Title: Electric Multipole Insulators

Speakers: Taylor Hughes
Date: February 01, 2021 - 12:30 PM

URL: http://pirsa.org/21020001

Abstract: In this talk I will present a general framework to distinguish different classes of charge insulators based on whether or not they insulate or conduct higher multipole moments (dipole, quadrupole, etc.). This formalism applies to generic many-body systems that support multipolar conservation laws. Applications of this work provide a key link between recently discovered higher order topological phases and fracton phases of matter.


ILLINOIS
1867
UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Electric Multipole Insulators
Taylor L. Hughes
UIUC
NSF
Office of Naval Research
ONR
Science & Technology


Taylor Hughes

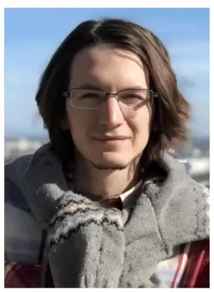
Collaborators

Wladimir Benalcazar
(Eberly Fellow,Penn State)

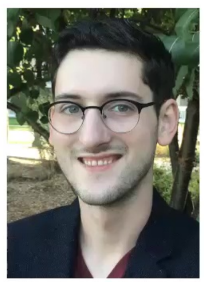
B. Andrei Bernevig
(Professor, Delocalized)

Yizhi You
(PCTS Fellow, Princeton)

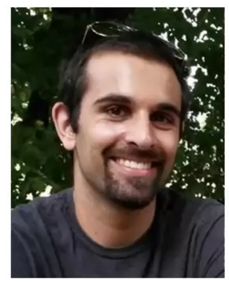
Fiona Burnell
(Professor, U Minnesota)



Oleg Dubinkin
(Grad Student, UIUC)



Julian May-Mann
(Grad Student, UIUC)



Will Wheeler
(Grad Student, UIUC)

Lucas Wagner
(Professor, UIUC)

Outline

Part 1: Classical Multipole Moments

Part 2: Quantized Multipole Moments in Crystals

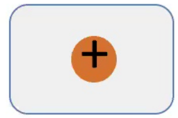
Part 3: Calculating Multipole Moments

Part 4: Many-body Multipole Insulators

Classical Multipole Moments of Point Charges

dim

$$\Phi = \sum_\alpha q_\alpha$$

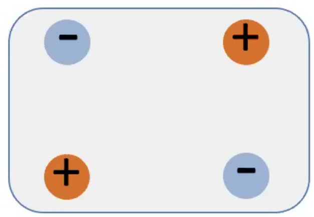


Monopole Moment (charge) 0

$$P^i = \sum_\alpha q_\alpha x^i_\alpha$$

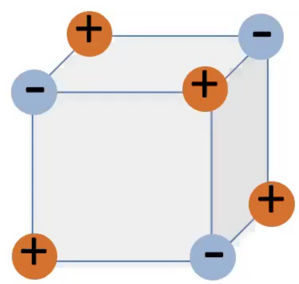
Dipole Moment 1

$$Q^{ij} = \sum_\alpha q_\alpha x^i_\alpha x^j_\alpha$$



Quadrupole Moment 2

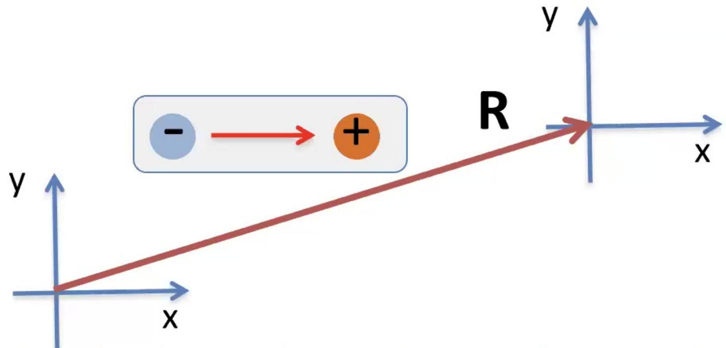
$$O^{ijk} = \sum_\alpha q_\alpha x^i_\alpha x^j_\alpha x^k_\alpha$$



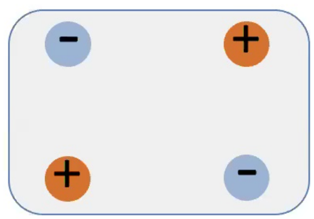
Octupole Moment 3

Classical Multipole Moments of Point Charges

Constraints for Independence of **Global** Origin of Coordinates



$$P^i \rightarrow \sum_\alpha q_\alpha (x^i_\alpha + R^i) = P^i + \Phi R^i$$

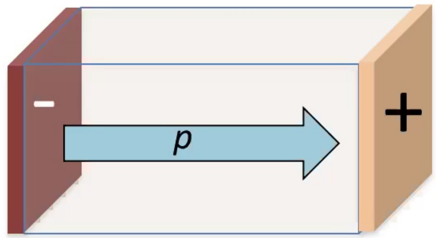


$$Q^{ij} \rightarrow Q^{ij} + P^i R^j + P^j R^i + \Phi R^i R^j$$

For *n*-th moment to be well-defined all lower moments must vanish.

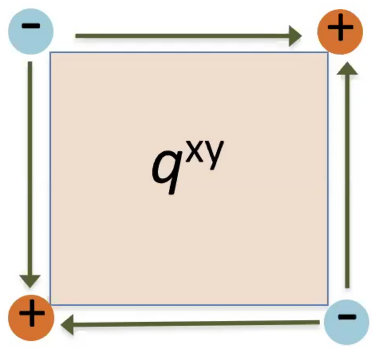
Multipole Moment Density in Materials

Polarization p^i (dipole moment per volume) is only well-defined in a neutral material



$$\sigma = \vec{p} \cdot \hat{n}$$

Quadrupole density q^{ij} is only well-defined in neutral, unpolarized materials

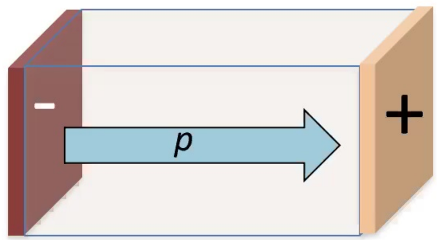


$$p^i_{edge} = q^{ij}\hat{n}_j$$

$$Q_{cor} = q^{ij}\hat{n}^{(1)}_i \hat{n}^{(2)}_j$$

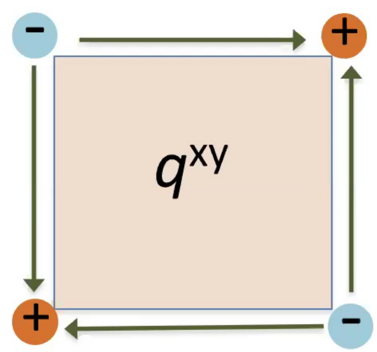
Multipole Moment Density in Materials

Polarization p^i (dipole moment per volume) is only well-defined in a neutral material



$$\sigma = \vec{p} \cdot \hat{n}$$

Quadrupole density q^{ij} is only well-defined in neutral, unpolarized materials



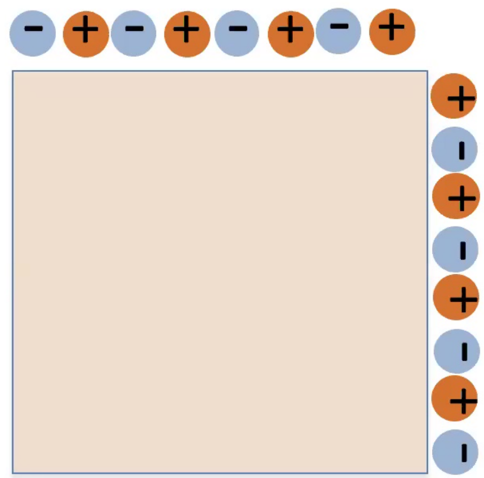
$$p^i_{edge} = q^{ij} \hat{n}_j$$

$$Q_{cor} = q^{ij} \hat{n}^{(1)}_i \hat{n}^{(2)}_j$$

Note: In their intrinsic dimensions each multipole moment density has units of charge.

An Operational Definition of Quadrupole Moment

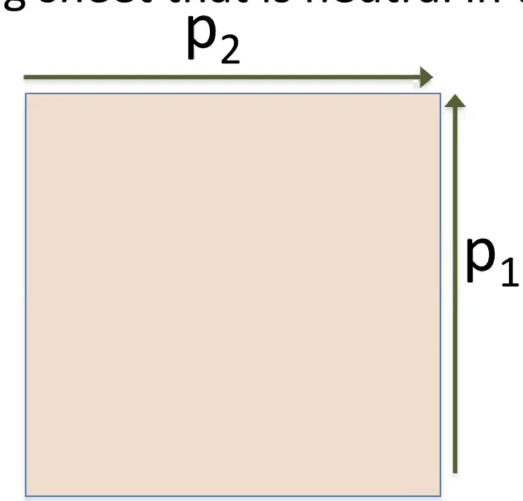
Take an insulating sheet that is neutral in the bulk, and unpolarized.



Attach polarization only on boundary.

