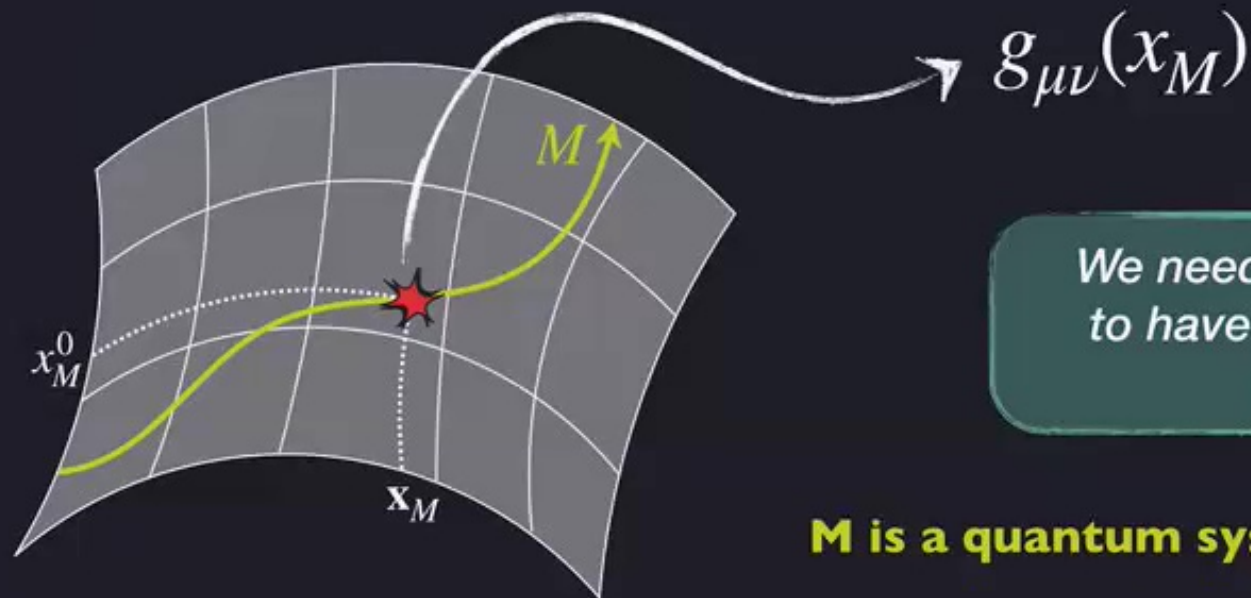


$$\hat{\Pi}_1 |\phi \triangleright \phi\rangle \longrightarrow \{x_0^{(1)}, y_1^{(2)}, z_1^{(3)}\}$$

$$\hat{\Pi}_2 |\phi \triangleright \phi\rangle \longrightarrow \{x_0^{(1)}, y_2^{(2)}, z_2^{(3)}\}$$

*Different identifications correspond to different measurement choices.*

# Physical coincidences



Coordinates of  $M$

$$x_M = (x_M^0, \mathbf{x}_M)$$

*We need a physical system  
to have information about  
the metric*

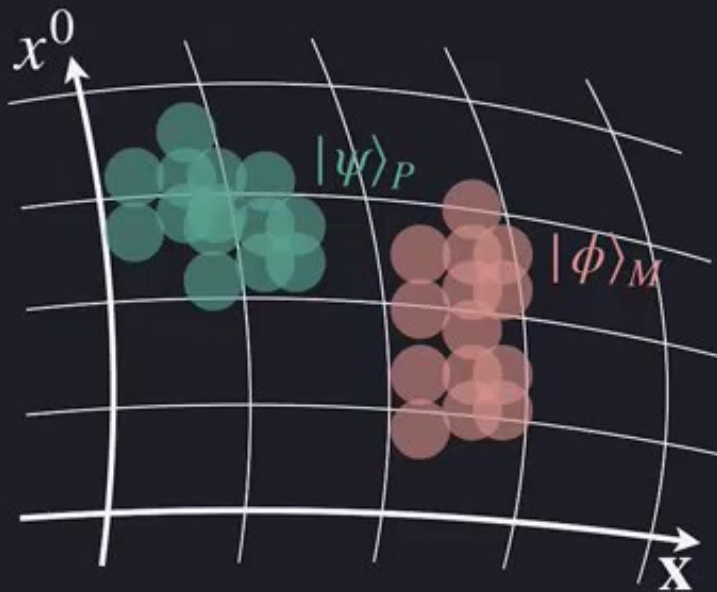
**$M$  is a quantum system in spacetime:  $|\phi\rangle_M$**

