Title: Approaches to Scattering in Quantum Gravity and Gauge Theory from Symmetry

Speakers: Monica Pate

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Abstract: The problem of quantum gravity -- i.e. to determine the microscopic structure underlying quantum mechanical theories that reproduce general relativity at long distances -- is a major outstanding problem in modern physics.  Solving the quantum gravitational scattering problem is one sharp way to address this question.  While in principle effective field theory (EFT) provides a systematic framework for solving scattering problems, in quantum gravity the complete answer requires an infinite number of measurements and thereby fails to predict details of the microscopic structure.

I will present two developments that provide new insight into the gravitational scattering problem.   The first is a class of infinite-dimensional symmetries generically found to arise in gauge and gravitational scattering.  The infinite number of constraints implied by the symmetries are equivalent to quantum field theoretic soft theorems, which prescribe the pattern of soft radiation produced during a scattering event.  The second development is a reformulation of the gravitational scattering problem in which Lorentz symmetry is rendered manifest and realized as the action of the global conformal group in two dimensions.  This reformulation, which involves scattering particles of definite boost weight as opposed to energy, offers a new approach precisely because it does not admit the decoupling of low and high-energy physics that underpins the traditional EFT approach.  I will describe new perspectives ensuing from these developments on various properties of the gravitational scattering problem, including collinear limits, infrared divergences and universal behavior associated to black hole formation. 

# APPROACHES TO SCATTERING IN QUANTUM GRAVITY AND GAUGE THEORY FROM SYMMETRY

MONICA PATE

HARVARD SOCIETY OF FELLOWS

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## THE PROBLEM OF QUANTUM GRAVITY:

*What is the microscopic structure underlying quantum mechanical theories that reduce to general relativity at long distances and low energies?*

# WILSONIAN EFFECTIVE FIELD THEORY

- Include all terms allowed by symmetry.

$$S = \frac{1}{16\pi G}\int d^n x\sqrt{-g}\left(R + a_0 R^2 + \cdots\right)$$

- Infinitely many terms, *organized* by energy.
- At low-energy with fixed precision can truncate to finite number of terms.
- No consistent truncation to finitely many terms, at energies comparable to Planck scale.

**Answer in Wilsonian format:**

*Microscopic structure of gravity is captured by an effective action with an infinite number of diffeomorphism-invariant terms.*

# A SMALL IMPROVEMENT ⇒ A CRITICAL INSIGHT:

- *What about field redefinitions?*
- Example: In four-spacetime dimensions, can remove all $R^2$ corrections:

$$S_{\text{eff}} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R + \alpha_1 R^2 + \alpha_2 R_{\mu\nu}R^{\mu\nu} + \alpha_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \mathcal{O}(R^3)\right]$$

$$\Longrightarrow \quad S_{\text{eff}} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R + \mathcal{O}(R^3)\right]$$

- *Manifest in Scattering Amplitudes:*
  - Only two distinct 3-graviton on-shell amplitudes

$$\mathcal{M}_{--+} = \frac{\langle 12\rangle^6}{\langle 23\rangle^2\langle 31\rangle^2} \sim \underbrace{\mathcal{O}(p^2)}_{\sim R}, \qquad \mathcal{M}_{---} = \langle 12\rangle^2\langle 23\rangle^2\langle 31\rangle^2 \sim \underbrace{\mathcal{O}(p^6)}_{\sim R^3}.$$

- **Takeaway:** phrase problem of quantum gravity in terms of observables (e.g. scattering amplitudes)

# CONSTRAINTS ON SCATTERING AMPLITUDES:

- **Causality, analyticity and unitarity**
  - Implies positivity conditions on coefficients in EFT expansion
    [Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi, hep-th/0602178; Huang, Huang, Arkani-Hamed, hep-th/2012.15849; ...]
- **Universality at high energies**
  - Black hole formation
    [Arkani-Hamed, Dubovsky, Nicolis, Trincherini, Villadoro, hep-th/0704.1814; Giddings, Porto, hep-th/0908.0004]
- **Universality at low energies**
  - Soft theorems ⇔ Infinite-dimensional (asymptotic) symmetries
    [He, Lysov, Mitra, Strominger, hep-th/1401.7026; Kapec, MP, Strominger, hep-th/1506.02906; Strominger, hep-th/1703.05448; ...]

- *Upshot: Problem appears highly constrained!*
- *Challenge: How to simultaneously impose all constraints?*

## A SHARP QUESTION IN QUANTUM GRAVITY:

*What is the space of consistent scattering amplitudes in quantum gravity?*

# OUTLINE

1. Introduction and Motivation
   - The problem of quantum gravity
   - Scattering in quantum gravity
2. Low-energy Constraints: Asymptotic Symmetries and Soft Theorems
   - *Asymptotic symmetries:* Why the scattering problem admits infinitely many symmetries
   - *Soft theorems:* How the scattering problem is constrained
3. Celestial Amplitudes: A Coalescence of UV and IR
   - Recast scattering problem: particles in boost eigenstates
   - Revisit constraints

# SYMMETRY CONSTRAINTS ON THE SCATTERING PROBLEM

# SYMMETRY IN SCATTERING PROBLEMS

$$\delta|\Psi\rangle = Q|\Psi\rangle$$

$$|\mathrm{out}\rangle = \mathcal{S}|\mathrm{in}\rangle$$

$$Q\mathcal{S} - \mathcal{S}Q = 0$$

- Symmetry
  - Transformation that leaves the laws of physics (i.e. equations of motion) unchanged.
  - Maps solutions to solutions.
- Scattering problem
  - Given initial state of system, what is the final state after time evolution?
  - Assume at early/late times, degrees of freedom are well-separated & non-interacting.
  - Non-trivial dynamics captured by map from initial to final states ("$\mathcal{S}$-matrix").
- Symmetry of scattering
  - Transformation of initial and final states that preserves the $\mathcal{S}$-matrix.

