

Title: Resource theories of communication

Speakers: HlÃ©r KristjÃ¡nsson

Series: Quantum Foundations

Date: January 29, 2021 - 12:00 PM

URL: <http://pirsa.org/21010022>

Abstract: A series of recent works has shown that placing communication channels in a coherent superposition of alternative configurations can boost their ability to transmit information. Instances of this phenomenon are the advantages arising from the use of communication devices in a superposition of alternative causal orders, and those arising from the transmission of information along a superposition of alternative trajectories. The relation among these advantages has been the subject of recent debate, with some authors claiming that the advantages of the superposition of orders could be reproduced, and even surpassed, by other forms of superpositions. To shed light on this debate, we develop a general framework of resource theories of communication. In this framework, the resources are communication devices, and the allowed operations are (a) the placement of communication devices between the communicating parties, and (b) the connection of communication devices with local devices in the parties' laboratories. The allowed operations are required to satisfy the minimal condition that they do not enable communication independently of the devices representing the initial resources. The resource-theoretic analysis reveals that the aforementioned criticisms on the superposition of causal orders were based on an uneven comparison between different types of quantum superpositions, exhibiting different operational features.

Ref. <https://iopscience.iop.org/article/10.1088/1367-2630/ab8ef7>

Resource theories of communication

$M_{BB} = \text{Tr}(AB^*AB)$ $\langle \alpha | \text{id} \rangle = 1$

$\|T\|^2 = \sum \alpha_k M_{BB}$
 $= \langle \alpha | M | \alpha \rangle$

Theorem of maximising $\|T\|$
 $\forall \lambda \in \mathbb{R}, \|T + \lambda I\|^2 = \sum_k \alpha_k M_{k,k}$
i.e. $M|\alpha\rangle = \|T\|^2 |\alpha\rangle$

Take the square root $M_{BB} = \sqrt{\alpha_k M_{k,k}}$
is equivalent to $\sqrt{\alpha_k} \rightarrow \max(\|T\|)$
It is reached for $\alpha_k = 1, \alpha_j = 0$

$= \text{Tr } T^*T$
 $= T^*T$
 $= \|T\|^2$

Resource theories of communication

Hlér Kristjánsson^{1,2}, Giulio Chiribella^{3,1,2},
Sina Salek^{4,3}, Daniel Ebler^{5,1,3} & Matthew Wilson^{1,2}

¹ Quantum Group, University of Oxford
² HKU-Oxford Joint Laboratory for Quantum Information and Computation
³ Department of Computer Science, The University of Hong Kong
⁴ Peng Cheng Laboratory, Shenzhen
⁵ SUSTech, Shenzhen
⁶ Wolfson College, University of Oxford

**New Journal of Physics 2020
(arXiv:1910.08197v3)**

Whiteboard notes (Student side):

$$M_{AB} := T_n(A_B^\dagger A_B) \quad \langle \alpha | \alpha \rangle = 1$$

$$\|T\|^2 = \sum_k \alpha_k M_{kk} = \langle \alpha | M | \alpha \rangle$$

Theorem: α maximizes $\|T\|^2$

$$\text{if } \forall m \quad \|T\|^2 \alpha_m = \sum_k \alpha_k M_{km}$$

i.e. $\alpha | \alpha \rangle = \|T\|^2 | \alpha \rangle$

Take the special case $M_{kk} = \delta_{kk} \sqrt{p_k}$
 \Rightarrow eigenvalues are the $\sqrt{p_k}$, so the max $\|T\|^2$
 It is needed for $\alpha = |0\rangle \rightarrow |2\rangle$

Whiteboard notes (Professor side):

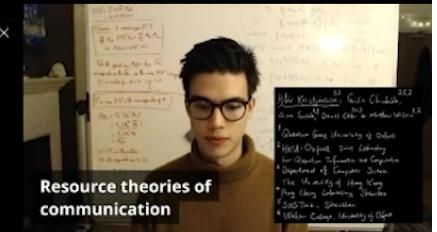
$$\begin{aligned} & \text{Tr } T^\dagger T \\ &= \text{Tr } \sum_k \alpha_k A_k^\dagger \sum_l \alpha_l A_l \\ &= \sum_k \alpha_k^2 \text{Tr}(A_k^\dagger A_k) \quad | M_{kk} = \text{Tr}(A_k^\dagger A_k) \\ &\text{constraint: } \sum_k \bar{\alpha}_k \alpha_k = 1 \quad \Rightarrow M | \alpha \rangle = \|T\|^2 | \alpha \rangle \\ & \|T\|^2 \text{ occurs at } \nabla X = 0, \quad X = \|T\|^2 - \lambda \left(\sum_k \bar{\alpha}_k \alpha_k \right) \\ & \sum_k \text{Tr}(A_k^\dagger A_k) - \lambda \sum_k \end{aligned}$$

