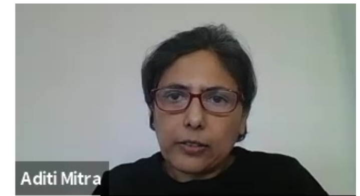
Title: Floquet spin chains and the stability of their edge modes

Speakers: Aditi Mitra
Date: February 15, 2021 - 12:30 PM

URL: http://pirsa.org/21020002

Abstract: In this talk I will begin by introducing symmetry protected topological (SPT) Floquet systems in 1D. I will describe the topological invariants that characterize these systems, and highlight their differences from SPT phases arising in static systems. I will also discuss how the entanglement properties of a many-particle wavefunction depend on these topological invariants. I will then show that the edge modes encountered in free fermion SPTs are remarkably robust to adding interactions, even in disorder-free systems where generic bulk quantities can heat to infinite temperatures due to the periodic driving. This robustness of the edge modes to heating can be understood in the language of strong modes for free fermion SPTs, and almost strong modes for interacting SPTs.

I will then outline a tunneling calculation for extracting the long lifetimes of these edge modes by mapping the Heisenberg time-evolution of the edge operator to dynamics of a single particle in Krylov space.



Floquet spin chains and the stability of their edge modes

Aditi Mitra
New York University

U.S. DEPARTMENT OF ENERGY | Office of Science

NSF

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Dr. Daniel Yates
(NYU PhD student)

Fabian Essler, Oxford
Sasha Abanov, Stony Brook
Dr. Yonah Lemonik, NYU

Daniel J. Yates, Alexander G. Abanov, and Aditi Mitra, *Dynamics of almost strong edge modes in spin chains away from integrability*, Phys. Rev. B **102**, 195419 (2020).

Daniel J. Yates, Alexander G. Abanov, and Aditi Mitra, *Lifetime of almost strong edge-mode operators in one dimensional, interacting, symmetry protected topological phases*, Phys. Rev. Lett. **124**, 206803 (2020).

Daniel J. Yates, Fabian H. L. Essler, and Aditi Mitra *Almost strong* $0, \pi$ *edge modes in clean, interacting 1D Floquet systems*, Phys. Rev. B **99**, 205419 (2019).

Daniel J. Yates, Yonah Lemonik, and Aditi Mitra, *Central charge of periodically driven critical Kitaev chains*, Phys. Rev. Lett. **121**, 076802 (2018).

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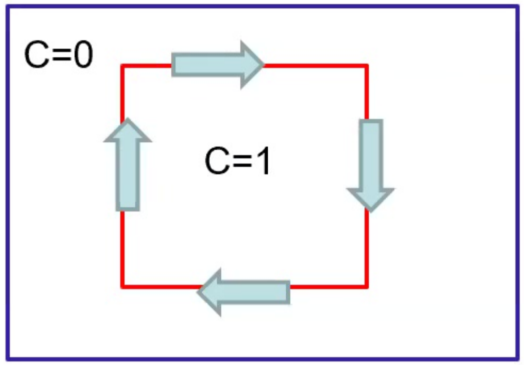
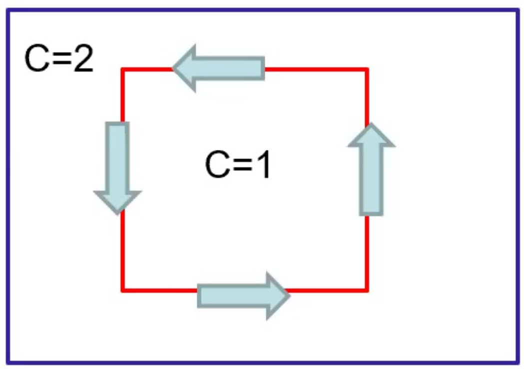
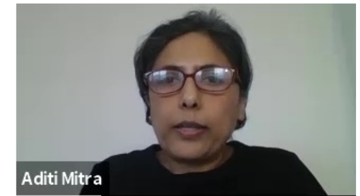
Wiki definition of symmetry protected topological (SPT) phase:

(a) *distinct SPT states with a given symmetry cannot be smoothly deformed into each other without a phase transition, if the deformation preserves the symmetry.*
(b) *however, they all can be smoothly deformed into the same trivial product state without a phase transition, if the symmetry is broken during the deformation.*







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Symmetry Protected Topological Phases for static (free) Fermions:

The notion of SPTs is powerful because the symmetries are difficult to remove (unlike crystal symmetries).

$TR : T^2 = 0, \pm,$
$PH : C^2 = 0, \pm,$ } Anti-Unit
$Chiral : S = 0, 1$ Unitary

$$HS = -SH$$

IQHE (T=0,C=0,S=0)

complex case:

Cartan\d	0	1	2	3	4	5	6	7	8	9	10	11	⋯
A	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	⋯
AIII	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	⋯

SSH

real case:

Cartan\d	0	1	2	3	4	5	6	7	8	9	10	11	⋯
AI	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	⋯
BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	⋯
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	⋯
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	⋯
AII	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	⋯
CII	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	⋯
C	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	⋯
CI	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	⋯

$T^2 = +,$
$C^2 = +,$
$S = 1$

This Talk:
Kitaev-type Chains: Floquet driving and interactions

4

Topological insulators and superconductors: ten-fold way and dimensional hierarchy
Shinsei Ryu, Andreas Schnyder, Akira Furusaki, Andreas Ludwig
New J. Phys. 12, 065010 (2010)

Periodic table for topological insulators and superconductors,
Alexei Kitaev (2009)

Kitaev chain:

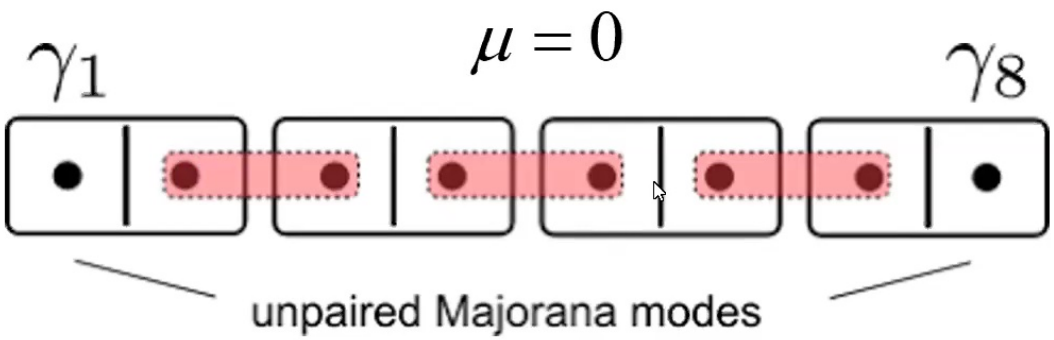
A. Y. Kitaev, Physics-Uspekhi **44**, 131 (2001).

$$\gamma_{j,1} = c_j + c_j^\dagger, \quad \gamma_{j,2} = i\left(c_j^\dagger + c_j\right),$$

$$H = -\mu \sum_j c_j^\dagger c_j + \sum_{j=0}^{N-1} \Big[-t\left(c_{j+1}^\dagger c_j + c_j^\dagger c_{j+1}\right) - |\Delta| \left(c_j c_{j+1} + c_{j+1}^\dagger c_j^\dagger\right) \Big].$$

Equivalent to the transverse field Ising model

$$H = J \sum_i \sigma_i^x \sigma_{i+1}^x - \mu \sum_i \sigma_i^z$$



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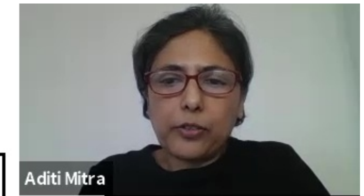
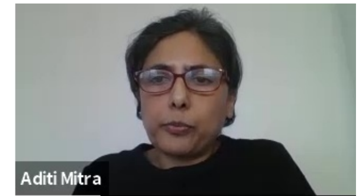


Generalized Kitaev chain



Spinless fermions with nn and nnn hopping, along with p-wave pairing

$$H = \sum_k \left(c_k^\dagger \;\; c_{-k} \right) H_{\text{BdG}}(k) \begin{pmatrix} c_k \\ c_{-k}^\dagger \end{pmatrix}$$

$$\begin{aligned} H_{\text{BdG}}(k) &= -\left(\Delta \sin(k) + \Delta' \sin(2k)\right)\sigma_y \\ &\qquad - \left(t_h \cos(k) + t_h' \cos(2k) + \frac{\mu}{2}\right)\sigma_z \\ &= \vec{h}_k \cdot \vec{\sigma}, \end{aligned}$$

