Title: Gravity Gradient Noise from Asteroids

Speakers: Michael Fedderke

Series: Particle Physics

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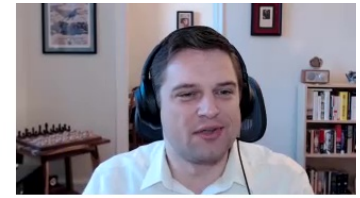
Abstract: The gravitational coupling of nearby massive bodies to test masses in a gravitational wave (GW) detector cannot be shielded, and gives rise to 'gravity gradient noiseâ€™ (GGN) in the detector. In this talk, I will discuss how any GW detector using local test masses in the Inner Solar System is subject to GGN from the motion of the field of 10^5 Inner Solar System asteroids, which presents an irreducible noise floor for the detection of GW that rises exponentially at low frequencies. This severely limits prospects for GW detection using local test masses for frequencies below (few) x 10^{-7} Hz. At higher frequencies, Iâ€™ll show that the asteroid GGN falls rapidly enough that detection may be possible; however, the incompleteness of existing asteroid catalogs with regard to small bodies makes this an open question around microHz frequencies, and Iâ€™ll outline why further study is warranted here. Additionally, Iâ€™ll mention some prospects for GW detection in the ~10 nHz-microHz band that could evade this noise source.



Gravity Gradient Noise from Asteroids

Perimeter Institute for Theoretical Physics
February 5, 2021
Remote seminar

Based on 2011.13833 (submitted to PRD)
with P.W. Graham and S. Rajendran

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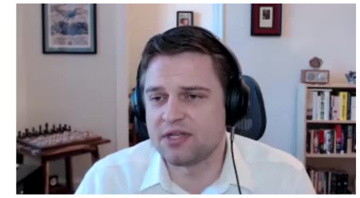
1

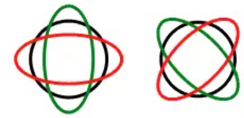


Gravitational Waves

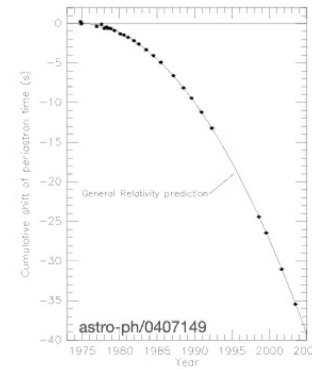
- Time-dependent quadrupolar mass distribution radiate metric disturbances as GW (e.g., compact objects mergers, phase transitions, etc.).

$$h \sim G_N \ddot{Q}/r$$

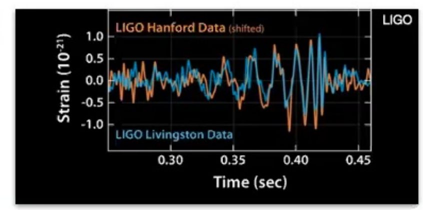
- GW cause a ring of test masses to oscillate in characteristic + or x patterns:



- Predicted by Einstein (1916).
- Indirect evidence from decay of Hulse-Taylor binary orbit (1974).

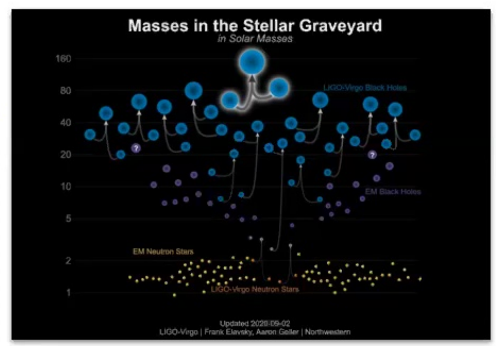


- Long history of efforts to directly detect GW (Weber bar, 1960s).
- First direct detection by LIGO Collaboration on 14 September 2015 of two $\sim 30\,M_\odot$ black holes [GW150914].



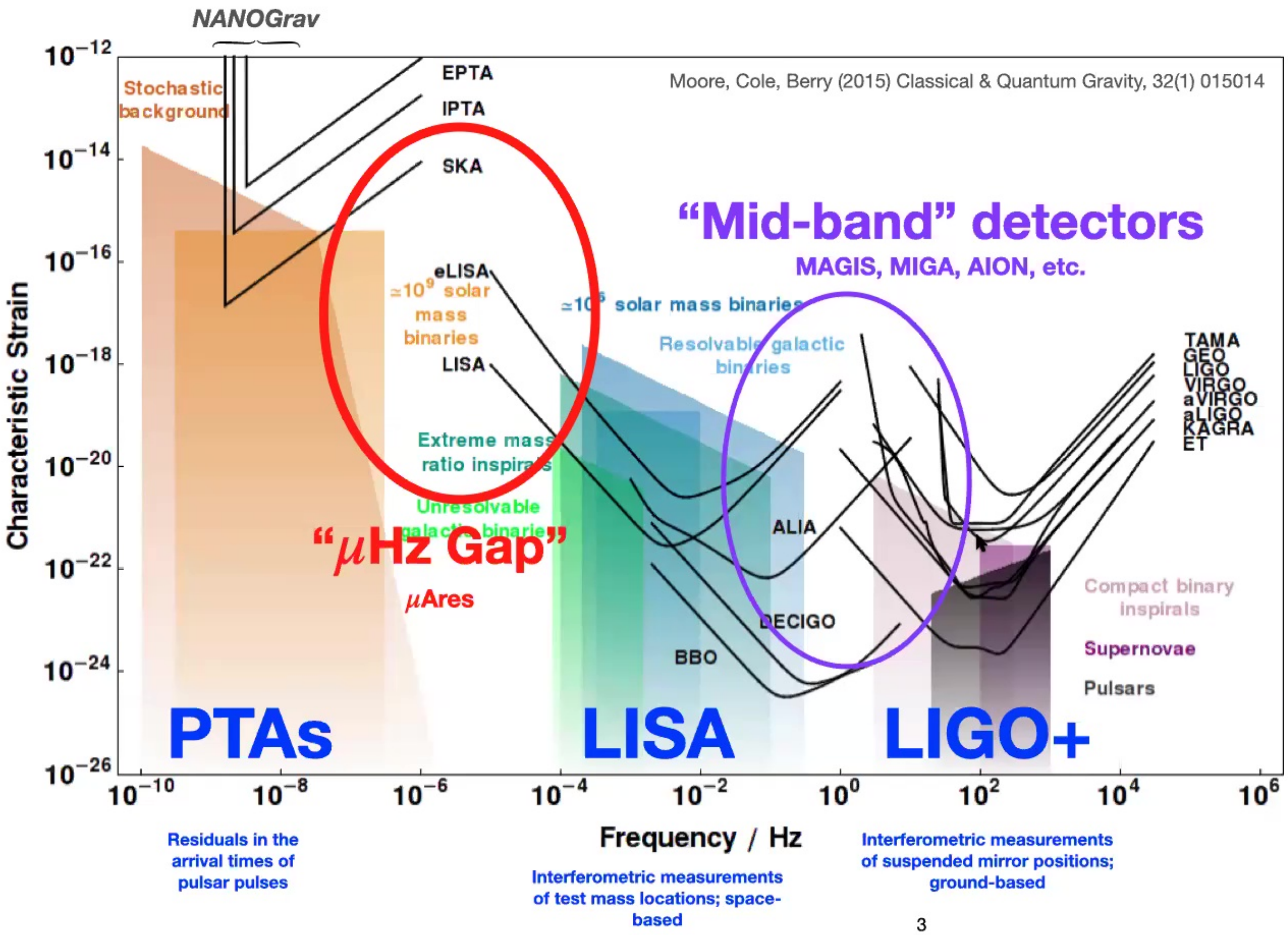
STATISTICS TIP: ALWAYS TRY TO GET DATA THAT'S GOOD ENOUGH THAT YOU DON'T NEED TO DO STATISTICS ON IT
xkcd

- ~ 50 merger detections by LIGO-Virgo, incl. a binary neutron star merger with EM signature [GW170817].



