Title: Are we Living in the Matrix?

Speakers: David Tong

Series: Colloquium

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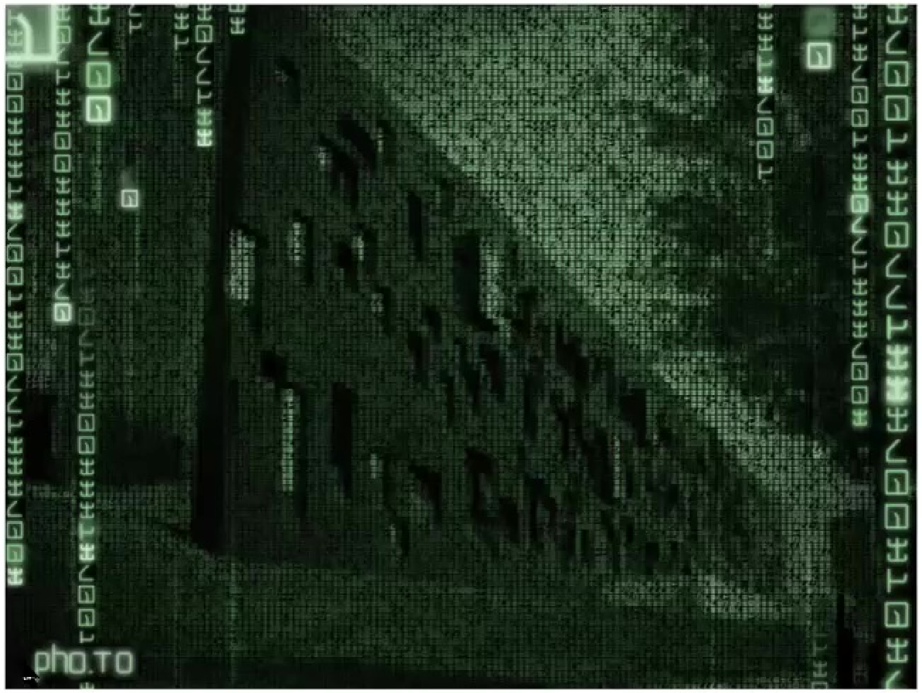
Abstract: No. Obviously not. It's a daft question. But, buried underneath this daft question is an extremely interesting one: is it possible to simulate the known laws of physics on a computer? Remarkably, there is a mathematical theorem, due to Nielsen and Ninomiya, that says the answer is no. I'll explain this theorem, the underlying reasons for it, and some recent work attempting to circumvent it.

Are we living in the Matrix?

David Tong



THE ROYAL SOCIETY

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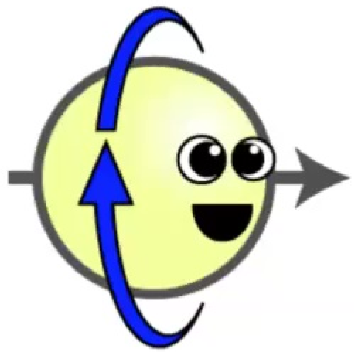
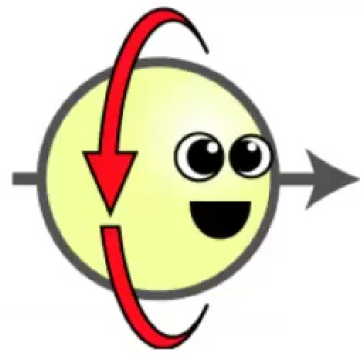
Parity Violation