PHS (C)

$$\sigma_x H_{\text{BdG}}^*(-k)\sigma_x = -H_{\text{BdG}}(k)$$

Chiral: $S = \sigma_z$

The winding of the spinor in y-z plane equals the number of Majorana modes

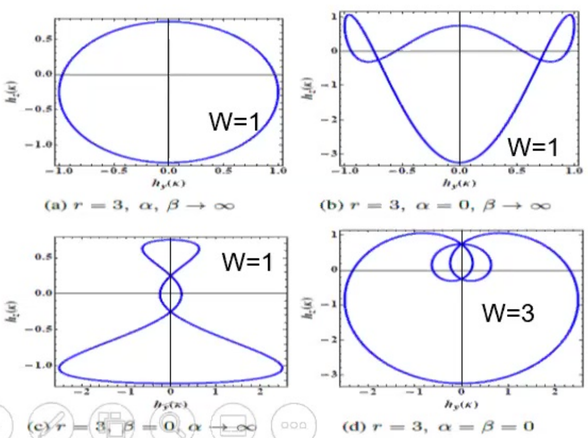


Photo from Alecce and Del'Anna, (PRB 2017) for long range chains

Cannot remove the winding W without either breaking BDI (introduce a third Pauli matrix), or by closing the gap (hk=0)

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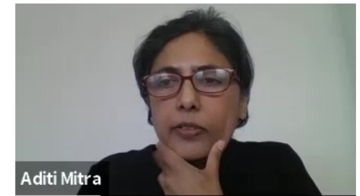
Floquet Systems

$$H_{el}(t+T) = H_{el}(t)$$



$|\psi_{k\alpha}(t)\rangle$ the exact solution of the Schrödinger equation

$$i\frac{d|\psi_{k\alpha}\rangle}{dt} = H_{el}(t)|\psi_{k\alpha}\rangle$$

Floquet theorem (analog of Bloch theorem for spatially periodic systems) wavefunction at t and t+T are same upto a phase.

$$|\psi_{k\alpha}(t)\rangle = e^{-i\epsilon_{k\alpha}t}|\phi_{k\alpha}(t)\rangle$$
$$\left[H_{\rm el} - i\partial_t\right]|\phi_{k\alpha}\rangle = \epsilon_{k\alpha}|\phi_{k\alpha}\rangle$$

Floquet quasi-modes $(|\phi_{k\alpha}(t+T)\rangle = |\phi_{k\alpha}(t)\rangle)$

quasi-energies $(\epsilon_{k\alpha})$

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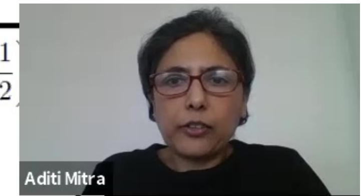
1D Kitaev chain with nn and nnn couplings
+ periodic driving

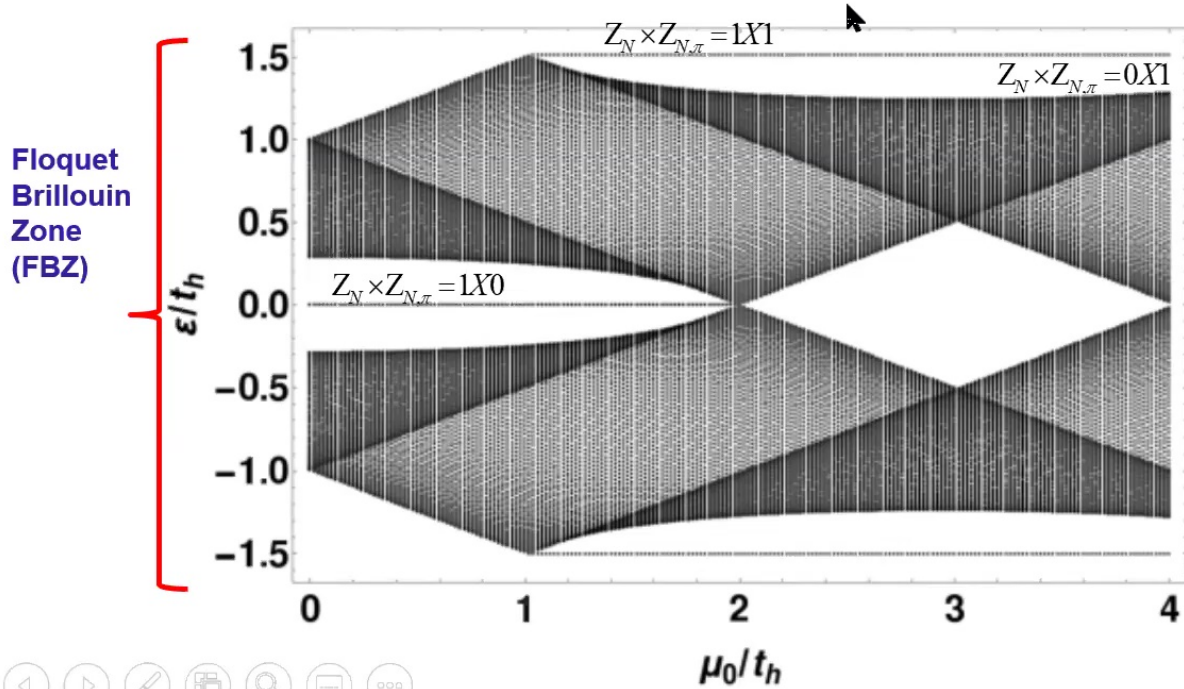
$$H = \sum_i \Big[-t_h c_i^\dagger c_{i+1} - \Delta(t) c_i^\dagger c_{i+1}^\dagger - \mu(t) \left(c_i^\dagger c_i - \frac{1}{2} \right)$$
$$- t_h' c_i^\dagger c_{i+2} - \Delta'(t) c_i^\dagger c_{i+2}^\dagger + h.c. \Big]$$
$$= \sum_k \left(c_k^\dagger \;\; c_{-k} \right) H_{\mathrm{BdG}}(k,t) \begin{pmatrix} c_k \\ c_{-k}^\dagger \end{pmatrix}. \quad (1)$$



Topological phase transitions
via gap-opening and closing at two
quasi-energies 0 and half-drive-frequency.

$$\mu(t) = \mu_0 + \xi \sin(\Omega t).$$

Quasi-energy-spectrum for open boundary conditions and nn only



$$
\begin{aligned}
H = \sum_i \Big[& -t_h c_i^\dagger c_{i+1} - \Delta(t) c_i^\dagger c_{i+1}^\dagger - \mu(t)\left(c_i^\dagger c_i - \frac{1}{2}\right) \\
& -t_h' c_i^\dagger c_{i+2} - \Delta'(t) c_i^\dagger c_{i+2}^\dagger + h.c. \Big] \\
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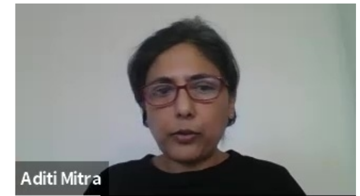
\end{aligned}
$$



Topology for time-periodic Kitaev chain

BDI $T^2 = +, C^2 = +, S = 1$

PHS (C) is still preserved even with a periodic drive

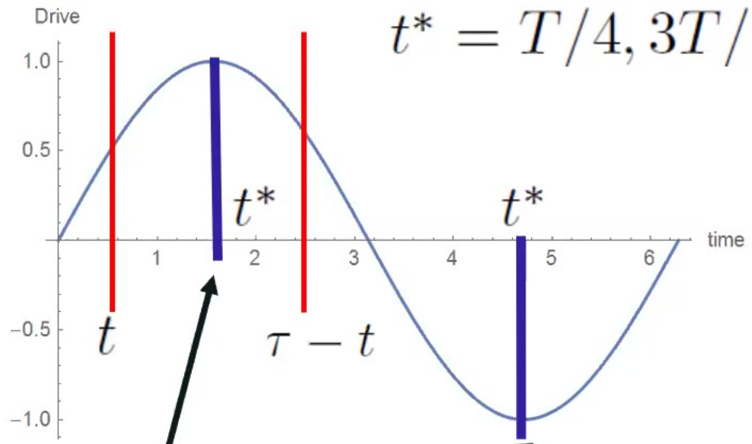
$$\sigma_x H^*_{\mathrm{BdG}}(-k)\sigma_x = -H_{\mathrm{BdG}}(k)$$

TRS (T) condition is now modified to

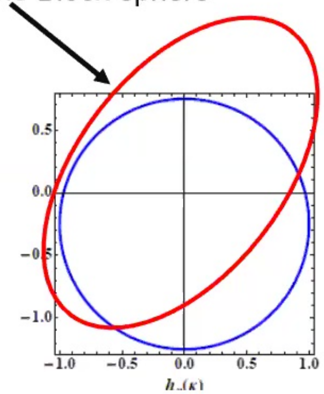
$$\mathcal{T} H(\tau - t)\mathcal{T}^{-1} = H(t), \text{ for some } \tau \text{ and for all } t$$

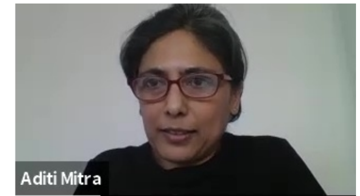
$\tau = \pi/\Omega = T/2$

$t^* = T/4, 3T/4$



At generic times the floquet quasi-modes are spinors on the Bloch sphere





$|\mathbb{Z}_0 - \mathbb{Z}_\pi|$

$\mathbb{Z}_0 + \mathbb{Z}_\pi$

State has well defined winding only at the two TRS points in time.
At these two times, Majorana modes are visible in the entanglement spectrum.