An Operational Definition of Quadrupole Moment

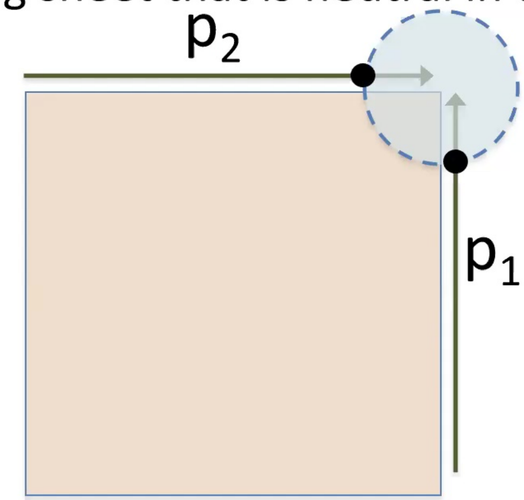
Take an insulating sheet that is neutral in the bulk, and unpolarized.



Attach polarization only on boundary.

An Operational Definition of Quadrupole Moment

Take an insulating sheet that is neutral in the bulk, and unpolarized.

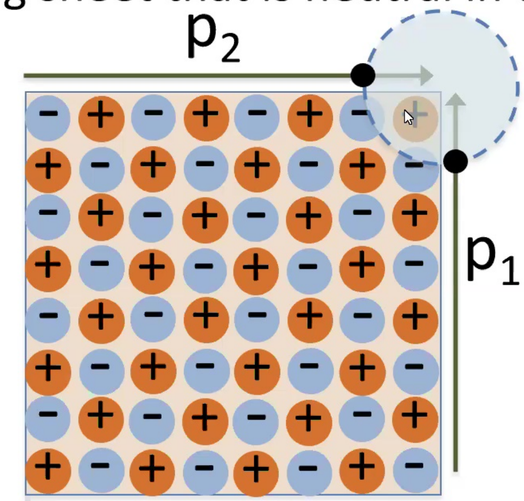


Attach polarization only on boundary.

$$Q_{cor} = p_1 + p_2$$

An Operational Definition of Quadrupole Moment

Take an insulating sheet that is neutral in the bulk, and unpolarized.



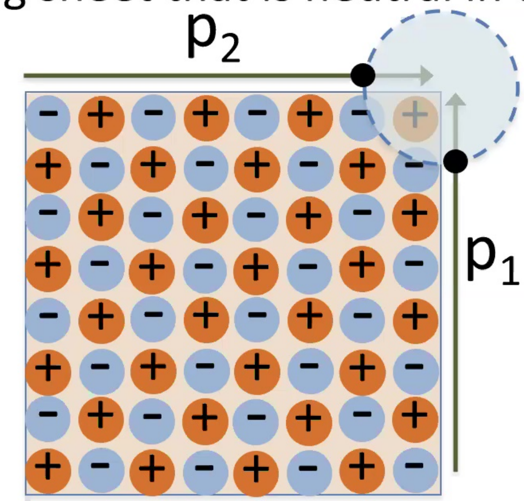
Attach polarization only on boundary.

$$Q_{cor} = p_1 + p_2$$

Bulk quadrupole moment captures the failure of this equation

An Operational Definition of Quadrupole Moment

Take an insulating sheet that is neutral in the bulk, and unpolarized.



Attach polarization only on boundary.

$$Q_{cor} = p_1 + p_2$$

Bulk quadrupole moment captures the failure of this equation

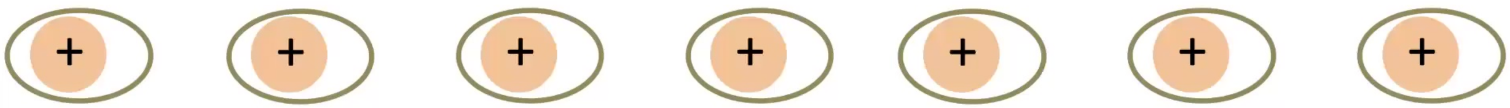
$$\boxed{Q_{cor} - p_1 - p_2 = -q^{xy}}$$

Part 2: Quantized Multipole Moments in Crystals

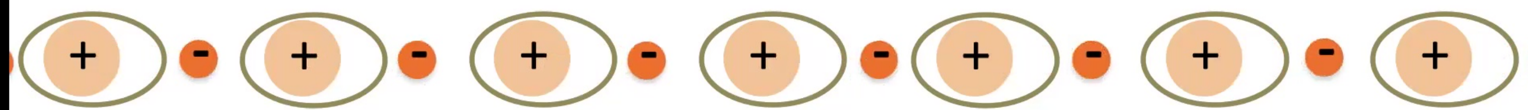
Wladimir Benalcazar, B. Andrei Bernevig, TLH (*Science* 2017)

Wladimir Benalcazar, B. Andrei Bernevig, TLH (*PRB* 2017)

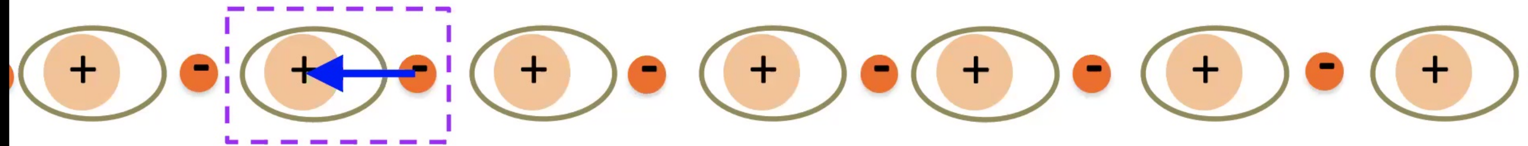
Electric Polarization in Crystals



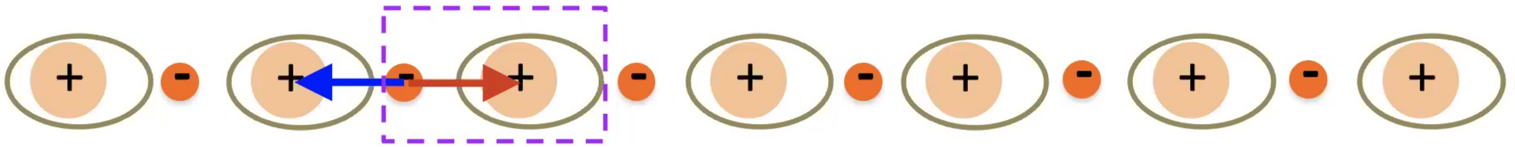
Electric Polarization in Crystals



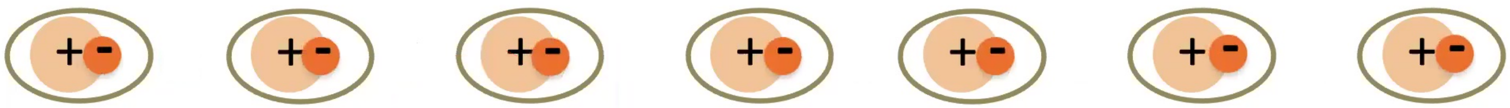
Electric Polarization in Crystals



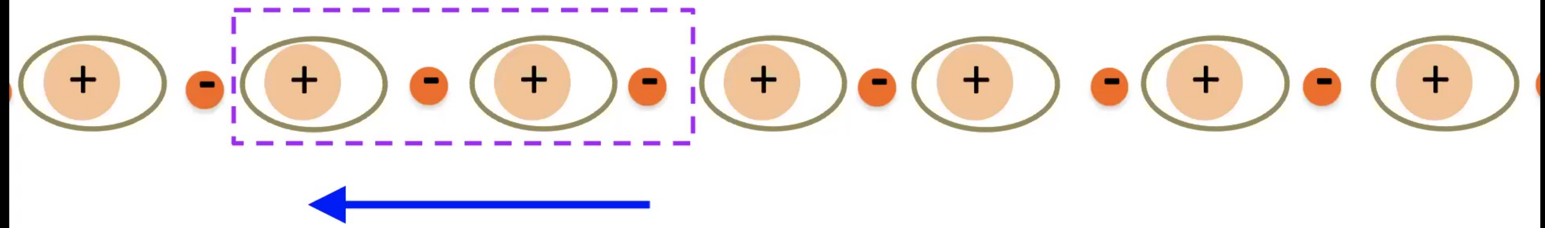
Electric Polarization in Crystals



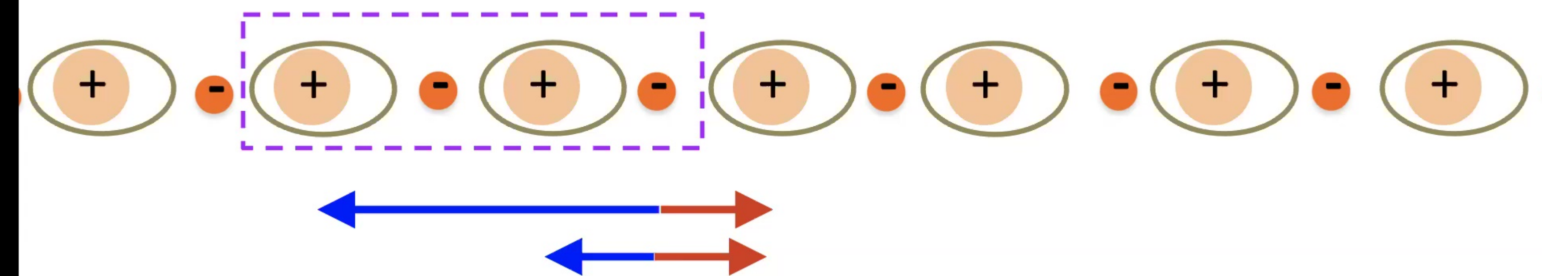
Electric Polarization in Crystals



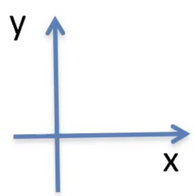
Electric Polarization in Crystals



Electric Polarization in Crystals

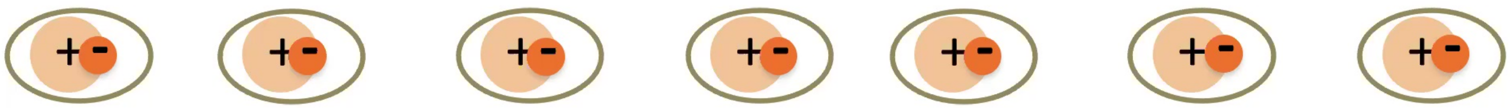


In each case the 1D polarization differs by an integer multiple of the charge *e*.