Chiral Gauge Theory



left-handed fermion

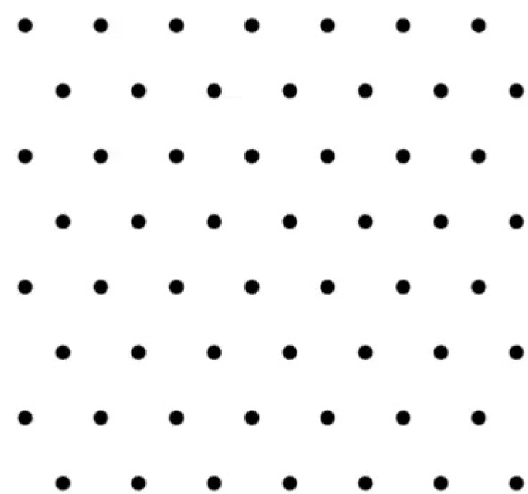


right-handed fermion

Parity violation ⟹ these experience different forces

The Nielson-Ninomiya Theorem

A chiral fermion, or chiral gauge theory, cannot live in a discrete spacetime



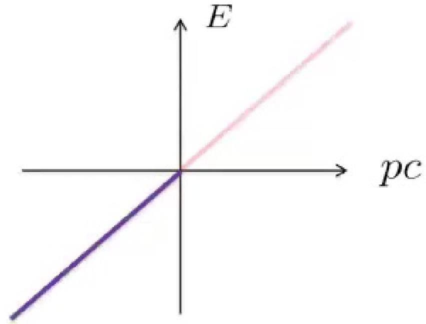
Chiral Fermions on the Lattice

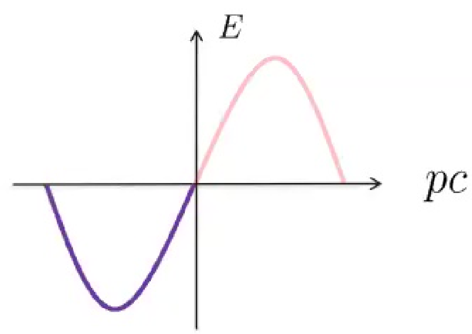
Take a massless, right-moving fermion

Continuum $E = +pc$

Lattice $E \sim \sin\left(\frac{pa}{2\hbar}\right)$

$$v = \frac{dE}{dp}$$





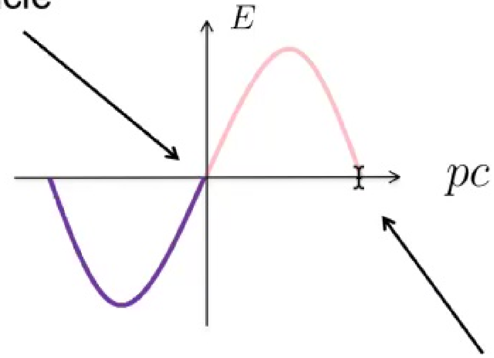
Space is discrete

Momentum is periodic

$$p \in \left[-\frac{\hbar\pi}{a}, \frac{\hbar\pi}{a}\right)$$

The Nielsen-Ninomiya Theorem

Start from right-moving particle



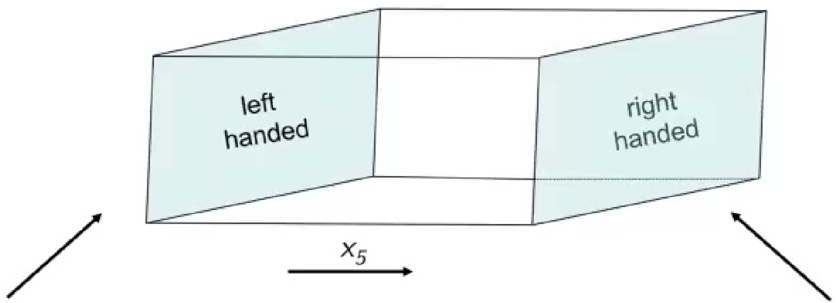
The lattice generates a new, low-energy left-moving state.

- Can't put a single right-moving fermion on a lattice. The lattice gives you back a left-moving *doubler* or *mirror fermion.*
- Moreover, left- and right-moving fermions experience same forces.

Nielsen and Ninomiya '81

A Different Perspective: Domain Wall Fermions

Take a *massive* fermion in one dimension higher. The extra dimension is an interval:



A massless chiral fermion appears on one edge…

…and its doubler on the other

Kaplan '92; Shamir '93

Why?

Quantum anomalies mean that most chiral theories do not make sense

Bell and Jackiw '69; Adler 69;
Bouchiat, Illiopoulos and Meyer '72; Georgi and Glashow '72; Gross and Jackiw '72

Quantum Anomalies

Consistency conditions need to be obeyed. Either...

