

Title: Towards Lorentzian quantum gravity via effective spin foams

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Series: Quantum Gravity

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Abstract: Euclidean quantum gravity approaches have a long history but suffer from a number of severe issues. This gives a strong motivation to develop Lorentzian approaches. Spin foams constitute an important such approach, which incorporate a rigorously derived discrete area spectrum. I will explain how this discrete area spectrum is connected to the appearance of an anomaly, which explains the significance of the Barbero-Immirzi parameter and forces an extension of the quantum configuration space, to also include torsion degrees of freedom. This can be understood as a defining characteristic of the spin foam approach, and provides a pathway to an (experimental) falsification.

All these features are captured in the recently constructive effective spin foam model, which is much more amenable to numerical calculations than previous models. I will present numerical results that a) show that spin foams do impose the correct equations of motion b) highlight the influence of the anomaly and c) underline the difference to Euclidean quantum gravity. I will close with an outlook on the features that can be studied with a truly Lorentzian model, e.g. topology change.



Towards Lorentzian quantum gravity via effective spin foams

**Bianca Dittrich,
Perimeter Institute**



for the second part:

**2004.07013, PRL
2011.14468**

**Seth Asante, BD, Hal Haggard,
Seth Asante, BD, Hal Haggard
and WIP**

Overview

A rush through history: From Euclidean to Lorentzian quantum gravity and a few surprises.

Effective spin foams.

Testing effective spin foams: Numerical results.

Emergence of quantum space time

Two view points leading to the same task:

- regulate path integral for quantum gravity via discretization, remove this regulator
- smooth, macroscopic geometry emergent from microscopic building blocks or quantum foam or shiny apples via a path integral

Path integral:

$$Z = \int \mathcal{D}\text{geom} \exp(iS(\text{geom}))$$

Lots of different choices:

- Space of “geometries”
- Measure on this space:
discrete, continuous, measure terms, ...

Here: (versions of)
Einstein Hilbert action

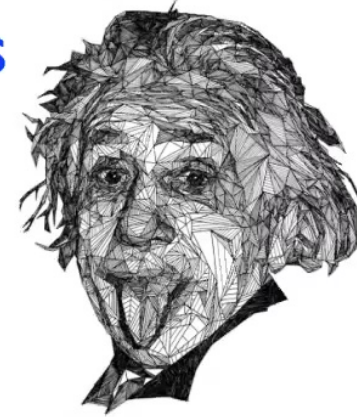
Remark: I do not think there is a fundamental tension to a suitable adapted canonical approach.
Just that it seems easier to derive the canonical description from the path integral
(including topology change).

Regge 1961

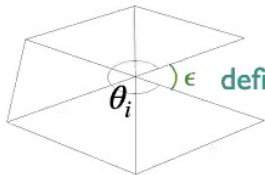
(Quantum) Regge calculus

geom =

- a (fixed) triangulation with piecewise flat geometry
- variables: lengths associated to the edges

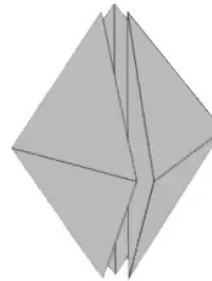


2D: (Euclidean geometry)



deficit angle $= 2\pi - \sum_i \theta_i$
a measure for curvature

3D:



$$S_E(\text{geom}) = - \sum_{\text{bones}} \text{Vol}_{\text{bone}} \cdot \epsilon_{\text{bone}}$$

Regge action, is a discretization of the Einstein-Hilbert action.

Boundary terms:



thin wedge

$$\psi = - \sum_i \theta_i$$



thick wedge

$$\psi = \pi - \sum_i \theta_i$$

Easier to get than
in continuum.

Euclideanization

Would like to compute: $Z = \int \mathcal{D}\text{geom} \exp(iS(\text{geom}))$

Integral over N edge lengths with a very complicated action
(involving inverse trigonometric functions of the lengths).
Practically impossible for N's needed for continuum limit.
(Additionally, the integration range can be infinite.)

Instead: Monte Carlo sampling:

But this works only for (positive) measures, e.g. $\exp(-S_E)$

In analogy to quantum field theory:

Wick rotate time (parameter) $t \rightarrow it$, $\exp(iS) \rightarrow \exp(-S_E)$

Hugely successful strategy in many areas of physics, e.g. lattice QCD.

Has been often the only available method for evaluating non-perturbative path integrals.

Euclideanization

But there are two key problems in the case of gravity:

Practical: The Euclidean Einstein-Hilbert action is unbounded from below: Conformal factor problem.
The system will be driven to configurations where $-S_E$ is very large.

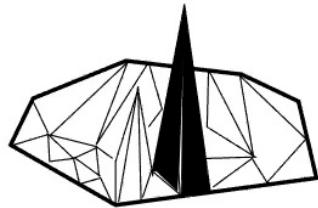
Conceptual: Space of Lorentzian geometries very different from space of Euclidean geometries,
no precise sense how these can be related via a (coordinate dependent) Wick rotation.



These problems killed (almost) all “Euclidean quantum gravity” approaches:
Regge calculus, (almost) dynamical triangulations, (Euclidean) lattice gauge formulations of gravity, ...

Open question: What do the Euclideanized versions tell us about
the ‘true i-quantum’ versions of the various path integral formulations.

Example: Spikes in Regge calculus



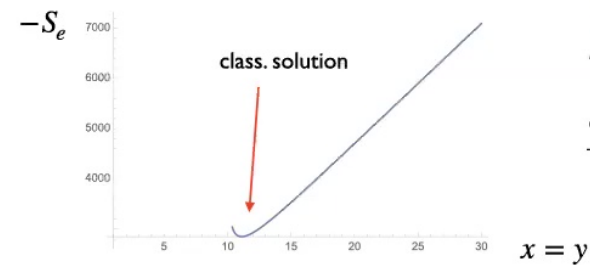
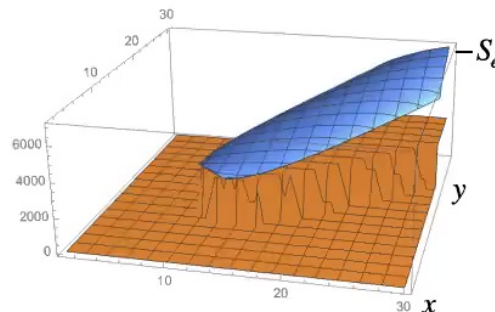
Weirdly infrared feature.