$$p \cong p + me$$

Quantization of Polarization in Crystals



Pick a symmetry under which the polarization is odd and enforce it

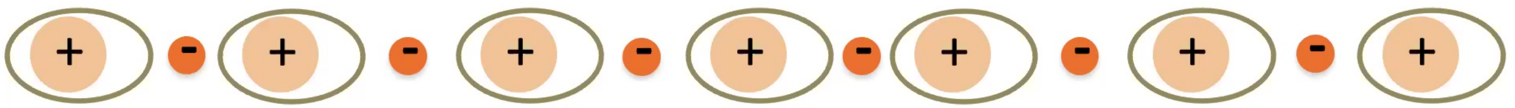
Ex: Inversion, Reflections, C2 rotations, Charge-conjugation

$$p = -p$$

$$2p = 0 \mod e$$

$$p = 0 \text{ or } e/2$$

Quantization of Polarization in Crystals



Pick a symmetry under which the polarization is odd and enforce it

Ex: Inversion, Reflections, C2 rotations, Charge-conjugation

$$p = -p$$

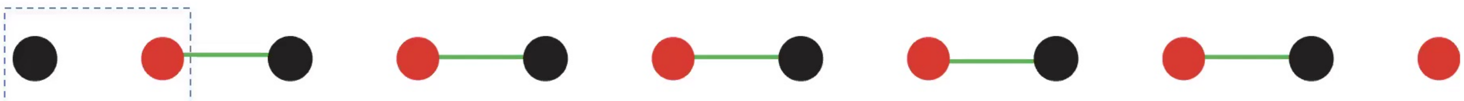
$$2p = 0 \mod e$$

$$p = 0 \text{ or } e/2$$

Example: 1D Insulators with Reflection Symmetry

p = 0

p = e/2



Calculation of polarization in momentum space (Vanderbilt, King-Smith 1993)

Given: $H(k)$

Construct: $A^{mn}(k) = -i\langle u_m(k)|\partial_k|u_n(k)\rangle$

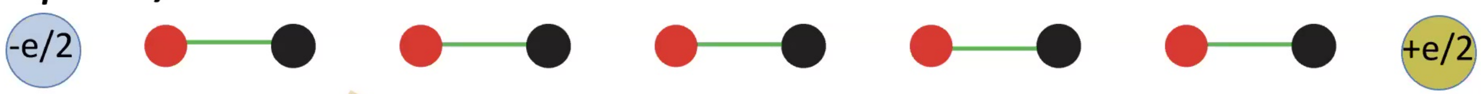
Calculate: $p = \frac{e}{2\pi}\int_{BZ} \mathrm{Tr}\left[A(k)\right] dk$

$$\boxed{\hat{x} \sim -i\partial_k}$$

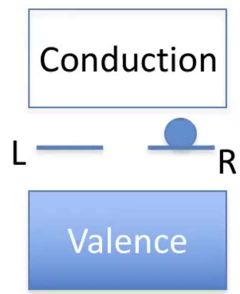
Polarization and Boundary Charge

$p = 0$

$p = e/2$



Connection between bulk polarization and boundary charge

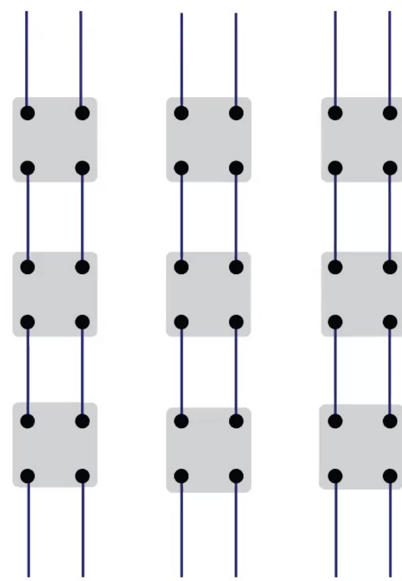


Make a quadrupole?

Quantized Quadrupole Model

Expectations:

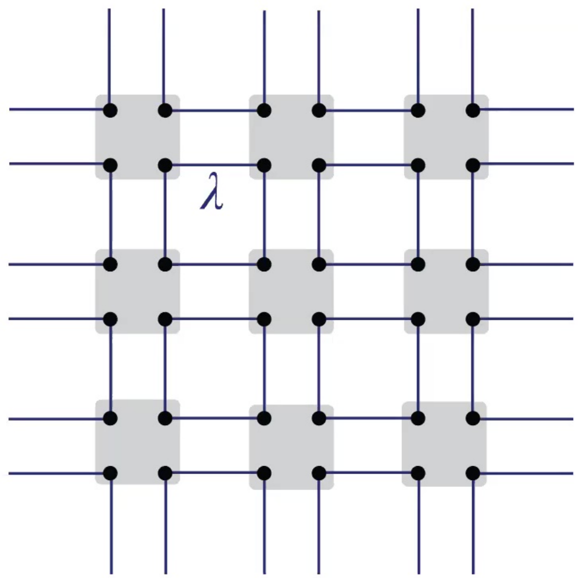
1. Quadrupole is formed from two dipoles, expect we need at least two occupied bands. Want quantized, vanishing polarization.
2. Need to choose symmetries under which quadrupole and polarization are odd. We will impose reflection symmetries M_x and M_y.



Quantized Quadrupole Model

Expectations:

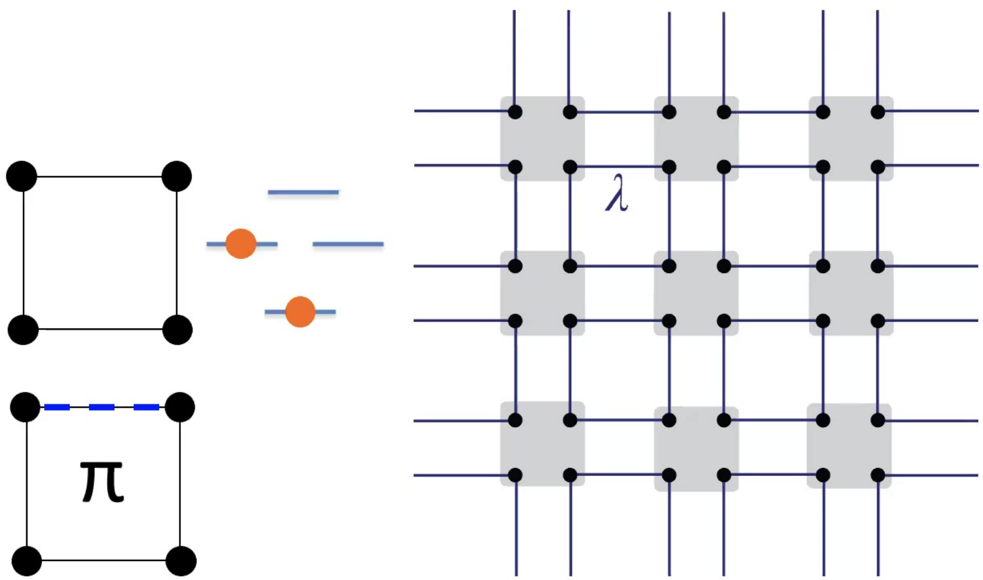
1. Quadrupole is formed from two dipoles, expect we need at least two occupied bands. Want quantized, vanishing polarization.
2. Need to choose symmetries under which quadrupole and polarization are odd. We will impose reflection symmetries M_x and M_y.



Quantized Quadrupole Model

Expectations:

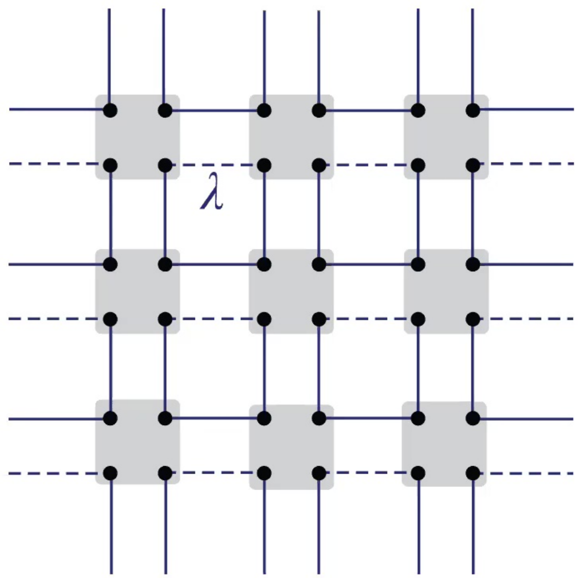
1. Quadrupole is formed from two dipoles, expect we need at least two occupied bands. Want quantized, vanishing polarization.
2. Need to choose symmetries under which quadrupole and polarization are odd. We will impose reflection symmetries M_x and M_y.