$$|\phi \triangleright g\rangle = \int d^4x_M \sqrt{-g(x_M)} \phi(x_M) |g(x_M)\rangle |x_M\rangle$$

The metric field “correlates” with the coordinates of  $M$

# Von Neumann-like measurement

*M probes the metric field anywhere it has support*



What is the metric field at  $P$ ?

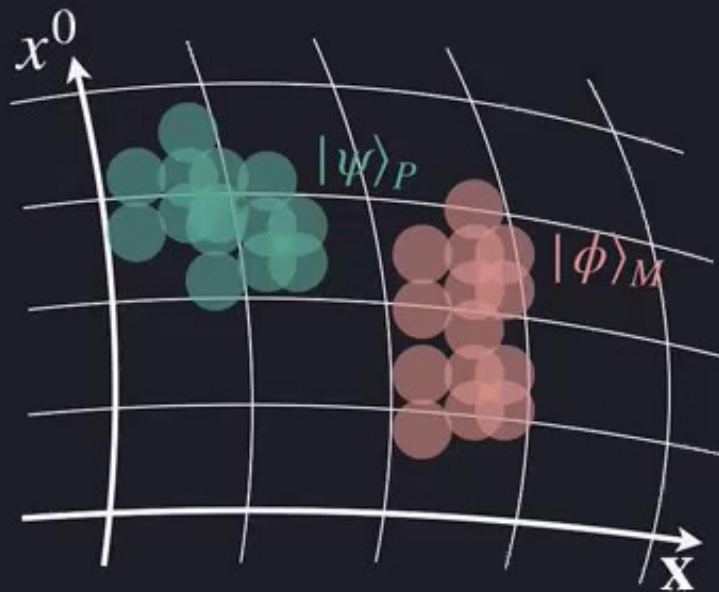
How do we know where  $P$  is?

Quantum Reference Frames





# Von Neumann-like measurement



*What is the metric field at P?*

$$|\psi; \phi \triangleright g\rangle = \sum_i |\psi_i\rangle_P |\phi_i \triangleright g_i\rangle$$

$$e^{\frac{i}{\hbar} \hat{x}_P \hat{p}_M} |x'\rangle_P |x\rangle_M = |x'\rangle_P |x - x'\rangle_M$$

$${}_M\langle 0 | e^{\frac{i}{\hbar} \hat{x}_P \hat{p}_M} |\psi; \phi \triangleright g\rangle$$

$$\sum_i \int d^4 x_i \sqrt{-g^i(x_i)} \psi_i(x_i) \phi_i(x_i) |g_i(x_i)\rangle |x_i\rangle_P$$





# EEP for Quantum Reference Frames

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*QRF as particle  $P$*

*What is a  
quantum state in  
spacetime?*

Covariant representation

*How to identify points  
across different  
spacetimes?*

Projection operation and  
notion of “physical point”