# ASYMPTOTIC SYMMETRIES PRIMER

## Why infinitely many?

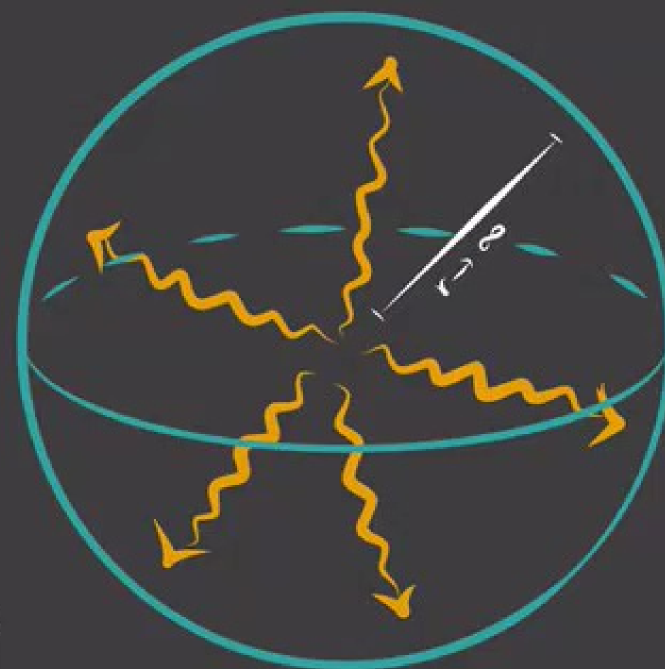
Points on "final" sphere out of causal contact

$\Rightarrow$ *Independent* time translational symmetry at each point

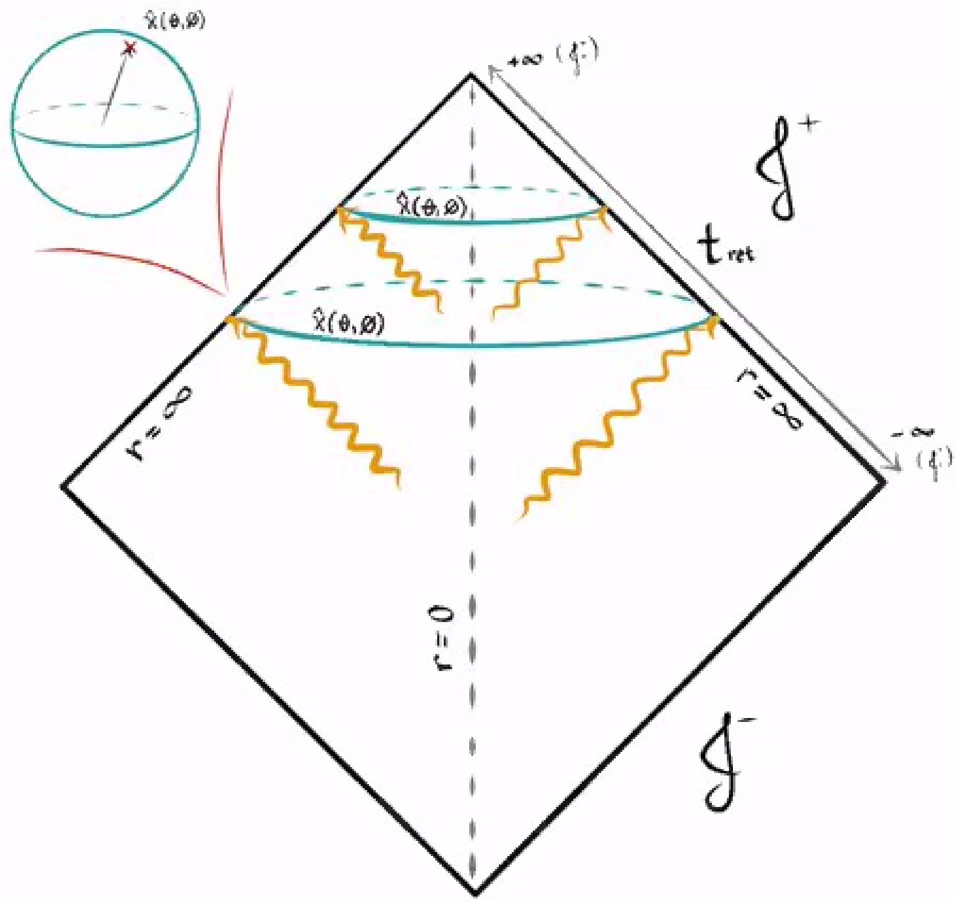
$\Rightarrow$ *Infinite-dimensional* enhancement of translation group

Supertranslations

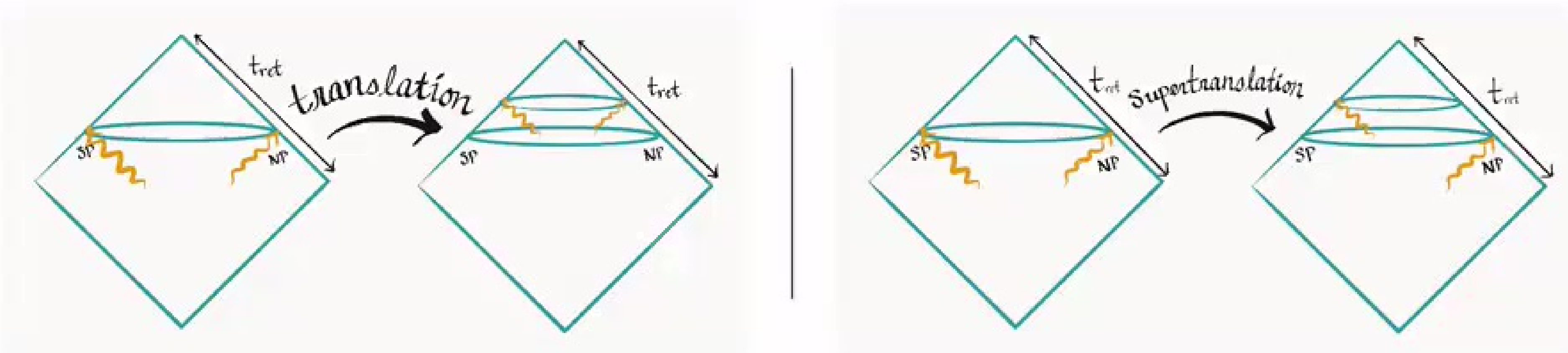
**(of BMS)**




*The Celestial Sphere*



# FUTURE NULL INFINITY




[Bondi, van der Burg, Metzner (1962); Sachs (1962)]