Quantized Quadrupole Model

Expectations:

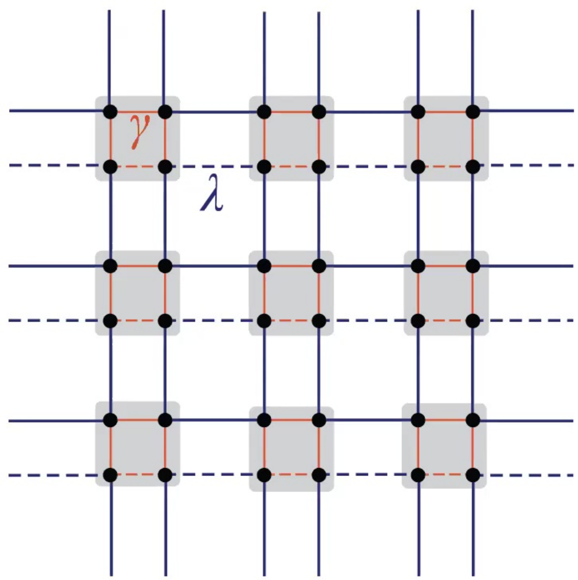
1. Quadrupole is formed from two dipoles, expect we need at least two occupied bands. Want quantized, vanishing polarization.
2. Need to choose symmetries under which quadrupole and polarization are odd. We will impose reflection symmetries M_x and M_y.



Quantized Quadrupole Model

Expectations:

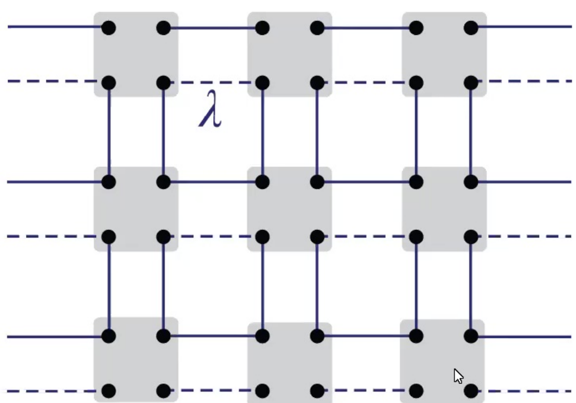
1. Quadrupole is formed from two dipoles, expect we need at least two occupied bands. Want quantized, vanishing polarization.
2. Need to choose symmetries under which quadrupole and polarization are odd. We will impose reflection symmetries M_x and M_y.



Quantized Quadrupole Model

Expectations:

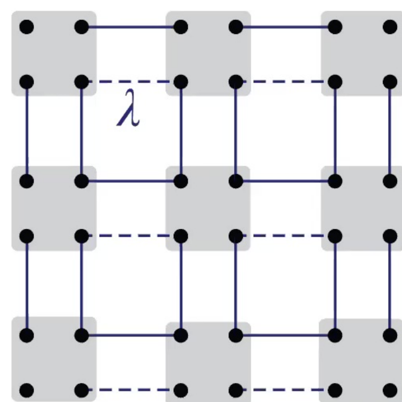
1. Quadrupole is formed from two dipoles, expect we need at least two occupied bands. Want quantized, vanishing polarization.
2. Need to choose symmetries under which quadrupole and polarization are odd. We will impose reflection symmetries M_x and M_y.



Quantized Quadrupole Model

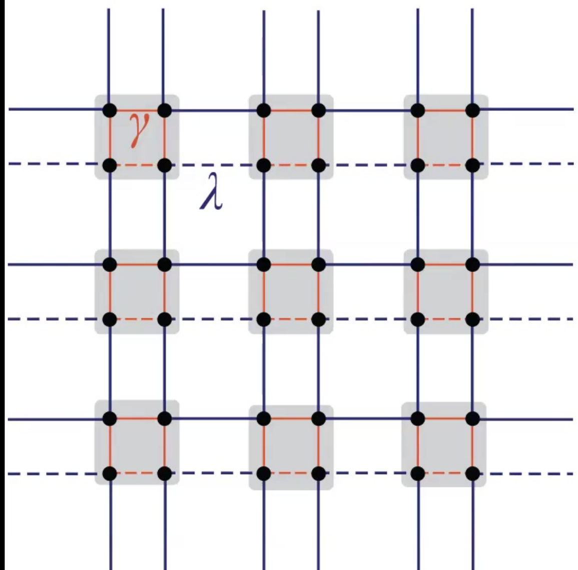
Expectations:

1. Quadrupole is formed from two dipoles, expect we need at least two occupied bands. Want quantized, vanishing polarization.
2. Need to choose symmetries under which quadrupole and polarization are odd. We will impose reflection symmetries M_x and M_y.