The chiral symmetry requires:

$SU(t^*)S = U^{-1}(t^*)$, where $U(t^*) = Te^{-i\int_{t^*}^{t^*+T} dt' H(t')}$ is the Floquet unitary

J. K. Asbóth, B. Tarasinski, and P. Delplace, Phys. Rev. B **90**, 125143 (2014).

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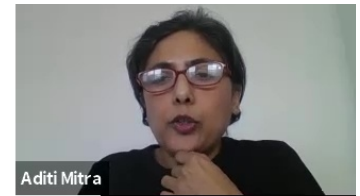

$$
\begin{aligned}
H = \sum_i \Big[& -t_h c_i^\dagger c_{i+1} - \Delta(t) c_i^\dagger c_{i+1}^\dagger - \mu(t) \left(c_i^\dagger c_i - \frac{1}{2} \right) \\
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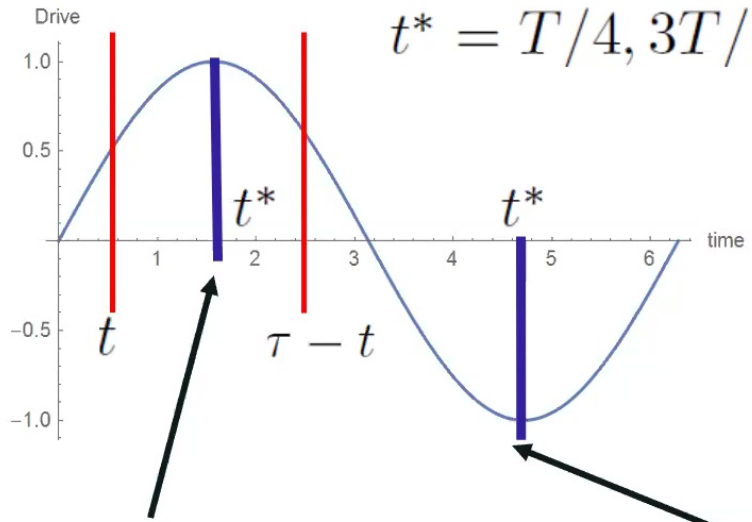

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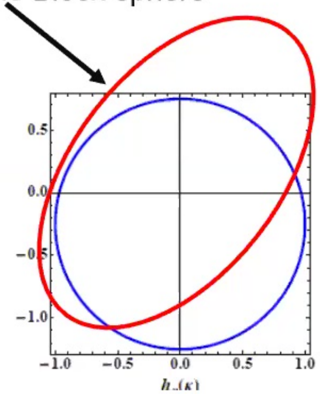

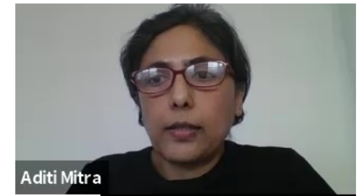

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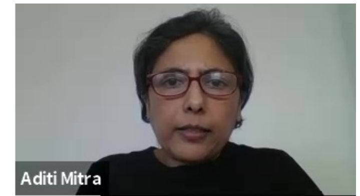
J. K. Asbóth, B. Tarasinski, and P. Delplace, Phys. Rev. B **90**, 125143 (2014).



Free fermions:

$$Z \to Z \times Z \text{ or } Z_2 \times Z_2$$
$$Z_2 \to Z \times Z \text{ or } Z_2 \times Z_2$$

T. Kitagawa, E. Berg, M. Rudner, and E. Demler, Phys. Rev. B **82**, 235114 (2010).
M. S. Rudner, N. H. Lindner, E. Berg, and M. Levin, Phys. Rev. X **3**, 031005 (2013).
J. K. Asbóth, B. Tarasinski, and P. Delplace, Phys. Rev. B **90**, 125143 (2014).
J. K. Asbóth and H. Obuse, Phys. Rev. B **88**, 121406 (2013).
R. Roy and F. Harper, Phys. Rev. B **96**, 155118 (2017).

V. Khemani, A. Lazarides, R. Moessner, and S. L. Sondhi, Phys. Rev. Lett. **116**, 250401 (2016).
C. W. von Keyserlingk and S. L. Sondhi, Phys. Rev. B **93**, 245145 (2016).
C. W. von Keyserlingk and S. L. Sondhi, Phys. Rev. B **93**, 245146 (2016).
A. C. Potter, T. Morimoto, and A. Vishwanath, Phys. Rev. X **6**, 041001 (2016).
D. V. Else and C. Nayak, Phys. Rev. B **93**, 201103 (2016).

R. Roy and F. Harper, Phys. Rev. B **94**, 125105 (2016).

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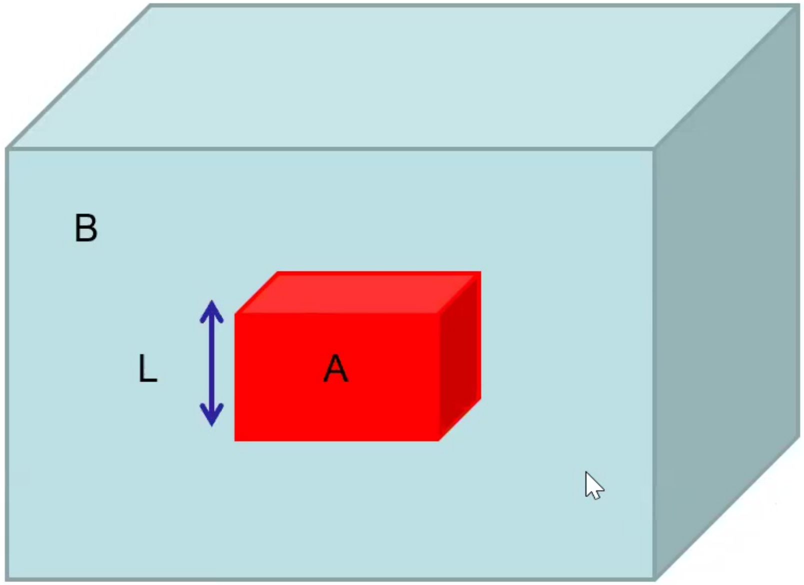
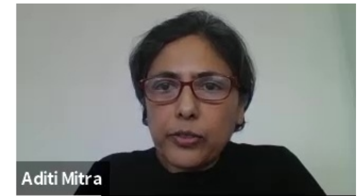
Entanglement (static system):





$$\rho_A = Tr_B\left[|\psi\rangle\langle\psi|\right]$$

$$S_A = -Tr_A\left[\rho_A \ln \rho_A\right]$$

J. Eisert, M. Cramer, and M. B. Plenio, Rev. Mod. Phys. **82**, 277 (2010).
B. Zeng, X. Chen, D.-L. Zhou, and X.-G. Wen, arXiv:1508.02595 (2015).
P. Calabrese and J. Cardy, Journal of Statistical Mechanics: Theory and Experiment **2005**, P04010 (2005).
P. Calabrese and J. Cardy, Journal of Statistical Mechanics: Theory and Experiment **2007**, P10004 (2007).
U. Schollwöck, Annals of Physics **326**, 96 (2011), january 2011 Special Issue.

Ground state of Hamiltonians:
Area law for short-range interaction, area law violated for long-range interactions.

$$S_A = O(L^{d-1})$$

d: spatial dimensions

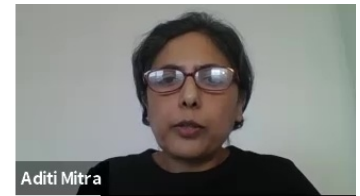
Excited states: Volume law

$$S_A = O(L^d)$$

SPT ground state show area law

Non-ergodic states (many body localized states) show area law entanglement entropy even for excited eigen-states.

Entanglement Spectrum has more information



Entanglement spectrum: Eigenvalues of the reduced density matrix

$$\rho_A(t) = Tr_B\left[\rho\right]$$

Reflects the bulk-boundary correspondence in topological systems where an entanglement cut in a spatially extended system now hosts "edge-states".