*Test particle  $M$  correlated with  $G$*

*How to write the state  
of a non-classical  
gravitational field?*

Superpositions of classical  
spacetimes

*What is the metric at  $P$ ?*

# EEP for Quantum Reference Frames

*Generalise Locally Inertial Frames transformations*

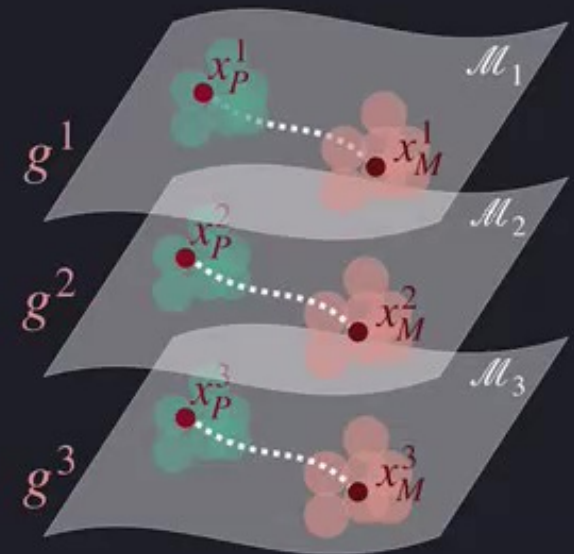
*Principle of linear superposition applied twice*

1. Within the same spacetime (positions)
2. Across different spacetimes (metrics)

$$(f^{(i)})^\mu_\alpha = \left. \frac{\partial x^\mu}{\partial \xi^\alpha} \right|_{x_P}^{(i)} \quad \text{16 free parameters}$$

$$\tilde{g}^i_{\alpha\beta}(\xi) = \left( \frac{\partial x^\mu}{\partial \xi^\alpha} \right)^{(i)} \left( \frac{\partial x^\nu}{\partial \xi^\beta} \right)^{(i)} g^i_{\mu\nu}(x_P^i + f^i \xi^i)$$

$$\tilde{g}^i_{\alpha\beta}(0) = (f^{(i)})^\mu_\alpha (f^{(i)})^\nu_\beta g^i_{\mu\nu}(x_P^i) \quad \text{10 quantities}$$



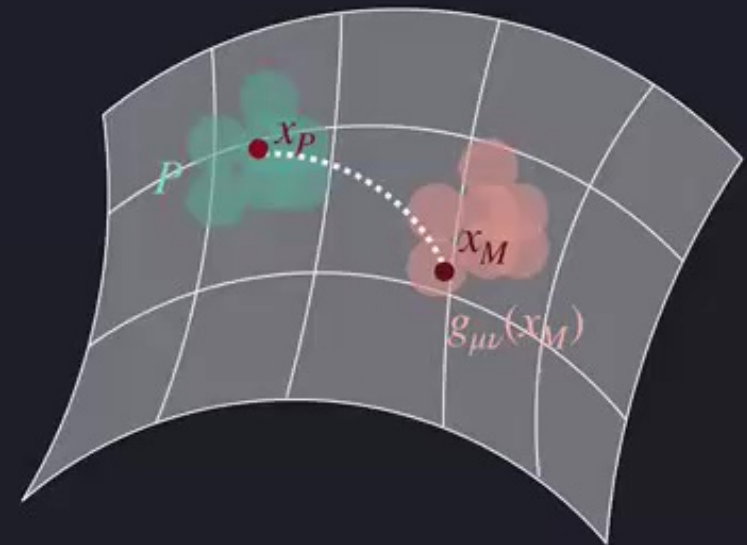
*At each point  $x_P$   
In each spacetime  $g^i$*

# Transformation to a QLIF

*Classical and curved spacetime*

*Different transformation at each point  $x_P$*

1. Centre origin in  $x_P$
2. Straigten coordinates around  $x_P$



$$|\psi; \phi \triangleright g\rangle = \int d^4x_P \sqrt{-g(x_M)} \psi(x_P) |x_P\rangle_P \int d^4x_M \sqrt{-g(x_M)} \phi(x_M) |g(x_M)\rangle |x_M\rangle$$

Unitary!  $\hat{S}^i$

$$\xi = f(x_P)(x_M - x_P) + \dots$$

Quantum-controlled (P)  
transformation on M.