## SUPERTRANSLATIONS

- Independent translation symmetry at every angle on the celestial sphere
- Desynchronization of events at different angles on the celestial sphere
- Infinite-dimensional symmetry group

  $\Rightarrow$ *Infinitely many constraints*

# SOFT THEOREMS

- In quantum field theory (QFT),

$$\text{initial/final state}: \quad |\text{in/out}\rangle = \underbrace{|p_1, p_2, \cdots\rangle}_{\text{particles}} = a^\dagger(p_1)a^\dagger(p_2)\cdots|0\rangle$$

- Solution to scattering problem: Scattering Amplitudes

$$\langle \text{out}|\mathcal{S}|\text{in}\rangle$$

- Soft Theorems: universal relation among scattering amplitudes

$$\lim_{E\to 0} E\langle \text{out}|a(E)\mathcal{S}|\text{in}\rangle = \left[\sum_{k\in\text{out}} S_k - \sum_{k\in\text{in}} S_k\right]\langle \text{out}|\mathcal{S}|\text{in}\rangle$$

- Physically, low energy particles only probe long-distance physics.
- *Goal:* soft theorem = Ward identity $\qquad \langle \text{out}|[Q, \mathcal{S}]|\text{in}\rangle = 0$

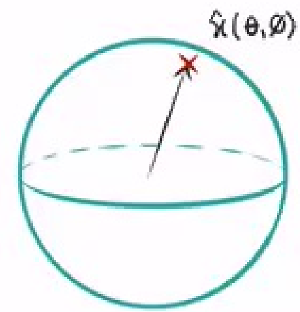
# WEINBERG'S SOFT GRAVITON THEOREM

$$\lim_{\omega\to 0}\omega\langle \text{out}|a_+(\omega\hat{x})\mathcal{S}|\text{in}\rangle = \sum_{k=1}^{n} S_k(\hat{x})\langle \text{out}|\mathcal{S}|\text{in}\rangle$$

Soft Factor: $S_k(\hat{x}) = \frac{\kappa}{2}\frac{\varepsilon^{+}_{\mu\nu}p_k^{\mu}p_k^{\nu}}{\hat{q}\cdot p_k}$

Graviton momentum:

$$q^{\mu} = \omega\hat{q}^{\mu} = \omega\big(1, \hat{x}(\theta,\phi)\big)$$



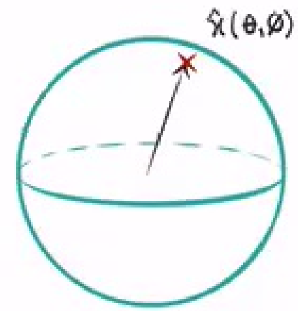

[Weinberg 1965]

# SOFT THEOREMS IMPLY SYMMETRIES

- *Can always interpret soft theorems as statements of invariance of the $\mathcal{S}$-matrix under an infinite-dimensional symmetry*

## Weinberg's Soft Graviton Theorem

$$\lim_{\omega\to 0} \omega \langle \text{out}|a_+(\omega\hat{x})\mathcal{S}|\text{in}\rangle = \sum_{k=1}^{n} S_k(\hat{x})\langle \text{out}|\mathcal{S}|\text{in}\rangle$$




- Regard soft factor $S_k$ as eigenvalue of single particle state under operator $Q_H$

$$S_k(\hat{x})|p_k\rangle = Q_H(\theta,\phi)|p_k\rangle = -i\delta_{(\theta,\phi)}|p_k\rangle$$

- RHS gives transformation of single particle states under $\delta_{(\theta,\phi)}$

$$\sum_{k=1}^{n} S_k\langle \text{out}|\mathcal{S}|\text{in}\rangle = \langle \text{out}|[Q_H,\mathcal{S}]|\text{in}\rangle$$

- Soft theorem implies that the $\mathcal{S}$-matrix is invariant under the transformation $\delta_{(\theta,\phi)}$ of single particle states provided that *a soft particle is added.*



[He, Lysov, Mitra, Strominger, hep-th/1401.7026; Strominger, hep-th/1703.05448]

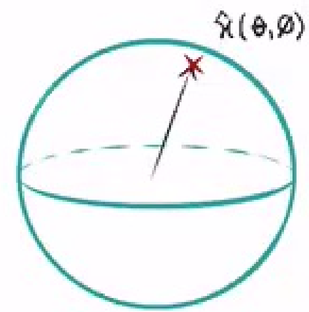
# SOFT THEOREMS IMPLY SYMMETRIES

## Weinberg's Soft Graviton Theorem

$$\lim_{\omega\to 0}\langle \text{out}|\omega a_+(\omega\hat{x})\mathcal{S}|\text{in}\rangle = \sum_{k=1}^{n} S_k\langle \text{out}|\mathcal{S}|\text{in}\rangle = \langle \text{out}|[Q_H,\mathcal{S}]|\text{in}\rangle$$

- Denote operator which **adds soft particles** $Q_S$

$$Q_S(\theta,\phi) \sim -\lim_{\omega\to 0}\omega\left[a_+(\omega\hat{x}) + a_-^\dagger(\omega\hat{x})\right]$$




- Then, the LHS can be written as

$$\lim_{\omega\to 0}\langle \text{out}|\omega a_+(\omega\hat{x})\mathcal{S}|\text{in}\rangle = -\langle \text{out}|[Q_S,\mathcal{S}]|\text{in}\rangle$$