M. Levin and X.-G. Wen, Phys. Rev. Lett. **96**, 110405 (2006).

A. Kitaev and J. Preskill, Phys. Rev. Lett. **96**, 110404 (2006).

H. Li and F. D. M. Haldane, Phys. Rev. Lett. **101**, 010504 (2008).

L. Fidkowski, Phys. Rev. Lett. **104**, 130502 (2010).

Characterizing Topology through entanglement statistics of unitarily time-evolving state is an active area of research:

See also:

Entanglement properties of Floquet Chern insulators
Daniel J. Yates, Yonah Lemonik, Aditi Mitra, PRB 2016

Topology of one dimensional quantum systems out of equilibrium
Max McGinley, Nigel R. Cooper, PRL 2018

Topological Entanglement-Spectrum Crossing in Quench Dynamics
Zongping Gong, Masahito Ueda, PRL 2018

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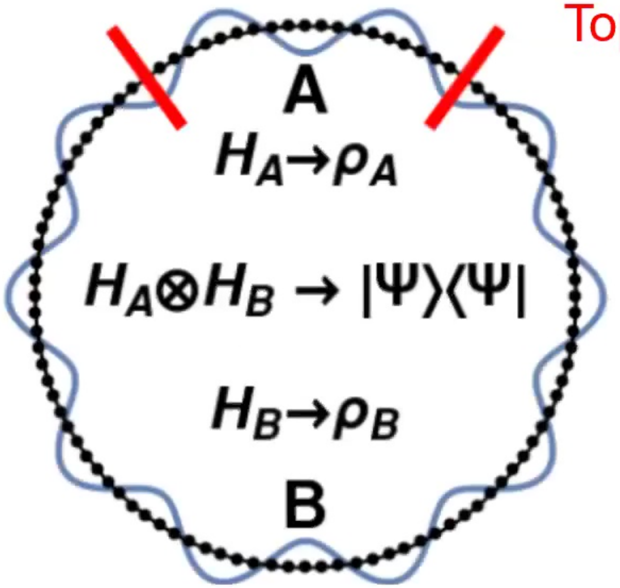

Topology from Entanglement Measu

I. Peschel and V. Eisler, Journal of Physics A: Mathematical and Theoretical **42**, 504003 (2009).

$$G_{i,j}(t) = \int_{-\pi}^{\pi} \frac{dk}{2\pi} e^{ik(i-j)} \mathcal{M}_k(t),$$

$$\mathcal{M}_{k,\text{static}} = \frac{\vec{d}(k) \cdot \vec{\sigma}}{|d(k)|}; \mathcal{M}_{k,\text{FGS}}(t) = \langle \phi(k,t)|\vec{\sigma}|\phi(k,t)\rangle \cdot \vec{\sigma}.$$

$$\boxed{S = -\frac{1}{2} \sum_{\alpha=\pm,\lambda_i} \left[\left(\frac{1-\alpha\lambda_i}{2} \right) \ln \left(\frac{1-\alpha\lambda_i}{2} \right) \right]}$$

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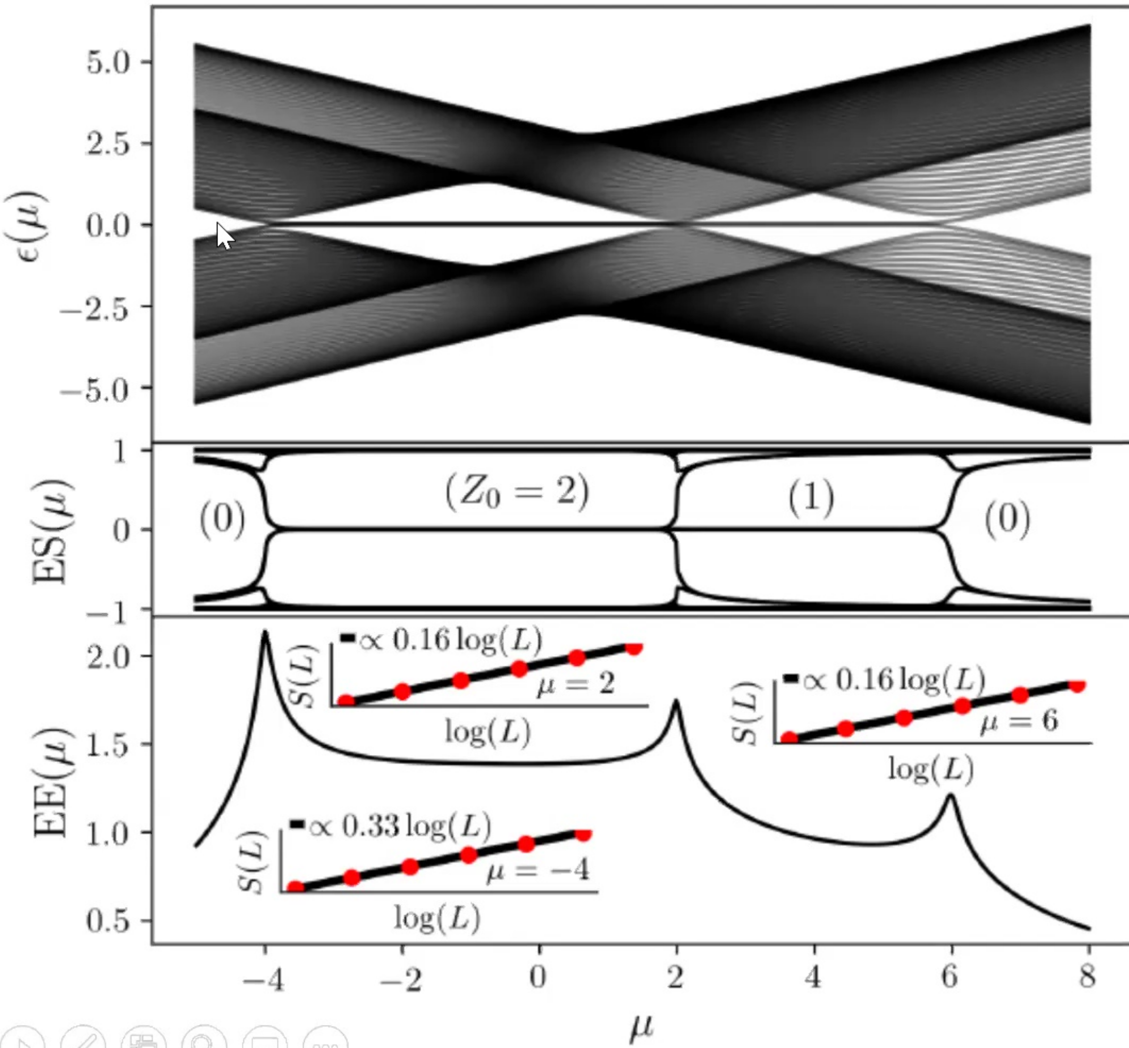
STATIC SYSTEM: Entanglement of half-filled many-body ground state (nn and nnn



energy
(open chain)

ES=Entanglement Spectrum or (tanh(ES/2))

EE=Entanglement entropy

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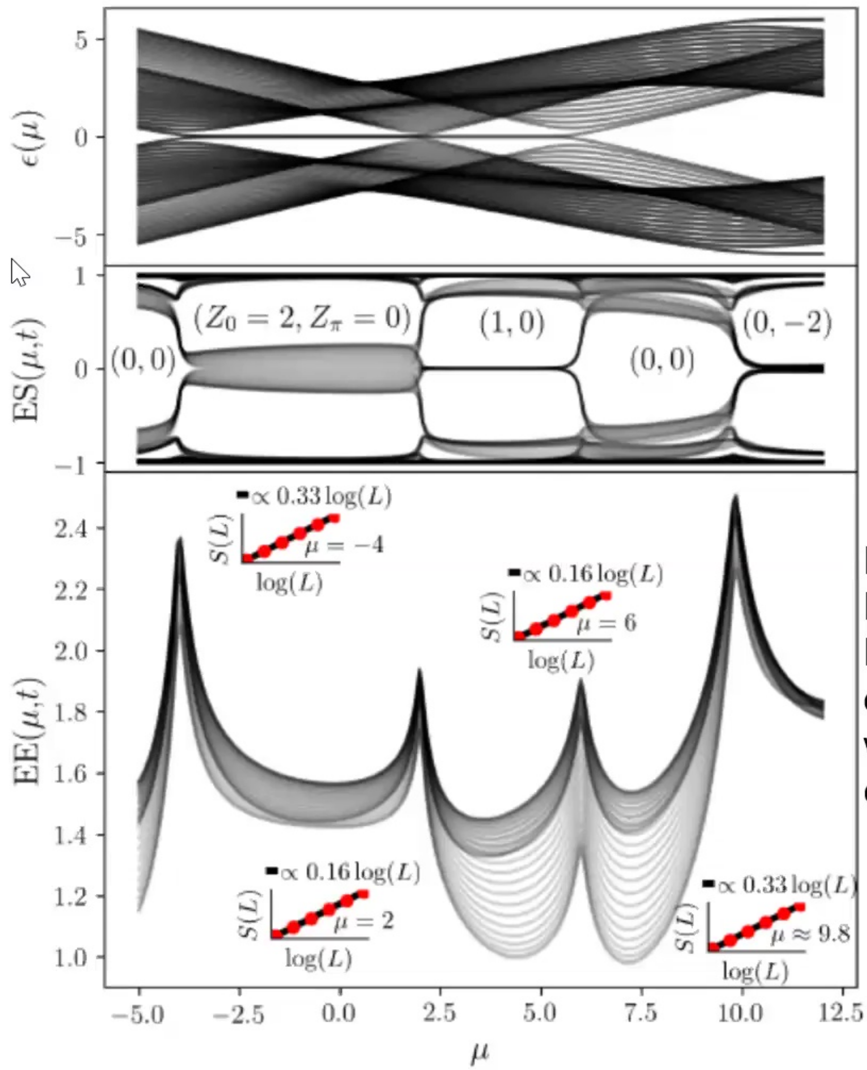
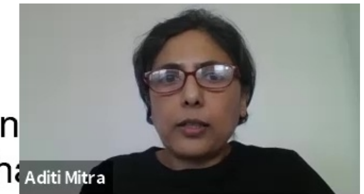
Aditi Mitra