[Asante, BD,
to appear]

Spikes: Lengths of all edges adjacent to the a vertex are very large.
Have large (negative) S_E (in 3D and 4D).

(Putting a positive cosmological constant does not help too much.)

Example: 4D triangulation with inner vertex and six adjacent edges, symmetry reduced to two length parameters x, y .



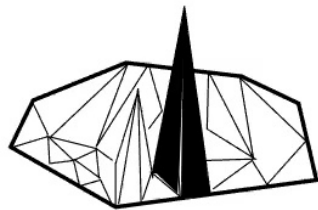
Ambjorn, Nielsen, Savvidy 1997:
Expectation value of
(sufficiently large power of)
lengths is infinite in
Euclidean Regge calculus.

Surprise:
Approx. linear growth
for larger lengths.

Toy model:

$$\frac{a(c_1) + \int_{c_1}^{c_2} dx x^{-N} x^k \exp(ix)}{b(c_1) + \int_{c_1}^{c_2} dx x^{-N} \exp(ix)}$$

Example: Spikes in Regge calculus

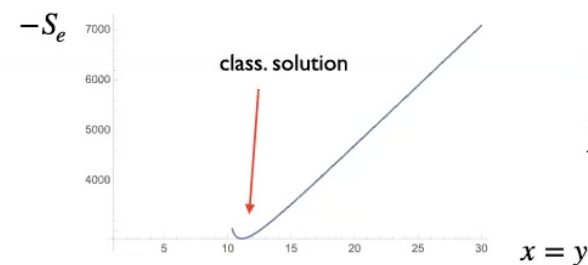
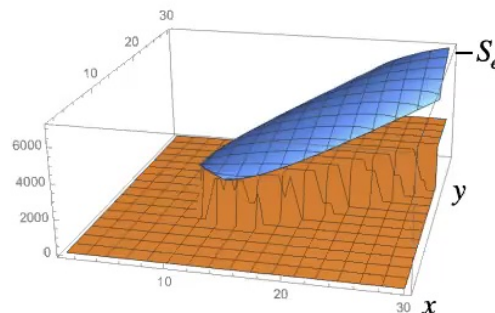


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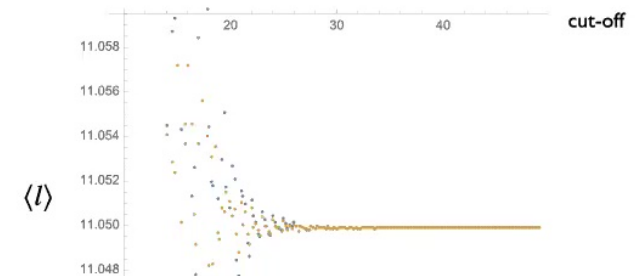
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With $\exp(-S_E)$: Even with l^{-N} measure
the lengths expectation value is given by the cut-off.

With $\exp(iS_E)$: (With appropriate measure)
the lengths^k expectation value converges and is finite.



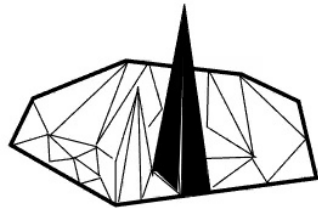
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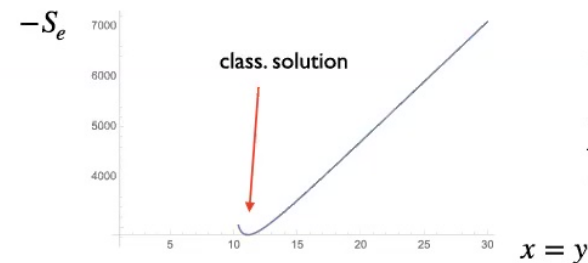
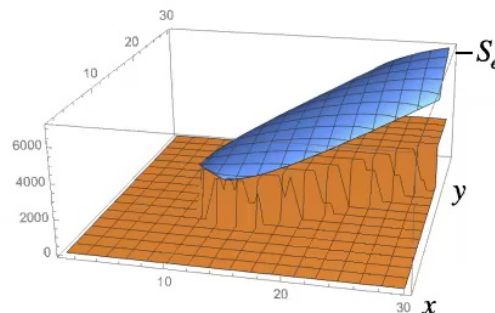


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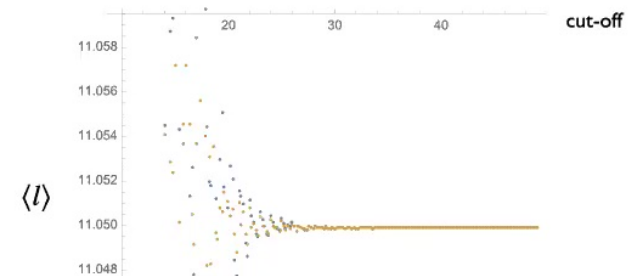
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What can the -Euclidean version tell us about the i-Euclidean version?

Can such spikes appear in Lorentzian geometries?



Dynamical Triangulations

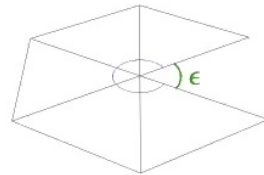
Agreement (largely): Euclidean Regge calculus does not lead to a suitable continuum limit.

One reason are spike configurations. Define a measure over (discrete) geometries that could suppress such configurations.



(Euclidean) Dynamical triangulations:

[Weingarten 1982,
Ambjorn, Jurkiewicz 1992:
first 4D simulations]



Sum over all triangulations with equal edge lengths.

Geometry is now encoded into how the simplices are glued together.