- Recent results from NANOGrav at very low frequency are a tantalising hint of what might become a signal of a stochastic GW background (jury still out!).
  [Z. Arzoumanian *et al* 2020 *ApJL* **905** L34]

GW Detection Landscape



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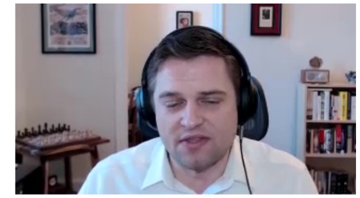
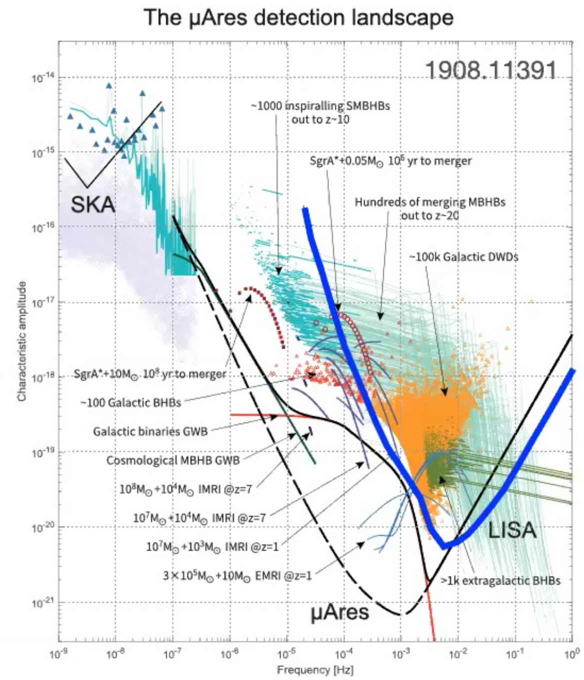
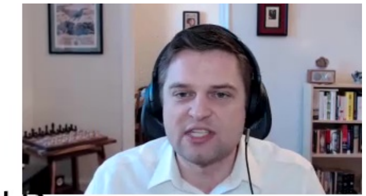
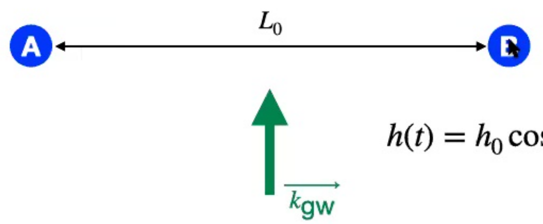




Local-TM—based GW Detection 101



- Measure the light travel time = proper distance between two or more test masses (TM) A and B (exact implementations differ: laser / atomic interferometry, pulse timing, etc.)



$$h(t) = h_0 \cos(\omega_{gw} t)$$

- Emitter (A) sends pulse at $t_A = t_0$; receiver (B) gets pulse at $t_B = t_0 + \Delta t$:

$$\Delta t = L_0 \left(1 - \frac{h_0}{2} \operatorname{sinc}(\omega_{gw} L_0/2) \cos[\omega_{gw}(t_0 + L_0/2)] \right) + \mathcal{O}(h_0^2) \xrightarrow{\omega_{gw} L_0 \ll 1} L_0 \left(1 - \frac{h_0}{2} \cos[\omega_{gw} t_0] \right) + \mathcal{O}(h_0^2)$$

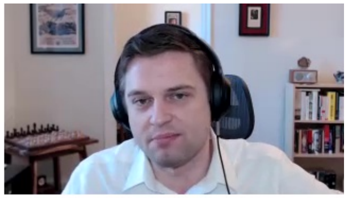
- Interpret this arrival timing offset as a strain h_L:

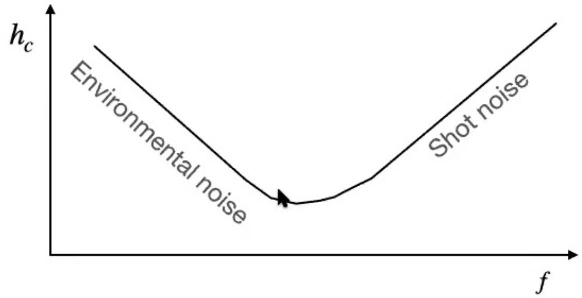
$$h_L = \frac{\Delta t - L_0}{L_0} \approx -\frac{h_0}{2} \cos[\omega_{gw} t_0]$$

- … or baseline-projected Newtonian acceleration a_L:

$$a_L = L_0 \partial_{t_0}^2 h_L(t) \approx \frac{h_0 \omega_{gw}^2 L_0}{2} \cos[\omega_{gw} t_0]$$

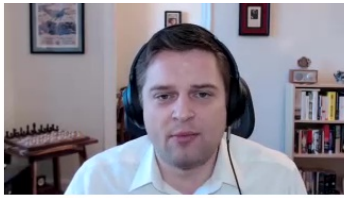
Typical Limitations / Dominant Noise



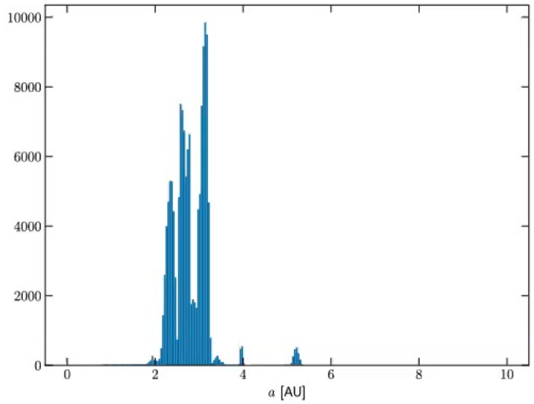


- High frequency: shot-noise / thermal noise [**"How well can the test mass (TM) position be measured?"**]
- Low frequency: environmental noise coupled to TMs [**"How well do the TMs stay where they should be?"**]
  - On the ground (e.g., LIGO-Virgo):
    - **Shield-able, to an extent:** mechanical seismic coupling to TM, etc.
    - **Un-shield-able:** gravitational seismic coupling to TM (gravity gradient noise [GGN] / Newtonian noise), etc.
  - In space @ mHz (e.g., LISA): residual gases in spacecraft, charging of TM, etc.
- Space-based local TM detection in the ~10 nHz to ~10 μHz range? Environmental noise?
- Observation: orbital motion of bodies in the Inner Solar System have frequencies in this range.
- **Question: Does the gravitational coupling of the orbital motion of Inner Solar System bodies cause in-band GGN for space-based TM when attempting GW detection?**