FLOQUET GROUND STATE (FGS): Slater Determinant constructed from all negative quasi-energy states of periodic chain



Quasi-en
(open ch



ES or tanh(ES/2) Entanglement spectrum at different times within a drive cycle

EE= Entanglement Entropy at different times within a drive cycle

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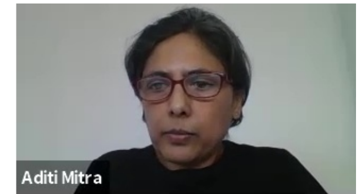
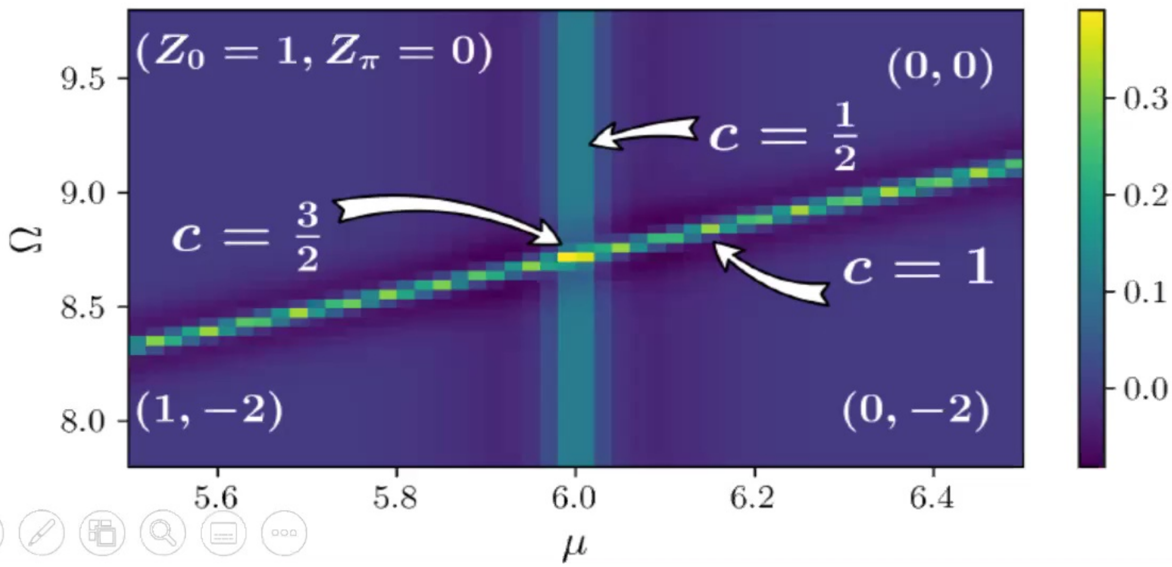


$$c = \left(|Z_0 - Z_0'| + |Z_\pi - Z_\pi'| \right) / 2$$

Number of Fermi-points at zero quasi-energy

Number of Fermi-points at pi quasi-energy

Multi-critical points also possible. Central charge = sum of central charge of intersecting lines



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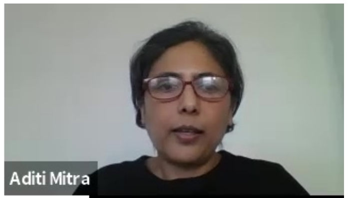
Robustness of edge modes?

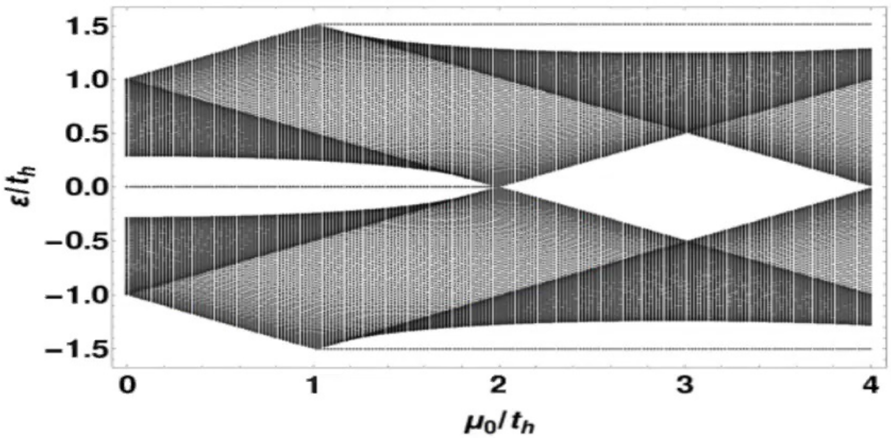


Static and Floquet systems:

It is believed that edge-modes vanish when "temperature" becomes comparable to single-particle gap.

NOT ALWAYS TRUE!

"temperature" related to how the single-particle states are occupied at half-filling.



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Strong zero modes Ψ_0

P. Fendley, Journal of Physics A: Mathematical and Theoretical **49**, 30LT01 (2016).

- $\mathcal{D}$ A discrete Z2 symmetry of the Hamiltonian (for example Fermion parity)
- Ψ_0 Commutes with the Hamiltonian in the thermodynamic limit

$$[H, D] = 0,$$
$$[H, \Psi_0] \approx 0,$$
$$H|n\rangle = \varepsilon_n |n\rangle$$

- $\{\Psi_0, \mathcal{D}\} = 0$ Anti-commutes with the discrete symmetry
- $\Psi_0^2 = \mathcal{O}(1)$ Normalizable

Above implies an eigenspectrum phase which is doubly-degenerate

$$\{|n\rangle, \Psi_0|n\rangle\}$$

Edge mode exists at all energies

$|n\rangle$: EVEN; $\Psi_0|n\rangle$: ODD

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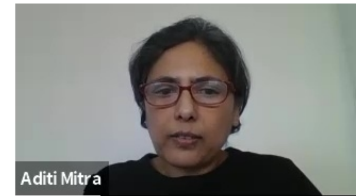
Example: Kitaev chain (transverse field Ising) with open boundary conditions



P. Fendley, Journal of Physics A: Mathematical and Theoretical **49**, 30LT01 (2016).

$$H = J \sum_i \sigma_i^x \sigma_{i+1}^x - \mu \sum_i \sigma_i^z$$

$$\mathcal{D} = \sigma_1^z \sigma_2^z \ldots \sigma_L^z$$ Or Fermion Parity

$$\Psi_0 = \sum_{s=0}^{\infty} (-\mu/J)^s \, \sigma_{s+1}^x \prod_{j=1}^{s} \sigma_j^z \quad \Rightarrow \quad \Psi_0(\mu = 0) = \sigma_1^x$$

Normalizable in the topologically non-trivial phase

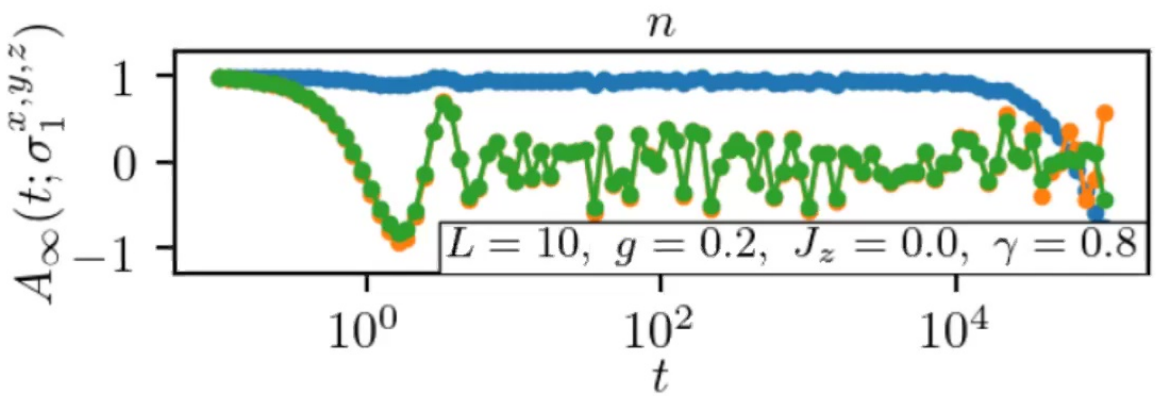
$$\Psi_0^2 = \frac{1}{1 - (\mu/J)^2}$$

20

Topological phase transition when operator is non-normalizable

Identification of strong modes: Infinite temperature correlation function of the boundary spin.

$$A_\infty(t) = \frac{1}{2^L}\mathrm{Tr}\left[\sigma_1^x(t)\sigma_1^x(0)\right].$$



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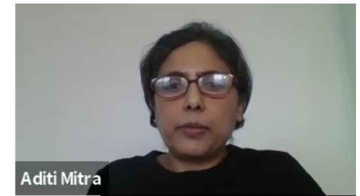

Adding interactions:

Almost strong zero modes:

Here the edge mode lives for times that grows exponentially with system size, eventually saturating at large enough system sizes. Yet these times are very long (>> 1/temperature)

J. Kemp, N. Y. Yao, C. R. Laumann, and P. Fendley, Journal of Statistical Mechanics: Theory and Experiment **2017**, 063105 (2017).
D. V. Else, P. Fendley, J. Kemp, and C. Nayak, Phys. Rev. X **7**, 041062 (2017).

$$H = J\sum_i \sigma_i^x \sigma_{i+1}^x - \mu \sum_i \sigma_i^z + J_z \sum_i \sigma_i^z \sigma_{i+1}^z$$

$$A_\infty(t) \sim e^{-\Gamma t} \qquad \Gamma \sim e^{-cJ/J_z}, c = O(1)$$

WHAT ABOUT INTERACTING FLOQUET CHAINS?

