Title: New routes to topological order: Toric code order in Rydberg atoms and fractional Chern insulators in moire materials

Speakers: Ashvin Vishwanath

Date: February 22, 2021 - 12:30 PM

URL: http://pirsa.org/21020003

Abstract: Despite decades of theoretical work, the physical realization of topological order, outside of the fractional quantum Hall effect, has proved to be an elusive goal. Even the simplest example of a time-reversal symmetric topological order, as encountered in the paradigmatic toric code, awaits experimental realization. Key challenges include the lack of physically realistic models in these phases, and of ways to probe their defining properties. I will discuss a simple `Rydberg blockade' model, and describe numerical results that point to (i) a ground state with toric code topological order that could potentially be realized in experiment and (ii) ``smoking gun'' signatures of the phase which be accessed using a dynamic protocol. I will also briefly discuss how a topological qubit can be constructed in this platform by tuning boundaries as well as implications for constructing fault-tolerant quantum memories. Time permitting, a different platform for realizing exotic phases, magic-angle graphene and the special features of its band structure will be described, which make it a prime candidate for realizing fractional quantum Hall topological order even in the absence of a magnetic field.

References: arXiv:2011.12310. and arXiv:1912.09634

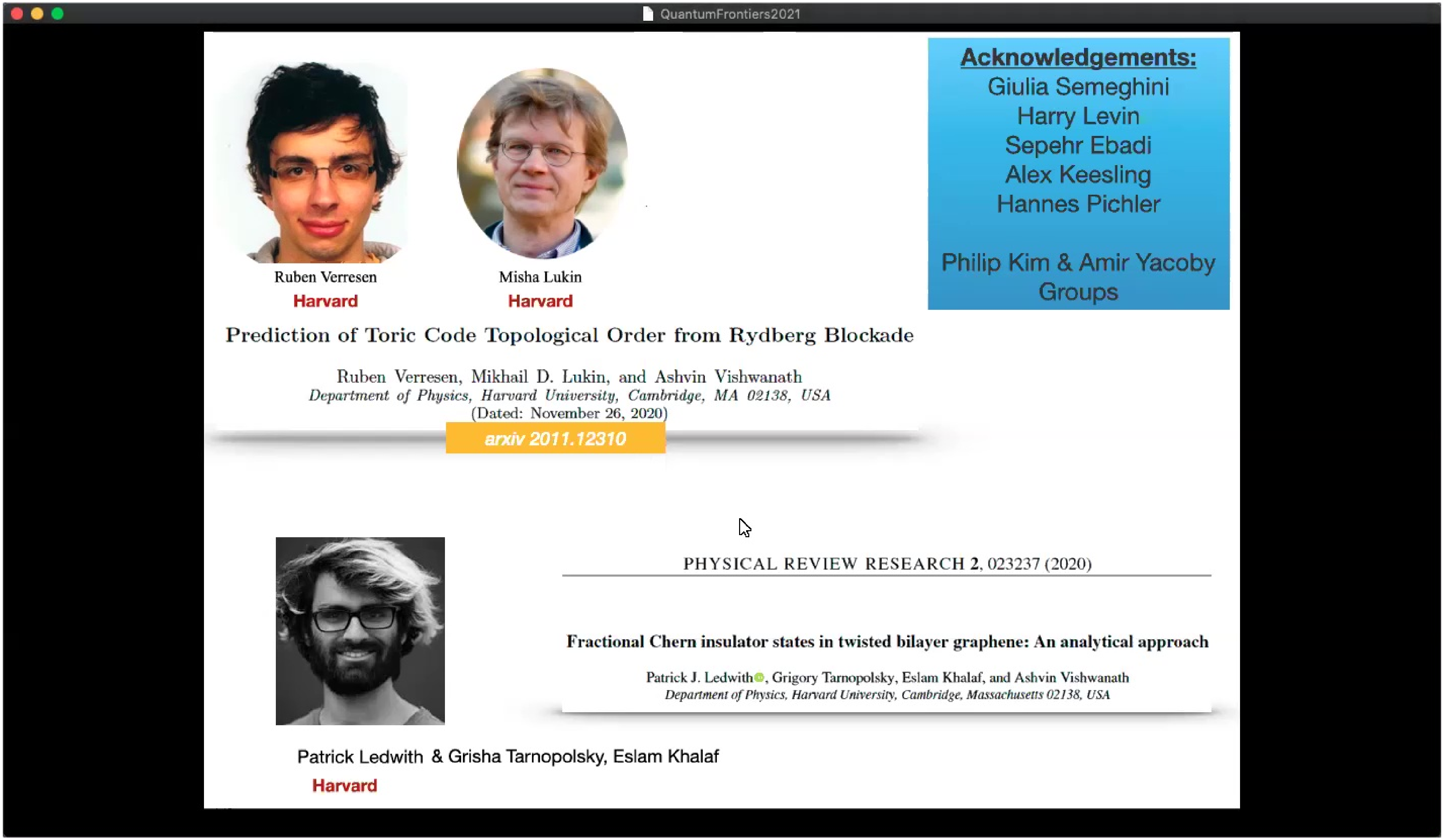
New routes to topological order:

Toric code in Rydberg atoms
and
Fractional Chern insulators in moire materials.

Ashvin Vishwanath

SIMONS FOUNDATION

Ultra-Quantum Matter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Ruben Verresen
Harvard
Misha Lukin
Harvard
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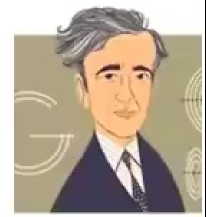
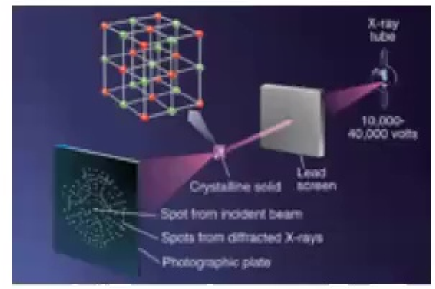
Sepehr Ebadi
Alex Keesling
Hannes Pichler
Philip Kim & Amir Yacoby Groups
Prediction of Toric Code Topological Order from Rydberg Blockade
Ruben Verresen, Mikhail D. Lukin, and Ashvin Vishwanath
Department of Physics, Harvard University, Cambridge, MA 02138, USA
(Dated: November 26, 2020)
arxiv 2011.12310
PHYSICAL REVIEW RESEARCH 2, 023237 (2020)
Fractional Chern insulator states in twisted bilayer graphene: An analytical approach
Patrick J. Ledwith, Grigory Tarnopolsky, Eslam Khalaf, and Ashvin Vishwanath
Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA
Patrick Ledwith & Grisha Tarnopolsky, Eslam Khalaf
Harvard

Symmetry Breaking States - Landau orders



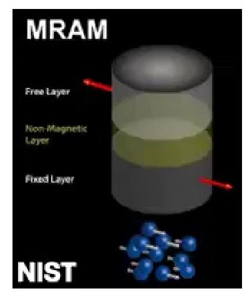
Landau Order Parameter: $\langle \varphi \rangle \neq 0$

(i) *Measure order parameter experimentally to diagnose a phase:*
Eg. X-rays on a Crystalline solid (Density(Q))



(ii) *Allows for a Classification of possible states*
Eg. 230 space groups of crystalline solids

(iii) *Many practical applications (eg. Magnetic memories):*



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1973:

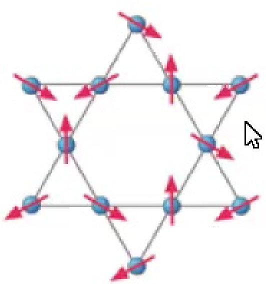
RESONATING VALENCE BONDS: A NEW KIND OF INSULATOR?*

P. W. Anderson
Bell Laboratories, Murray Hill, New Jersey 07974
and
Cavendish Laboratory, Cambridge, England

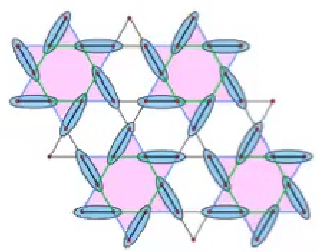
(Received December 5, 1972; Invited**)

ABSTRACT

The possibility of a new kind of electronic state is pointed out, corresponding roughly to Pauling's idea of "resonating valence bonds" in metals. As observed by Pauling, a pure state of this type would be insulating; it would represent an alternative state to the Néel antiferromagnetic state for $S = 1/2$. An estimate of its energy is made in one case.

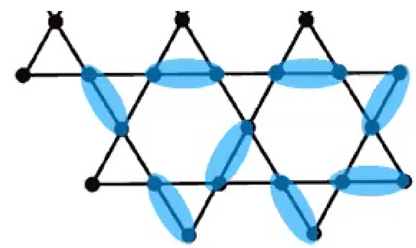


Neel order



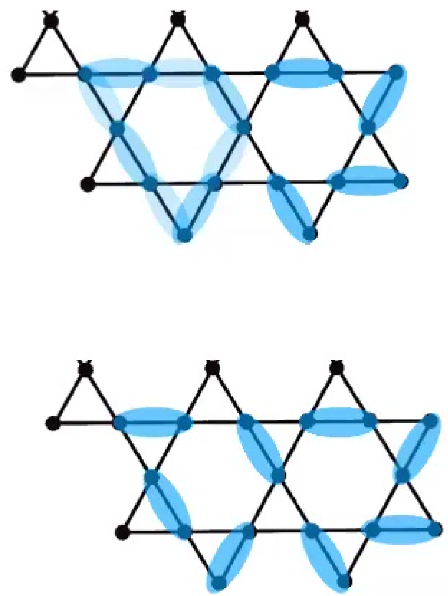
$\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$

Valence Bond Crystal



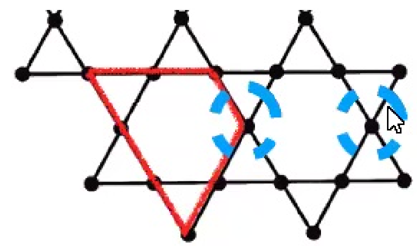
Resonating Valence Bonds

Condensates of Loops - Topological Order



Reference State

States represented by closed loops



$$\Omega = |\bigcirc\rangle + |\circ\;\circ\rangle + \ldots$$

Deconfined phase of an Emergent Gauge theory.

$$\nabla \cdot E = 0 \pmod{2}$$

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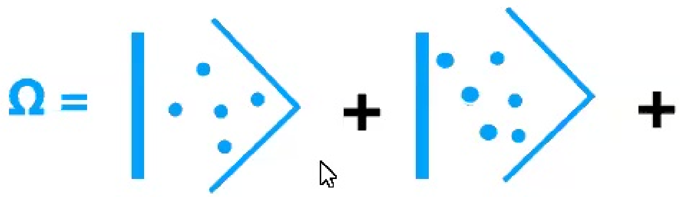
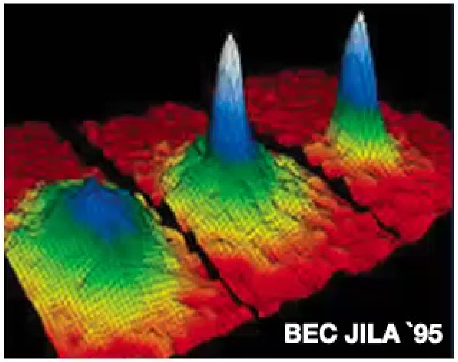
Particle Condensates - Symmetry Breaking Orders

Landau Order Parameter:

$$\langle \varphi \rangle \neq 0$$

Condensate of particles:



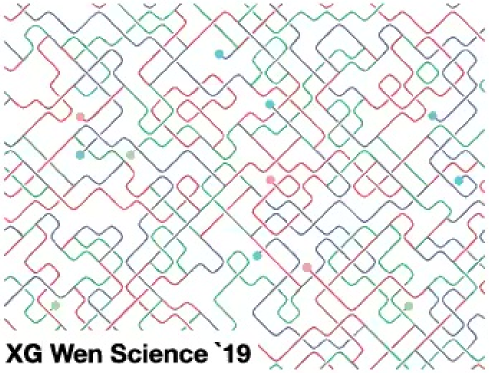


Beyond Landau Orders:

Topological Order

Condensate of closed loops:

$$\Omega = |\,\square\,\rangle + |\,{}^{\circ}{}_{\circ}\,\rangle + \dots$$



XG Wen Science `19

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Realization of Topological Order?