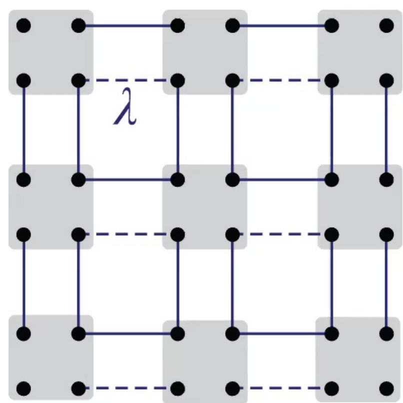


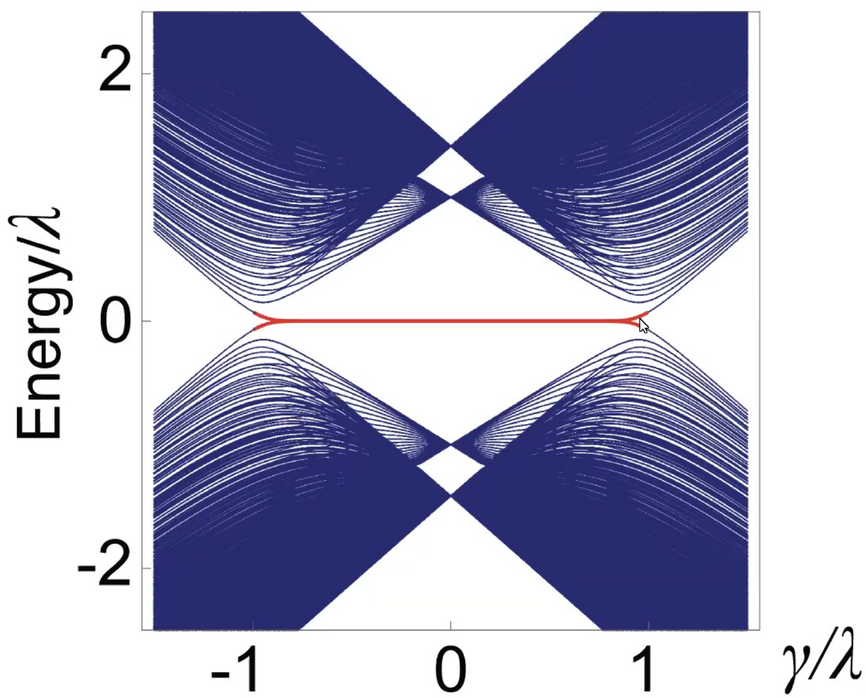
Bulk quadrupole density: $q^{xy} = e/2$

Properties of Quadrupole Model

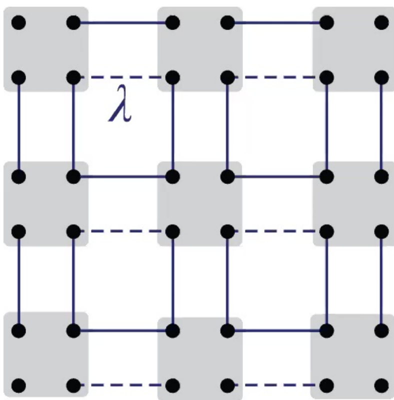


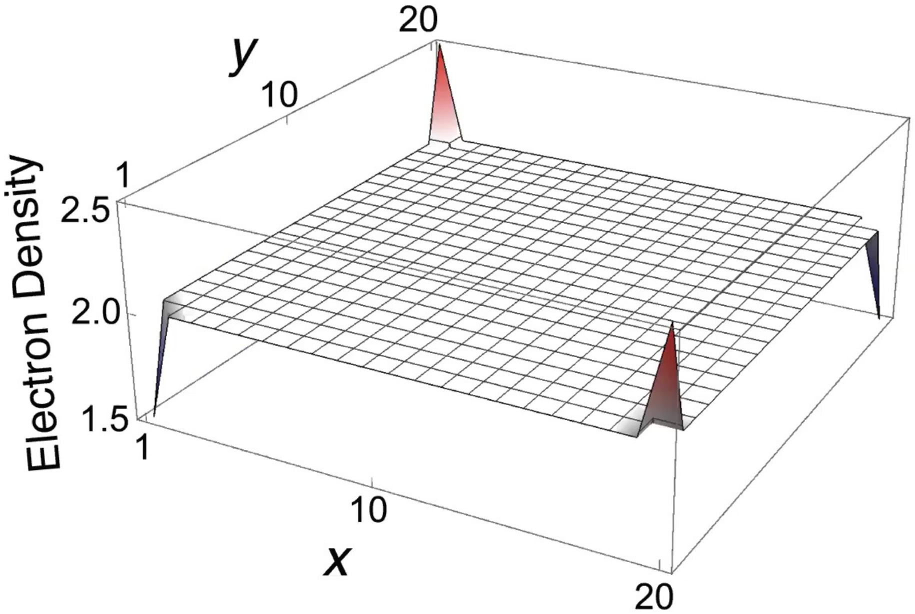
Properties of Quadrupole Model



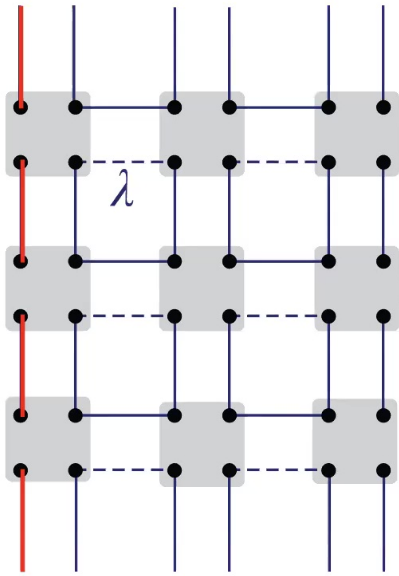


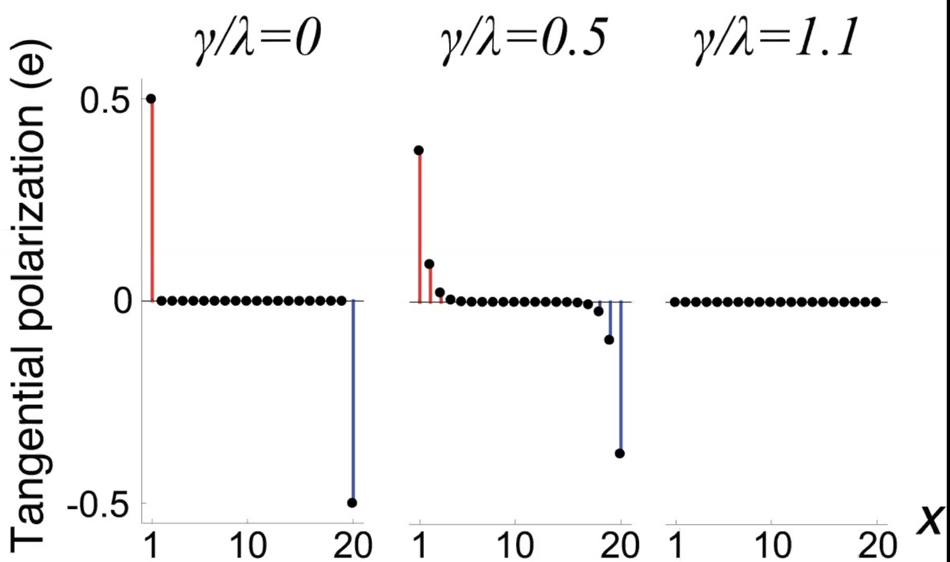
Properties of Quadrupole Model





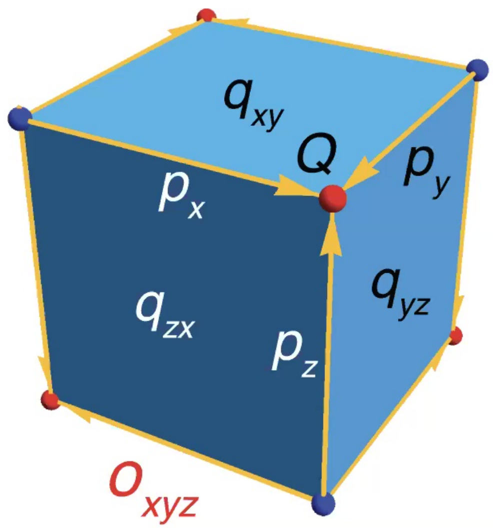
Properties of Quadrupole Model

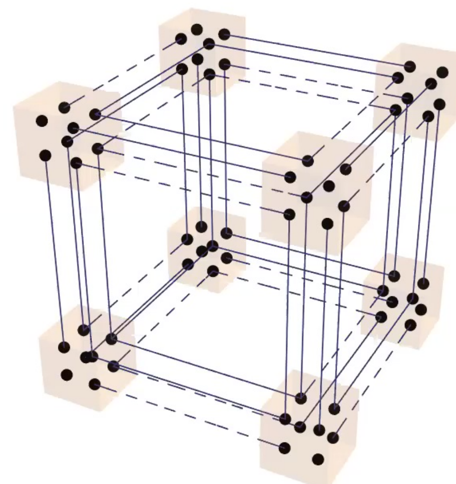




Octupole Moment in 3D systems

Simplest extension to 3D is with octupole moment o_{xyz}





Need four occupied bands (two cancelling quadrupoles).

Quantized by keeping all three reflections m_x, m_y, m_z

Part 3: Calculating Multipole Moments

Bulk Characterization of Quadrupole

Quadrupole is just a product of two coordinates, e.g., X*Y. The problem is that the effective electron position operators in the lattice do not commute with each other and cannot be measured simultaneously.

Can calculate corner charge and edge polarization:

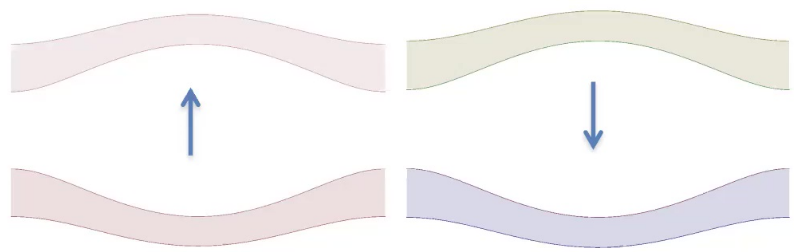
$$\boxed{Q_{cor} - p_1 - p_2 = -q^{xy}}$$

Instead we want to be able to diagnose the multipole moments from the bulk. For the quantized cases we can look for topological invariants.

Hidden Topology

Sometimes when a topological invariant is zero, it can be non-zero in some subsector of the occupied states.

An example is the Quantum Spin Hall effect where the total Chern number is zero, but we can resolve the invariant further by looking at each spin component:

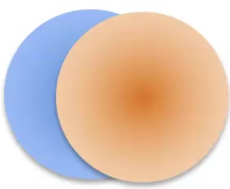


Chern Number = 0 = +1 (spin up) -1 (spin down)

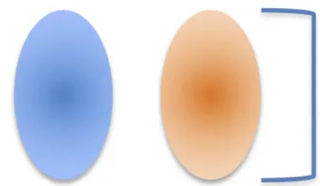
However, in our case we do not have a nice quantum number like spin to resolve the topology.

Resolving Topology in Real-Space

Key idea: Resolve the electrons in real-space

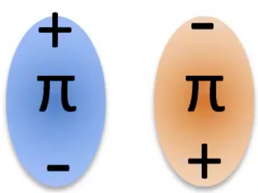


Resolve in x-direction:



Two separate clouds/sectors

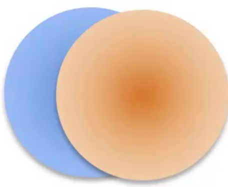
If we look at each cloud individually is it topological?



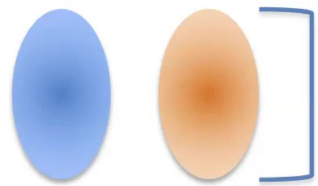
Calculate Berry Phase

Resolving Topology in Real-Space

Key idea: Resolve the electrons in real-space

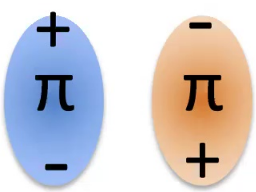


Resolve in x-direction:

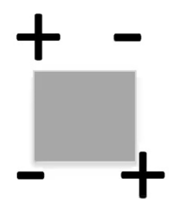


Two separate clouds/sectors

If we look at each cloud individually is it topological?