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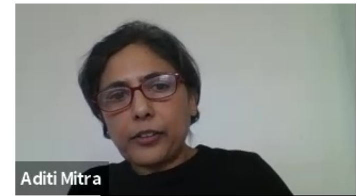
GENERAL VIEW: CLOSED, DISORDER-FREE FLOQUET SYSTEMS HEAT TO INFINITE TEMPERATURE.



A. Lazarides, A. Das, and R. Moessner, Phys. Rev. E **90**, 012110 (2014).
H. Kim, T. N. Ikeda, and D. A. Huse, Phys. Rev. E **90**, 052105 (2014).
L. D'Alessio and M. Rigol, Phys. Rev. X **4**, 041048 (2014).
P. Ponte, A. Chandran, Z. Papi, and D. A. Abanin, Annals of Physics **353**, 196 (2015).
A. Haldar, R. Moessner, and A. Das, Phys. Rev. B **97**, 245122 (2018).

SO NAÏVE EXPECTATION: NO PROTECTED EDGE MODES.

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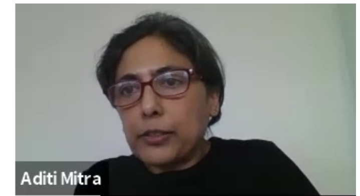
LONG LIVED EDGE MODES IN INTERACTING FLOQUET SPT PHASES ARE KNOWN TO PRIMARILY OCCUR WITH DISORDER (MBL PREVENTS HEATING).



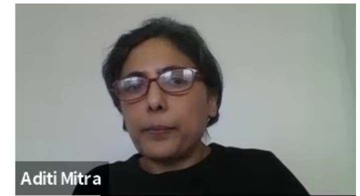
Y. Bahri, R. Ronen, E. Altman, and A. Vishwanath, Nature Communications **6**, 7341 (2015).

V. Khemani, A. Lazarides, R. Moessner, and S. L. Sondhi, Phys. Rev. Lett. **116**, 250401 (2016).

G. J. Sreejith, A. Lazarides, and R. Moessner, Phys. Rev. B **94**, 045127 (2016).

I.-D. Potirniche, A. C. Potter, M. Schleier-Smith, A. Vishwanath, and N. Y. Yao, Phys. Rev. Lett. **119**, 123601 (2017).

A. Kumar, P. T. Dumitrescu, and A. C. Potter, Phys. Rev. B **97**, 224302 (2018).



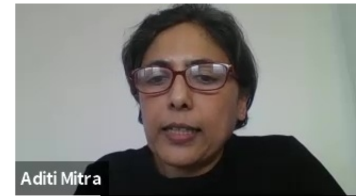
QUESTION WE POSED: STRONG/ALMOST STRONG MODES IN CLEAN FLOQUET SYSTEMS?

YES!

NO NEED TO BE IN THE HIGH-FREQUENCY DRIVING REGIME

Floquet System: Almost Strong zero/pi modes: Two different eigenspectrum phases

$$\Psi_{0,\pi}$$



$U(T)$: Time-evolution operator over one period.

$$\Rightarrow \quad \Psi_{0,\pi}^2 = O(1)$$

$$\Rightarrow \quad \{\Psi_{0,\pi}, \mathcal{D}\} = 0$$

See also:

M. Thakurathi, A. A. Patel, D. Sen, and A. Dutta, Phys. Rev. B **88**, 155133 (2013).

G. J. Sreejith, A. Lazarides, and R. Moessner, Phys. Rev. B **94**, 045127 (2016).

Eigenspectrum Phase-I

$$[\Psi_0, U(T)] \approx 0$$

$$\{|n\rangle, \Psi_0|n\rangle\}$$

Degenerate pairs of states

Eigenspectrum Phase-II

$$\{\Psi_\pi, U(T)\} \approx 0$$

$$\{|n\rangle, \Psi_\pi|n\rangle\}$$

Pairs with quasi-energy separated by π/T

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Edge mode diagnostic: A measure of edge decoherence

$$A(nT) = \frac{1}{2^L}\mathrm{Tr}\left[\sigma_1^x(nT)\sigma_1^x\right] = \frac{1}{2^L}\mathrm{Tr}\left[a_1(nT)a_1\right]$$

$$A_\psi(nT) = \langle\psi|\sigma_1^x(nT)\sigma_1^x|\psi\rangle$$

For detecting Pi mode: $A^-(nT) = \left[A(nT) - A((n+1)T)\right]/2$

For detecting 0 mode: $A^+(nT) = \left[A(nT) + A((n+1)T)\right]/2$

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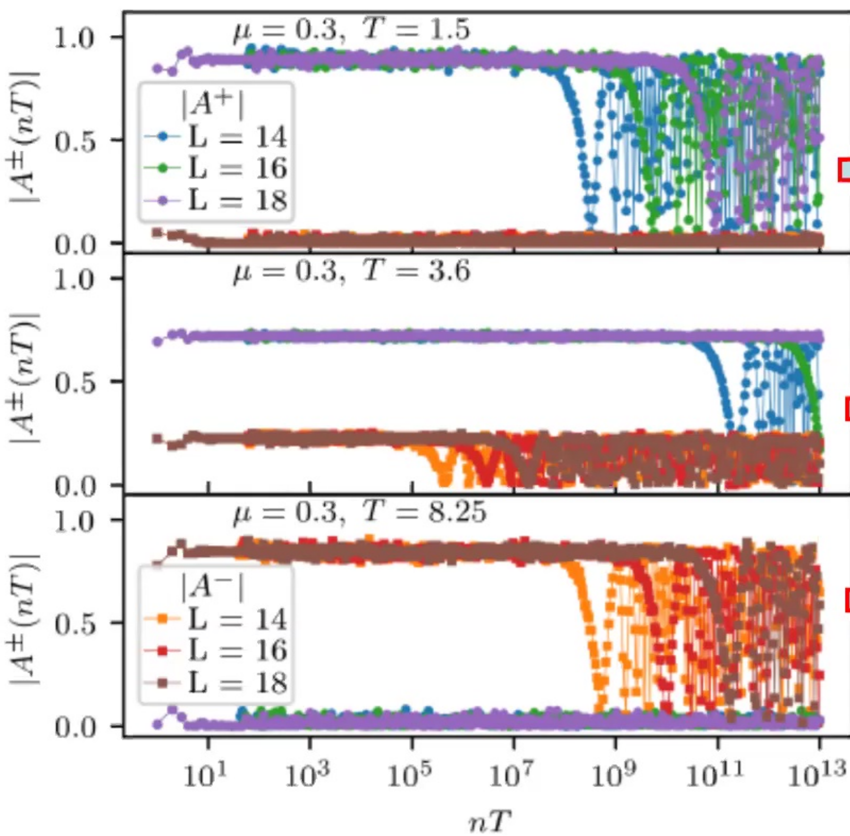
Free Floquet System: Strong zero/pi modes

$$U(T) = e^{\frac{-iTJ_x}{2} H_{xx}} e^{\frac{-iT\mu}{2} H_z} ,$$

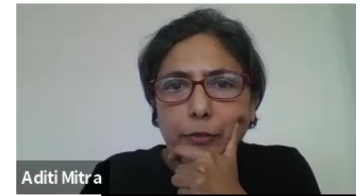
$$H_{xx} = \sum_{i=1}^{L} \sigma_i^x \sigma_{i+1}^x , \quad H_z = \sum_{i=1}^{L} \sigma_i^z .$$