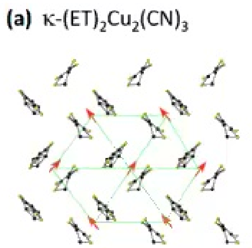
"Unfortunately, we have yet to unambiguously identify any topologically ordered phases in any real material (outside of the quantum Hall effect). This is a dire situation, and frankly is 'egg on the face' for the theoretical community."

– Matthew P. A. Fisher (~2010?)

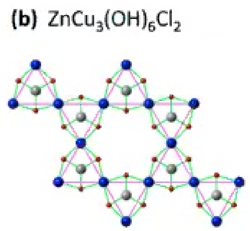
Topological Order

(Gapped) Quantum spin liquids?
Loop condensates

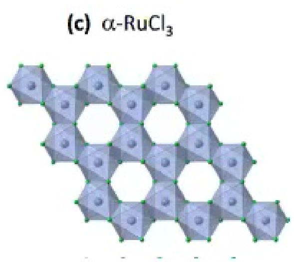
Promising candidates - but no clearcut example



Triangular Lattice Organic



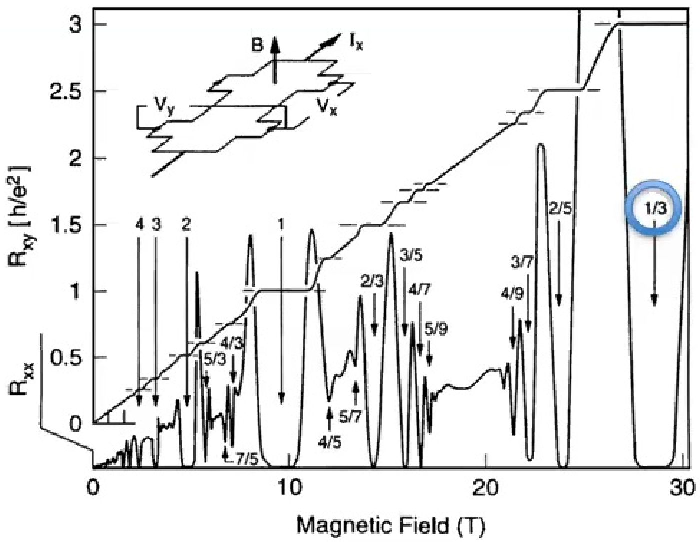
Kagome Lattice

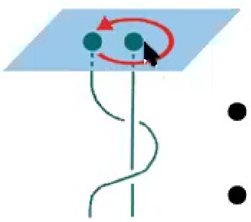


Kitaev-Honeycomb+ B field

From: Broholm et al. Science 2020

Fractional Quantum Hall





Gapped states with anyon excitations -

- Self statistics -unlike boson/ fermion
- *Mutual* statistics between excitations

Classical Orders versus Topological orders

Landau Order: $\langle \varphi \rangle \neq 0$

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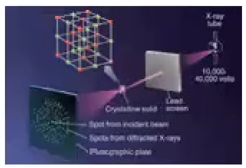

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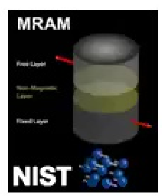
Eg. 230 space groups of crystalline solids

(iii) Many practical applications (eg. Magnetic memories):

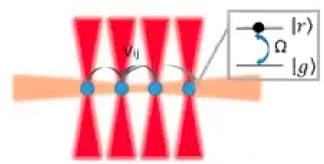


Topological Order:

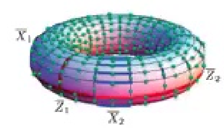
- How to measure?

(beyond Quantum Hall - edge states and quantized conductance)

- New Platforms for realization?



- quantum memories/computing



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Classical Orders versus Topological orders

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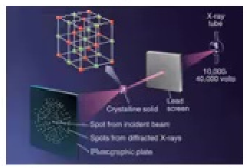
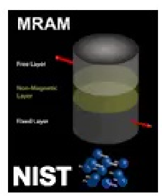


(ii) Allows for a Classification of possible states
Eg. 230 space groups of crystalline solids
All are realized in nature!

(iii) Many practical applications (eg. Magnetic memories):



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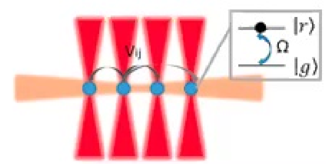

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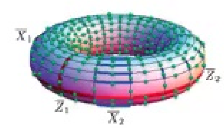
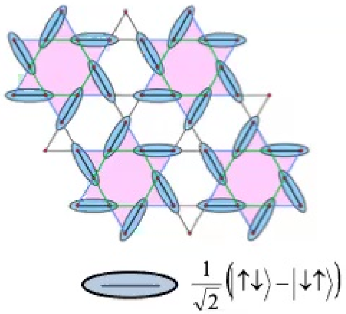
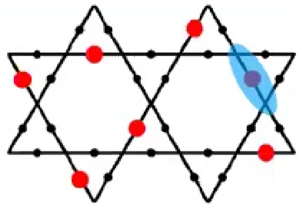


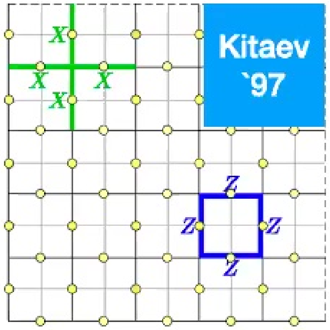
Explicit Realization of Gauge Fields

- Seek NEW platforms and models where gauge fields are *explicit* microscopic degrees of freedom eg. dimer model/ toric code





Allowed state



Toric code -
With 4 body interactions

Wegner `71; Fradkin&Shenker `79
Rokhsar and Kivelson `88; Read&Sachdev `90; Sachdev `92; Wen `91
Moessner and Sondhi 2000; Misguich, Serban and Pasquier 2001
Balents Fisher Girvin `01; Motrunich Senthil `02

Realization in Rydberg atom Array?

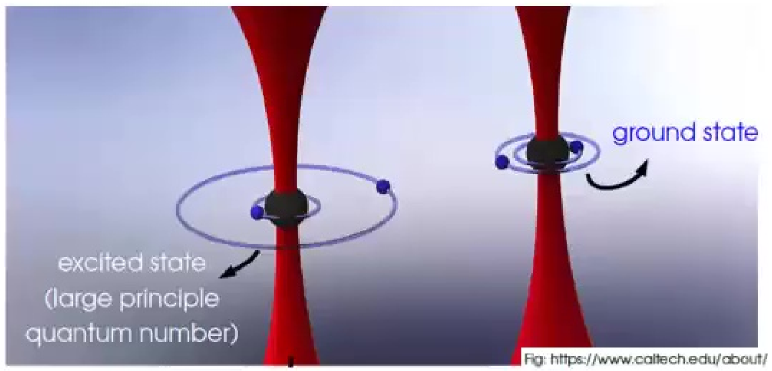
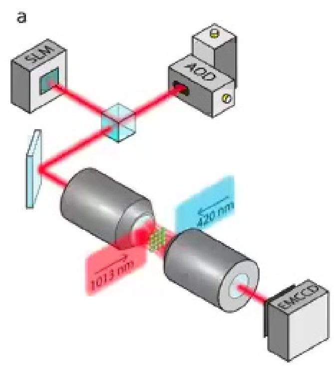
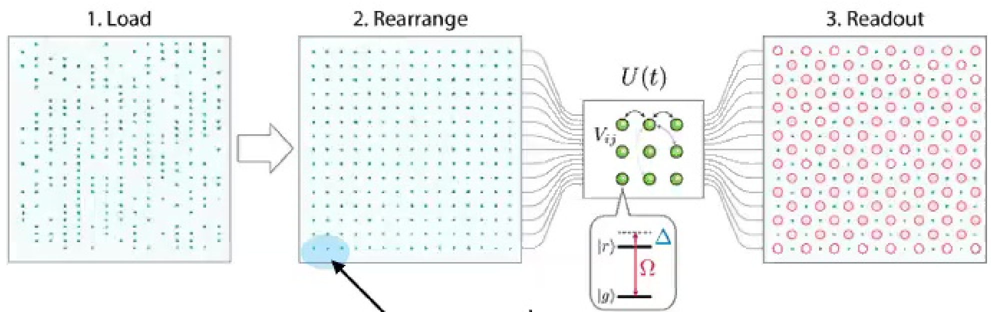
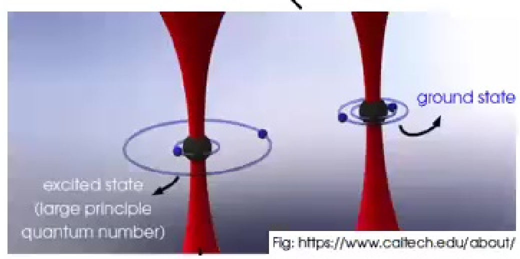


Fig: https://www.caltech.edu/about/

Rydberg atom array





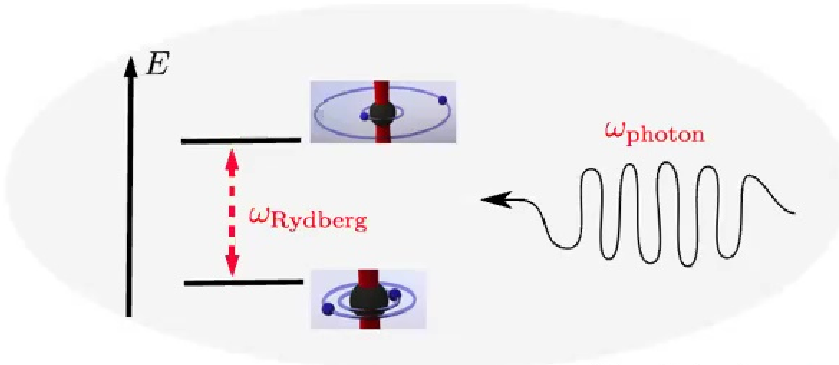


optical tweezer (traps atom)