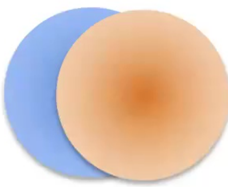


Calculate Berry Phase

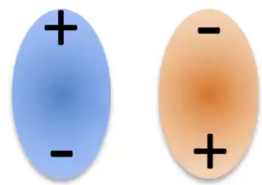


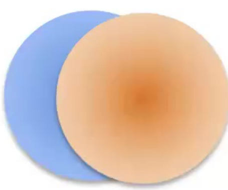
Resolving Topology in Real-Space

Key idea: Resolve the electrons in real-space

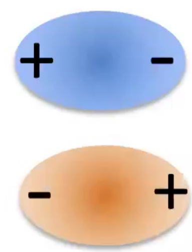


Resolve in x-direction:



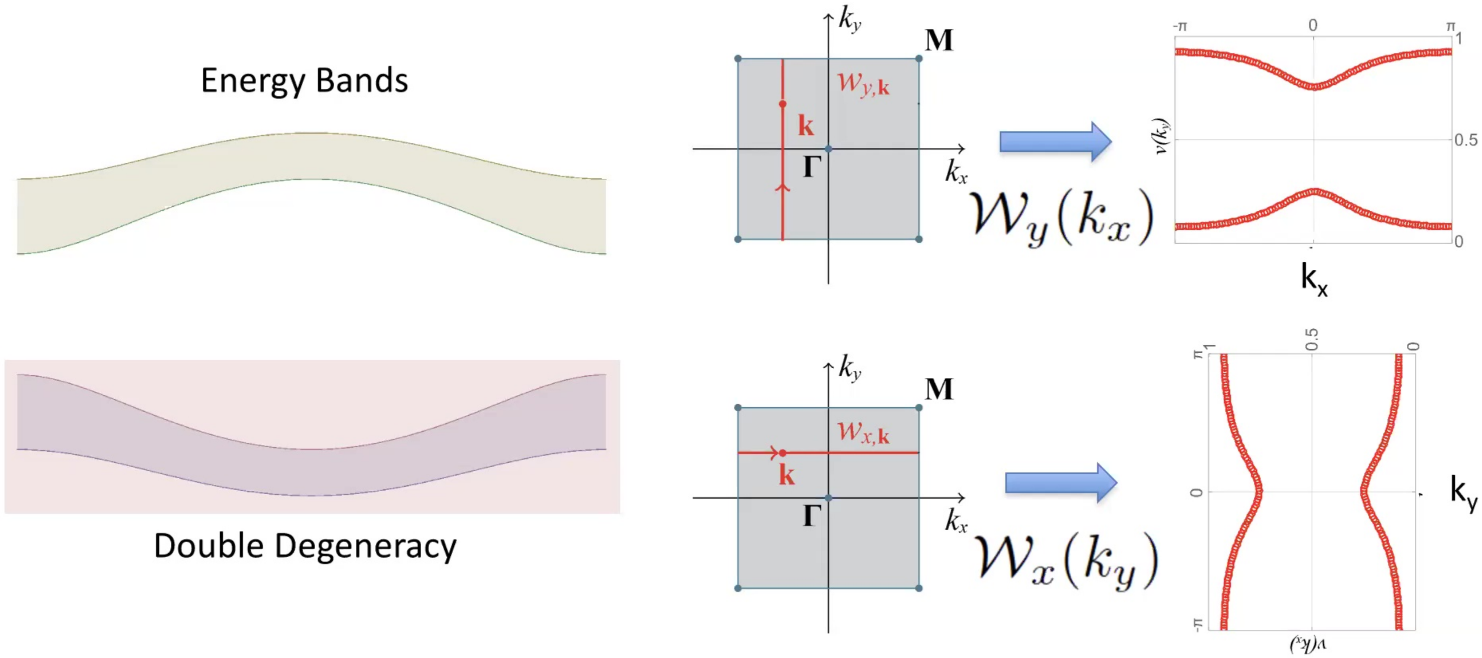


Resolve in y-direction:



If both sets of Berry phases are non-trivial, then it has a topological quadrupole moment!

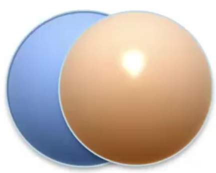
Topological Characterization of Quadrupole



Quadrupole phase characterized by π Berry phase of Wannier bands in both directions.

Resolving Topology in Real-Space

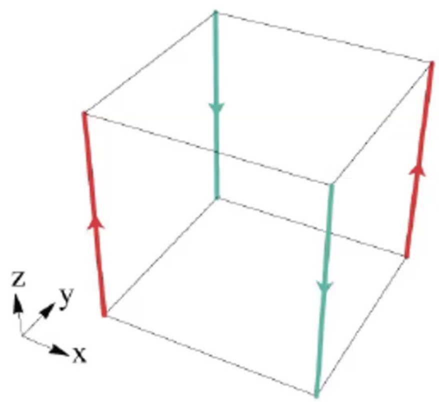
What else can we do? Let's look at 3D



Resolve in z-direction:

Quasi-2D
Electron Clouds

If the Chern number of each cloud is non-zero you find a chiral hinge insulator



Other Methods Needed

Different symmetry classes?
- Can use other symmetry indicators.

Non-quantized quadrupole?
- Can go back to surface polarization and corner charge.
  Still need vanishing bulk dipole moment.

How can we generically diagnose from the bulk?

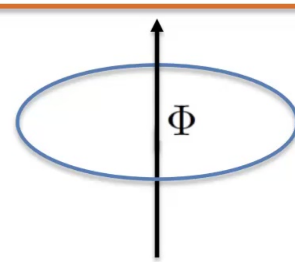
What about many-body systems?

Many-body calculation of electric polarization

Given: $|\Psi_0\rangle$

In 1D

Berry Phase:



$$A_x = \frac{\Phi}{L_x}$$

$$P = \frac{1}{2\pi}\mathrm{Im}\int d\Phi \langle \Psi_0 | \partial_\Phi | \Psi_0 \rangle$$

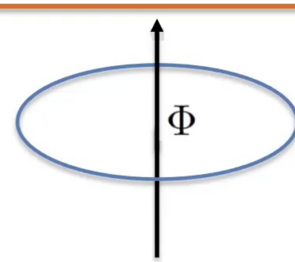
Ortiz, Martin (1994)
Resta (1999)

Many-body calculation of electric polarization

Given: $|\Psi_0\rangle$

In 1D

Berry Phase:



$$A_x = \frac{\Phi}{L_x}$$

$$P = \frac{1}{2\pi}\mathrm{Im}\int d\Phi \langle \Psi_0 | \partial_\Phi | \Psi_0 \rangle$$

Many-body
Twist Operator:

$$U_X = \exp\left[\frac{2\pi i \hat{X}}{L_x}\right] \qquad \hat{X} = \sum_{\alpha=1}^{N_e} \hat{x}_\alpha$$

$$U_X|\Psi_0\rangle = e^{2\pi i P}\left(|\Psi_0\rangle + O(1/L)\right)$$

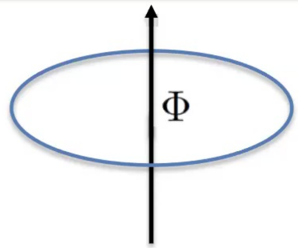
Ortiz, Martin (1994)
Resta (1999)

Many-body calculation of electric polarization

Given: $|\Psi_0\rangle$

In 1D

Berry Phase:



$$A_x = \frac{\Phi}{L_x}$$

$$P = \frac{1}{2\pi}\mathrm{Im}\int d\Phi \langle\Psi_0|\partial_\Phi|\Psi_0\rangle$$

Many-body Twist Operator:

$$U_X = \exp\left[\frac{2\pi i\hat{X}}{L_x}\right] \qquad \hat{X} = \sum_{\alpha=1}^{N_e}\hat{x}_\alpha$$

$$U_X|\Psi_0\rangle = e^{2\pi i P}\left(|\Psi_0\rangle + O(1/L)\right)$$

$$z_X \equiv \langle\Psi_0|U_X|\Psi_0\rangle$$

Ortiz, Martin (1994)
Resta (1999)

$$P = \lim_{N_e\to\infty}\frac{1}{2\pi}\mathrm{Im}\log z_X$$

Many-body calculation of electric quadrupole

Given: $|\Psi_0\rangle$

In 2D

Nested
Berry Phase?

No obvious way to do this. Will come back to it later.

Many-body
Twist Operator:

$$U_{XY} = \exp\left[\frac{2\pi i \widehat{XY}}{L_x L_y}\right] \qquad \widehat{XY} = \sum_{\alpha=1}^{N_e} \hat{x}_\alpha \hat{y}_\alpha$$

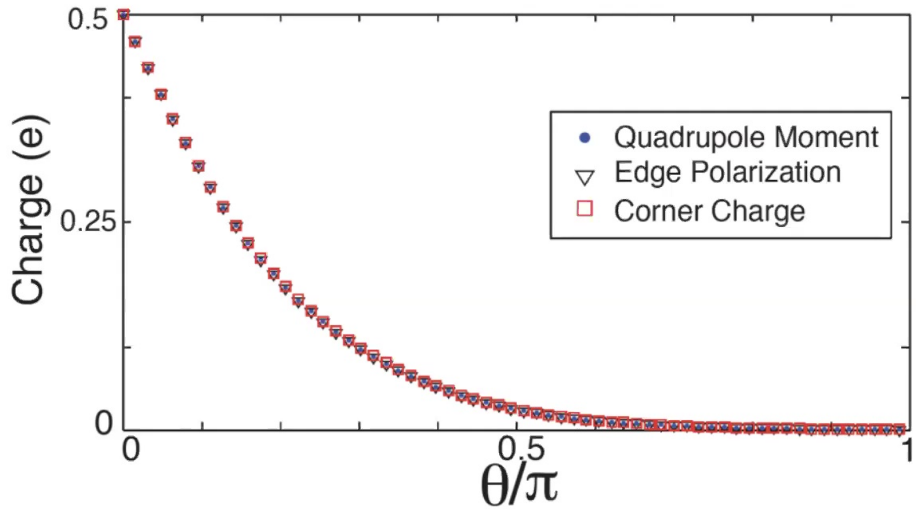
$$z_{XY} \equiv \langle \Psi_0 | U_{XY} | \Psi_0 \rangle$$

$$q_{xy} = \lim_{N_e \to \infty} \frac{1}{2\pi} \mathrm{Im} \log z_{XY}$$

Wheeler, Wagner, TLH (2018)
Kang, Shiozaki, Cho (2018)