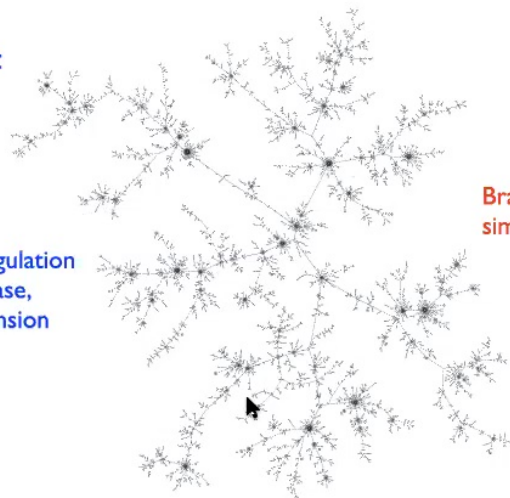
E.g. curvature around a bone is given by the number of adjacent top-dimensional simplices.

Does clearly avoid very long edges.

But the empire strikes back:

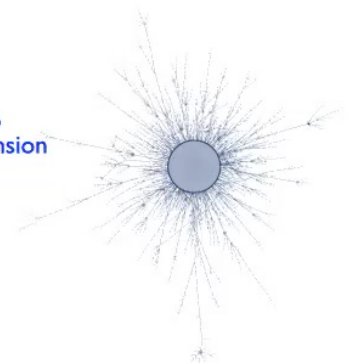
[from Rindlisbacher,
de Forcrand 2015]

typical 4D triangulation
in elongated phase,
Hausdorff dimension
around 2



Branching of baby universes,
similar to spikes.

crumpled phase,
Hausdorff dimension
infinity



First order phase transition.

Causal Dynamical Triangulations

[Ambjorn, Loll 1998, ...,
4D simulations: 2004 w/ Jurkiewicz, ...]

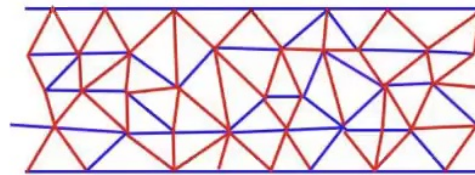
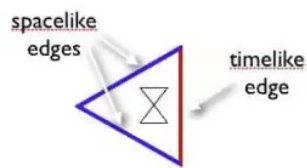
Key input: A regular causal structure suppresses baby universes.

(For such causally regular (dynamical) triangulations one can also define a Wick rotation.)

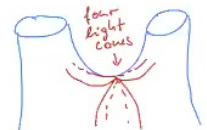
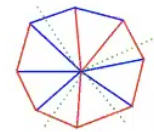
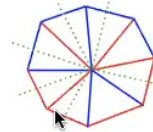
Still use Monte Carlo simulations, that is the Euclideanized action on causally regular triangulations.

(Similar philosophy: Causal sets.)

More general version [Jordan, Loll 2013: 3D]



Condition: exactly two light cones adjacent at each space-like bone.



Very encouraging results:

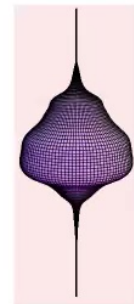
- phase with 'smooth' geometry: deSitter phase
- dimensional reduction of spectral dimension from approx. 4 to 2
- strong indications of 2nd order phase transition

Questions:

- Horava-Lifshitz or GR (at end point of phase transition line)
- 4D simulation with more general version
- interpretation of Wick rotation, in particular for black hole space times

Remarks:

- One has three coupling constants
- Laiho (et al): EDT with baby universe suppressing measure (and three coupling constants)



Lesson: Lorentzian (causally regular) configuration space can be very different from Euclidean configuration space.
Can even overcome the conformal factor problem, allowing for interesting Monte-Carlo simulation results.

What about Lorentzian Regge geometries?

Lorentzian Regge calculus

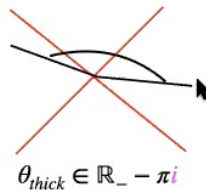
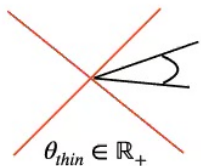
[Sorkin 1975, Sorkin 2019]

[Sorkin 2019] Surprise: The Regge action has imaginary parts if light cone structure is irregular.
Suppresses baby-universes (with the appropriate choice of root).

[coincides with 2D continuum calculation by Louko, Sorkin 1995]

In the following I restrict to Lorentzian d-simplices which have only space-like (d-1) sub-simplices.

Lorentzian angles



deficit angle

(for spacelike bones)

$$\epsilon = -2\pi i - \sum_j \theta_j$$

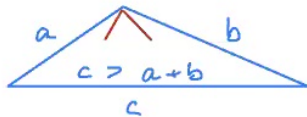
Imaginary parts cancel iff there are two thick wedges.

With this definition the Gauss-Bonnet theorem holds: $S_{2D} = -2\pi i \chi$

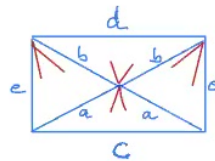
The beauty of Regge calculus: get boundary, corner and singular terms with much more ease.

Lorentzian triangulations

Lorentzian triangle with spacelike edges: violates (Euclidean) triangle inequ.

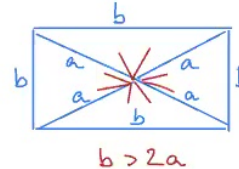


A triangulation with regular light cone structure.



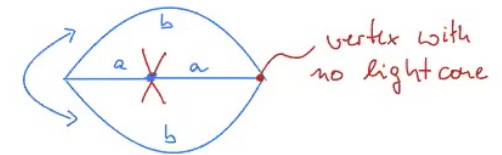
$\mathcal{F}(S) = 0$
(if you declare appropriately the corners as thick and thin)

A triangulation with irregular light cone structure.

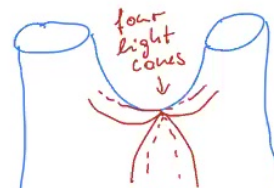


$\mathcal{F}(S) = +2\pi i$
(if you declare all corners as thin)

A triangulation with two "Euclidean vertices".