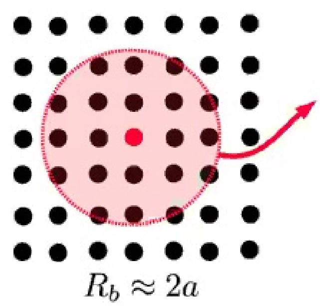
Ebadi et al. arXiv:2012.12281
Scholl et al. arXiv:2012.12268
Samajdar et al. PRL `20
Samajdar et al. PNAS `21
Ruben Verresen, Lukin, AV arxiv 2011.12310

Quantum number n~70 of Rb atom - large van-der-Waals interactions

Rydberg Hamiltonian



Described by $H = \frac{\Omega}{2}\sigma^x - \frac{\delta}{2}\sigma^z$ with
- Ω = Rabi frequency
- $\delta = \omega_{\text{photon}} - \omega_{\text{Rydberg}}$ = "detuning"



two nearby atoms cannot both be excited into Rydberg state

= Rydberg blockade mechanism

Jaksch et al., PRL (2000); Lukin et al., PRL (2001)

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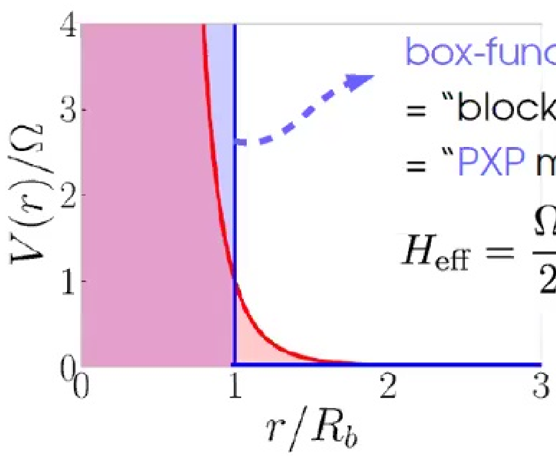
Rydberg Hamiltonian

$$n_i := \frac{\sigma_i^z + 1}{2}$$

$$H = \frac{\Omega}{2} \sum_i \sigma_i^x - \delta \sum_i n_i + \frac{1}{2} \sum_{i,j} V(|\vec{r}_i - \vec{r}_j|) n_i n_j$$

van der Waals: $V(r) \sim \left(\frac{R_b}{r}\right)^6$ $\quad R_b = \text{blockade radius}$



box-function approximation

= "blockade model"

= "PXP model" (see literature on 1D quantum scars)

$$H_{\text{eff}} = \frac{\Omega}{2} \sum_i P \sigma_i^x P - \delta \sum_i n_i$$

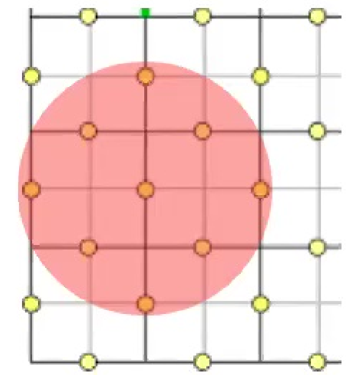
projects out double-occupation in blockade radius

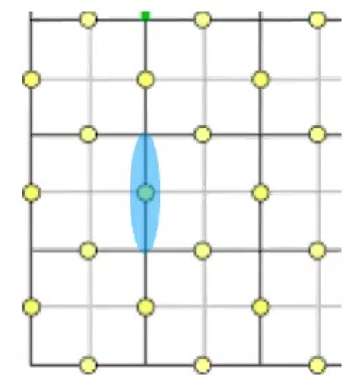
Dimer Model From Blockade?

$$H = \frac{\Omega}{2}\sum_i P\sigma_i^x P - \delta\sum_i n_i$$

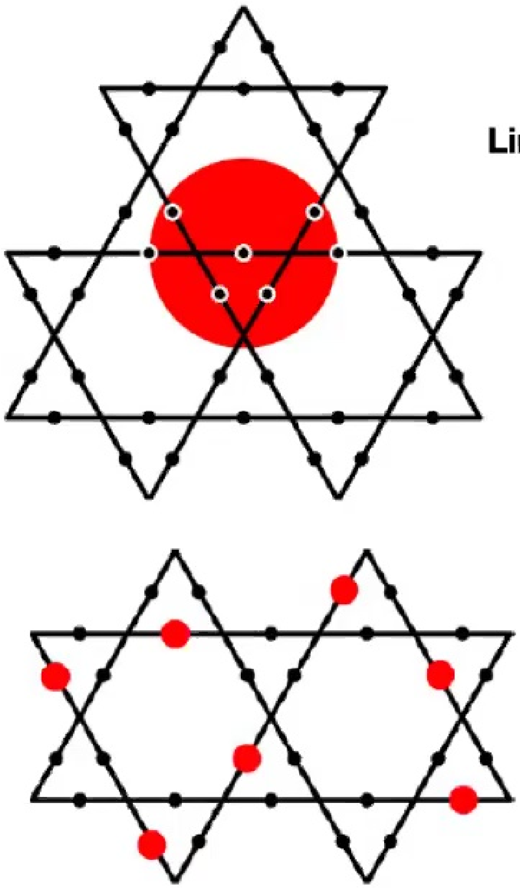
projects out double-occupation

Within radius R_b





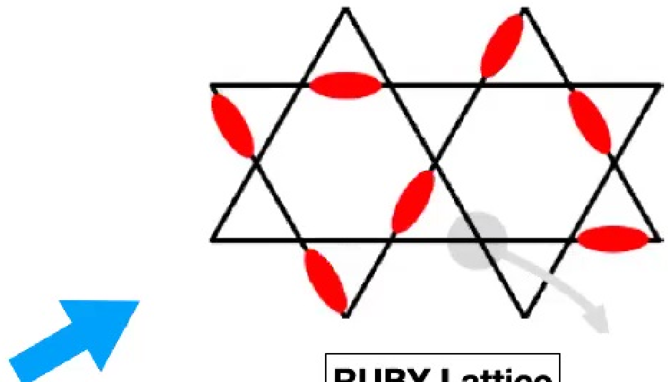
Dimer Model From Blockade



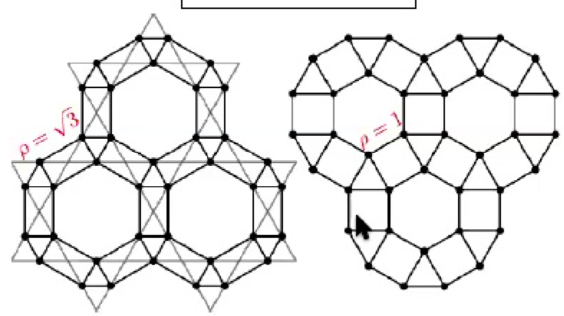
$$H = \frac{\Omega}{2}\sum_i P\sigma_i^x P - \delta \sum_i n_i$$

Links of the Kagome

projects out double-occupation



RUBY Lattice



Other 2D lattices: *Samajdar et al. PRL `20. Samajdar* et al. PNAS `21

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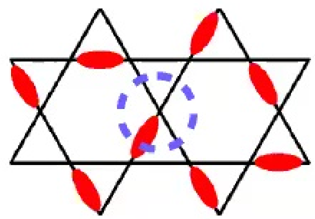
Z_2 topological order?

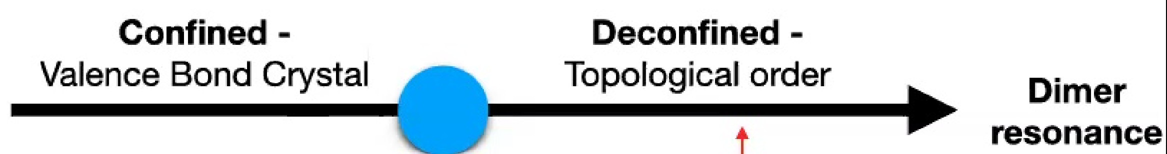
- Hilbert space - Z_2 gauge theory

The hardcore dimer constraint = Gauss law $\nabla \cdot E = 1 \mod 2$

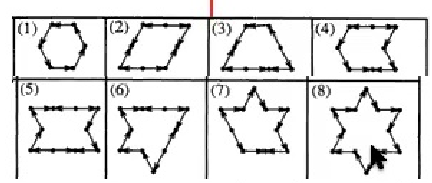


The parity around any vertex = -1

- *Dynamics* - deconfinement?



<u>Exact Solution:</u>
32 resonance terms with equal weight
Misguich, Serben, Pasquier '01
Zeng & Elser '95



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Pure Dimer Model

$$H = \frac{\Omega}{2}\sum_i P\sigma_i^x P - \delta \sum_i n_i$$

$\delta \to +\infty$: degenerate dimer states

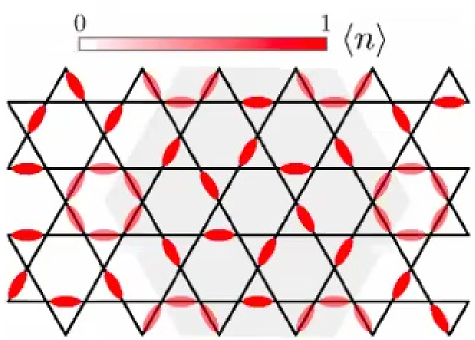
perturbation theory gives:

$$H_{\text{eff}} = -\frac{3\Omega^6}{32\delta^5}\sum_{\hexagon}\left(|\hexagon\rangle\langle\hexagon| + h.c.\right)$$

$$\frac{\Omega}{\delta} \to 0$$

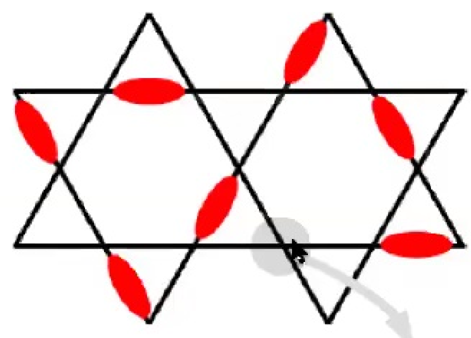
… Crystal

12 unitcells



(Nikolic and Senthil, Singh and Huse, …)

$\delta \downarrow$ **Introduce vacancies - Topological order?**



Dimer state with one monomer

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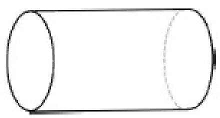
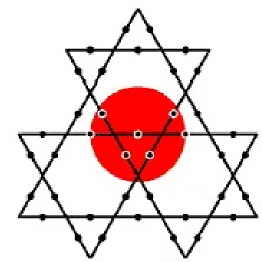
Phase Diagram of Blockade Model