- Rearranging the soft theorem

$$\langle \text{out}|[Q,\mathcal{S}]|\text{in}\rangle = 0, \qquad Q = Q_H + Q_S$$

⇒ Obtain statement of invariance under symmetry generated by $Q$

- *Can always interpret soft theorems as statements of invariance of the $S$-matrix under an infinite-dimensional symmetry*

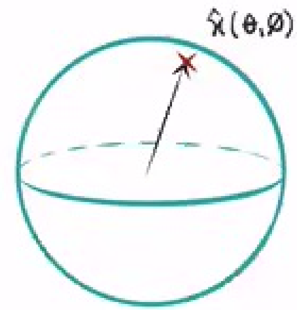


[He, Lysov, Mitra, Strominger, hep-th/1401.7026; Strominger, hep-th/1703.05448]

# SUPERTRANSLATIONS & WEINBERG'S SOFT THEOREM

## Construction of local charges

- Parametrize symmetry transformations by functions $f(\theta, \phi)$ rather than points $(\theta, \phi)$.
- Obtain charges with *local* action (*i.e.* action only depends on $f$ at point $(\theta_k, \phi_k)$)




$$-i\delta_f|p_k\rangle = Q_H[f]|p_k\rangle = \omega_k f(\theta_k, \phi_k)|p_k\rangle$$

## Equivalence with supertranslations

- When $f = 1$, find total energy

$$-i\delta_{f=1}|p_k\rangle = \omega_k|p_k\rangle$$

$\Rightarrow$ Q[$f$ = 1] generates ordinary time translations

- Generic $f = f(\theta, \phi)$:

$$-i\delta_f|p_k\rangle = \omega_k f(\hat{x}_k)|p_k\rangle$$

$\Rightarrow$ $f$ parametrizes *amount* of translation at every angle



[He, Lysov, Mitra, Strominger, hep-th/1401.7026; Strominger, hep-th/1703.05448]

# CONSTRAINTS FROM SUPERTRANSLATIONS

- To fully appreciate constraints from supertranslations, reexamine the soft charge:

$$Q_S(\theta,\phi) \sim \lim_{\omega\to 0} \omega \left[a(\omega\hat{x}) + a^\dagger(\omega\hat{x})\right] \sim \lim_{\omega\to 0} \int dt\ \varepsilon^{\mu\nu} e^{i\omega t} \partial_t h^{\mathrm{rad}}_{\mu\nu}$$

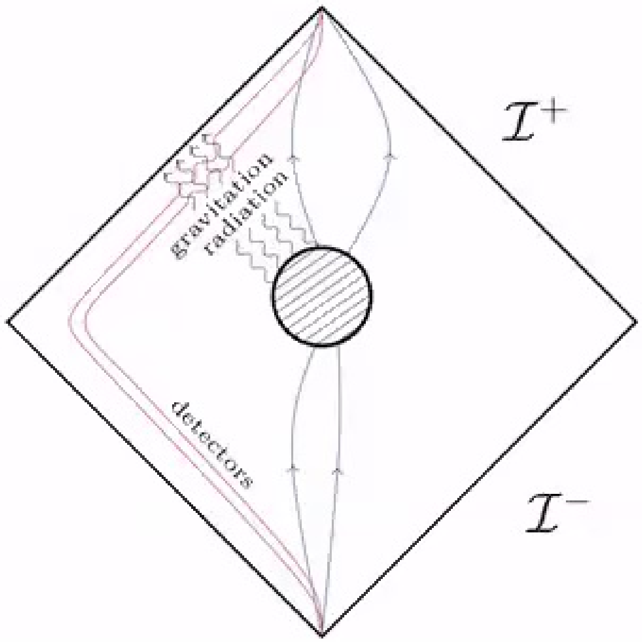
- The soft charge is the *zero-frequency component* of the *radiative gravitational field*.

$$Q_S(\theta,\phi) \sim \varepsilon^{\mu\nu} \underbrace{\Delta h_{\mu\nu}}_{h_{\mu\nu}(t_f) - h_{\mu\nu}(t_i)}$$

Follows from

$$\int d\omega\ \frac{e^{i\omega t}}{\omega} \sim \Theta(t)$$

- The soft charge is also equal to the *net shift* in the *asymptotic metric*.




NATURE VOL. 327 14 MAY 1987

## Gravitational-wave bursts with memory and experimental prospects

Vladimir B. Braginsky* & Kip S. Thorne†

permanent change in the gravitational-wave field (the burst's memory) $\delta h_{ij}^{TT}$ is equal to the 'transverse, traceless (TT) part'[36] of the time-independent, Coulomb-type, $1/r$ field of the final system minus that of the initial system. If $\mathbf{P}^A$ is the 4-momentum of mass $A$ of the system and $P_i^A$ is a spatial component of that 4-momentum in the rest frame of the distant observer, and if $\mathbf{k}$ is the past-directed null 4-vector from observer to source, then $\delta h_{ij}^{TT}$ has the following form:

$$\delta h_{ij}^{\mathrm{TT}} = \delta\left(\sum_A \frac{4P_i^A P_j^A}{\mathbf{k}\cdot\mathbf{P}^A}\right)^{\mathrm{TT}}$$

# GRAVITATIONAL MEMORY

[Zel'dovich & Polnarev 1974]

# GRAVITATIONAL MEMORY & SUPERTRANSLATIONS

- Measures net shift in radiative gravitational field!
- Answers the question of how to measure soft radiation!
  - ⇒ Tile the celestial sphere with inertial observers and measure the change in their relative displacements.