- Left-handed and right-handed fermions feel the same force

Or

- More delicate balancing act between left-handed and right-handed

↑ Only this second option violates parity

An Example: The Standard Model

strong weak hypercharge

$$G = SU(3) \times SU(2) \times U(1)$$

Take one generation of quarks and leptons to have usual properties under strong and weak force but arbitrary *integer* charges under hypercharge

After a change of variables, consistency conditions (ignoring gravity) require

$$X^3 + Y^3 = Z^3$$

with X, Y and Z integers. Unique solution, e.g $1^3 + 0^3 = 1^3$ gives observed charges.

Lohitsiri and Tong '19

Quantum Anomalies

Most chiral theories do not make sense. We must have either

The lattice chooses this option

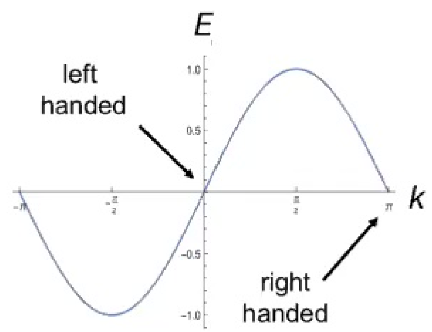
- Left-handed and right-handed fermions feel the same force
- More delicate balancing act between left-handed and right-handed

Nature chose this option

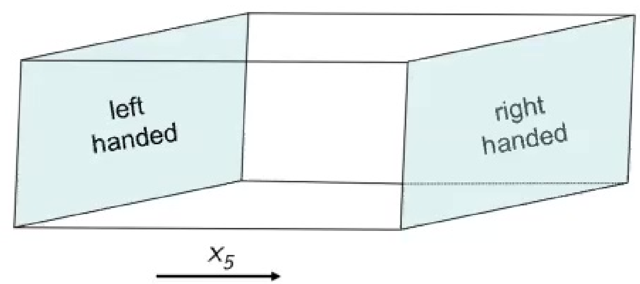
Evading the Nielsen-Ninomiya Theorem

Take your fermions and their doublers…

Either separated in momentum space…



…or in an extra spatial dimension



Evading the Nielsen-Ninomiya Theorem

Take your fermions and their doublers…

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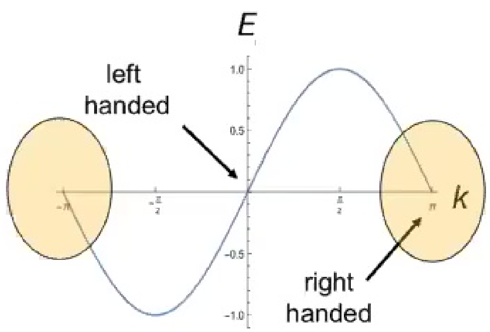
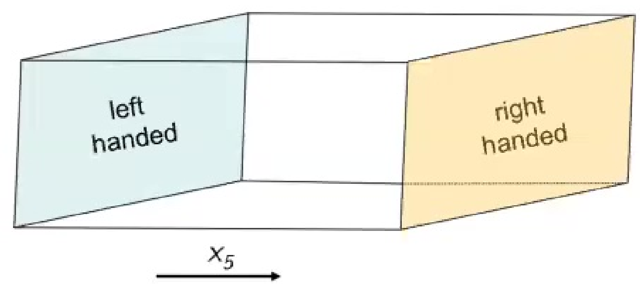




An Old Idea: Find a way to gap the doublers, leaving the original fermions untouched

Challenges:

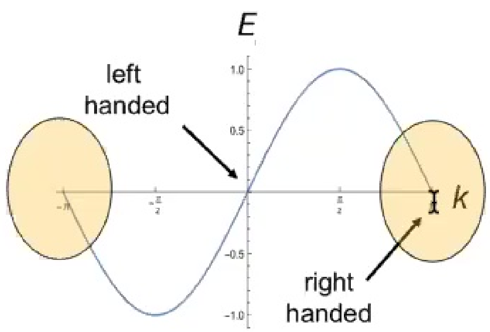
- Ensure that only the mirror fermions experience the interactions
- Find interactions that gap chiral fermions *without* breaking gauge symmetry!