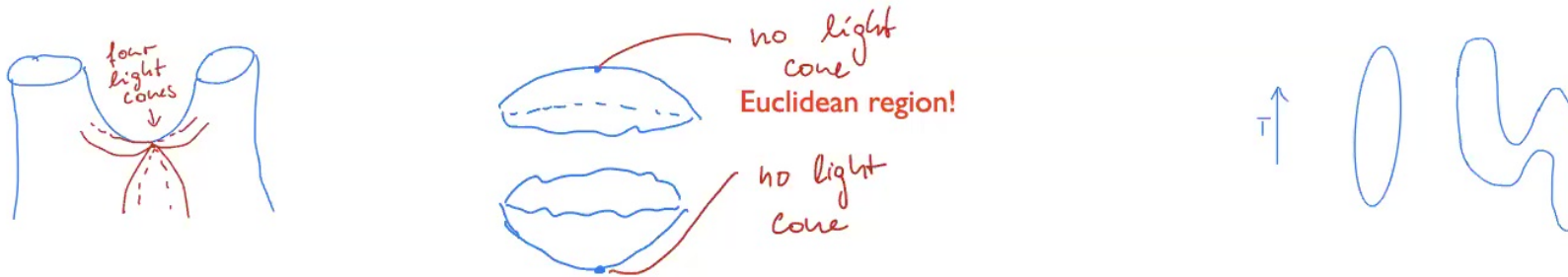


$S = -2 \times 2\pi i$
(Exact result for 2D sphere.)



In 4D: We have constructed a triangulation (with only spacelike lengths), which can have either regular or irregular causal structure. Separated by region where (Lorentzian) triangle inequalities are violated.
Is this generic? (For triangulation with only spacelike edges: yes)

Topology change already there



In a Lorentzian quantum- Regge path integral trousers (baby universes) are suppressed. 'Yarmulkes' are enhanced.

Requires the choice of the appropriate root for (-1) .

[Sorkin 2019]

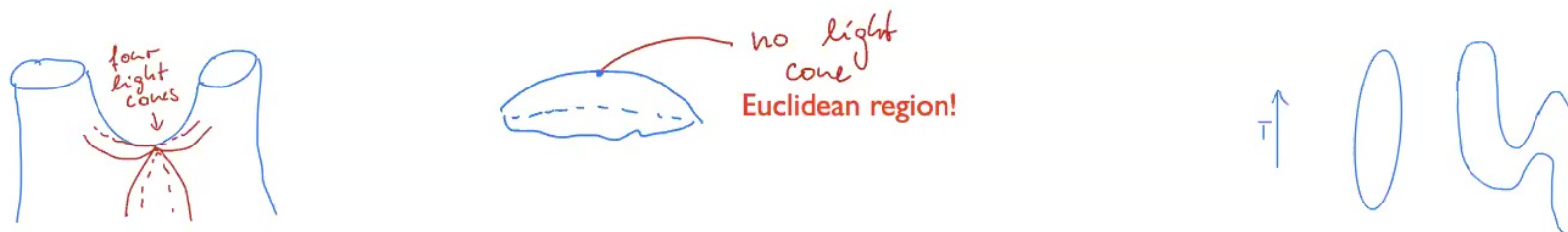
You see the difference between real curvature and irregular causal structures only in the Lorentzian geometry, keeping the i .

Topology change is already included if we decide to sum over all Lorentzian geometries allowed by the (Lorentzian) triangle inequalities.

We can decide to **restrict** to a causal regular structure or to allow for causal irregularities.

The restricting version would introduce a bit of non-locality.

Topology change already there



One can also build spin foam configuration with irregular causal structure.

Are there such imaginary terms in the asymptotics of (EPRL/FK) spin foams (and what happens with the sum over orientations)?

Not obvious in the literature (I looked at). But it could be due to some implicit assumption of having a regular causal structure.

Issue: Spin foam asymptotics gives Regge action with boundary or corner term for one simplex. Matching the type of corner term to the type of angle, there will be no imaginary terms.

[Regge action of Barret, Foxon 1994 leaves out imaginary term, but there seem to be “imaginary action contribution” in the weights for the bones.]

Remark: For (EPRL/FK) spin foam models I expect it to be hard to put causality restriction.
Much more straightforward in effective spin foam models.

Lorentzian (really-) quantum gravity

There has been lots of work on (non-perturbative) Euclidean quantum gravity.
A number of approaches have 'failed' — but that is likely due to the Euclideanization and rather an 'infra-red' problem!

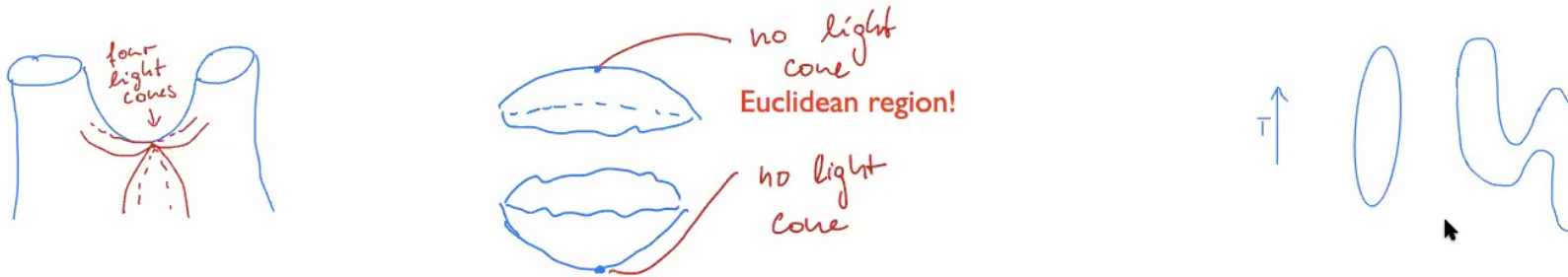
Introducing Lorentzian structures (e.g. CDT) can change the picture drastically.

Many open (quite basic) questions regarding Lorentzian configuration space and (quantum) path integral, which are also highly relevant for spin foams:

- Properties of Lorentzian configuration space with and without regular causality?
- What are the non-compact directions? Are there spikes?
- Can we deal with all the 'infra-red' divergencies in the quantum path integral?

[Tate, Visser:
study in lower dimensions
on Lorentzian triangle
inequalities]

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Why spin foams?

(A reformulation into a lattice gauge theory)

Inform your choices for the path integral by a rigorous notion of quantum geometry (aka quantization of geometry).
(Can be seen as a prescription for the UV properties.)

Deep connection to TQFT's, among other things, allowing for a kinematical 'flat' vacuum / phase. [BD, Geiller 2014, ...]
[Baez, BD, Freidel, Girelli, Smolin, ...]