Zero-mode

Zero/Pi-mode

Pi-mode



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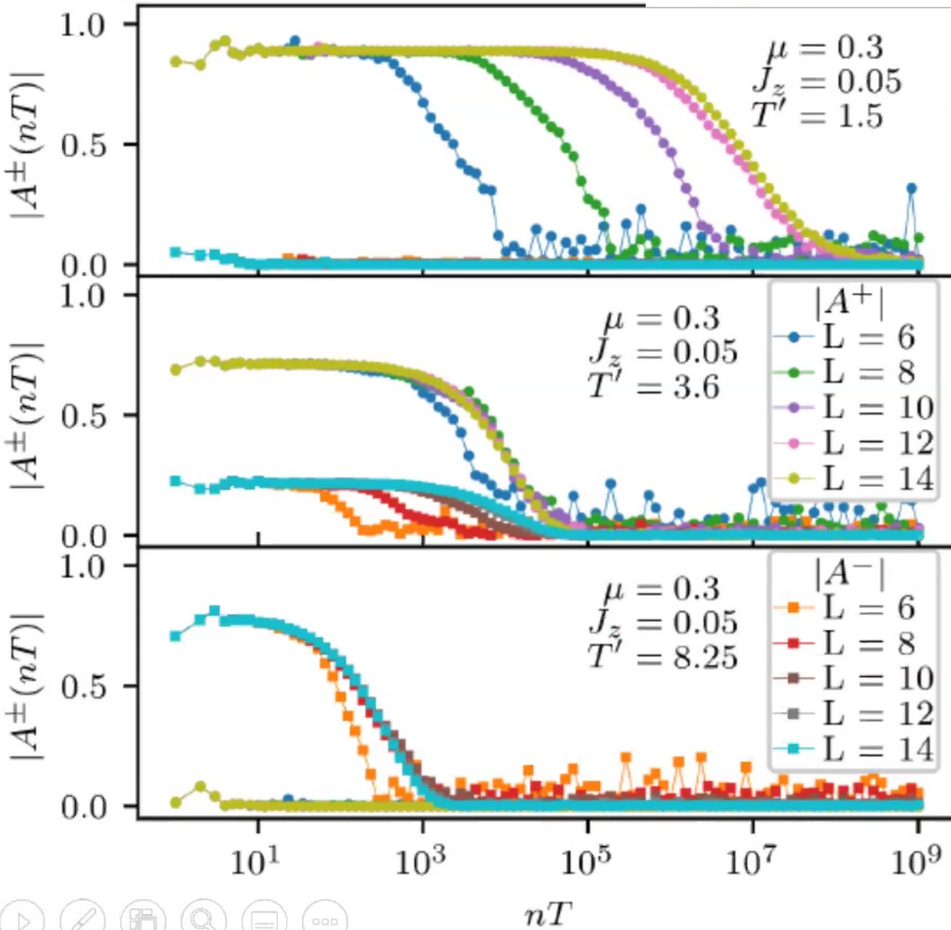
Interacting Floquet System: Almost-Strong zero/pi modes

$$U(T) = e^{-i\frac{TJ_z}{3}H_{zz}} e^{-i\frac{TJ_x}{3}H_{xx}} e^{-i\frac{T\mu}{3}H_z}$$

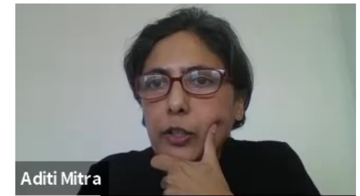
$$H_{zz} = \sum_{i=1}^{L-1} \sigma_i^z \sigma_{i+1}^z$$



Almost strong zero-mode

Almost strong Zero/pi-mode

Almost strong pi-mode



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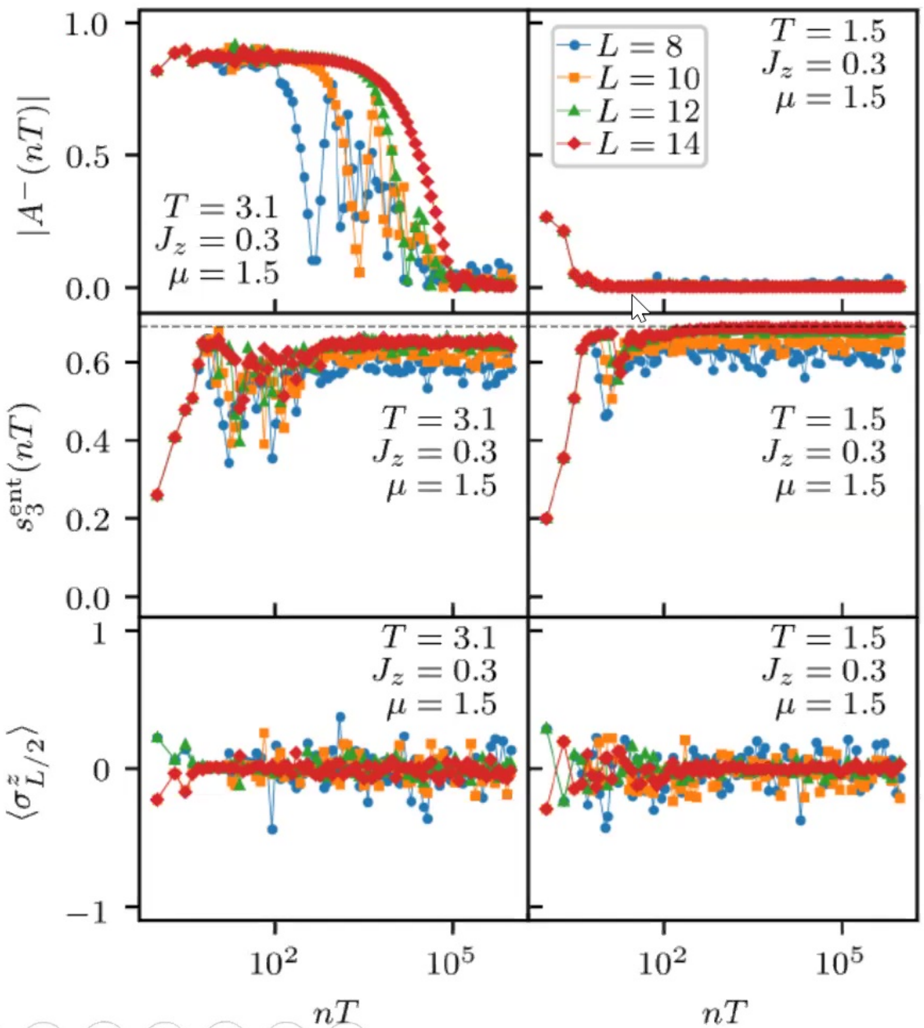

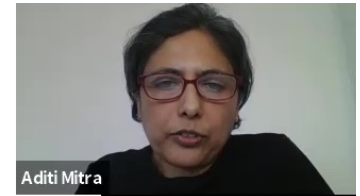

Lifetime of almost strong modes exceeds thermalization times by many orders.

Initial state: classical Neel state

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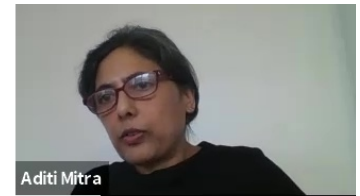
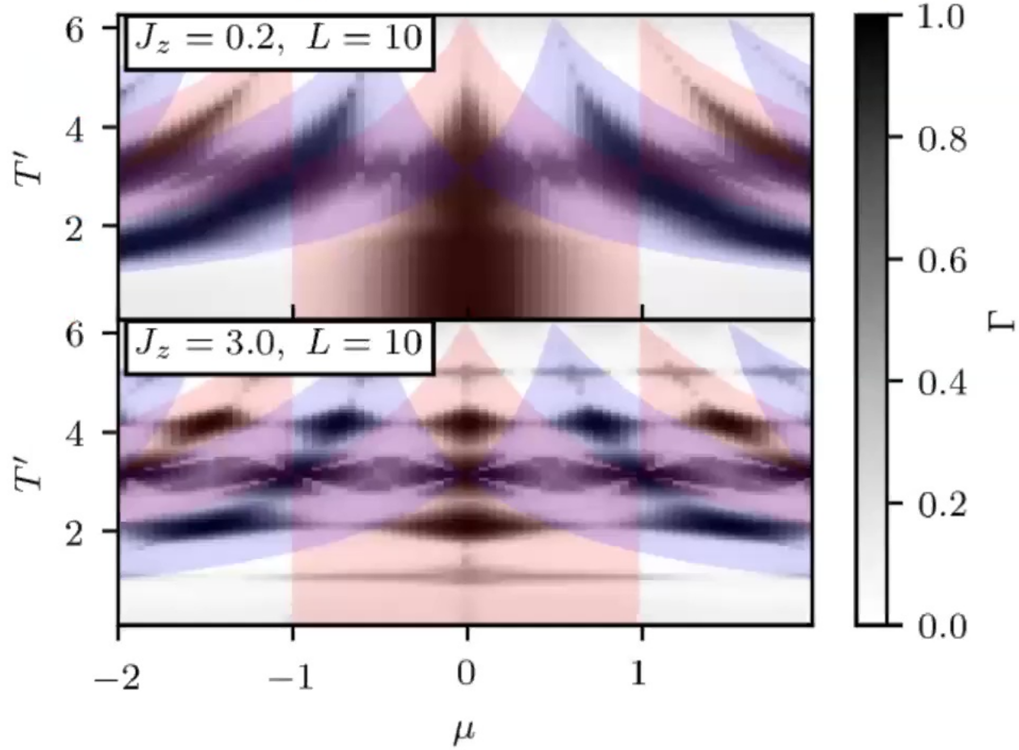