Eichten and Preskill '86

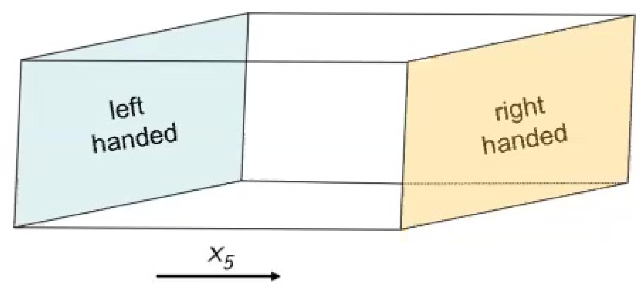
Evading the Nielsen-Ninomiya Theorem

Take your fermions and their doublers…

Either separated in momentum space…



…or in an extra spatial dimension



Most attempts work with irrelevant multi-fermion operators, cranked up to the lattice scale

$$\mathcal{L}_{4-\text{fermi}} \sim \psi\psi\psi\psi$$

Sadly, so far, to no avail.

Eichten and Preskill '86, Golterman, Petcher and Rivas '93; Creutz, Rebbi, Tytgat, Xue '96;
Poppitz and Shang '10; Chen, Giedt and Poppitz '12; Wen '13; Wang and Wen '13; Kikukawa '17'; Wang and Wen '18

Gapping Chiral Fermions

Goal: Find a way to give a mass to chiral fermion without breaking symmetries

e.g. give a mass to one generation of the Standard Model
without breaking electroweak symmetry

Razamat and Tong '20

Rules of the Game

Start from free massless fermions realising a non-anomalous chiral symmetry *G*

- Can't write down a mass term as it breaks G
- Can't write down 4-fermion terms as irrelevant.
- Must introduce new degrees of freedom.

Idea: Add new massive matter that is *vector-like* under G and allow it to interact.

- This can include gauge fields for another symmetry *H* provided
  - $[H,G] = 0$
  - There are no mixed anomalies with *H*
  - You include scalar fields that can decouple *H* gauge bosons

An Example: The Standard Model

$G = SU(3) \times SU(2) \times U(1)$

Razamat and Tong '20

(left-handed)^c		right-handed		
leptons	quarks	electron	up quark	down quark
$(\mathbf{1}, \mathbf{2})_{-3}$	$(\bar{\mathbf{3}}, \mathbf{2})_{+1}$	$(\mathbf{1}, \mathbf{1})_{+6}$	$(\mathbf{3}, \mathbf{1})_{-4}$	$(\mathbf{3}, \mathbf{1})_{+2}$

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Razamat and Tong '20

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$(\mathbf{1},\mathbf{2})_{-3}$				$(\mathbf{3},\mathbf{1})_{+2}$	$(\mathbf{1},\mathbf{1})_{0}$
$(\mathbf{1},\mathbf{2})_{+3}$				$(\bar{\mathbf{3}},\mathbf{1})_{-2}$	

- Add three further pairs of fermions
- Gauge the H = SU(2) symmetry

An Example: The Standard Model

$G = SU(3) \times SU(2) \times U(1)$

Razamat and Tong '20

(left-handed)^c — leptons, quarks; right-handed — electron, up quark, down quark

leptons	quarks	electron	up quark	down quark	neutrino
$(\mathbf{1},\mathbf{2})_{-3}$	$(\bar{\mathbf{3}},\mathbf{2})_{+1}$	$(\mathbf{1},\mathbf{1})_{+6}$	$(\mathbf{3},\mathbf{1})_{-4}$	$(\mathbf{3},\mathbf{1})_{+2}$	$(\mathbf{1},\mathbf{1})_{0}$
$(\mathbf{1},\mathbf{2})_{-3}$				$(\mathbf{3},\mathbf{1})_{+2}$	$(\mathbf{1},\mathbf{1})_{0}$
$(\mathbf{1},\mathbf{2})_{+3}$				$(\bar{\mathbf{3}},\mathbf{1})_{-2}$	

- Add three further pairs of fermions
- Gauge the H = SU(2) symmetry
- Supersymmetrize.
  - Add scalar superpartners for all fermions, and a H = SU(2) gaugino