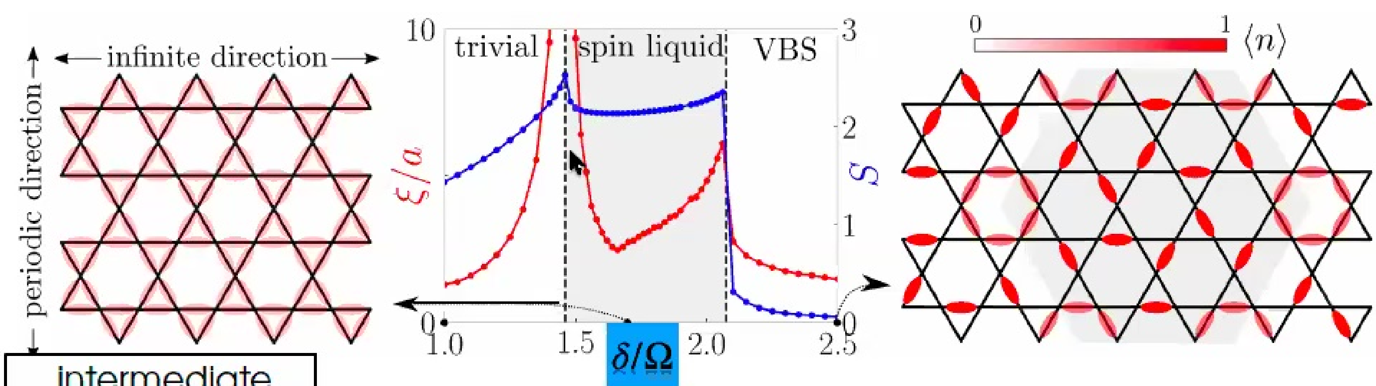
$$H = \frac{\Omega}{2}\sum_i P\sigma_i^x P - \delta \sum_i n_i$$



we put the model on an infinitely-long cylinder

→ use density matrix renormalization group (DMRG)

(White '92, Stoudenmire '13, Hauschild '18)



intermediate featureless phase

Ruben Verresen, Lukin, AV arxiv 2011.12310

→ numerical confirmation of spin liquid

* topological entanglement entropy
* topological string operators
* ground state degeneracy and modular matrices

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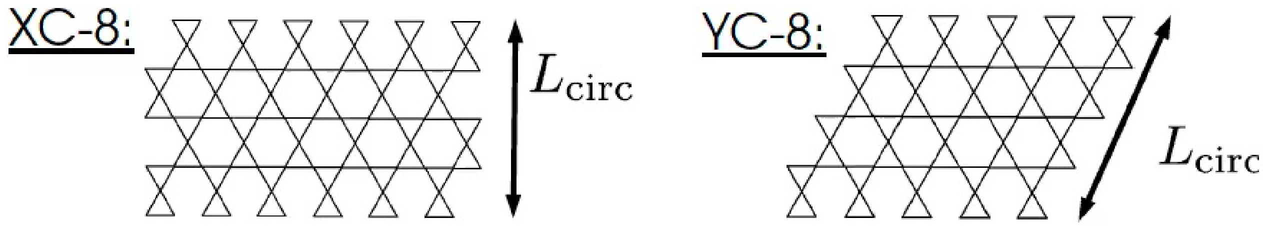
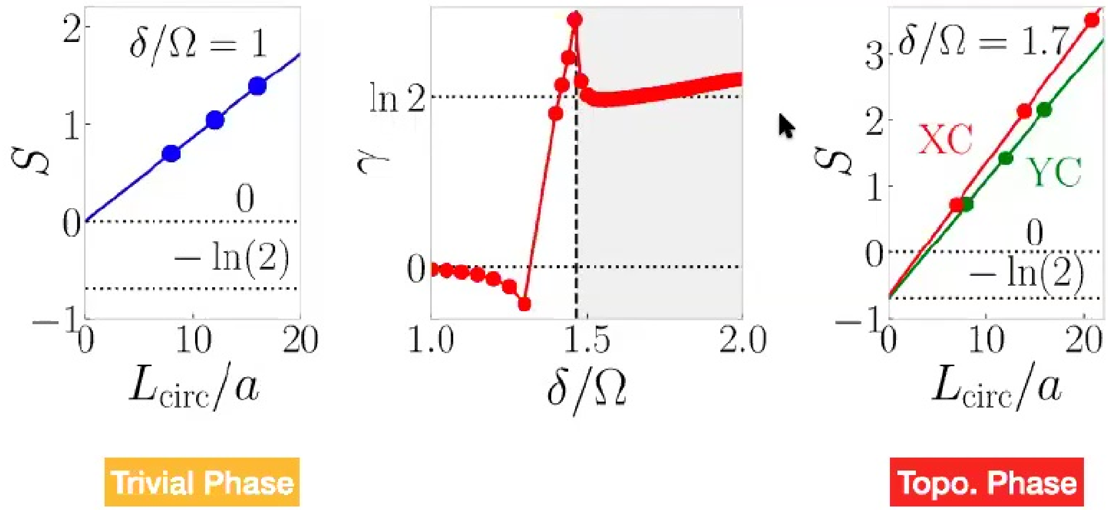
1.Topological Entanglement Entropy

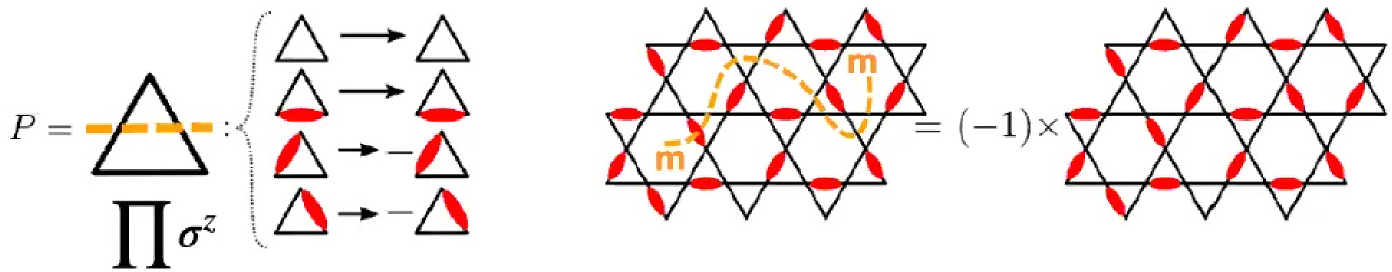
$$S_{\text{ent}} = \alpha L_{\text{circ}} - \gamma$$ (Kitaev and Preskill, '05)



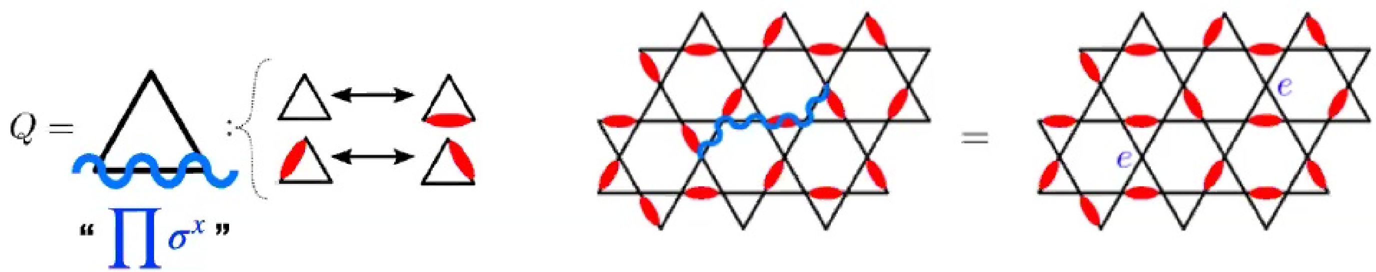


Line Operators (BFFM)

't Hooft line operator $e^{i\pi \int E}$ is diagonal in the dimer basis



Wilson line operator $e^{i \int A}$ has to anticommute → off-diagonal



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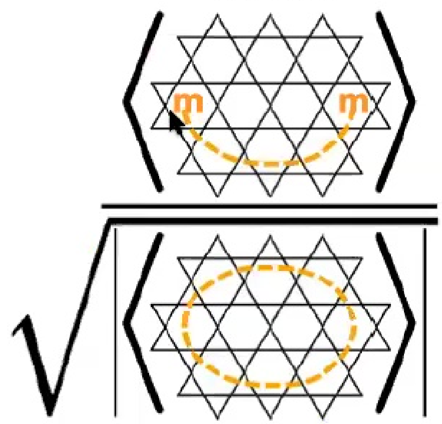
Line Operators (BFFM)

confined phase (= VBS)
= condensate of m-anyons

Higgs phase
(= trivial phase)
= condensate of e-anyons

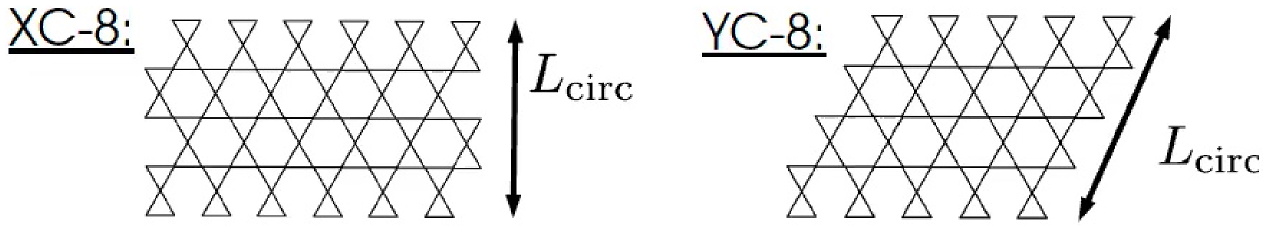
$\langle P \rangle_{\mathrm{BFFM}} =$

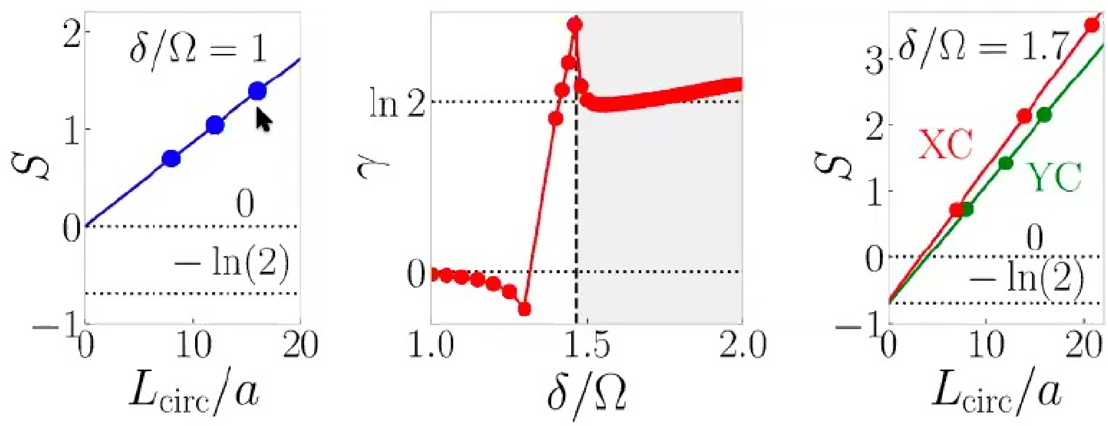


(Bricmont and Frohlich '83; Fredenhagen and Marcu '83)
K. Gregor et al. `10

1.Topological Entanglement Entropy

$$S_{\text{ent}} = \alpha L_{\text{circ}} - \gamma$$ (Kitaev and Preskill, '05)