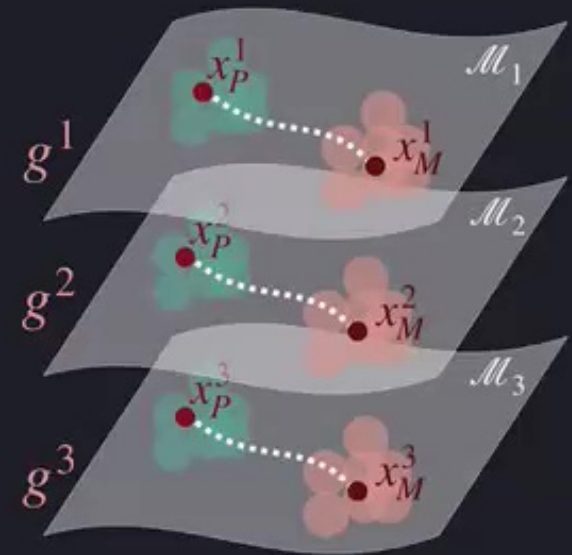


# Transformation to a QLIF

*Superposition of classical spacetimes*

*Different transformation at each point  $x_P$  and for each  $g^i$*

1. Centre origin in  $x_P^i$  for each  $g^i$
2. Straigten coordinates around  $x_P^i$  for each  $g^i$



$$|\psi; \phi \triangleright g\rangle = \sum_{i=1}^N \int d^4 x_P^i \sqrt{-g^i(x_M^i)} \psi_i(x_P^i) |x_P^i\rangle_P \int d^4 x_M^i \sqrt{-g^i(x_M^i)} \phi_i(x_M^i) |g^i(x_M^i)\rangle |x_M^i\rangle$$

Unitary!  $\hat{S}$

$\xi^i = f^i(x_P^i, i)(x_M^i - x_P^i) + \dots$  Quantum-controlled (P, G) transformation on M.



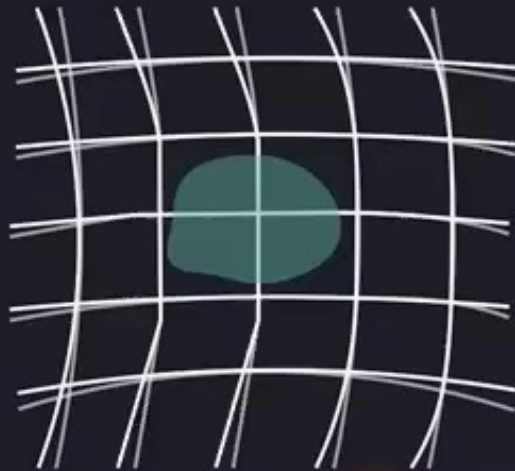
# Metric in the QLIF

*State in the Quantum Locally Inertial Frame (QLIF)*

$$\hat{S}|\psi; \phi \triangleright g\rangle$$

*Probe the metric at origin of QLIF*

$${}_M\langle 0 | e^{\frac{i}{\hbar} \hat{x}_P \hat{p}_M} \hat{S} |\psi; \phi \triangleright g\rangle$$



$$\tilde{g}_{\alpha\beta}^i(\xi) = \eta_{\alpha\beta} + O(\xi^2)$$

Locally Minkowski  
for all  $i$ 's!