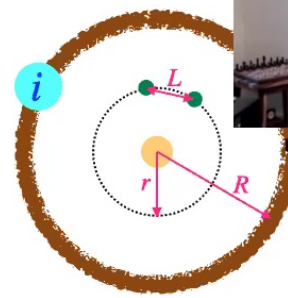
Possible sources of in-band GGN

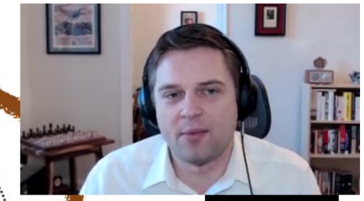


- The Sun, Planets (and Pluto ☹️), Moons
  - Relatively few (1 Sun, 8 planets + Pluto, ~200 moons)
  - Masses (or $G_N M$) and locations (ephemerides) known
  - TMs response can reasonably be modelled out.
  - Optimistic viewpoint: do **not** consider these to be noise.
- The Inner Solar System asteroids
  - $\mathcal{O}(10^6)$ objects with semi-major axes less than a few AU
  - Except for most massive objects (Ceres, Vega, ...), masses generally poorly and indirectly determined: radii mostly indirectly determined from their absolute magnitude and estimated reflectance.
  - Ephemerides are known to some extent, but they are noisy
  - **NOT** reasonable to assume that one can successfully fit out $\mathcal{O}(10^6)$ waveforms of poorly determined normalization and somewhat poorly determined shape from the TM response, and still be able to **robustly** claim detection of a GW.
  - This is noise!



Rough estimate





- 2 detectors at $r = 1\mathrm{AU}$ (circular orbit with ang. freq. Ω); baseline L
- N asteroids of mass M in co-rotating, co-planar circular orbits with ang. freq. ω at $R > r$
- Asteroids randomly distributed around the orbit.
- Relative orbital frequency $\varpi \equiv \Omega - \omega$
- Differential acceleration component projected onto the baseline, Δa_i due to each asteroid $i = 1,\ldots,N$ is **approximately**
  (see paper appendices for full expressions; this is formally wrong, but it captures qualitative features)

$$\Delta a_i \sim \frac{G_N M L}{\left[R^2 + r^2 - 2rR\cos(\varpi t + \alpha_i)\right]^{3/2}}; \qquad \alpha_i \sim \mathcal{U}[0,2\pi)$$

- For $R > r$, multipole expansion gives: $$\Delta a_i \sim \frac{G_N M L}{R^3}\left[1 + \sum_{j=1}^{\infty} c_j \left(\frac{r}{R}\right)^j \cos^j(\varpi t + \alpha_i)\right]$$
- Terms with $\omega' \sim q\varpi$ will appear at leading order from terms $\sim \cos^q(\varpi t + \alpha_i) \sim \cos[q(\varpi t + \alpha_i)] + \cdots$, with magnitude

$$\Delta a_i(\omega' \sim q\varpi) \propto \frac{G_N M L}{R^3}\left(\frac{r}{R}\right)^q$$

- Summing over asteroids gives, on average, $\langle \Delta a(\omega') \rangle \propto \dfrac{G_N M L \sqrt{N}}{R^3}\left(\dfrac{r}{R}\right)^{\omega'/\varpi}$

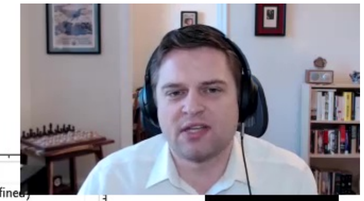
EXPONENTIAL IN FREQUENCY

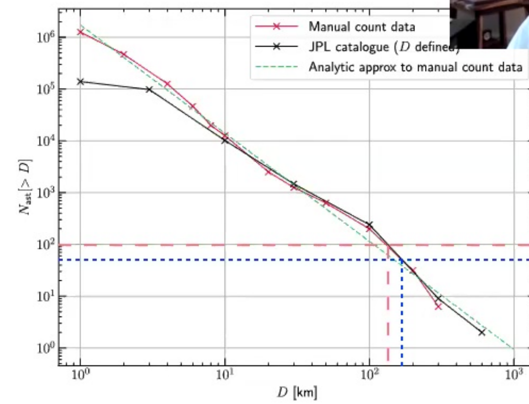
7

Rough estimate — contd.

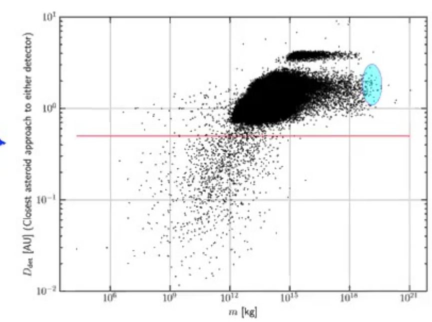
- Compare $\langle \Delta a(\omega') \rangle$ to monochromatic GW, $\Delta a(\omega') \sim h_c^{\rm ggn} L(\omega')^2$:

$$h_c^{\rm ggn}(\omega') \propto \frac{G_N M \sqrt{N}}{(\omega')^2 R^3} \left(\frac{r}{R} \right)^{\omega'/\varpi}$$





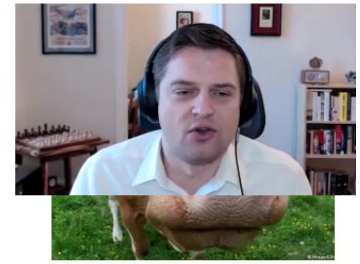
- Hard to remove $N \sim 50$ asteroids.
- 50 asteroids with $d \gtrsim 170\,\mathrm{km}$ [per JPL-SBD].
- $M(d) \sim 10^{19}\,\mathrm{kg}$ assuming $\rho \sim 3\mathrm{g/cm}^3$ (~ Ceres).
- In the bulk of the asteroid belt at $R \sim 3\mathrm{AU}$.