Trivial Phase

Topo. Phase

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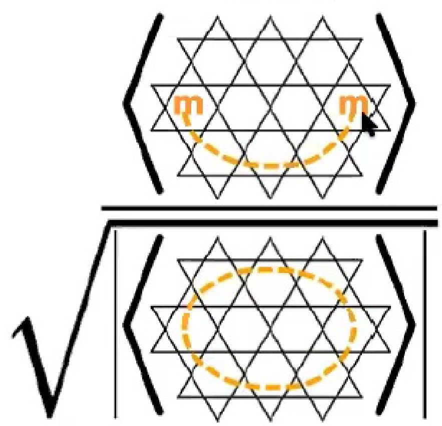
Line Operators (BFFM)

confined phase (= VBS)
= condensate of m-anyons

Higgs phase
(= trivial phase)
= condensate of e-anyons

$\langle P \rangle_{\mathrm{BFFM}} =$



(Bricmont and Frohlich '83; Fredenhagen and Marcu '83)
K. Gregor et al. `10

Diagnosing Phases

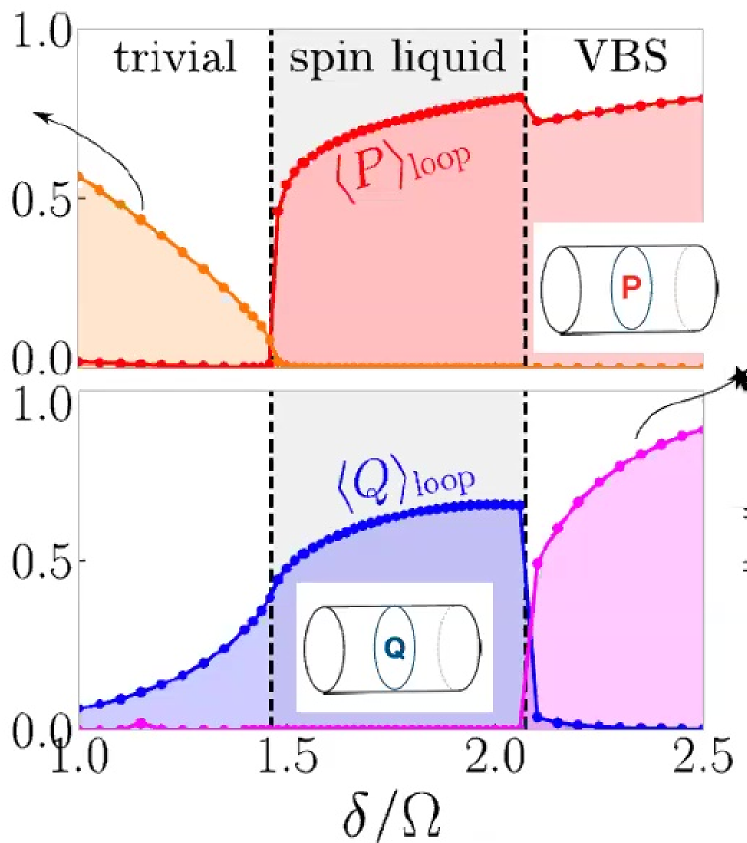
$\langle Q \rangle_{\mathrm{BFFM}} \neq 0$

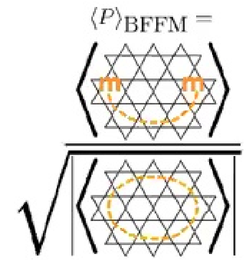
↓

trivial phase
$= e$-condensate
$=$ Higgs phase

↓

suppresses $\langle P \rangle$





$\langle P \rangle_{\mathrm{BFFM}} \neq 0$

↓

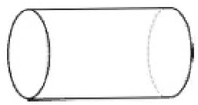
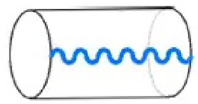
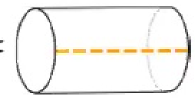
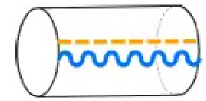
VBS phase
$= m$-condensate
$=$ confined phase

↓

suppresses $\langle Q \rangle$

Modular Matrices

An infinitely-long cylinder has four topological ground states

$|1\rangle =$  $|e\rangle =$  $|m\rangle =$  $|f\rangle =$ 

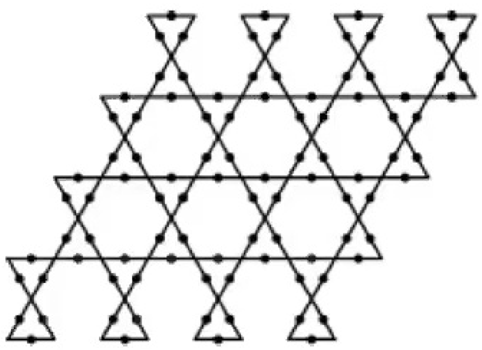
Can be diagnosed by P and Q strings around circumference

Yi Zhang et al. `12
Cincio & Vidal `12

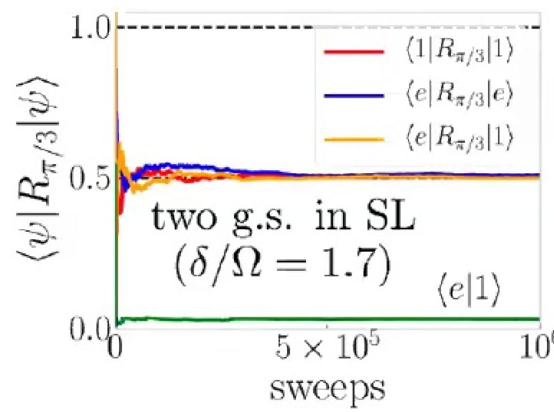
$$\begin{pmatrix} \langle 1|R_{\pi/3}|1\rangle & \langle 1|R_{\pi/3}|e\rangle \\ \langle e|R_{\pi/3}|1\rangle & \langle e|R_{\pi/3}|e\rangle \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

(XC-8)

$$\frac{m,f}{1,e} \approx 0.0001\Omega$$



(YC-8)



Towards an experimental realization

→ stability of spin liquid to van der Waals interactions

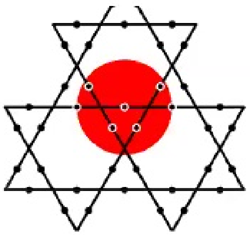
→ how to measure

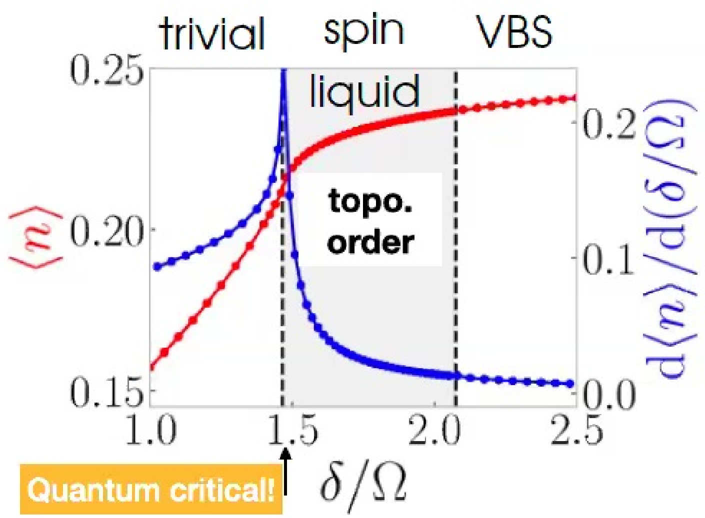
Dimer density signature

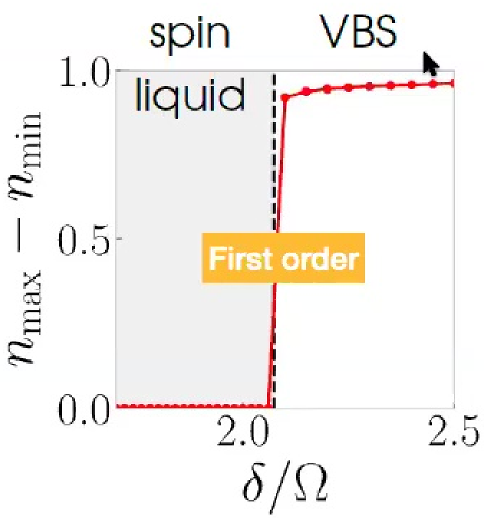
$$H = \frac{\Omega}{2}\sum_i P\sigma_i^x P - \delta \sum_i n_i$$

dimer state: $\langle n \rangle = 1/4$

with monomers: $\langle n \rangle < 1/4$







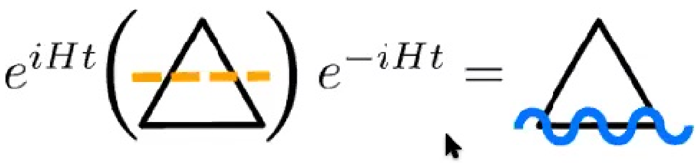
Ebadi et al. arXiv:2012.12281

Ruben Verresen, Lukin, AV arxiv 2011.12310

Measuring String Operators?

→ Diagonal string P measured from snapshots of atoms

→ Off-diagonal string Q can be reduced to P after rotation



for $\Omega t = \frac{4\pi}{3\sqrt{3}}$

using $H = \frac{\Omega}{2} \sum_i P \sigma_i^y P$ where P only enforces blockade inside triangles!

Measuring String Operators?