Problem: It is very hard to rigorously quantize directly the Regge configuration space
— it has a very complicated boundaries due to triangle inequalities and positivity requirements.
(Similar problem for continuum metric.)

Solution: Reformulation of the theory using triads or tetrads and associated connection.

Manifestly (semi-) positive: $g = e \cdot e$

(Only lower-dimensional) triangle inequalities implemented with Gauss-constraint / local rotation symmetry.

Works wonderfully for (2+1)D gravity. (Chern-Simons, BF, Ponzano-regge, Turaev-Viro, ...)

A price to pay: But adds sum over orientations, degenerate sector, even Lorentzian geometries in Euclidean signature and vice versa. [Barret, Foxon 1994]

An even bigger enlargement of the configuration space in (3+1)D!

(3+1)D quantum geometries

$$g = e \cdot e \xrightarrow{\text{Hodge dualization}} E^a$$

su(2) algebra valued: components are **non-commutative!**

Give the **normals** to 2D surfaces (e.g. the triangles).

Simplicity constraints: ensure that these normals are arising from $n = e \times e$.

But spatial geometry is **non-commutative**, leading to an **anomaly** in the constraint algebra.

- ➡ Part of the simplicity constraints **are not implemented** on the LQG Hilbert space.
- ➡ The LQG Hilbert space describes generalized geometries, which include a certain type of torsion.

[BD, Ryan 2008+ ; Freidel, Speziale 2010, Asante, BD, Girelli, Riello, Tsimiklis 2019]

But to get a suitable (non-flat) dynamics, we have to implement the constraints in some form into the path integral.

➡ **Coherent state/ weak implementation in the path integral.**

2007: EPRL [Engle, Pereira, Rovelli, Livine], FK [Freidel, Krasnov], [Livine-Speziale]
Weak implementation of (primary) simplicity constraints for fluxes

2020: Effective spin foams [Asante, BD, Haggard 2020]: weak implementation
of secondary simplicity constraints for gauge invariant variables,
based on Area-Angle Regge action [BD, Speziale 2008]
Much greater numerical accessibility

Remark: [Magueijo, Zlosnik 2020] argue that including a coherent state average of (a certain type of) torsion makes the Kodama state normalizable.

Effective spin foam models

Effective spin foam model amplitude:

Regge action in terms of areas + constraints (3D angles, areas)

$$\exp\left(\frac{i}{\hbar}S(A)\right)$$

coherent states in angle variables



integrate out angles

$$\exp\left(\frac{i}{\hbar}S(A)\right) \times \text{Gaussian}$$

- peaked on area constraints

- with spread $\sigma^2 = \frac{\gamma \ell_P^2}{\text{Area}}$

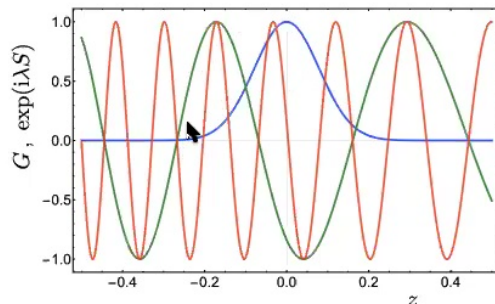
Barbero-Immirzi parameter parametrizes the anomaly and thus minimal uncertainty in the constraints

Amplitude given by **Oscillating factor** and **Gaussian**.

[Asante, BD, Haggard 2020]:

- For such systems with weakly implemented constraints we need to adjust the notion of semi-classical regime.
- $\hbar \rightarrow 0$ is not sufficient. We need also to demand that the anomaly γ is sufficiently small (scaled with \hbar).

Gaussian and **oscillating factor**



Oscillations can wash out the Gaussian.
Condition for reproducing critical value:
#oscillation over deviation interval $\leq \mathcal{O}(1)$

Scaling in \hbar :

$$[\sqrt{[\text{action}]}]/\sqrt{\hbar} \leq \mathcal{O}(1)$$

(Naive) Estimate for spin foams:

$$\frac{\sqrt{\gamma}}{\ell_P} \sqrt{A} \epsilon \lesssim \mathcal{O}(1)$$

Confirmed by numerical simulations for simplest case with curvature.

But more involved behaviour for more complicated triangulations.

[Asante, BD, Haggard 2020]

To get equations of motions we need small \hbar — highly oscillating amplitude. But this threatens to wash out the constraints.

Is there a semi-classical regime which reproduces the equations of motion of Regge calculus?

Effective spin foam models: numerical results

[Asante, BD, Haggard 2020.04 and 2020.11 and to appear]

Triangulation with one bulk area: testing implementation of constraints (but not equation of motion)

- for Euclidean geometry and Lorentzian geometry (with spacelike tetrahedra, causally regular sector): some features are qualitatively the same
- quantum geometry: area spectra are discrete (another way to see that area constraints cannot be imposed sharply: Diophantine equations)

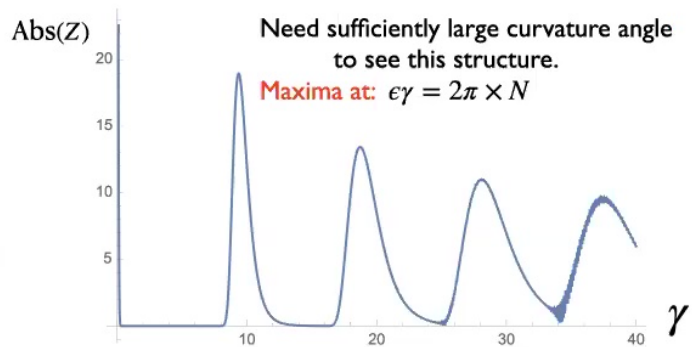
$$A = \gamma \ell_P^2 \sqrt{j(j+1)} \sim \gamma \ell_P^2 j \quad \longrightarrow \quad S \sim \gamma j \epsilon, \quad \sigma^2 \sim \frac{1}{j}$$

Path integral is actually a sum over discrete area values (spins).