An Example: The Standard Model

$$G = SU(3) \times SU(2) \times U(1)$$

Razamat and Tong '20

L	Q	E	U	D	N
leptons	quarks	electron	up quark	down quark	neutrino
$(\mathbf{1},\mathbf{2})_{-3}$	$(\bar{\mathbf{3}},\mathbf{2})_{+1}$	$(\mathbf{1},\mathbf{1})_{+6}$	$(\mathbf{3},\mathbf{1})_{-4}$	$(\mathbf{3},\mathbf{1})_{+2}$	$(\mathbf{1},\mathbf{1})_{0}$
$(\mathbf{1},\mathbf{2})_{-3}$				$(\mathbf{3},\mathbf{1})_{+2}$	$(\mathbf{1},\mathbf{1})_{0}$
$L' \longrightarrow (\mathbf{1},\mathbf{2})_{+3}$				$D' \longrightarrow (\bar{\mathbf{3}},\mathbf{1})_{-2}$	

- The H = SU(2) gauge theory is coupled to six doublets.
- This confines *without* breaking the global symmetry. Seiberg '94
- The low-energy physics consists of 15 free mesons:

$$\epsilon_{ab}L^aL^b \qquad \epsilon_{ijk}D^iD^j \qquad L^aD^i \qquad L^aN \qquad D^iN$$

An Example: The Standard Model

$$G = SU(3) \times SU(2) \times U(1)$$

Razamat and Tong '20

L	Q	E	U	D	N
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If we add the superpotential

$$\mathcal{W}_{UV} = \epsilon_{ab} L^a L^b E + \epsilon_{ijk} D^i D^j U^k + \epsilon_{ab} L^a D^i Q^b_i + \epsilon_{ab} L^a N L'^b + D^i N D'_i$$

But, in the infra-red, this becomes

$$\mathcal{W}_{IR} = \widetilde{E} E + \widetilde{U}_k U^k + \widetilde{Q}^i_b Q^b_i + \widetilde{L}^b L'^b + \widetilde{D}_i D'_i$$

Comments

- Supersymmetry is merely a crutch to understand strong coupling
- Process robust against soft breaking of supersymmetry

An Example: The Standard Model

$$G = SU(3) \times SU(2) \times U(1)$$

Razamat and Tong '20

L	Q	E	U	D	N
leptons	quarks	electron	up quark	down quark	neutrino
$(\mathbf{1}$					$\mathbf{1})_0$
$(\mathbf{1}$					$\mathbf{1})_0$

The theory is gapped while preserving G = SU(3) x SU(2) x U(1)!

$L' \longrightarrow (\mathbf{1}, \mathbf{2})_{+3}$

$D' \longrightarrow (\bar{\mathbf{3}}, \mathbf{1})_{-2}$

If we add the superpotential

$$\mathcal{W}_{UV} = \epsilon_{ab} L^a L^b E + \epsilon_{ijk} D^i D^j U^k + \epsilon_{ab} L^a D^i Q_i^b + \epsilon_{ab} L^a N L'^b + D^i N D_i'$$

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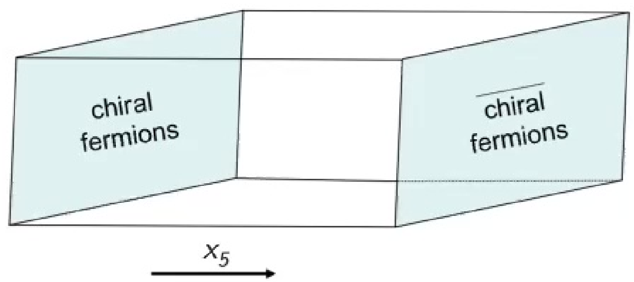
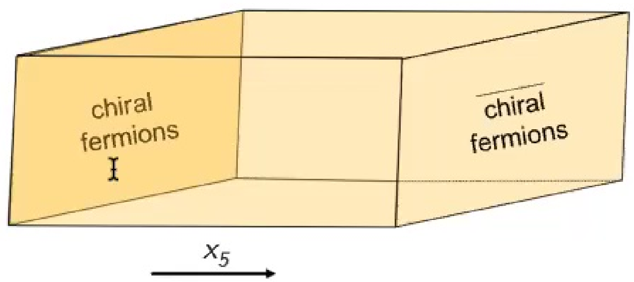
Back to the Lattice (sort of!)