*Interpretation:* **Vacuum Transition**

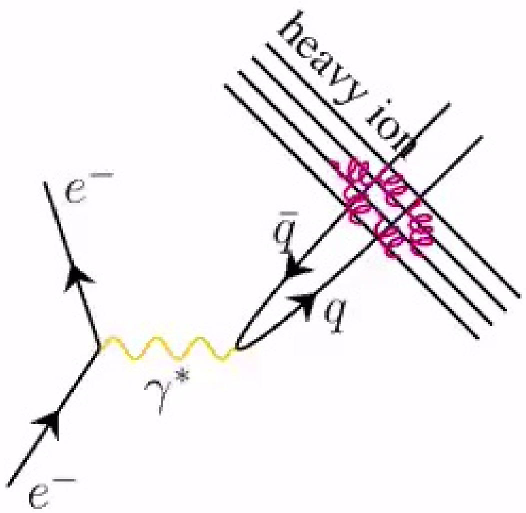
- Change in displacements
  - ⟶ Change in connection on celestial sphere

[Strominger & Zhiboedov, hep-th/1411.5745]

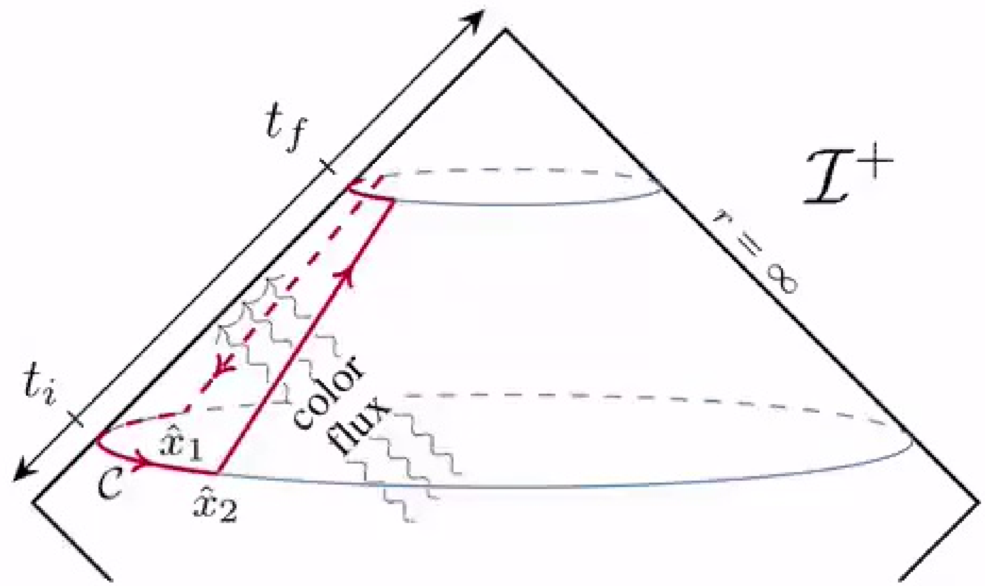
Relative displacement induced by radiative gravitational field:

$$\Delta s^i \sim \Delta h^{ij} s_j$$
$$\sim \lim_{\omega \to 0} \int dt \; e^{i\omega t} \partial_t h^{ij} s_j$$

(Geodesic Deviation Equation)




Deep Inelastic Scattering in Regge Limit




$$\mathcal{W}_{\mathcal{C}} = \mathcal{P} \exp\left( ig \oint_{\mathcal{C}} A \right)$$

Net relative color rotation

# COLOR MEMORY IN EXPERIMENT

[MP, Raclariu, & Strominger hep-th/1707.08016;
Ball, MP, Raclariu, Strominger & Venugopalan, hep-ph/1805.12224]

# CELESTIAL AMPLITUDES: A COALESCENCE OF UV AND IR

# HOLOGRAPHY
## *INSPIRED BY ASYMPTOTIC SYMMETRIES*

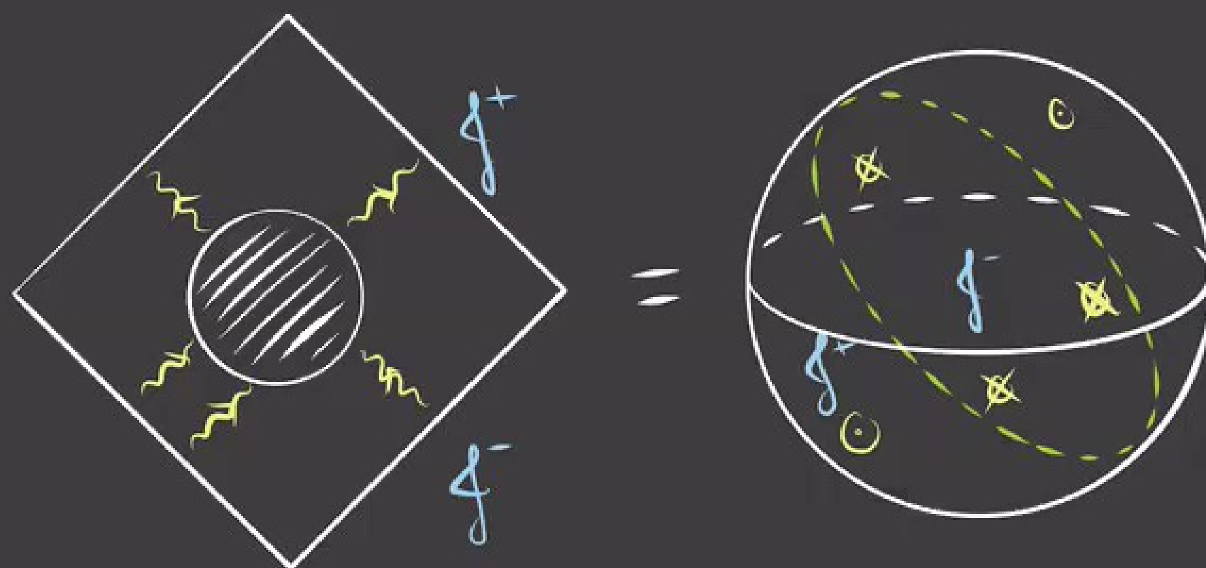
- Subleading soft graviton theorem

  ⇔ Virasoro Ward identity

  [Kapec, Lysov, Pasterski, Strominger, hep-th/1406.3312]

→ *Could scattering in 4D quantum gravity be captured holographically by a 2D conformal theory?*

- Rigid structure of ordinary 2D CFTs ⇒ New insights for quantum gravity?
- No verdict yet, still enlightening to recast amplitudes in manifestly conformally covariant form.