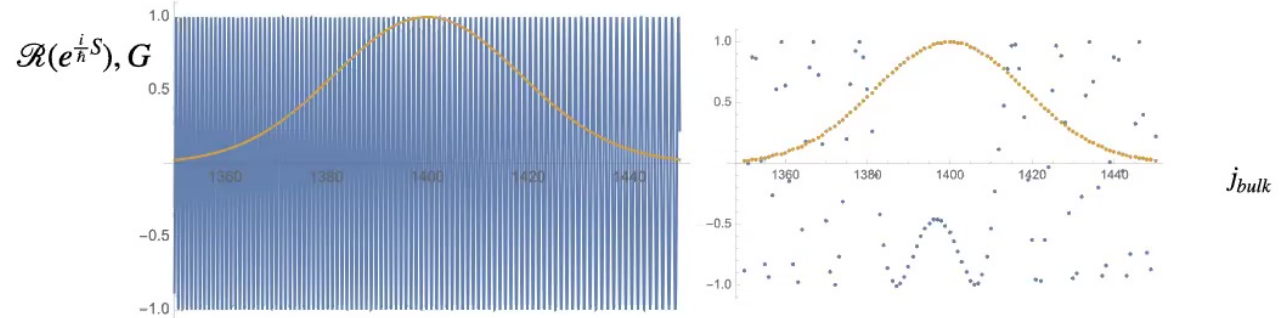
Effective spin foam models: numerical results

Triangulation with one bulk area: testing implementation of constraints (but not equation of motion)

- Lorentzian, $\epsilon = 0.67$, $j_{bulk}^{class} = 2600$,



These maxima arise from **pseudo stationary points**, due to the discrete sum:

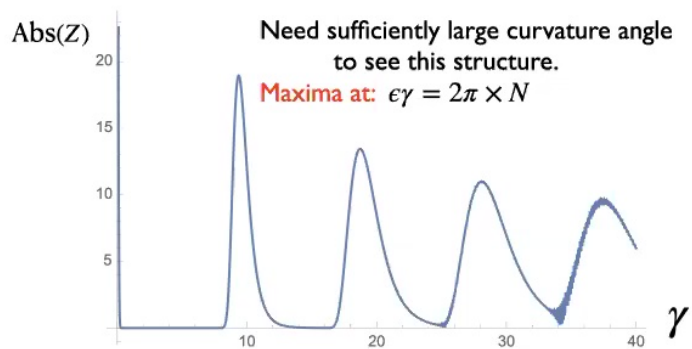


But these maxima are well outside the semi-classical regime.

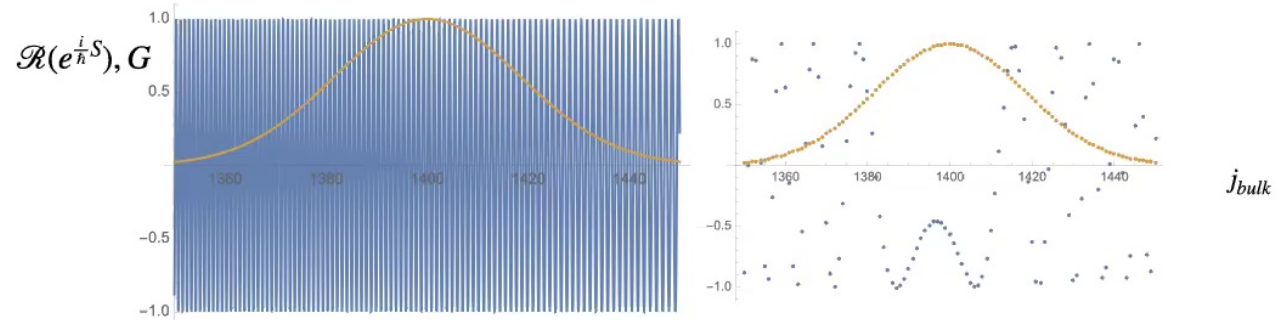
Effective spin foam models: numerical results

Triangulation with one bulk area: testing implementation of constraints (but not equation of motion)

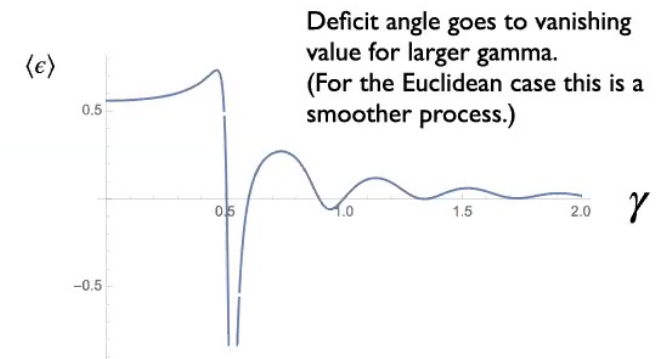
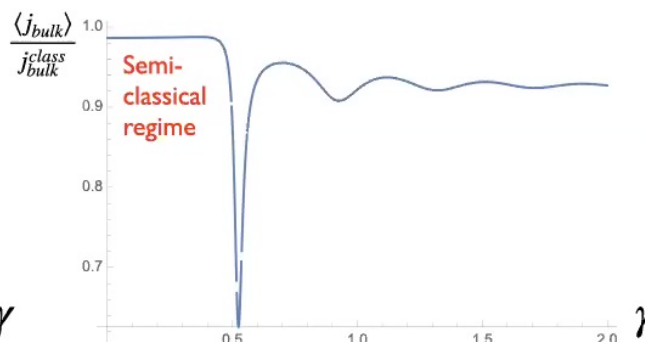
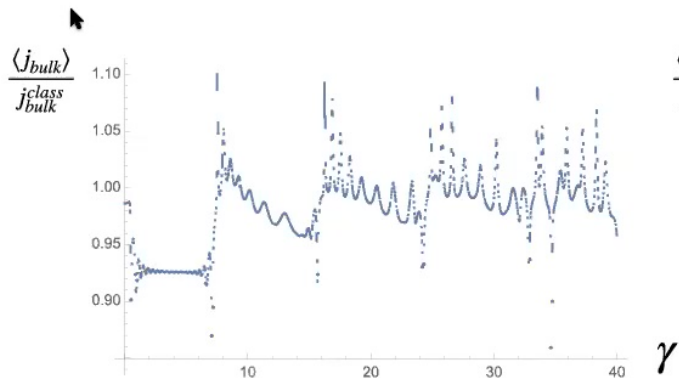
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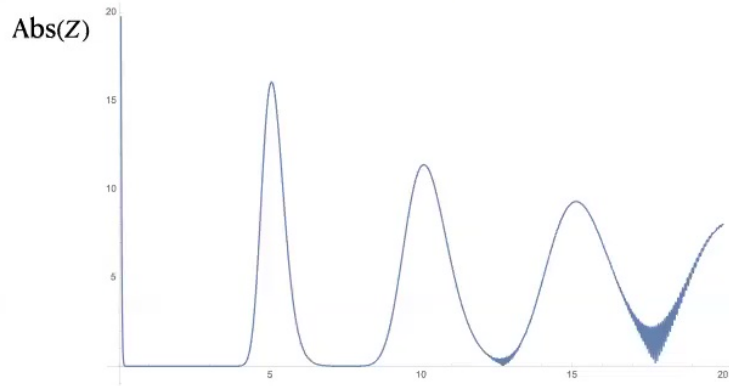