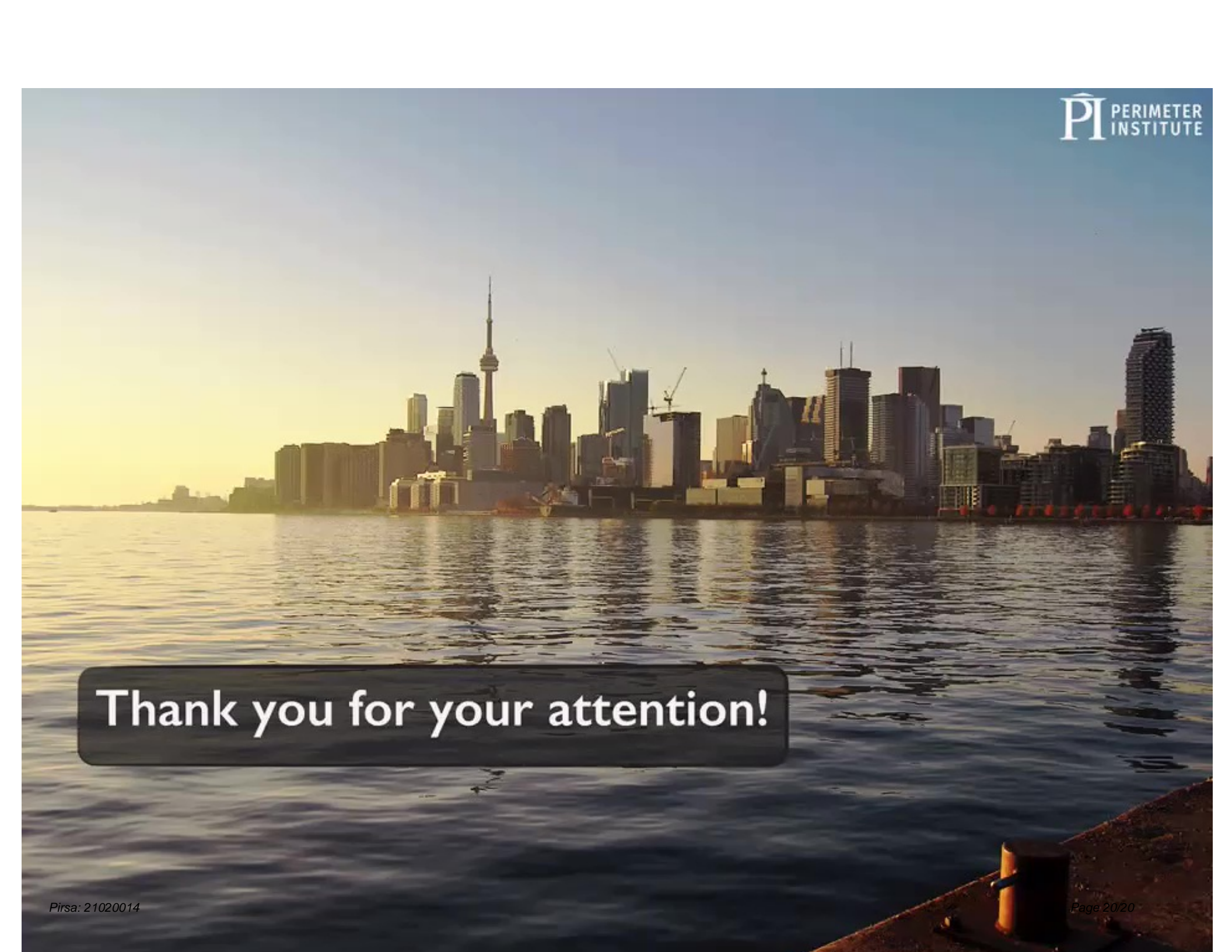
# Conclusions

*Generalisation of Quantum Reference Frames to curved spacetimes and superposition of spacetimes.*

*Introduction of “Quantum Locally Inertial Frames”*

*For every quantum state of a system  $P$  living in a superposition of classical spacetimes, one can find a Quantum Locally Inertial Frame transformation to the Quantum Reference Frame of  $P$  such that the metric is locally Minkowski at the origin of the Quantum Reference Frame.*

*Not covered: Freely-falling quantum systems*

A wide-angle photograph of the Toronto skyline across Lake Ontario at sunset. The sun is low on the horizon to the left, casting a warm orange glow over the water and the city. The skyline includes the CN Tower and various skyscrapers. The water in the foreground shows gentle ripples and reflects the city lights and the sunset. A dark, rounded rectangular box is superimposed over the lower part of the image, containing the text 'Thank you for your attention!'. In the bottom right corner, a portion of a ship's hull and a metal fitting are visible.

Thank you for your attention!