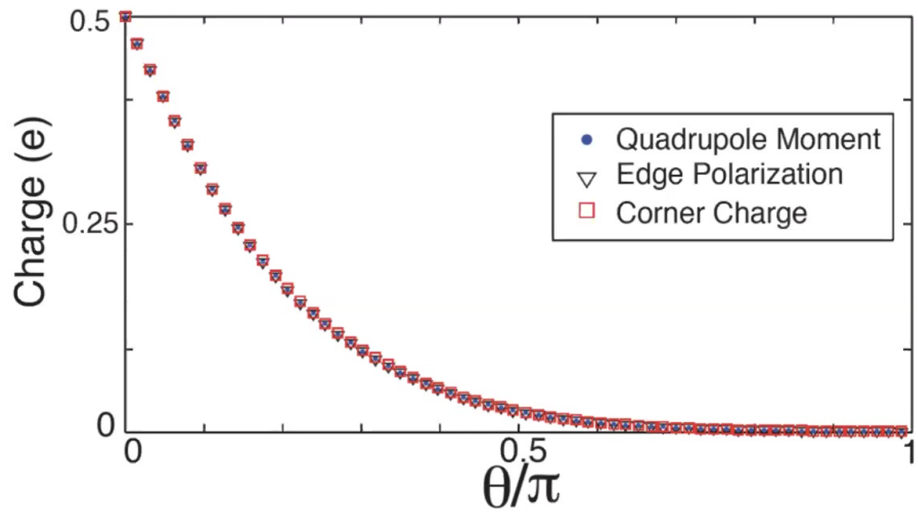
Many-Body Calculation of Quadrupole

It works. Example here is a pumping process going from topological to trivial phase:



Many-Body Calculation of Quadrupole

It works. Example here is a pumping process going from topological to trivial phase:



But something unusual happens in thermodynamic limit

Insulator: $U_X|\Psi_0\rangle = e^{2\pi i P}\left(|\Psi_0\rangle + O(1/L)\right) \longrightarrow \lim_{N\to\infty}|z_X| = 1$

But we find: $\lim_{N\to\infty}|z_{XY}| = 0$

So the phase, i.e., quadrupole moment is not well-defined in thermodynamic limit?

Charge localization and dipole fluctuations

The approach of $|z_X|$ to unity in the thermodynamic limit has a physical interpretation.

Charge localization length $$\xi^2 = -\lim_{N\to\infty} \frac{L_x^2}{4\pi^2 N} \log |z_X|^2$$

For finite size $|z_X| = 1$

Only if $\xi = 0$

$$\xi^2 = \lim_{N\to\infty} \frac{1}{N} \left(\langle \hat{X}^2 \rangle - \langle \hat{X} \rangle^2 \right)$$ Dipole fluctuations

$$U_X |\Psi_0\rangle = e^{2\pi i P} \left(|\Psi_0\rangle + O(1/L) \right)$$

Leads to a criterion:

$\xi \to \infty$	Metal	Resta, Sorella (1999)
		Aligia, Ortiz (2000)
$\xi \to const.$	Insulator	Souza, Wilkens, Martin (2000)

Even if dipole moment vanishes on average, the fluctuations can seemingly cause issues with the calculation of the quadrupole moment.

Dipole Conserving Hamiltonians

Want to study systems having $\xi = 0$

- Free fermion insulators: Wannier functions must be delta-functions. Too restrictive.
- Many-body insulators: Just need U(1) charge and dipole conservation

Charge: $e^{i\alpha}$ $\quad$ $e^{i\alpha}c_i^\dagger e^{-i\alpha}c_j = c_i^\dagger c_j$

Dipole: $e^{i\alpha\hat{X}}$

$$e^{i\alpha x}c^\dagger_{(x,y)}e^{-i\alpha(x+1)}c_{(x+1,y)}e^{i\alpha(x+1)}c^\dagger_{(x+1,y+1)}e^{-i\alpha x}c_{(x,y+1)}$$

$$= c^\dagger_{(x,y)}c_{(x+1,y)}c^\dagger_{(x+1,y+1)}c_{(x,y+1)}$$

One example: ring exchange

Pretko, PRB (2018)

Dipole Conserving Hamiltonians

Want to study systems having $\xi = 0$

Consequence 1:

$$U_X|\Psi_0\rangle = e^{2\pi i P_x}|\Psi_0\rangle \qquad U_Y|\Psi_0\rangle = e^{2\pi i P_y}|\Psi_0\rangle$$

Consequence 2:

Can couple the system to a rank-2 gauge field, e.g., A_{xy} [Pretko, PRB (2018)]

$$c^\dagger_{(x,y)}c_{(x+1,y)}c^\dagger_{(x+1,y+1)}c_{(x,y+1)} \rightarrow \boxed{e^{iA_{xy}}}c^\dagger_{(x,y)}c_{(x+1,y)}c^\dagger_{(x+1,y+1)}c_{(x,y+1)}$$

Heuristically: $A_{xy} = \frac{1}{2}(\partial_x A_y + \partial_y A_x)$

Dipole Insulator and Dipole Metal Criterion

Generalize Kohn's Criterion (sensitivity to shifts in gauge field): $A_{xy} \rightarrow A_{xy} + \mathfrak{q}$

$$D_d = -\frac{\pi}{V}\frac{\partial^2 E_0}{\partial \mathfrak{q}^2}\bigg|_{\mathfrak{q}=0}$$ Dipole stiffness (aka dipole Drude weight)

Dipole localization: $$\lambda_d^2 = -\lim_{N_d\to\infty}\frac{L_x^2 L_y^2}{4\pi^2 N_d}\log|z_{XY}|^2$$ $[\lambda_d] = Length^2$

Quadrupole fluctuations: $$\lambda_d^2 = \lim_{N_d\to\infty}\frac{1}{N_d}\left(\langle\widehat{XY}^2\rangle - \langle\widehat{XY}\rangle^2\right)$$

Leads to a criterion:

$\lambda_d = const,\ D_d = 0$ Dipole Insulator

$\lambda_d = \infty,\ D_d = const$ Dipole Metal

Dubinkin, May-Mann, TLH (2019)

Rank 2 Berry Phase

For ground states obeying

$$U_X|\Psi_0\rangle = |\Psi_0\rangle$$
$$U_Y|\Psi_0\rangle = |\Psi_0\rangle$$

Polarization vanishes

$$U_{XY}|\Psi_0\rangle = e^{2\pi i q_{xy}}\left(|\Psi_0\rangle + O(1/L)\right)$$

Rank-2 Berry phase gives quadrupole moment!

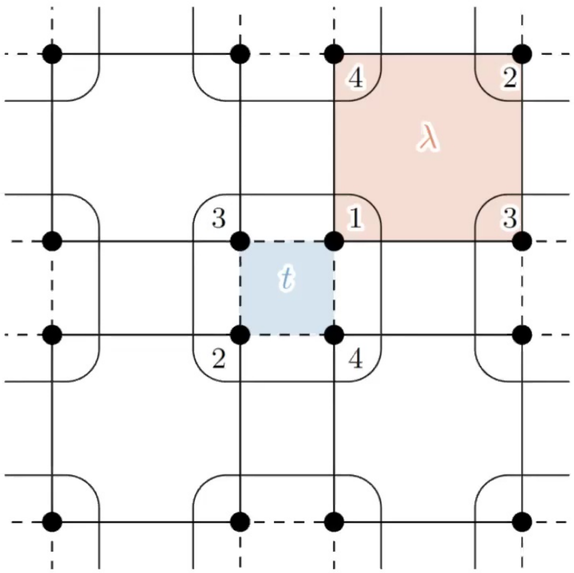
$$A_{xy} \to A_{xy} + \frac{2\pi}{L_x L_y}$$

$$q_{xy} = \frac{1}{2\pi}\mathrm{Im}\int_0^{2\pi/L_x L_y} d\mathfrak{q}\langle\Psi_0|\partial_{\mathfrak{q}}|\Psi_0\rangle$$

Connection to Araki, Mizoguchi, Hatsugai? (1906.00218)

Dubinkin, May-Mann, TLH (2019)

An Example Model:



$$H = \lambda \sum_{\mathbf{p}} |\square\rangle\langle\square| + t \sum_{\mathbf{s}} |\square\rangle\langle\square| + h.c.$$

Let t=0

$$|\psi\rangle_{\mathbf{p}} = \frac{1}{\sqrt{2}}\left(|\square\rangle_{\mathbf{p}} - |\square\rangle_{\mathbf{p}}\right)$$