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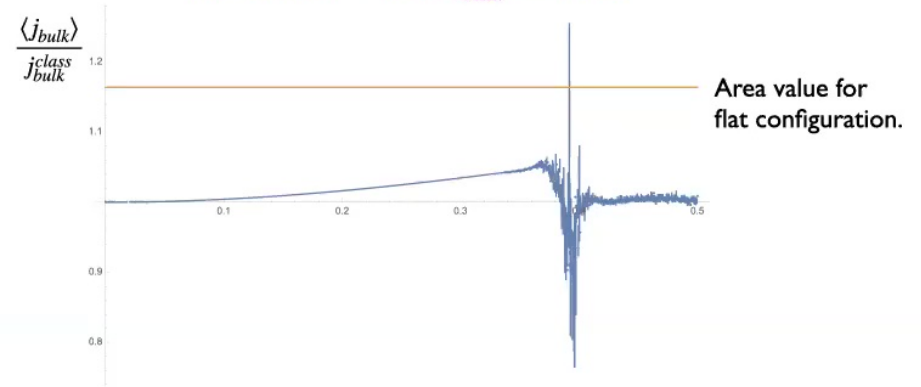


Effective spin foam models: numerical results

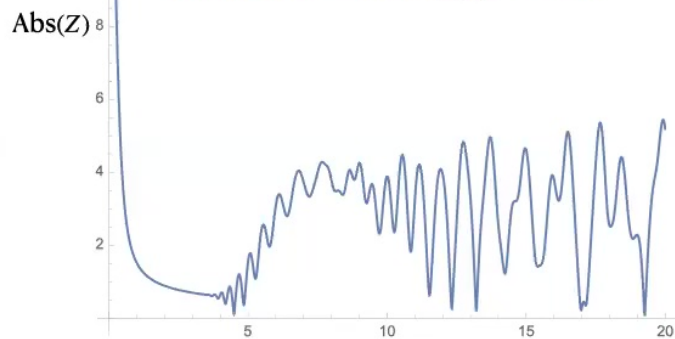
- Euclidean, $\epsilon = 1.23$, $j_{bulk}^{class} = 1400$,



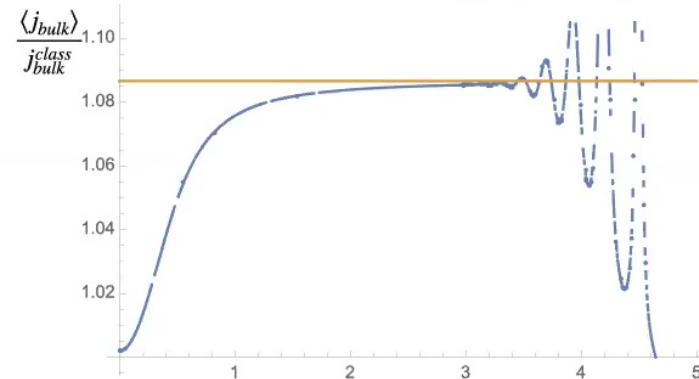
- Euclidean, $\epsilon = 1.23$, $j_{bulk}^{class} = 1400$,