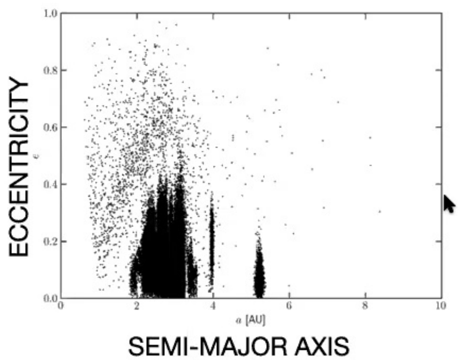
- Taking $f' \sim 1/\mathrm{year}$ gives $h_c^{\rm ggn} \sim 10^{-12}$.
- Strain sensitivity for interesting sources of $h_c \sim 10^{-15}$. **GGN noise too large!**
- At higher frequency $f' \sim 10/\mathrm{year}$, then $h_c^{\rm ggn} \sim 10^{-19}$ and required $h_c \sim 10^{-17}$. **GGN noise is OK.**
- **<u>PRELIMINARY</u> CONCLUSION: ASTEROID GGN IS A PROBLEM AT LOW FREQUENCIES, BUT DROPS RAPIDLY, SO HIGHER FREQUENCIES COULD BE OK.**

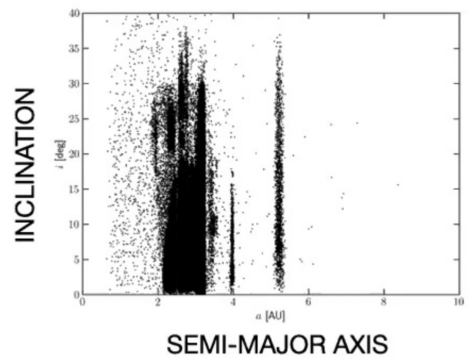
8

Cows aren't spherical

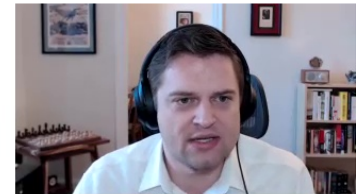


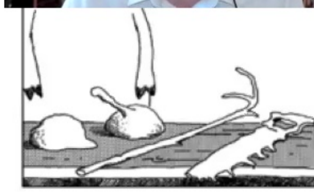
- Rough estimate naïve in many ways:
  - Ad hoc choice of N
  - Only took one M, estimated from ad hoc N
  - Asteroids were taken on radius-R circular, co-planar, co-rotating orbits
- Real asteroids have correlated distributions of orbital parameters (inclined, elliptical), masses





- N-body dynamics.
- Must capture *some* of this richness to give a reasonable estimate of the noise spectrum.




Cow tools

Simulation Setup

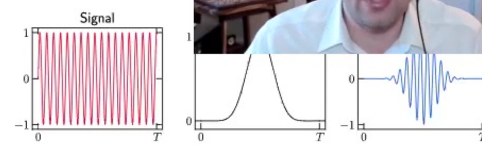
- Detectors A and B on same circular orbits at radius r and baseline L at locations $\vec{x}_I(t)$; $I = A, B$
- Asteroids: use **real asteroids in Solar System**.
  - JPL Small-Body Database (JPL-SBD).
  - Approximate approach based on 2-body dynamics (N-body unwarranted):
    - Asteroid locations $\vec{X}_i(t)$; $i = 1, \ldots, N$ assumed to be given by (osculating) elliptical orbits.
- Only $N = 140{,}299$ have a diameter estimate d_i.
  - Mass estimate $M_i(d_i) \sim \rho \pi d_i^3/6$ assuming spherical asteroids, uniform density $\rho \sim 2.5 \mathrm{g/cm}^3$.
  - Use only these objects $N = 140{,}299$ with masses $M_i(d_i)$ in our simulations.
- Compute baseline-projected differential accelerations $\Delta a_i(t_n)$ for each object $i = 1, \ldots, N$

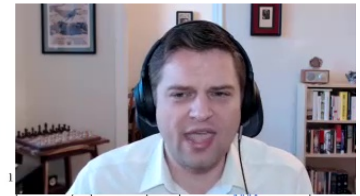
$$\vec{a}_{i,I}(t_n) = \frac{G_N M_i \left(\vec{X}_i(t_n) - \vec{x}_I(t_n) \right)}{|\vec{X}_i(t_n) - \vec{x}_I(t_n)|^3}; \; i = 1, \ldots, N; \; I = A, B; \qquad \Delta a_i(t_n) \equiv \left(\vec{a}_{i,A}(t_n) - \vec{a}_{i,B}(t_n) \right) \cdot \frac{\left(\vec{x}_A(t_n) - \vec{x}_B(t_n) \right)}{\left| \vec{x}_A(t_n) - \vec{x}_B(t_n) \right|}$$

- Sum over all asteroids: $\Delta a(t_n) = \sum_i \Delta a_i(t_n)$
- Simulate a mission duration of $T = 10\,\mathrm{yrs}$.
- $t_n = n\Delta T = nT/\mathcal{N}$ for $\mathcal{N} = 3000$. Sufficient frequency coverage: $2\,\mathrm{nHz} \lesssim f \lesssim 5\,\mu\mathrm{Hz}$.

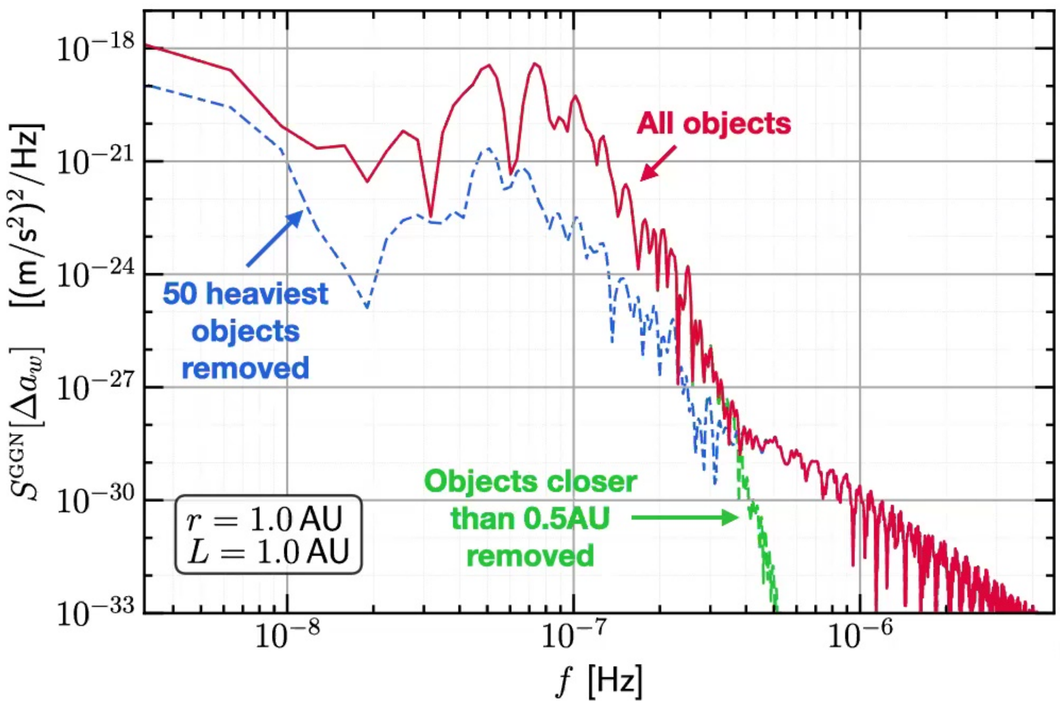
Windowed noise PSD

- Technical point: must window before taking the PSD (high dynamic range):
  $\Delta a_w(t_n) \equiv \Delta a(t_n) \times w(t_n); \qquad w(t) \equiv \sin^8(\pi t/T)$

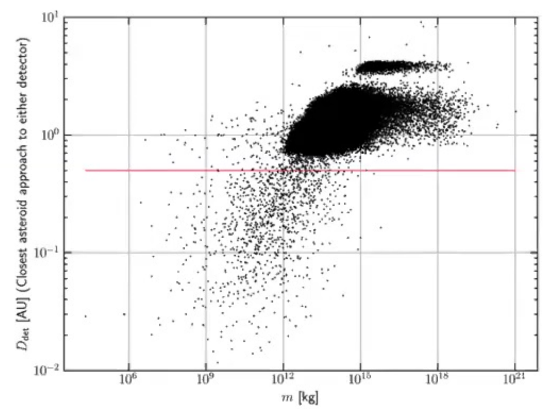




- Take the FFT and get the asteroid GGN noise PSD (windowed)



- Acceleration; want strain
- Windowed: impacts normalization
- Frequency here is **NOT** the GW frequency: modulation by the rotating baseline!