# 2D CONFORMAL SYMMETRY OF SCATTERING AMPLITUDES

- *Why did we find the conformal group of two dimensions?*
  - 4D Lorentz group $SO(3,1) \cong$ 2D global conformal group $SL(2,\mathbb{C})$
- *How can the conformal symmetry be made manifest?*
  - Construct observables that transform simply under dilations.
- *What type of scattering states transform simply under dilations?*
  - Dilations in 2D identified with Lorentz transformation in 4D.
  - ? Momentum eigenstates: diagonalize translations
  - ✓ Boost eigenstates: diagonalize boost ⇔ dilation on celestial sphere

# CONSTRUCTION OF BOOST EIGENSTATES


- Identify massless momentum with point on $S^2$:

$$p^\mu = \omega\big(1, \hat{x}(\theta,\phi)\big)$$

- Identify boost in direction $p^\mu$ with dilation on sphere:

$$p^\mu \to p'^\mu = \lambda p^\mu, \qquad \hat{x} \to \lambda \hat{x}$$

- Generator of boosts along $p^\mu$:

$$K = \omega \partial_\omega$$

⇒ $K$ is diagonalized by the *Mellin transform*

$$|\Delta, \hat{x}\rangle \equiv \int_0^\infty \frac{d\omega}{\omega} \omega^\Delta |p(\omega,\hat{x})\rangle$$

[Kapec, Mitra, Raclariu, & Strominger, hep-th/1609.00282; Cheung, de la Fuente & Sundrum, hep-th/1609.00732]

## Celestial Amplitude

$$\mathcal{A}(\Delta_i, \hat{x}_i) = \Big( \prod_{k=1}^{n} \int_0^\infty d\omega_k \, \omega_k^{\Delta_k - 1} \Big) \mathbf{A}(p_i)$$

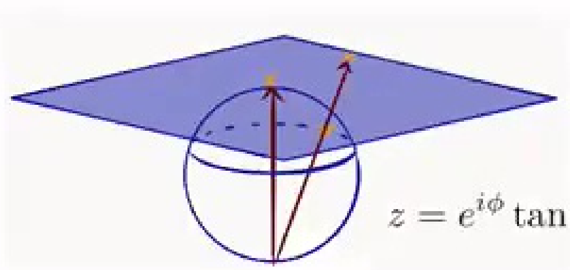
## Conformal Covariance

$$z \to \frac{az+b}{cz+d}, \qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{C})$$

$$\mathcal{A}(\Delta_i, z_i, \bar{z}_i) \to \Big( \prod_{k=1}^{n} (cz_k + d)^{\Delta_k + s_k} \left(\bar{c}\bar{z}_k + \bar{d}\right)^{\Delta_k - s_k} \Big) \mathcal{A}(\Delta_i, z_i, \bar{z}_i)$$

(transformation of correlation functions in 2D CFT)

## Stereographic Projection



$$z = e^{i\phi} \tan\left(\tfrac{1}{2}\theta\right)$$

# CELESTIAL AMPLITUDES AS CONFORMAL CORRELATORS

# CONSTRAINING CELESTIAL AMPLITUDES

- *Having recast our question in the language of celestial amplitudes, next revisit constraints.*

  - Kinematics – Poincaré
  - Symmetries – internal, asymptotic, ...
  - Dynamics – unitarity, causality, soft behavior at high energy, ...

- ***Upshot:*** constraints take simple, manifestly Lorentz-covariant form in celestial amplitude

# $2 \to 2$ SCATTERING ON THE CELESTIAL SPHERE

- Conformal (Lorentz) symmetry fixes $z_i$ dependence

$$\mathcal{A}(\Delta_i, z_i, \bar{z}_i) = \Big(\prod_{i<j} z_{ij}^{\frac{1}{3}h - h_i - h_j} \bar{z}_{ij}^{\frac{1}{3}\bar{h} - \bar{h}_i - \bar{h}_j}\Big) \hat{\mathcal{A}}(\Delta_i; z, \bar{z})$$

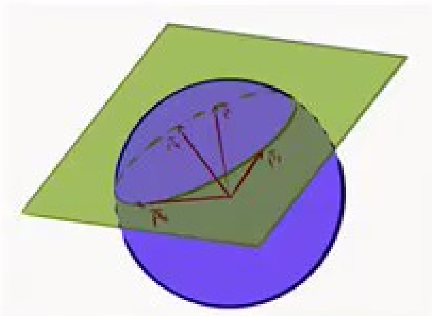
$$z_{ij} = z_i - z_j,$$
$$h_i = \frac{1}{2}(\Delta_i + s_i),$$
$$\bar{h}_i = \frac{1}{2}(\Delta_i - s_i),$$
$$h = \sum_{i=1}^{4} h_i$$

$\Rightarrow$ Depends non-trivially only on the conformal cross ratio

$$z \equiv \frac{z_{13} z_{24}}{z_{12} z_{34}}$$

- Translation symmetry $\Rightarrow$ momentum conservation
  - Spatial momenta lie in a plane
  - Plane intersects celestial sphere on a circle
  - Four points $z_i$ must lie on circle
  - Cross ratio must be real

$$\hat{\mathcal{A}}(\Delta_i; z, \bar{z}) \propto \delta(z - \bar{z})$$



[Arkani-Hamed, MP, Raclariu, & Strominger, hep-th/ 2012.04208]