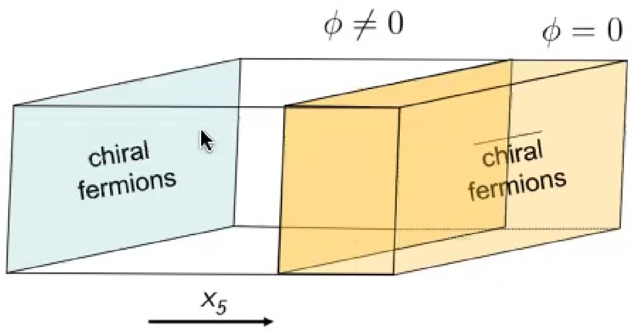
- Put gauge field that you care about everywhere in the fifth dimension.
  - e.g. G= *SU(3) x SU(2) x U(1)*
  - It couples to chiral fermions + their conjugates in a vector-like manner

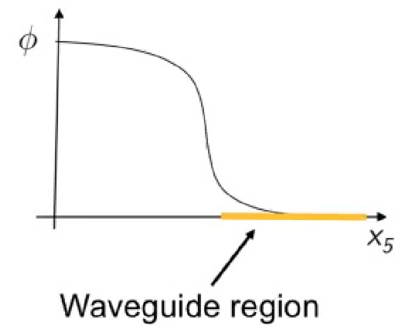
Gapping Domain Wall Fermions



- Put gauge field that you care about everywhere in the fifth dimension.
  - e.g. G= *SU(3) x SU(2) x U(1)*
  - It couples to chiral fermions + their conjugates in a vector-like manner
- Put the auxiliary gauge field everywhere.
  - e.g. *H = SU(2)*
  - It too couples to chiral fermions and their conjugates

Gapping Domain Wall Fermions





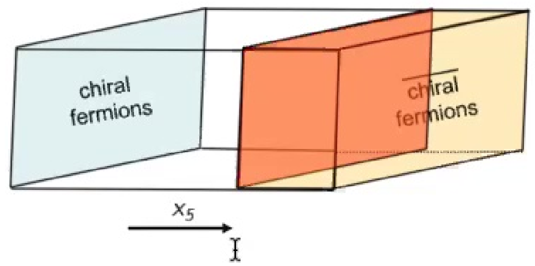
- Put gauge field that you care about everywhere in the fifth dimension.
  - e.g. G= *SU(3) x SU(2) x U(1)*
  - It couples to chiral fermions + their conjugates in a vector-like manner

- Put the auxiliary gauge field everywhere.
  - e.g. *H = SU(2)*
  - It too couples to chiral fermions and their conjugates

- Add Higgs fields for *H* with a profile in the fifth dimension.
  - Add extra fermions coupled to H

A Pitfall

You might get extra degrees off freedom on the edge of the waveguide…

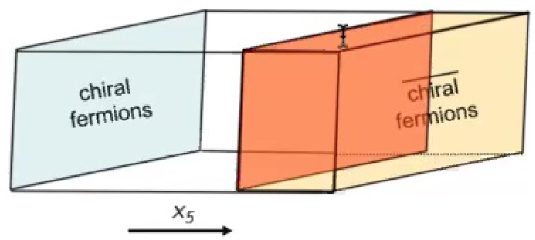


…this could ruin the whole thing

Golterman, Jansen, Petcher, Vink ’93; Golterman and Shamir ‘94

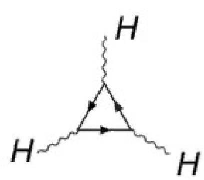
A Pitfall

When do these extra degrees of freedom arise?



Answer: when the fermions on one edge transform in an anomalous representation of *H*

$$\sum_{\text{5d Fermions}}$$



$$= \frac{k}{24\pi^2} \mathrm{tr} A \wedge F \wedge F + \dots$$

To see cancellation *must* sum over fermions. But this is hard as there is a *sign problem!*

Golterman, Jansen, Petcher, Vink '93; Golterman and Shamir '94

Thank you for your attention