(See paper for more parameter points L, r)

11

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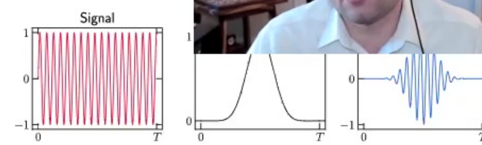

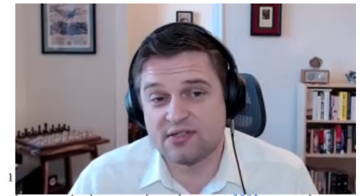

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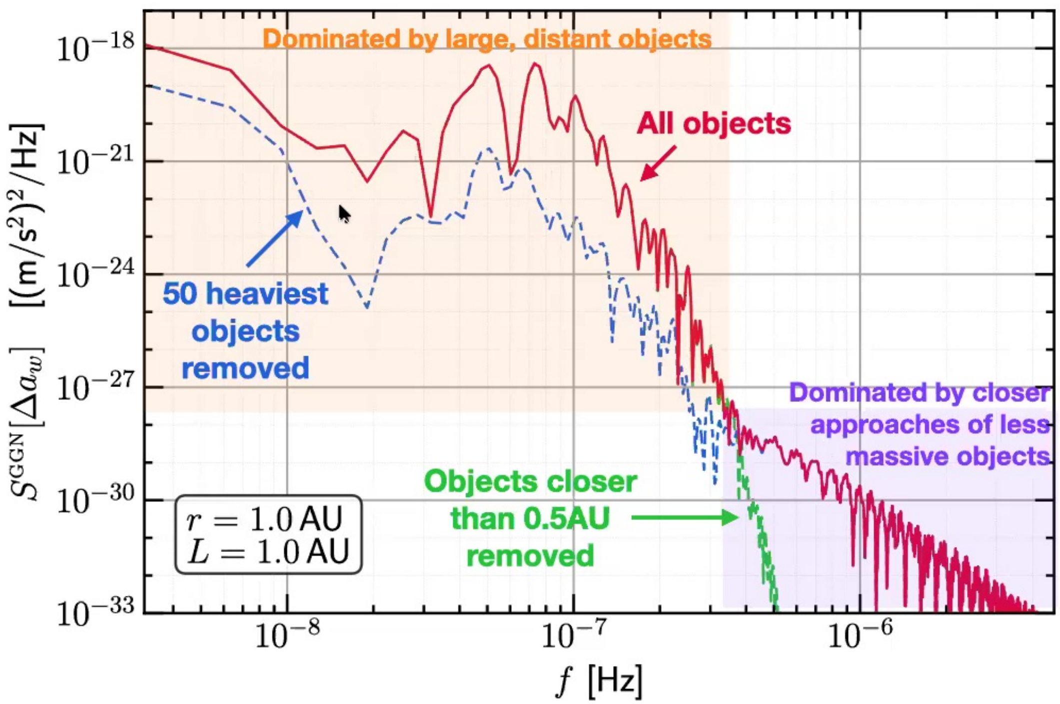

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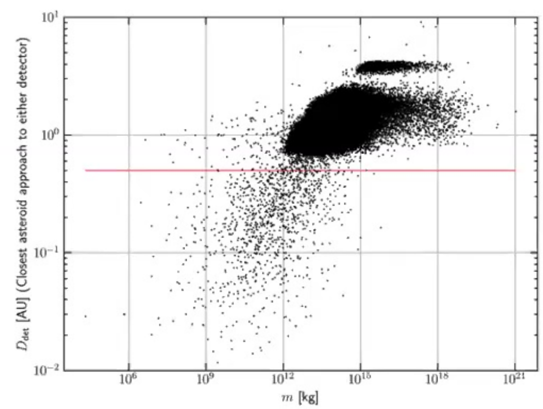

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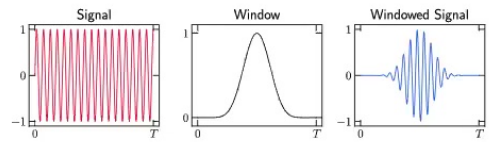

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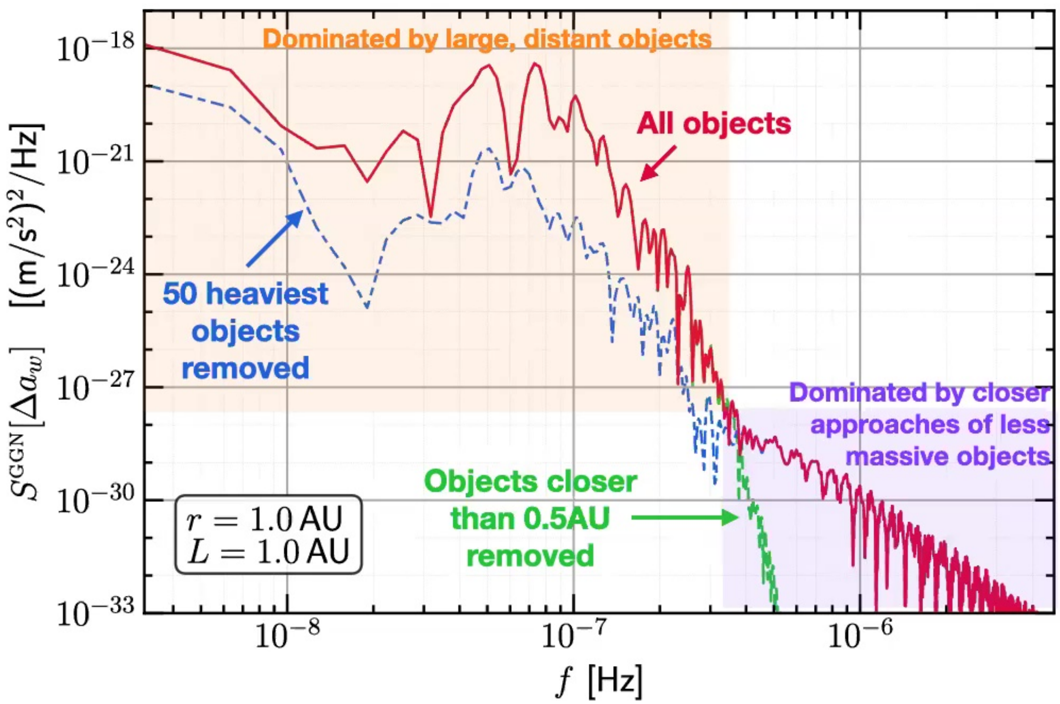