# $2 \to 2$ SCATTERING ON THE CELESTIAL SPHERE

- *Expect Poincaré symmetry to be more constraining*
- Recall massless scalar $2 \to 2$ scattering in momentum space
  - Lorentz symmetry

$$\mathbf{A} = \mathbf{A}(p_i \cdot p_j)$$

  - Translations $\Rightarrow$ *only two* $p_i \cdot p_j$ are independent

$$\mathbf{A} = \mathbf{M}(s,t)\, \delta^{(4)}\Big(\sum_{i=1}^{4} p_i\Big),$$

Center of mass energy: $s = -(p_1 + p_2)^2 = -2p_1 \cdot p_2,$

Momentum transfer: $t = -(p_1 + p_3)^2 = -2p_1 \cdot p_3.$

- *Does something similar apply to celestial amplitudes?*
  - Can identify *ratio* of invariants with conformal cross ratio
  - *What replaces the overall energy scale?*

$$z = -\frac{t}{s}$$ (scattering angle)

## 2 → 2 SCATTERING ON THE CELESTIAL SPHERE

- Revisit massless scalar 2 → 2 celestial amplitude:

Parametrize Mandelstam invariants by center of mass energy $\omega$ and conformal cross ratio $z$:

$$s = \omega^2, \quad t = -z\omega^2.$$

$$\mathcal{A}(\Delta_i, z_i, \bar{z}_i) = \left(\prod_{j=1}^{4} \int_0^\infty \frac{d\omega_j}{\omega_j} \omega_j^{\Delta_j}\right) \mathbf{M}(s,t)\ \delta^{(4)}\left(\sum_{k=1}^{4} p_k\right)$$

$$\sim \int_0^\infty \frac{d\omega}{\omega}\omega^\beta \prod_{j=2}^{4} \int_0^\infty \frac{d\sigma_j}{\sigma_j}\sigma_j^{\Delta_j}$$

$$\sim \delta(z-\bar{z}) \prod_{i=2}^{4} \delta\big(\sigma_i - f_i(z_j, \bar{z}_j)\big), \quad \sigma_i = \frac{\omega_i}{\omega_1}.$$

$$\beta = \left(\sum_{i=1}^{4} \Delta_i - 1\right)$$

⇒ Momentum-conserving delta function localizes three of four integrals

[Arkani-Hamed, MP, Raclariu, & Strominger, hep-th/2012.04208]

## $2 \to 2$ SCATTERING ON THE CELESTIAL SPHERE

*Arrive at following decomposition:*

$$\mathcal{A}(\Delta_i, z_i, \bar{z}_i) \sim \underbrace{\left(\prod_{i<j} z_{ij}^{\frac{1}{3}h-h_i-h_j} \bar{z}_{ij}^{\frac{1}{3}\bar{h}-\bar{h}_i-\bar{h}_j}\right)\delta(z-\bar{z})}_{\text{Fixed by kinematics}} \underbrace{\int_0^\infty \frac{d\omega}{\omega}\omega^\beta \mathbf{M}(\omega^2, -z\omega^2)}_{\equiv \mathcal{M}(\beta, z)}$$

(captures dynamics)

$$\beta = \left(\sum_{i=1}^{4} \Delta_i - 1\right)$$

- Center of mass energy $\omega$ traded for sum of conformal dimensions $\beta + 4$
- M (dynamical content) related to momentum space matrix element (at fixed angle) by *single* Mellin transform!

[Arkani-Hamed, MP, Raclariu, & Strominger, hep-th/2012.04208]

# BEYOND THE WILSONIAN PARADIGM

- **Wilsonian paradigm:** *low-energy physics is insensitive to the details of the UV*
  - Realized explicitly by consistent truncation of EFT expansion at low energies
- *Celestial amplitudes are sensitive to UV physics*
  - Drastic difference if truncate EFT expansion
  - Really only exist for consistent UV complete theories
  - UV sensitivity is consequence of scattering boost eigenstates, which contain contributions of arbitrarily high energy.

# BEYOND THE WILSONIAN PARADIGM

- Consider scattering of massless scalars mediated by massive exchange

$$\mathbf{M}(s) \sim \lambda \frac{M^2}{s - M^2}$$

- In momentum space, admits low-energy (EFT) expansion

$$\mathbf{M}(\omega) \sim -\lambda\left(1 + \frac{\omega^2}{M^2} + \frac{\omega^4}{M^4} + \cdots\right), \qquad s = \omega^2$$

- Consider celestial amplitude for leading term

$$\mathcal{M}(\beta) \sim \int_0^\infty \frac{d\omega}{\omega}\omega^\beta(-\lambda) \overset{\beta=ib}{=} -2\pi\lambda\delta(b)$$

- Subleading corrections ruins marginal convergence (diverges at upper limit).
- Celestial amplitudes don't exist for *truncated* EFT's!

[Arkani-Hamed, MP, Raclariu, & Strominger, hep-th/2012.04208]

# BEYOND THE WILSONIAN PARADIGM

- Simple result for full (not truncated) amplitude

$$\mathbf{M}(s) = \lambda \frac{M^2}{s - M^2} \quad \Longrightarrow \quad \mathcal{M}(\beta) = \lambda M^\beta \frac{i\pi e^{i\pi\beta}}{1 - e^{i\pi\beta}}$$

- Celestial amplitudes don't really exist for truncated EFT expanded amplitudes.

➢ *Do we expect celestial amplitudes to exist in theories of quantum gravity?*

✓ Yes! Universal soft behavior at high energy is a crucial ingredient.