New Journal of Physics
 The open access journal at the forefront of physics

PAPER • OPEN ACCESS
 Resource theories of communication
 Hlér Kristjánsson^{1,2}, Giulio Chiribella^{3,4,5}, Sina Salek^{6,1}, Daniel Ebler^{7,8,3} and Matthew Wilson^{1,2}

Published 15 July 2020 • © 2020 The Author(s). Published by IOP Publishing Ltd on behalf of the Institute of Physics and Deutsche Physikalische Gesellschaft
New Journal of Physics, Volume 22, 1 July 2020

[Article PDF](#) [Article ePub](#)



Resource theories of communication

Plan:

- Second-quantised Shannon theory
- New: resource-theoretic formulation
- Applications
- Taster: routed quantum circuits

from Shannon theory to quantum Shannon theory

$$M_{kl} := T_n(A_k^\dagger A_l) \quad \langle \alpha | \alpha \rangle = 1$$

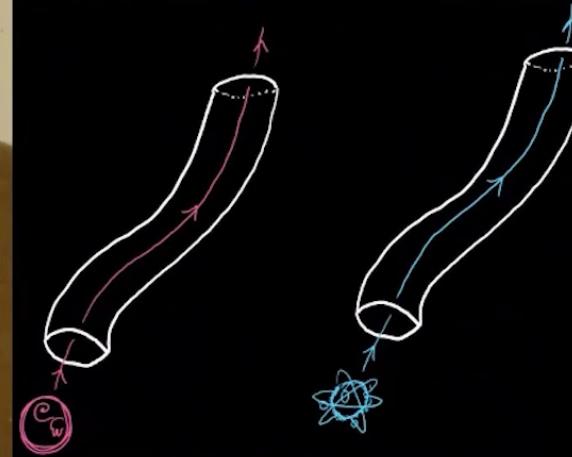
$$\|T\|^2 = \sum_k \alpha_k M_{kk} = \langle \alpha | M | \alpha \rangle$$

Theorem: α maximizes $\|T\|^2$
iff $\forall m \quad \|T\|^2 \alpha_m = \sum_k \alpha_k M_{km}$
i.e. $M|\alpha\rangle = \|T\|^2 |\alpha\rangle$

Take the special case $M_{kl} = \delta_{kl} \sqrt{p_k}$
 \Rightarrow eigenstate on the (p_k) to the max $\|T\|^2$

It is needed for $\alpha = 10^{-20} \dots$

$$\begin{aligned} \text{Tr } T^{\dagger} T &= \sum_k \alpha_k A_k^\dagger A_k \\ &= \text{Tr } \sum_k \alpha_k A_k^\dagger \sum_\ell A_\ell \\ &\leq \sum_k \alpha_k \|A_k\|^2 = \text{Tr } M^2 = \text{Tr } M^2 \alpha_m \end{aligned}$$



Resource theories of communication

Max-Entropy Coding Class
Stan Gudder, Dept. of Mathematics
Resource Theory of Quantum Information
Hiroki Hayashi, Dept. of Mathematics
Resource Theory of Quantum Cryptography
Department of Computer Science
The University of Hong Kong
Ping-Cheng Zhang, Shantou
Sergei Strelchuk, University of Oxford
Walter Rudolf, University of Oxford

$$M_{kl} := T_n(A_k^* A_l) \quad \langle \alpha | \alpha \rangle = 1$$

$$\|T\|^2 = \sum_k \alpha_k M_{kk} = \langle \alpha | M | \alpha \rangle$$

Theorem: α maximizes $\|T\|^2$
 $\Leftrightarrow \forall m \quad \|T\|^2 \alpha_m = \sum_k \alpha_k M_{km}$
 i.e. $M|\alpha\rangle = \|T\|^2 |\alpha\rangle$

Take the special case $M_{kk} = \delta_{kk} \sqrt{P_k}$
 \Rightarrow signals on the (P_k) to the max!!
 It is needed for $\alpha = |0\rangle \rightarrow |1\rangle$

second level of quantisation of Shannon theory

$$\begin{aligned}
 & \text{Tr } T^\dagger T = \sum_{k,l} \alpha_k \alpha_l T_k^* T_l \\
 &= \text{Tr} \left(\sum_k \alpha_k A_k^* \sum_l A_l \right) \\
 &= \sum_k \alpha_k \text{Tr}[A_k^* A_k] \quad | \quad M_{kk} = T_n(A_k^* A_k) \\
 &\text{constraint: } \sum_k \alpha_k \alpha_k = 1 \quad \Rightarrow \quad M|\alpha\rangle = \|T\|^2 |\alpha\rangle \\
 &\|T\|^2 \text{ occurs at } \nabla X = 0, \quad X = \|T\|^2 - \lambda \left(\sum_k \alpha_k \alpha_k \right)
 \end{aligned}$$