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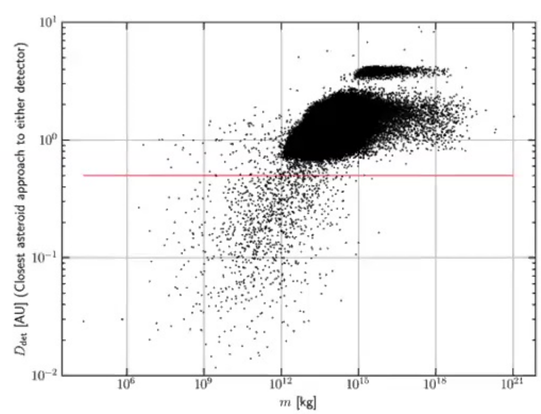
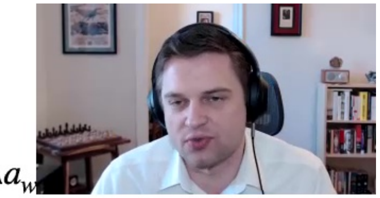


(See paper for more parameter points L, r)

11

Implications for GW detection - I



- Have net baseline-projected differential acceleration noise curve after applying a particular window: $S^{\rm ggn}[\Delta a_w$
- Apples-to-apple comparison: need GW signal processed through the same pipeline!
- Monochromatic plane GW incident on Solar System, perpendicular to the plane of the ecliptic (maximally optimistic!)
  - In the long-wavelength limit ($\omega_{\rm gw} L \ll 1$), effective Newtonian acceleration:

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \ddot{h}_+ & \ddot{h}_\times \\ \ddot{h}_\times & -\ddot{h}_+ \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}, \qquad h_{+,\times}(t, z=0) \equiv h^{(0)}_{+,\times} \cos(\omega_{\rm gw} t + \alpha_{+,\times})$$

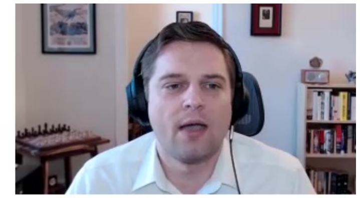
  - Perturbatively solve $\vec{x}(t) = \vec{x}_0(t) + \overrightarrow{\delta x}(t)$.
  - Compute

$$\Delta a^{\rm gw}_+(t) \approx \frac{1}{2} L \omega^2_{\rm gw} h^{(0)}_+ \cos(\omega_{\rm gw} t + \alpha_+) \cos(2\Omega t) = \frac{1}{4} L \omega^2_{\rm gw} h^{(0)}_+ \left\{ \cos\left[(\omega_{\rm gw} + 2\Omega)t + \alpha_+\right] + \cos\left[(\omega_{\rm gw} - 2\Omega)t + \alpha_+\right] \right\}$$

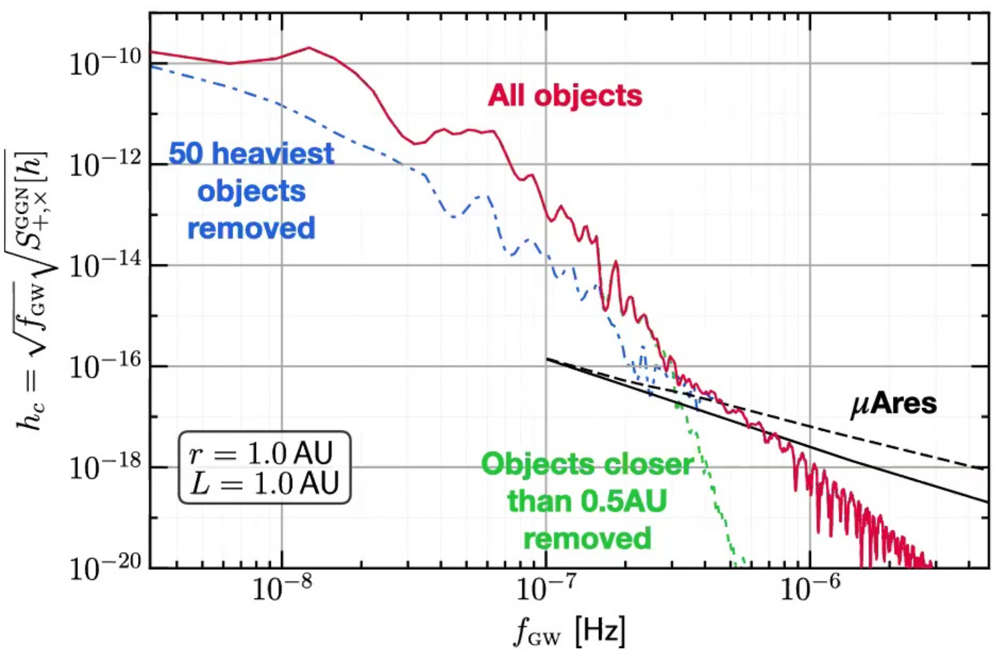
  - Window, FFT
- Matched filter search on the windowed baseline-projected differential acceleration: $\rho^2 = \frac{1}{T} \frac{|\widetilde{\Delta a^{\rm gw}_w}(f_0)|^2}{S^{\rm ggn}_0[\Delta a_w]} + \frac{1}{T} \frac{|\widetilde{\Delta a^{\rm gw}_w}(f_{\mathcal{N}/2})|^2}{S^{\rm ggn}_{\mathcal{N}/2}[\Delta a_w]} \delta_{0,\mathcal{N} \bmod 2} + \frac{4}{T} \sum_{k=1}^{\lfloor(\mathcal{N}-1)/2\rfloor} \frac{|\widetilde{\Delta a^{\rm gw}_w}(f_k)|^2}{S^{\rm ggn}_k[\Delta a_w]}$
- Find GW amplitude such that SNR = 1: $\rho(\hat{h}^{(0)}_{+,\times}) \equiv 1$

12

Implications for GW detection - II



- Extract an effective noise ASD curve for GW searches: $\sqrt{S^{\rm ggn}_{+,\times}[h](f_{\rm gw})} \equiv \sqrt{T} \cdot \hat{h}^{(0)}_{+,x}(f_{\rm gw})$
- This is the “standard noise curve” in strain per root-Hz; can use this result without regard to windowing, modulation, etc.
- Characteristic strain is $h_c(f_{\rm gw}) \equiv \sqrt{f_{\rm gw} \cdot S^{\rm ggn}_{+,\times}[h](f_{\rm gw})} \quad \left[= \sqrt{N_{\rm cycles}} \hat{h}^{(0)}_{+,\times} \right]$