- Euclidean, $\epsilon = 0.74$, $j_{bulk}^{class} = 150$,



- Euclidean, $\epsilon = 0.74$, $j_{bulk}^{class} = 150$,

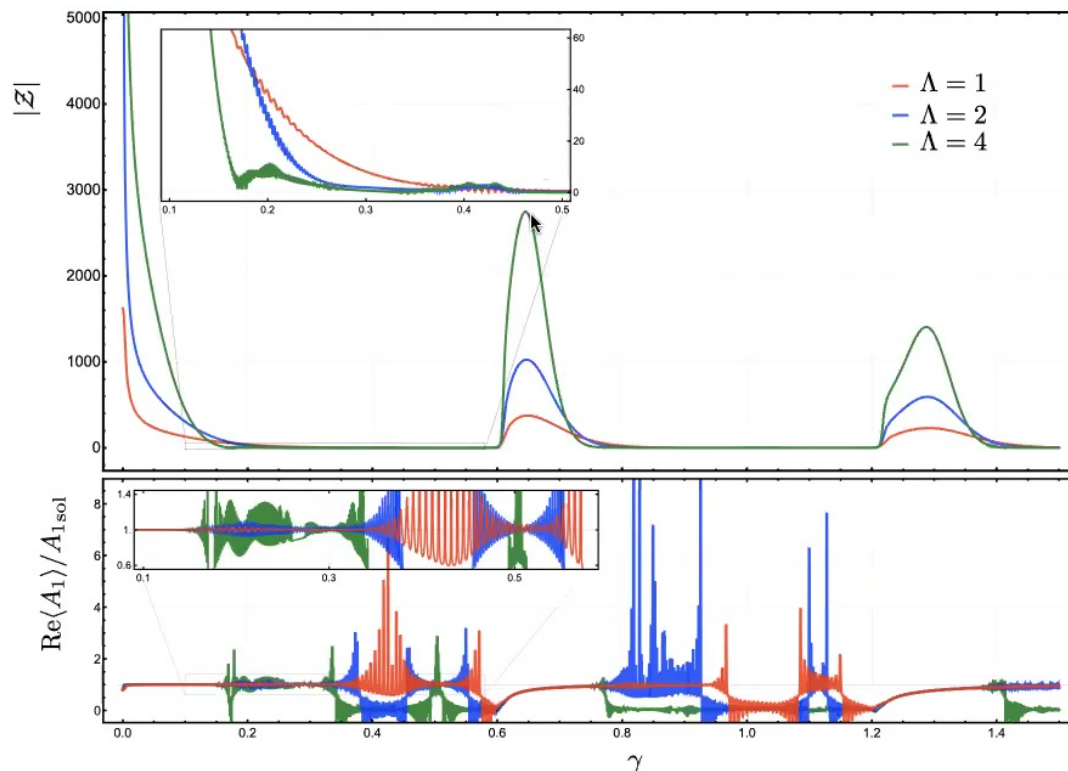


Effective spin foam models: numerical results

Triangulation with bulk edge: testing the equation of motion: need minimal gamma

[Asante, BD, Haggard: 2020.11]

Very large curvature $\epsilon_1(t_{\text{sol}}) = 4.193$, $\epsilon_2(t_{\text{sol}}) = -1.790$, $\epsilon_3(t_{\text{sol}}) = -1.1432$



Not clear yet:
characterization of maxima
(Pseudo-stationarity in three directions)

It is NOT $\epsilon\gamma = 2\pi \times N$.

Semiclassical regime:
Green scale forces gamma to be
quite small: gamma < 0.15.
For smaller (red, blue) scales:
gamma < 0.3

Effective spin foam models: numerical results

Reproduce classical solutions for a regime specified by:

- the area values of classical solutions — here determined by boundary data → Determines "lattice constant".
- curvature (per building block) of classical solution — here determined by boundary data → Should be small also classically.
- the Barbero-Immirzi parameter → Determines spectral gap.

Allowed range for γ gets larger for smaller curvature angles.
For refinement limit we have smaller and smaller curvature angles.

Indication: Semi-classical regime gets 'larger' for triangulations with more building blocks (with small curvature).

Crucial question for continuum limit: How does weak constraint implementation interact with coarse graining?

Summary and Outlook

Lorentzian quantum geometry and quantum gravity

- A priori include configurations with irregular light cone structure — but exponentially suppressed
- Could help in reaching a continuum limit where smooth manifolds arise
- Could avoid longstanding problem of (infra-red) divergences, particular appearing in Euclidean models
- Lots of open issues: non-compact directions in configuration space and spikes, sum over orientation, ...

Effective spin foams:

- Numerical computations are several magnitudes more efficient than for other models.
- Can test some long standing open issues: sum over orientations, including degenerate geometries, topology change, ...
- Weak implementation of constraints challenges existence of semi-classical regime.
- But could identify a semi-classical regime for all examples considered so far.
- Selection principle for various (measure) choices: consistent coarse graining flow.

(3+1)D quantum geometries

$$g = e \cdot e \xrightarrow{\text{Hodge dualization}} E^a$$

su(2) algebra valued: components are **non-commutative!**

Give the **normals** to 2D surfaces (e.g. the triangles).

Simplicity constraints: ensure that these normals are arising from $n = e \times e$.

But spatial geometry is **non-commutative**, leading to an **anomaly** in the constraint algebra.

➡ Part of the simplicity constraints **are not implemented** on the LQG Hilbert space.

➡ The LQG Hilbert space describes generalized geometries, which include a certain type of torsion.

[BD, Ryan 2008+ ; Freidel, Speziale 2010, Asante, BD, Girelli, Riello, Tsimiklis 2019]

But to get a suitable (non-flat) dynamics, we have to implement the constraints in some form into the path integral.

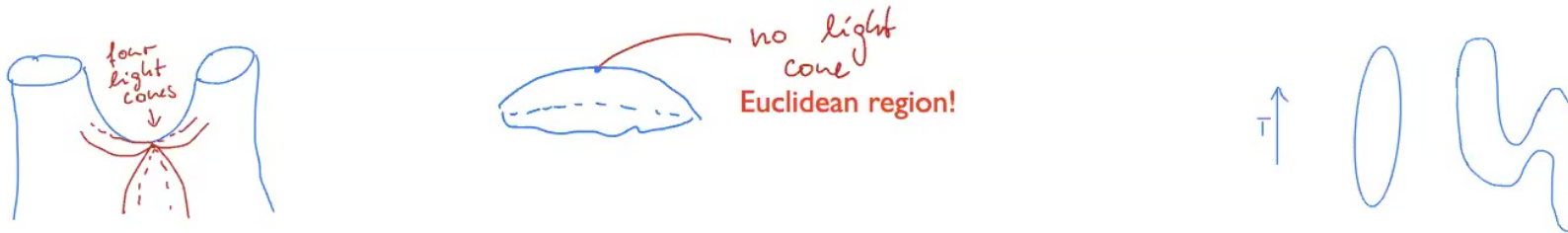
➡ **Coherent state/ weak implementation in the path integral.**

2007: EPRL [Engle, Pereira, Rovelli, Livine], FK [Freidel, Krasnov], [Livine-Speziale]
Weak implementation of (primary) simplicity constraints for fluxes

2020: Effective spin foams [Asante, BD, Haggard 2020]: weak implementation
of secondary simplicity constraints for gauge invariant variables,
based on Area-Angle Regge action [BD, Speziale 2008]
Much greater numerical accessibility

Remark: [Magueijo, Zlosnik 2020] argue that including a coherent state average of (a certain type of) torsion makes the Kodama state normalizable.

Topology change already there



One can also build spin foam configuration with irregular causal structure.
Are there such imaginary terms in the asymptotics of (EPRL/FK) spin foams (and what happens with the sum over orientations)?

Not obvious in the literature (I looked at). But it could be due to some implicit assumption of having a regular causal structure.
Issue: Spin foam asymptotics gives Regge action with boundary or corner term for one simplex. Matching the type of corner term to the type of angle, there will be no imaginary terms.

[Regge action of Barret, Foxon 1994 leaves out imaginary term, but there seem to be “imaginary action contribution” in the weights for the bones.]

Remark: For (EPRL/FK) spin foam models I expect it to be hard to put causality restriction.
Much more straightforward in effective spin foam models.