[Arkani-Hamed, MP, Raclariu, & Strominger, hep-th/2012.04208]

# SOFT UV BEHAVIOR IN QUANTUM GRAVITY

- High-energy scattering at energies $\gg M_{Planck}$ is dominated by black hole production
- Black holes are produced with order 1 probability

$$1 \sim \sum_I |\underbrace{\langle \text{in}|\text{BH}_I\rangle}|^2 \quad \Longrightarrow \quad \langle \text{BH}_I|\text{in}\rangle \sim e^{-S_{BH}/2} e^{i\phi_I^{\text{in}}}$$

Amplitude for producing black hole microstate $I$

$e^{S_{BH}}$ black hole microstates

- Implies exponentially soft UV behavior in exclusive $2 \to 2$ amplitude

$$\mathbf{A}_{2\to 2} \overset{\omega \gg M_P}{\sim} \sum_I \langle \text{out}|\text{BH}_I\rangle\langle \text{BH}_I|\text{in}\rangle \sim e^{-S_{BH}} \underbrace{\sum_I e^{i(\phi_I^{\text{in}} - \phi_I^{\text{out}})}}_{\sim e^{S_{BH}/2}} \qquad S_{BH} = 4\pi G_N \omega^2$$

[Arkani-Hamed, Dubovsky, Nicolis, Trincherini & Villadoro, hep-th/0704.1814; Giddings & Porto, hep-th/0908.0004]

# CELESTIAL AMPLITUDES IN QUANTUM GRAVITY

- **Quantum gravity:**
  - Exponentially soft behavior at high-energy in momentum space
  - ⇒ Convergence at upper limit of integration
  - Analyticity in $\beta$ along positive real axis

$$\int^{\infty} \frac{d\omega}{\omega} \omega^{\beta} e^{-\omega^2/M_p^2}$$

- **Quantum field theory:**
  - Power-law fall-off at high energies
  - ⇒ Converges for $\beta < 2m$
  - Poles at positive even integer $\beta$

$$\int^{\infty} \frac{d\omega}{\omega} \omega^{\beta} \omega^{-2m} \sim \frac{1}{\beta - 2m}$$

Recall toy example:

$$\mathcal{M}(\beta) = \int_0^{\infty} \frac{d\omega}{\omega} \omega^{\beta} \lambda \frac{M^2}{\omega^2 - M^2} = \lambda M^{\beta} \frac{i\pi e^{i\pi\beta}}{1 - e^{i\pi\beta}}$$

[Arkani-Hamed, MP, Raclariu, & Strominger, hep-th/2012.04208]

# SOFT THEOREMS ON THE CELESTIAL SPHERE

- Do soft theorems constrain celestial amplitudes (where states are energy eigenstates)?
  - *Symmetry interpretation* supports the existence of a *celestial avatar of soft theorems.*

- Yes, celestial amplitudes are constrained by asymptotic symmetries!
  - *Strategy:* Apply similar logic used to deduce QFT UV behavior (i.e. power law fall-off) directly from celestial amplitudes.
  - Behavior of celestial amplitudes due to soft expansion is determined by focusing on *lower range of integration.*

$$f(\omega) = \frac{1}{\omega}a_{(-1)} + a_{(0)} + \omega a_{(1)} + \cdots$$

$f$ admits Laurent expansion about $\omega = 0$

$\Rightarrow$

$$\widetilde{f}(\Delta) \equiv \int_0^\infty \frac{d\omega}{\omega}\omega^\Delta f(\omega)$$

$$\supset \int_0 \frac{d\omega}{\omega}\omega^\Delta \sum_n \omega^n a_{(n)} \sim \sum_n \frac{a_{(n)}}{\Delta + n}$$

$\tilde{f}$ admits simple poles at $\Delta = 1, 0, -1, -2, \ldots$

- Residues of poles at integer $\Delta \leq 1$ give coefficients of Laurent expansion about $\omega = 0$.

# SYMMETRY CONSTRAINTS ON CELESTIAL AMPLITUDES

- Mellin transformation of Laurent expansion:

$$f(\omega) = \frac{1}{\omega}a_{(-1)} + a_{(0)} + \omega a_{(1)} + \cdots \quad \Rightarrow \quad \widetilde{f}(\Delta) \sim \sum_n \frac{a_{(n)}}{\Delta + n}$$

$f$ admits Laurent expansion about $\omega = 0$

$\tilde{f}$ admits simple poles at $\Delta = 1, 0, -1, -2, \ldots$

- Soft theorems:

$$\mathbf{A}_{n+1}(\omega_i) = \left(\frac{1}{\omega_i}S_{(0)} + S_{(1)} + \omega_i S_{(2)}\right)\mathbf{A}_n + \cdots \quad \Rightarrow \quad \mathrm{Res}[\mathcal{A}_{n+1}(\Delta_i)]_{\Delta_i=1} = S_{(0)}\mathcal{A}_n$$

Expansion in energy $\omega_i$ of *single* external particle

Residues of poles in $\Delta_i$

- Asymptotic symmetries ⇒ Leading coefficients in soft expansion are universal.

⇒ Simple poles at graviton $\Delta_i = 1, 0, -1$ with *universal residues*.

[Cheung, de la Fuente,& Sundrum, hep-th/1609.00732;
Fan, Fotopoulos & Taylor, hep-th/1903.01676;
MP, Raclariu, & Strominger, hep-th/1904.10831;
Adamo, Mason, & Sharma, hep-th/1905.09224; Puhm, hep-th/1905.09799]

## SUMMARY AND OUTLOOK

- Scattering amplitudes are central to the study of quantum gravity.
- Scattering in gravitational theories is constrained by infinite-dimensional symmetries.
    - ⇔ Soft theorems for scattering amplitudes
- 4D scattering enjoys Virasoro symmetry of 2D CFT's.
    - ⇒ Construction of celestial amplitudes, representing scattering of particles in boost eigenstates.
- Universal constraints on high and low-energy behavior of momentum space scattering amplitudes translate into powerful constraints on the analytic structure of celestial amplitudes.

- Focused on analytic structure in $\beta$. Expect interesting dependence on conformal cross ratio $z$

  (Apply conformal bootstrap techniques?)
- No Wilsonian UV/IR decoupling ⟶ What can we learn about UV completions by asking standard CFT questions?

EXTRA SLIDES BELOW

$Q_H = \int T_{uu}$

$Q_S \sim \int$

EXTRA SLIDES BELOW

$$Q_H^{(\hat{x})} = \int T_{uu}(\hat{x}) \qquad Q = Q_H + Q_S.$$

$$Q_S \sim \underbrace{\int \partial_t h_{mn}}_{\lim_{\omega \to 0} \omega a^+}$$

$$\underline{Q_S|\Omega\rangle} = |\Omega\rangle.$$