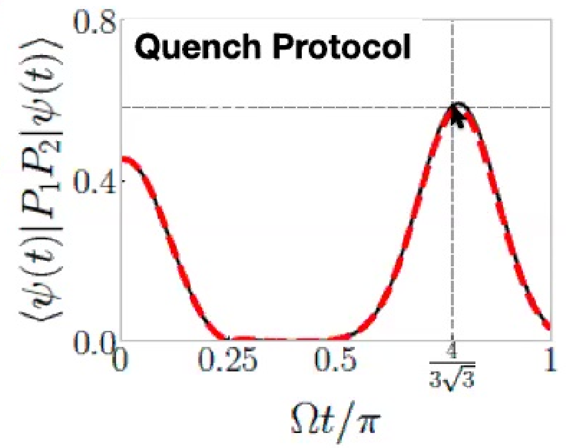
→ Diagonal string P measured from snapshots of atoms

→ Off-diagonal string Q can be reduced to P after rotation

$$e^{iHt}\left(\triangle\right)e^{-iHt} = \wedge \quad \text{for } \Omega t = \frac{4\pi}{3\sqrt{3}}$$

using $H = \frac{\Omega}{2}\sum_i P\sigma_i^y P$

b



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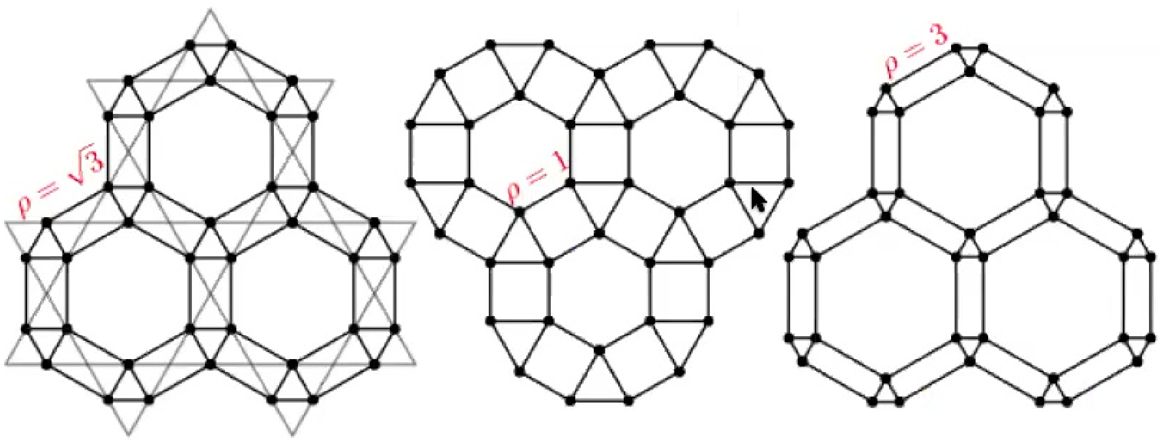
Spin liquid with van der Waals Interactions

$$H = \frac{\Omega}{2}\sum_i \sigma_i^x - \delta \sum_i n_i + \frac{1}{2}\sum_{i,j} V(|\vec{r}_i - \vec{r}_j|) n_i n_j$$

$$V(r) = \Omega \times \left(\frac{R_b}{r}\right)^6$$

Ruby lattices with $\rho > 1/\sqrt{2}$
all give same blockade model



QuantumFrontiers2021

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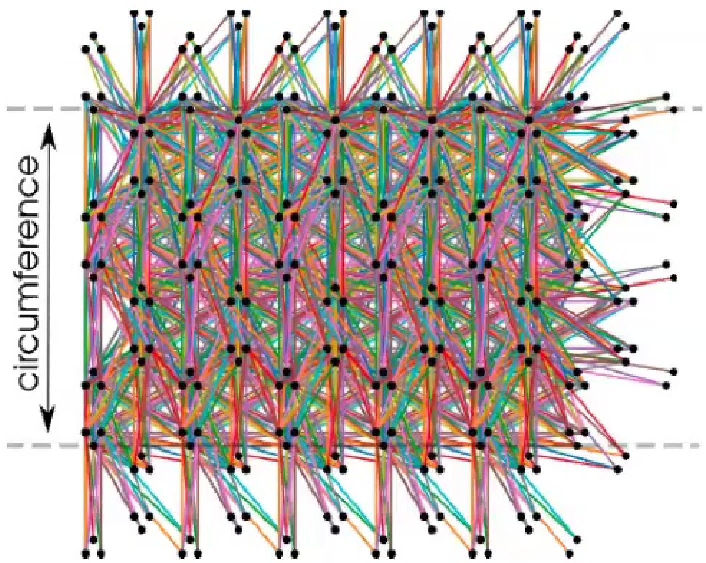
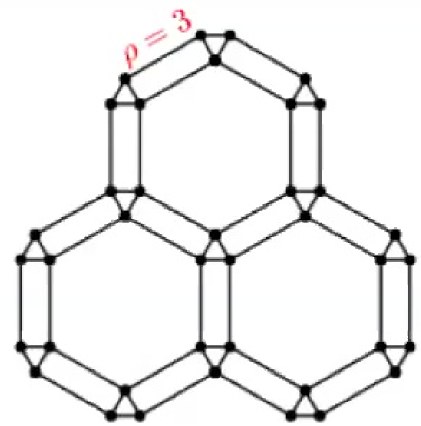


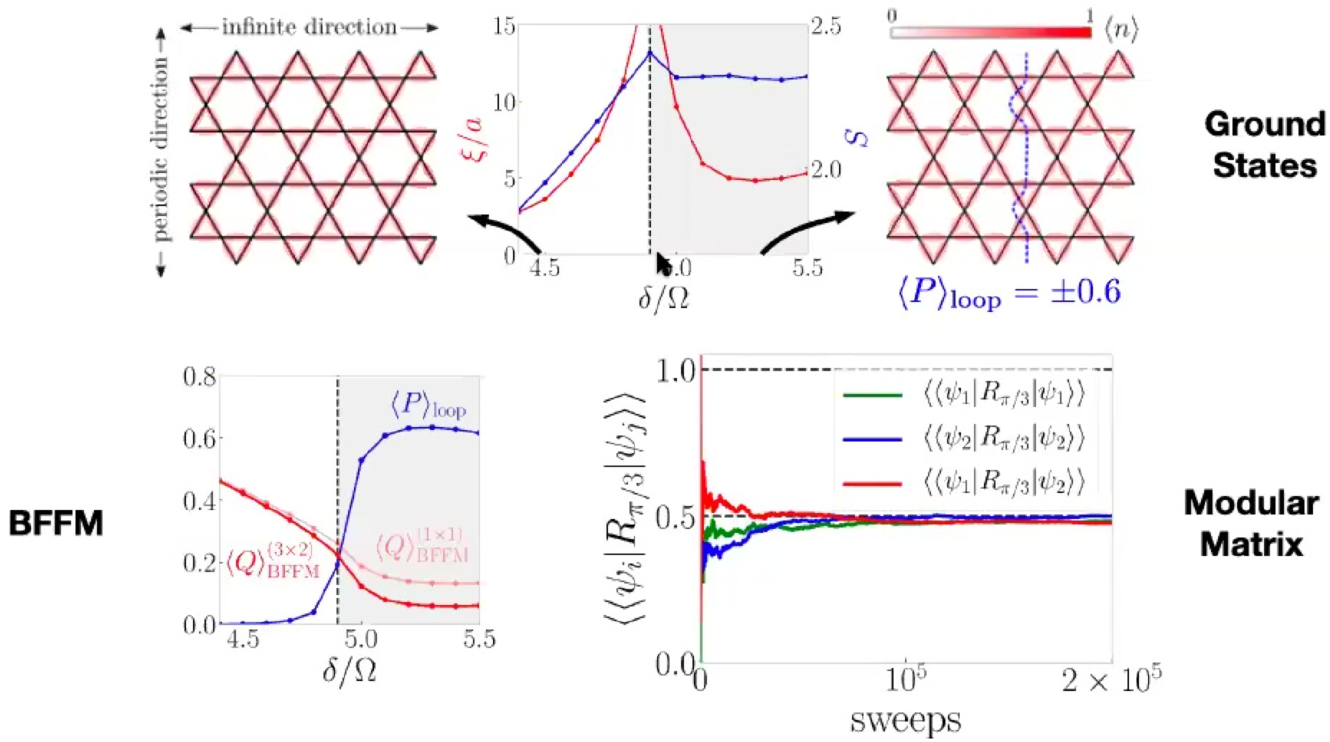


Retain: r_1, r_2, r_3, r_4, ... r_{16}

Each atom couples to 44 other sites

Spin liquid with van der Waals Interactions

$V(r) = \Omega(R_b/r)^6$ with $R_b = 3.8a$ on ruby lattice with $\rho = 3$



Ruben Verresen, Lukin, AV arxiv 2011.12310

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Towards a topological qubit

→ capturing anyons

→ topological boundary conditions

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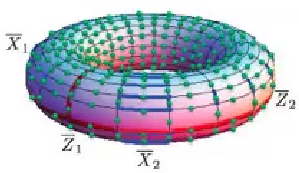
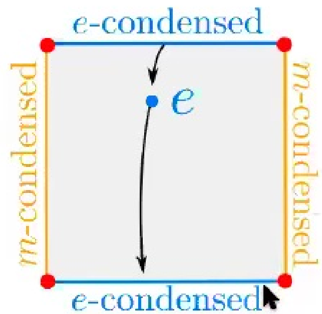
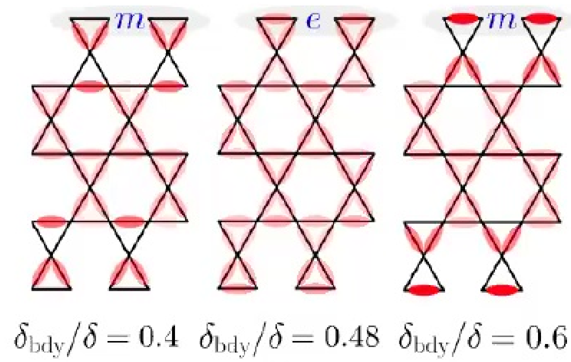
Towards Topological Quantum Bit

Alexei Kitaev

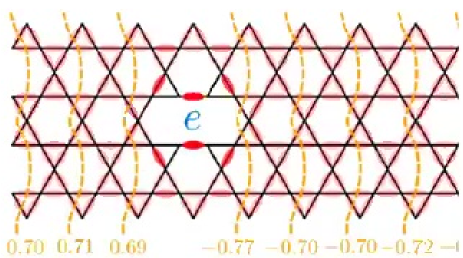


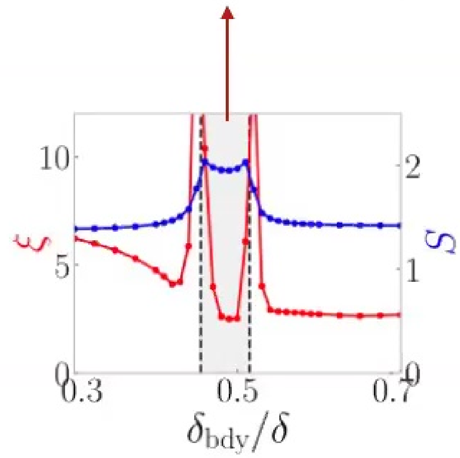


Tune boundaries
Two-fold degenerate



Trapping an anyon:





Boundary phase transition
On changing *detuning* δ

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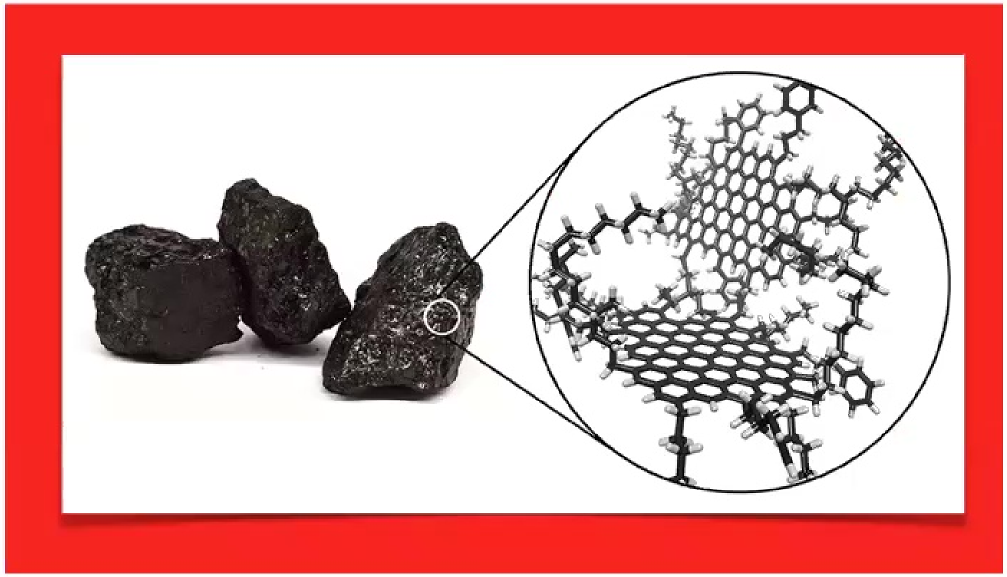
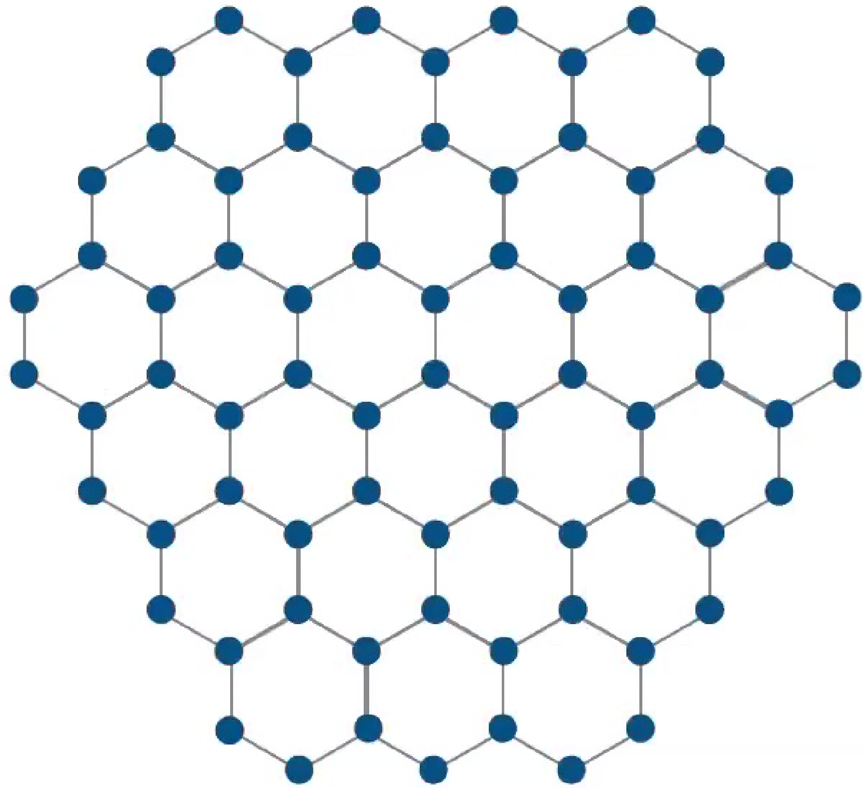


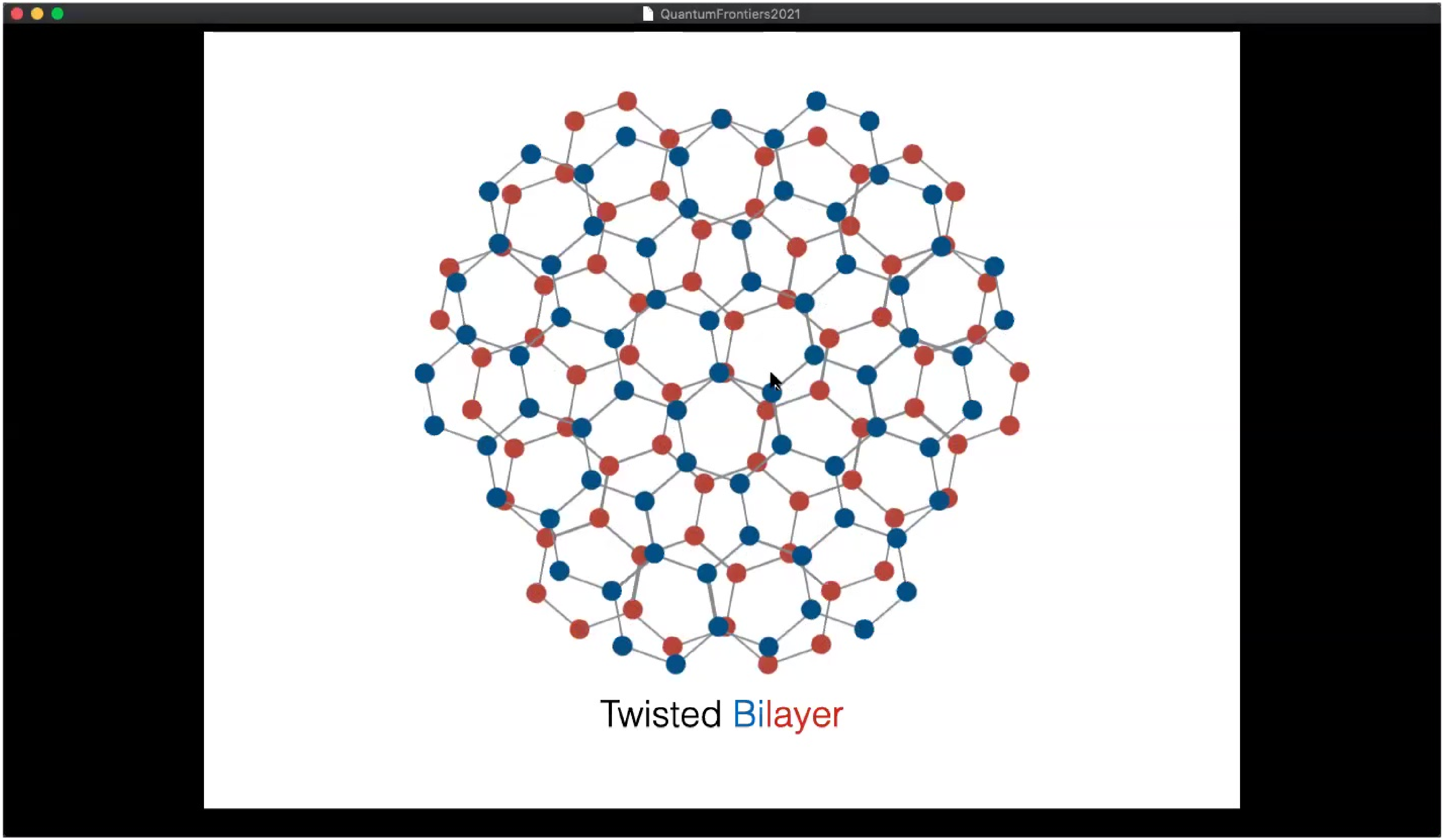

PHYSICAL REVIEW RESEARCH **2**, 023237 (2020)

Fractional Chern insulator states in twisted bilayer graphene: An analytical approach

Patrick J. Ledwith, Grigory Tarnopolsky, Eslam Khalaf, and Ashvin Vishwanath
Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA



Bilayer

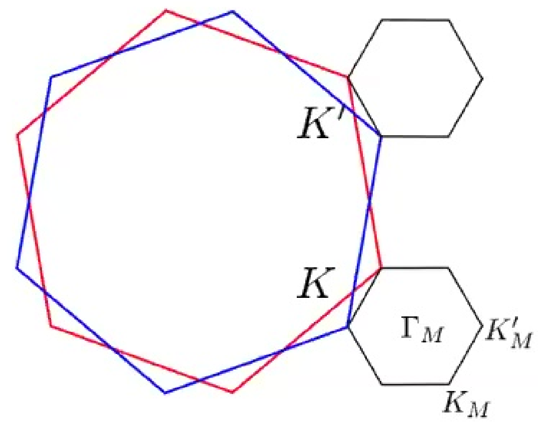

QuantumFrontiers2021
Twisted Bilayer

QuantumFrontiers2021

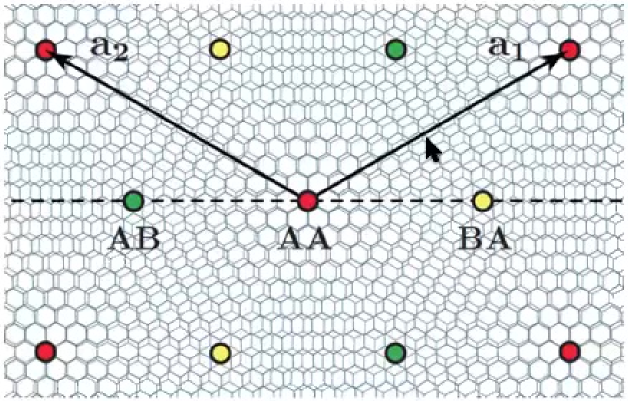
Continuum model

- Larger unit cell → smaller BZone
- Bistrizer-Macdonald (BM) model (2011)

$$\mathcal{H}_K = \begin{pmatrix} -iv_F \boldsymbol{\sigma}_{\theta/2} \cdot \nabla & T(\boldsymbol{r}) \\ T^\dagger(\boldsymbol{r}) & -iv_F \boldsymbol{\sigma}_{-\theta/2} \cdot \nabla \end{pmatrix}_{12},$$



- Lattice relaxation: AB stacking favored to AA stacking (Carr et al. '19, Nam, Koshino `17) $\implies w_0/w_1 \approx 0.7$

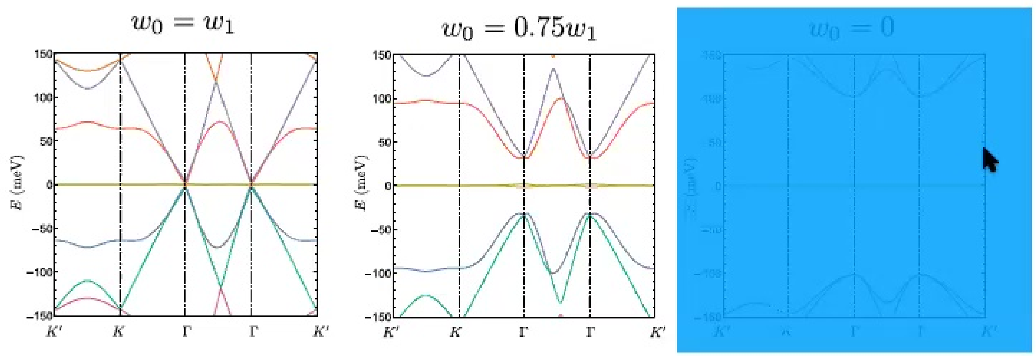


Chiral Model

Tarnopolski, Kruchkov, AV
PRL 2019

Switch off AA coupling. Only AB coupling

$$T(\boldsymbol{r}) = \begin{pmatrix} w_0 U_0(\boldsymbol{r}) & w_1 U(\boldsymbol{r}) \\ w_1 U^*(-\boldsymbol{r}) & w_0 U_0(\boldsymbol{r}) \end{pmatrix}$$