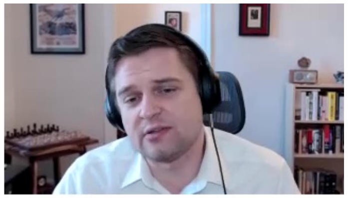


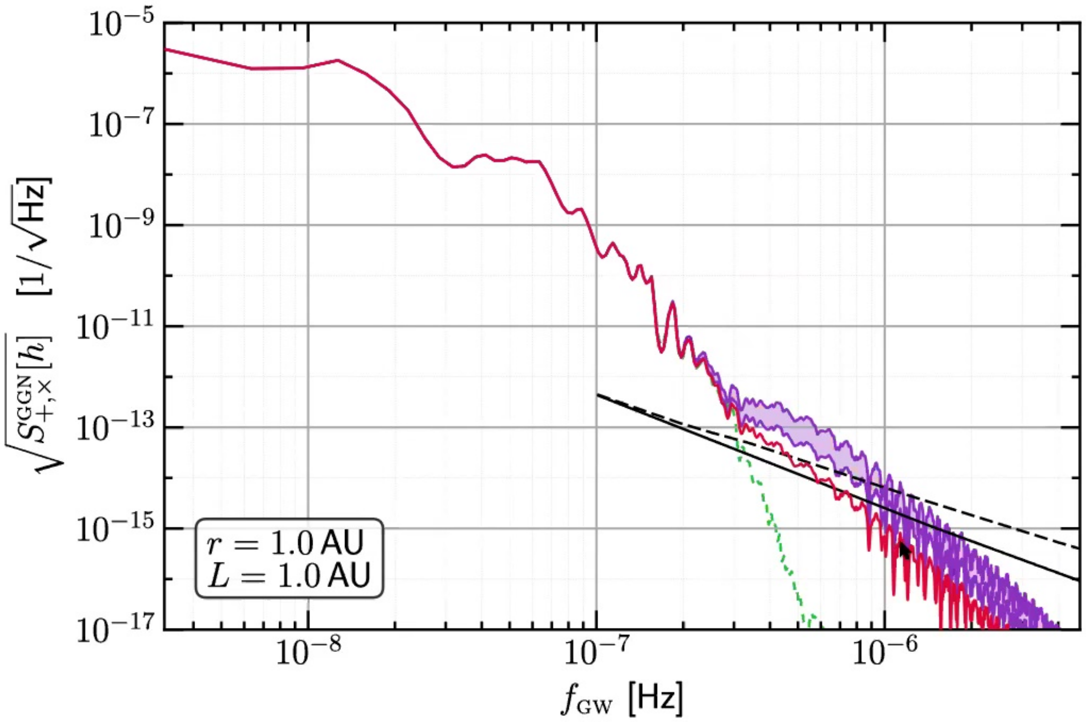
- Confirms earlier ad hoc estimate at low frequency
- But higher noise at higher frequency than in ad hoc estimate
- Potentially problematic as compared to μAres required sensitivity* up to frequencies (few) $\times\, 10^{-7}$ Hz
- Even removing heavy distant objects does not change this conclusion
- At higher frequency, noise drops off… but, is this complete?

*Slight mismatch in comparison, as this is a 1AU baseline, and μAres uses $L \sim 3$ AU

High frequency tail

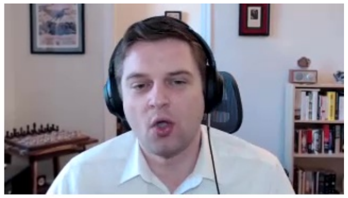
- Approximate estimate of noise in high frequency tail
- Shape here not computed. Estimate: undercounting by 3-10; rescale computation upward.



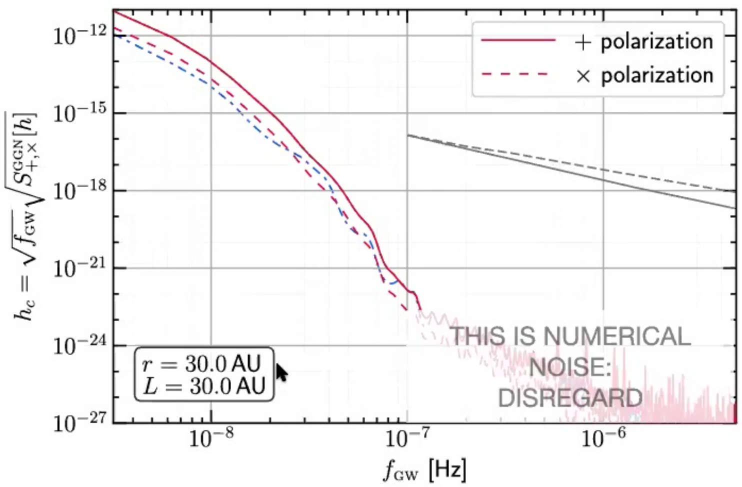


- Qualitative conclusions unchanged.
- Actual noise near $\mu\mathrm{Hz}$ not very well known; above that, looks OK.
- Below that, seems very hard to make progress in Inner Solar System.

Some ideas on the path forward

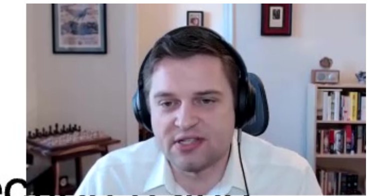


- Go to the Outer Solar System?
  - Technically challenging



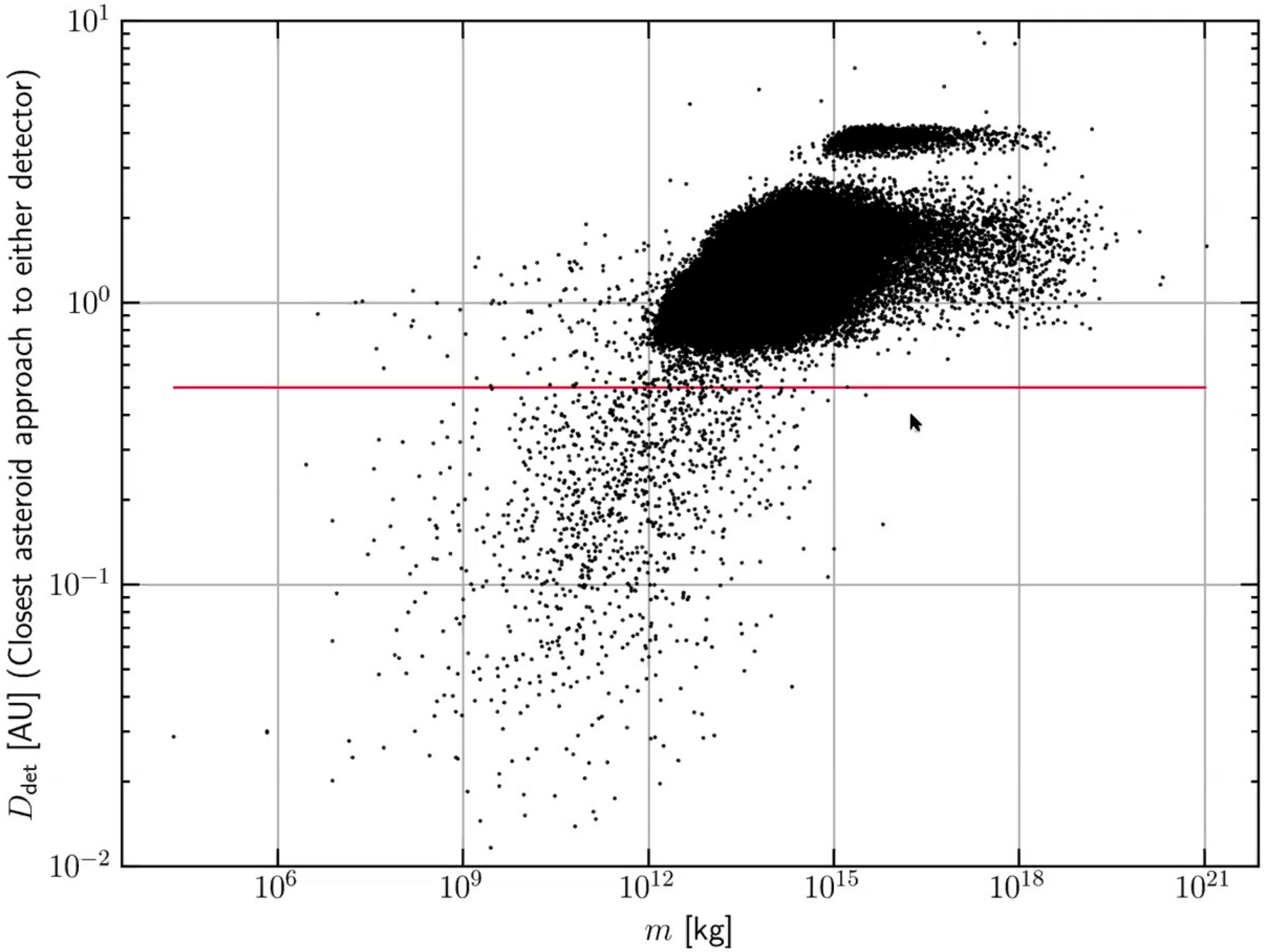
- Use baselines that extend outside the Inner Solar System:
  - Precision astrometry?
  - Work in progress here