Large regions of parameters support almost strong modes:

$$U(T) = e^{-i\frac{TJ_z}{3}H_{zz}}e^{-i\frac{TJ_x}{3}H_{xx}}e^{-i\frac{T\mu}{3}H_z}$$

$$\Gamma = \frac{1}{2^L}\sum_s \max_{s'} |\langle s|\sigma_1^x|s'\rangle|^2$$

Blue/Red regions: Strong zero/pi mode of the free Floquet system



T' Effective drive period introduced in order to compare the binary and ternary drive results

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The recursion method

$$e^{iHt}Oe^{-iHt} = \sum_{n=0}^{\infty} \frac{(it)^n}{n!}\mathcal{L}^n O, \qquad \mathcal{L}O = [H, O].$$

Heisenberg time-evolution can be mapped to motion of a free particle in a Krylov basis

$$|O_0) = |\sigma_1^x) \longrightarrow |A_1) = \mathcal{L}|O_0) \longrightarrow b_1 = \sqrt{(A_1|A_1)} \longrightarrow |O_1) = |A_1)/b_1$$

$$\mathcal{L}|O_n) = b_n|O_{n+1}) + b_{n-1}|O_{n-1}),$$

bn: Lanczos coefficients

$$\mathcal{L} = \begin{pmatrix} & b_1 & & \\ b_1 & & b_2 & \\ & b_2 & & \ddots \\ & & \ddots & \end{pmatrix}.$$

V. Vishwanath and G. Müller, *The Recursion Method: Applications to Many-Body Dynamics*, Springer, New York (2008).
D. E. Parker, X. Cao, A. Avdoshkin, T. Scaffidi, and E. Altman, Phys. Rev. X **9**, 041017 (2019).

$$A_\infty(t) = (e^{i\mathcal{L}t})_{1,1}.$$

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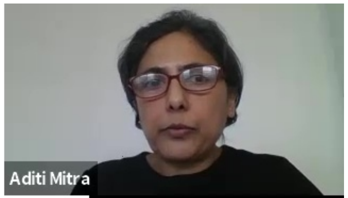
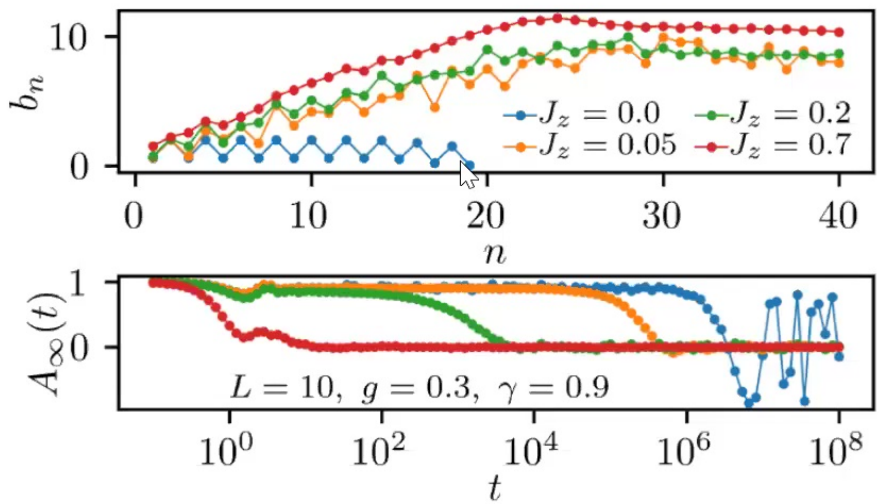
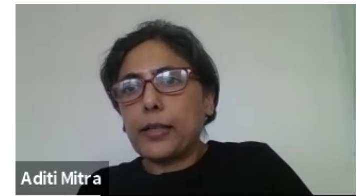
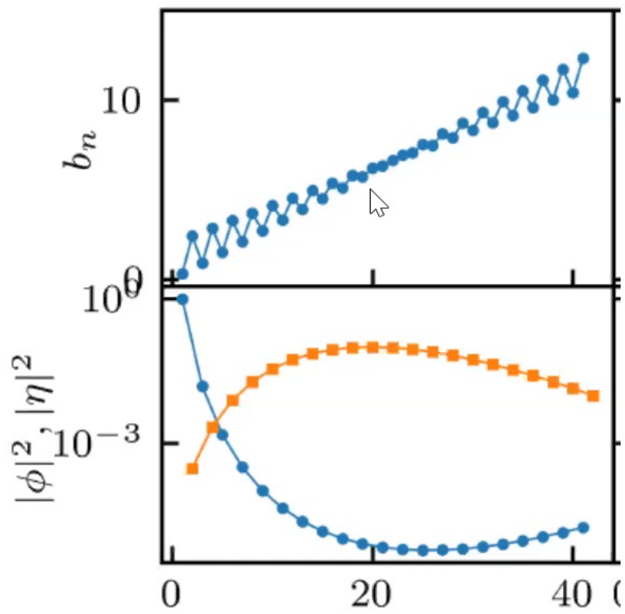




Krylov basis: Generalized SSH-type model, with increasing hopping decreasing dimerization.



A linear slope alone is not sufficient to remove edge modes:



$$b_n = \begin{cases} \alpha_1 n + \delta_1 & n \text{ odd} \\ \alpha_2 n + \delta_2 & n \text{ even.} \end{cases}$$

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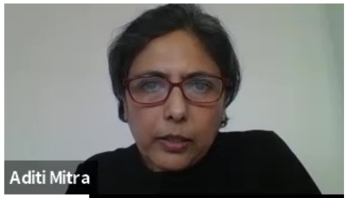
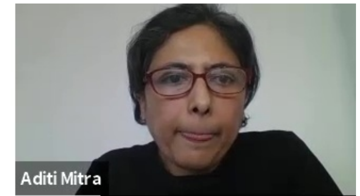
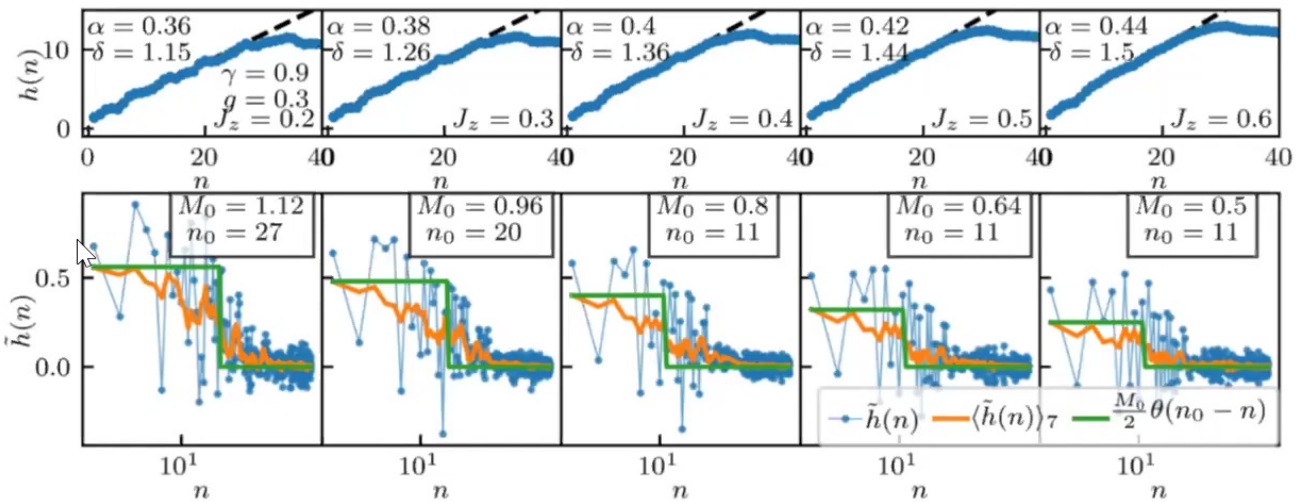


Map the model to a continuum model: Dirac model on half-line





$$
\begin{aligned}
b_n &= h_n + (-1)^n \tilde{h}_n, \\
h_n &= \alpha n + \delta, \\
\tilde{h}_n &= \frac{M_0}{2\left[\left(\frac{n}{n_0}\right)^{\beta} + 1\right]}.
\end{aligned}
$$

$$
i\partial_t \chi = \left[\sigma^z i\partial_X + \sigma^y m(X)\right]\chi\,,
$$

$$
m(X) = 2\tilde{h}(X) - \frac{\partial_X \tilde{h}(X)}{2h(X)} \approx 2\tilde{h}(X)\,.
$$

$$
\Gamma_A \sim 4M_0 e^{-\frac{M_0}{\alpha}\log\left(\frac{\alpha x_0}{\delta}\right)}\,.
$$

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M_0 is dimerization at the left end of the wire, and $\alpha \propto J_z$ is the linear slope.

Conclusions

Although clean interacting Floquet systems heat to infinite temperature, if an almost strong mode is present, edge modes survive heating. They live for a time that exceeds thermalization times by many orders of magnitude.

Good news for realizing non-abelian edge modes by Floquet driving.

Developed an analytic and computational scheme to extract lifetimes of topological edge-modes in the thermodynamic limit. This method can be generalized to any dimensions. It can be applied to both static and Floquet systems.

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