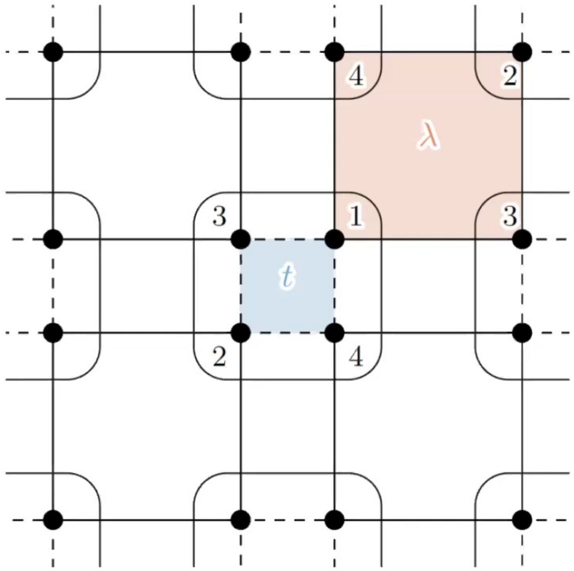
$$|GS\rangle = \bigotimes_{\mathbf{p}} |\psi\rangle_{\mathbf{p}}$$

$\lambda_d = a^2/2$ Dipole insulator

$$|\psi(\mathfrak{q})\rangle_{\mathbf{p}} = \frac{1}{\sqrt{2}}\left(e^{-i\frac{\mathfrak{q}}{2}}|\square\rangle_{\mathbf{p}} - e^{i\frac{\mathfrak{q}}{2}}|\square\rangle_{\mathbf{p}}\right) \longrightarrow \mathrm{Im}\sum_{\mathbf{p}=1}^{L_xL_y}\int_0^{2\pi/L_xL_y} d\mathfrak{q}\langle\psi(\mathfrak{q})|\partial_{\mathfrak{q}}|\psi(\mathfrak{q})\rangle_{\mathbf{p}} = \pi$$

Dubinkin, May-Mann, TLH (2019); You, Burnell, TLH (2019)

An Example Model:



$$H = \lambda \sum_{\mathbf{p}} |\square\rangle\langle\square| + t \sum_{\mathbf{s}} |\square\rangle\langle\square| + h.c.$$

Let t=0

$$|\psi\rangle_{\mathbf{p}} = \frac{1}{\sqrt{2}}\left(|\square\rangle_{\mathbf{p}} - |\square\rangle_{\mathbf{p}}\right)$$

$$|GS\rangle = \bigotimes_{\mathbf{p}} |\psi\rangle_{\mathbf{p}}$$

$\lambda_d = a^2/2$ Dipole insulator

$$|\psi(\mathfrak{q})\rangle_{\mathbf{p}} = \frac{1}{\sqrt{2}}\left(e^{-i\frac{\mathfrak{q}}{2}}|\square\rangle_{\mathbf{p}} - e^{i\frac{\mathfrak{q}}{2}}|\square\rangle_{\mathbf{p}}\right) \longrightarrow \mathrm{Im}\sum_{\mathbf{p}=1}^{L_xL_y}\int_0^{2\pi/L_xL_y} d\mathfrak{q}\langle\psi(\mathfrak{q})|\partial_{\mathfrak{q}}|\psi(\mathfrak{q})\rangle_{\mathbf{p}} = \pi$$

Two dipole insulator phases, t=0 and λ = 0 differ by rank 2 Berry phase (quadrupole)

Dubinkin, May-Mann, TLH (2019); You, Burnell, TLH (2019)

2D Topological "Dipole" Insulator

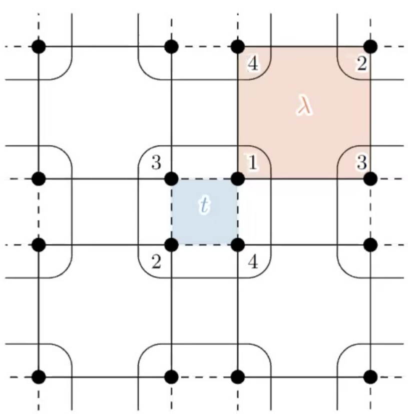
1D topological charge insulator

$$S_{eff} = \frac{1}{4\pi}\int dx dt\, \theta \epsilon^{\mu\nu} F_{\mu\nu} = \frac{1}{2\pi}\int dx dt\, \theta E_x$$

$$\rho = -\frac{1}{2\pi}\partial_x \theta \qquad \boxed{P_x = \frac{\theta}{2\pi}}$$

$$j_x = \frac{1}{2\pi}\partial_t \theta$$

2D topological dipole insulator



$$S_{eff} = \frac{1}{2\pi}\int dx dy dt\, \theta E_{xy} \qquad E_{xy} = \partial_x \partial_y A_0 - \partial_t A_{xy}$$

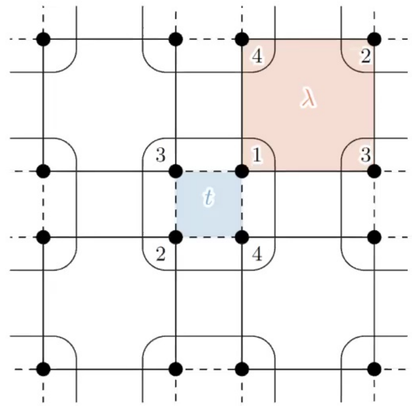
$$\rho = \frac{1}{2\pi}\partial_x \partial_y \theta \qquad \boxed{q_{xy} = \frac{\theta}{2\pi}}$$

$$j_{xy} = \frac{1}{2\pi}\partial_t \theta$$

You, Burnell, TLH (2019)

2D Topological "Dipole" Insulator

2D topological dipole insulator

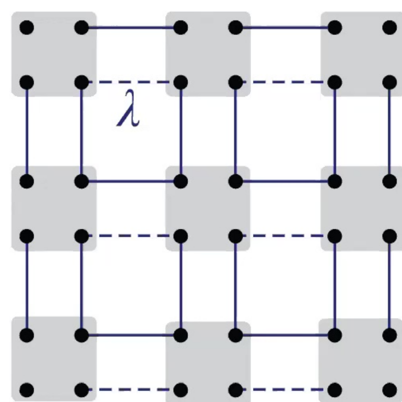


$$S_{eff} = \frac{1}{2\pi}\int dxdydt\,\theta E_{xy}$$

Break dipole U(1),
But keep total
polarization zero

$$E_{xy} = \frac{1}{2}(\partial_x E_y + \partial_y E_x)$$

2D higher order TI

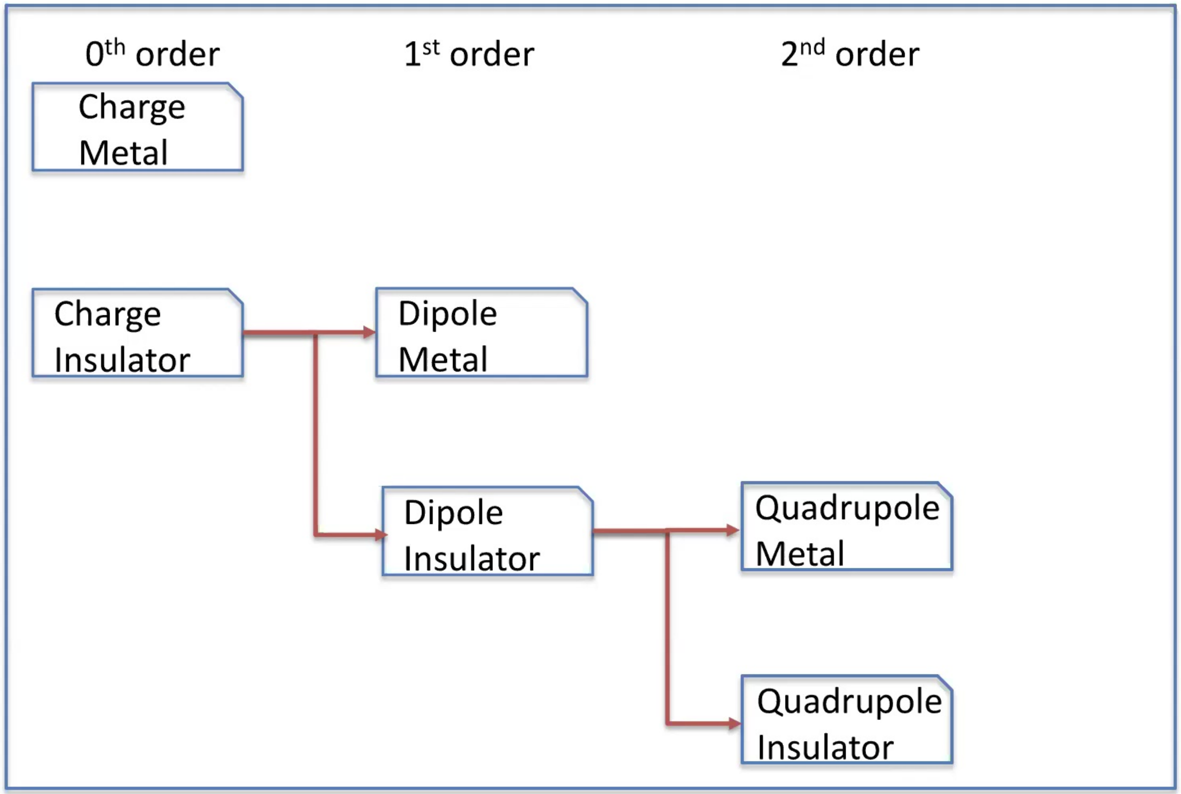


$$S_{eff} = \frac{1}{2}\int dxdydt\, q_{ij}\partial_i E_j$$

You, Burnell, TLH (2019)

Summary: Multipole Metals and Insulators

We propose a refinement of charge insulators



Dubinkin, May-Mann, TLH (2019)