Chiral Symmetry

$\{\sigma_z \otimes 1, \mathcal{H}\} = 0$

sublattice

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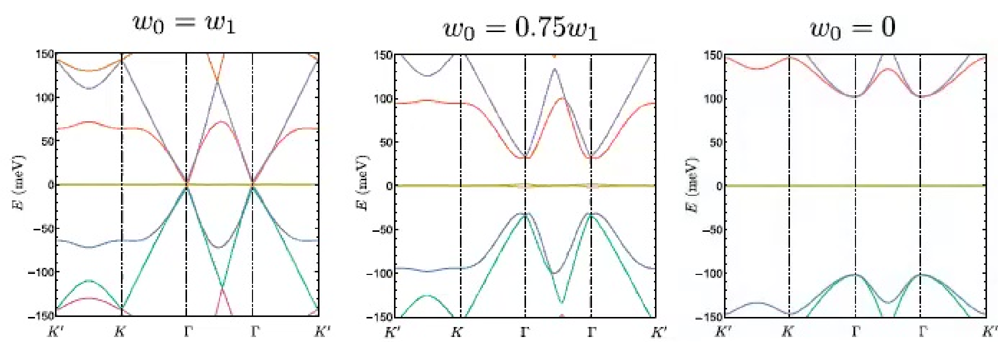


Chiral Symmetry

$\{\sigma_z \otimes 1, \mathcal{H}\} = 0$

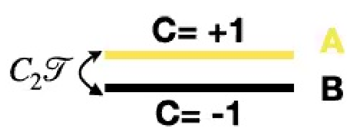
sublattice

Chiral Limit Wavefunctions

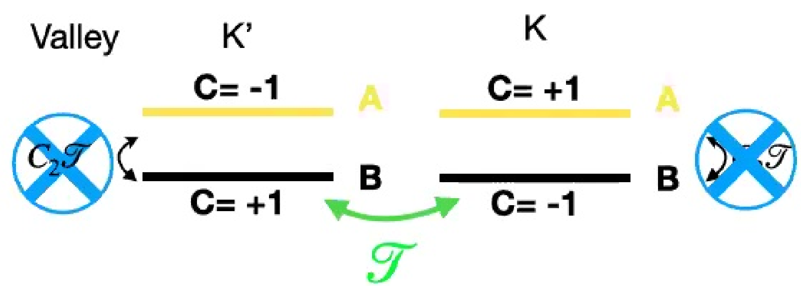
Many special properties:

(i) Chern number+ sublattice polarized

$$\psi_A(x,y) = \frac{\psi_K(x,y)}{\theta_1(\frac{z-z_0}{a_1}|\omega)} f(x+iy)$$



Chern =±1;
Sublattice polarized



Ledwith, Tarnopolsky, Khalaf, AV `20
Wang, Zheng, Millis, Cano `20
Becker, Embree, Wittsten, Zworski `20

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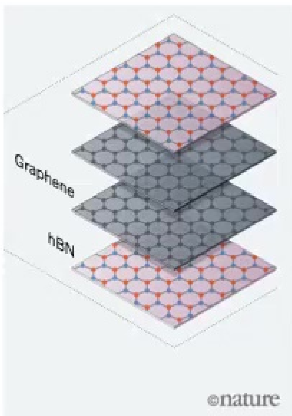
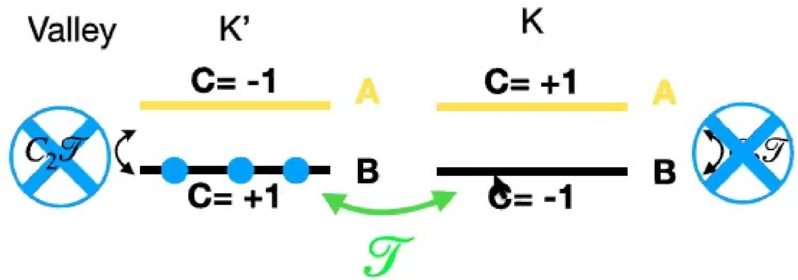
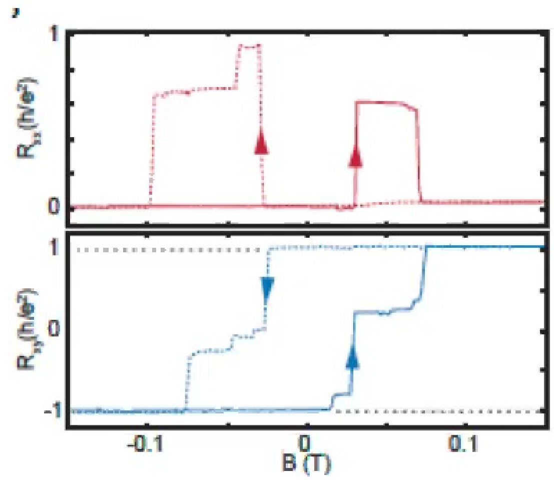
Intrinsic quantized anomalous Hall effect in a moiré heterostructure

M. Serlin,[1,*] C. L. Tschirhart,[1,*] H. Polshyn,[1,*] Y. Zhang,[1] J. Zhu,[1] K. Watanabe,[2] T. Taniguchi,[2] L. Balents,[3] and A. F. Young[1,†]







Aligned h-BN substrate.
Break A-B symmetry

A. Sharpe…D. G-Gordon (2019)
M. Serlin… Young (2019)

QuantumFrontiers2021

Chiral Limit Wavefunctions

Many special properties:

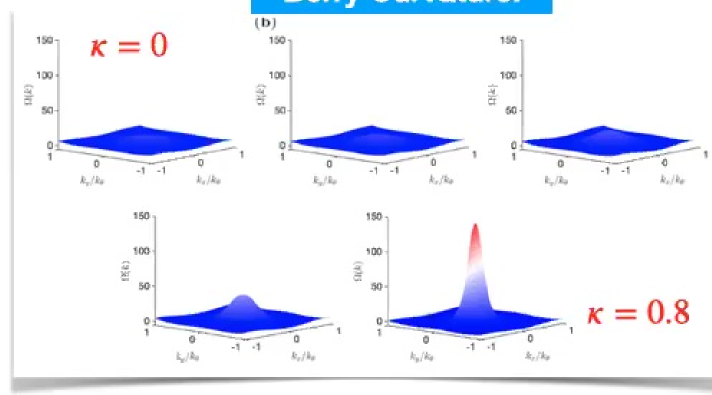
Ledwith, Tarnopolsky, Khalaf, AV `20

(i) Bloch wavefunctions - *analytic* in $k = k_x + ik_y$

$$u_{\mathbf{k}}(\mathbf{r}) = e^{-2\pi i r_2 k/b_2} \frac{\vartheta_1\left(\frac{z-z_0}{a_1} - \frac{k}{b_2}|\omega\right)}{\vartheta_1\left(\frac{z-z_0}{a_1}|\omega\right)} \psi_K(\mathbf{r})$$

(ii) nearly *uniform* Berry curvature

Berry Curvature:



(iii) Isotropic ideal droplet condition -

Ideal for Fractional Chern insulators

quantum metric:

$$\eta(k) = \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} \Omega(k)$$

Roy Phys. Rev. B **90**, 165139 (2014)
Claassen et. al. Phys. Rev. Lett. **114**, 236802 (2015)
Jackson, Möller, Roy Nat. Comm. **6** 8629 (2015).
Ledwith, Tarnopolsky, Khalaf, AV `20

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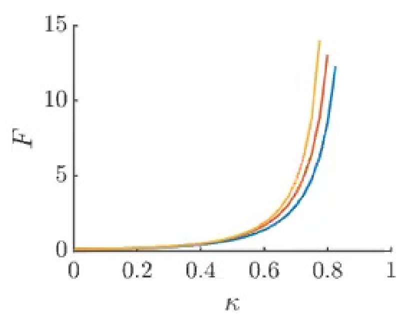
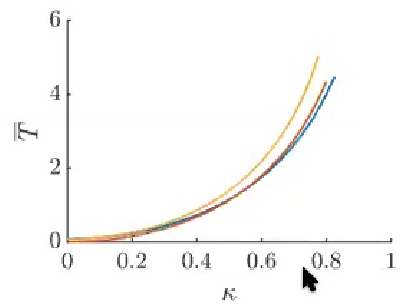
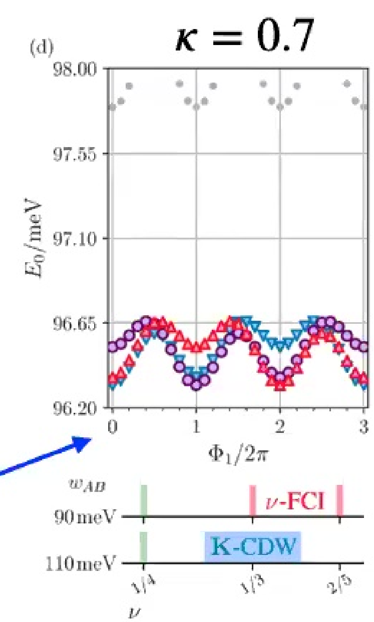
AA/BB Tunneling => non-ideal band geometry

- Berry curvature variation: $F = \left\langle \left(\frac{\Omega}{2\pi} - C \right)^2 \right\rangle_{BZ}$

- Violation of ideal droplet condition: $\bar{T} = \left\langle \mathrm{tr}\, g - |\Omega| \right\rangle_{BZ}$







- FCI states often still appear in exact diagonalization

Abouelkomsan, Liu, Bergholtz Phys. Rev. Lett. **124** 106803 (2020)
Repellin, Senthil Phys. Rev. Research. **2** 023238 (2020)
Wilhelm, Lang, Läuchli arXiv:2012.09829 (2020)

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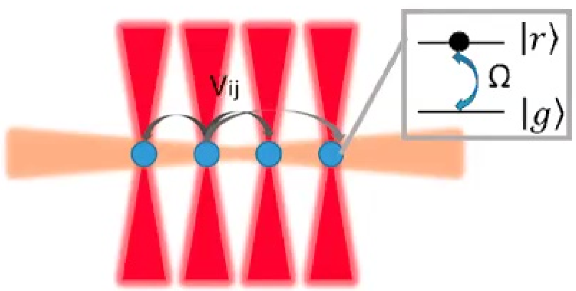
Conclusions:

Topological order, emergent gauge fields, non-local quantum entanglement could be more widespread than previously thought.

Need to explore new platforms and detection schemes.

Rydbergs:
Low symmetry
Long range interactions
Tunability



Bernien *et al.*

Magic angle graphene:
Remarkable (and mysterious) similarity to lowest Landau level