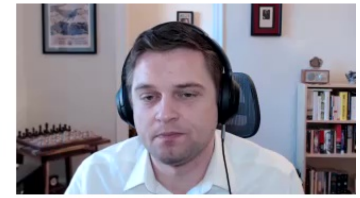
15

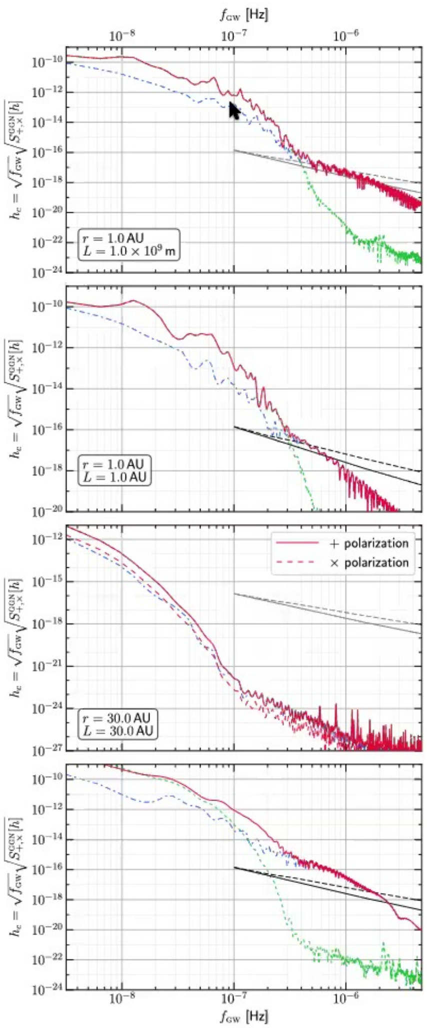


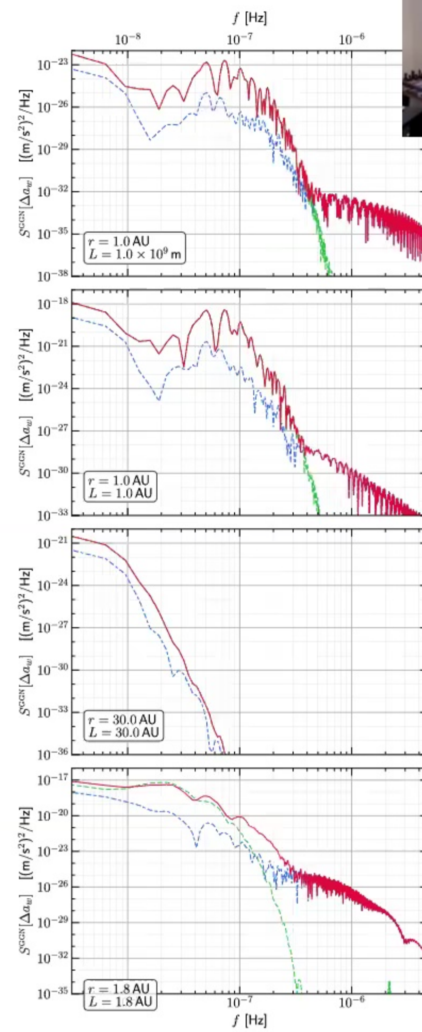
Conclusions

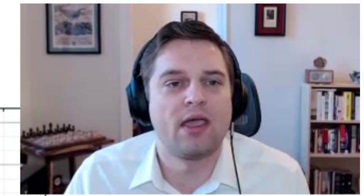
- Asteroid GGN is a limiting noise source for low-frequency GW detection using local test masses in the Inner Solar System
- Simulations robustly that this is limiting for $f \lesssim (\mathrm{few}) \times 10^{-7}\,\mathrm{Hz}$.
- Noise in this band dominated by large distant objects.
- Above $f \gtrsim \mu\mathrm{Hz}$, asteroid GGN drops rapidly, and detection again clearly OK.
- Situation around $f \sim \mu\mathrm{Hz}$ not completely clear: noise dominated by smaller, close-passing asteroids. Possible catalog incompleteness makes estimate uncertain by factor of 3-10.
- To do GW detection below $f \lesssim (\mathrm{few}) \times 10^{-7}\,\mathrm{Hz}$ requires approaches that utilize baselines that are not fully contained in the Inner Solar System.


10^1
10^0
10^{-1}
10^{-2}
10^6
10^9
10^{12}
10^{15}
10^{18}
10^{21}
m [kg]
$D_{\rm det}$ [AU] (Closest asteroid approach to either detector)




$f_{\rm GW}$ [Hz]
$r = 1.0$ AU
$L = 1.0 \times 10^9$ m
$r = 1.0$ AU
$L = 1.0$ AU
+ polarization
× polarization
$r = 30.0$ AU
$L = 30.0$ AU
$f_{\rm GW}$ [Hz]


f [Hz]
[(m/s^2)^2/Hz]
$r = 1.0$ AU
$L = 1.0 \times 10^9$ m
$r = 1.0$ AU
$L = 1.0$ AU
$r = 30.0$ AU
$L = 30.0$ AU
$r = 1.8$ AU
$L = 1.8$ AU
f [Hz]



28