Configuration of transmission lines in a quantum superposition

superposition of trajectories

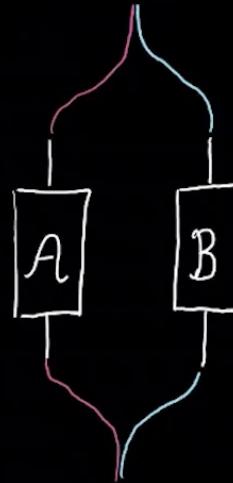
$$M_{k\ell} := T_n(A_k^\dagger A_\ell) \quad \langle \alpha | \alpha \rangle = 1$$

$$\|T\|^2 = \sum_{k,\ell} M_{k\ell}$$

Theorem: α maximizes $\|T\|^2$
 $\Leftrightarrow \forall m \quad \|T\|^2 \alpha_m = \sum_k \alpha_k M_{km}$
i.e. $M|\alpha\rangle = \|T\|^2 |\alpha\rangle$

Take the special case $M_{k\ell} = \delta_{k\ell} \sqrt{P_k}$
 \Rightarrow eigenstate on the (P_k) , i.e. the
IC is needed for $\alpha = 1_0$

$$\begin{aligned} \|T\|^2 &= \\ &= T \\ &= \\ &\text{Constraint:} \\ &\text{MAX } \|T\|^2 \end{aligned}$$



Y. Aharonov et al., Phys. Rev. Lett. 64 (1990)

J. Åberg, Annals of Physics 313 (2004)

D. Oi, Phys. Rev. Lett. 91 (2003)

A. Abbott et al., Quantum 4 (2020)

G. Chiribella & H.K., Proc. R. Soc. A 475 (2019)

Hilfer Kristjánsson^{1,2}, Giulio Chiribella^{3,4,5},
Ojan Sarek^{6,3}, Daniel Eberle^{2,6} & Matthew Wilson^{2,2}
¹Quantum Group, University of Oxford
²HKU-Oxford Joint Laboratory
for Quantum Information and Computation
³Department of Computer Science
The University of Hong Kong
⁴Ping Cheung Laboratory, Shekou
⁵SUSTech, Shenzhen
⁶Wolson College, University of Oxford

$$M_{k\ell} := T_n(A_k^\dagger A_\ell) \quad \langle \alpha | \alpha \rangle = 1$$

$$\|T\|^2 = \sum_{k,\ell} M_{k\ell}$$

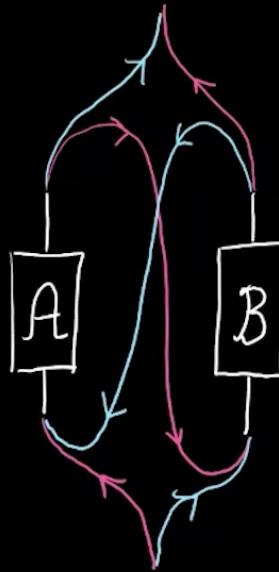
$$\begin{aligned} &= \langle \alpha | M | \alpha \rangle \\ &\text{Theorem: } \alpha \text{ maximizes } \|T\|^2 \\ &\text{iff } \forall m \quad \|T\|^2 \alpha_m = \sum_k \alpha_k M_{km} \\ &\text{i.e. } M|\alpha\rangle = \|T\|^2 |\alpha\rangle \end{aligned}$$

Take the special case $M_{k\ell} = \delta_{k\ell} \sqrt{p_k}$
⇒ signals on the (p_k) go to the α

IC is needed for $\alpha = 0$.

superposition of causal orders: quantum SWITCH

$$\begin{aligned} \|T\|^2 &= T \\ &= \text{Tr} \\ &= \\ &\text{Constraint:} \\ &\text{MAX } \|T\|^2 \text{ or} \\ &\leq \end{aligned}$$



- G. Chiribella et al., Phys. Rev. A 88 (2013)
D. Ebler et al., Phys. Rev. Lett. 120 (2018)
S. Sálek et al., arXiv: 1809.06655 (2018)
G. Chiribella et al., arXiv: 1810.10457 (2018)
...

Hilfer Kristjánsson^{1,2}, Giulio Chiribella^{3,4,5},
Ojan Sálek^{6,3}, Daniel Ebler^{2,3} & Matthew Wilson^{2,2}
¹Quantum Group, University of Oxford
²HKU-Oxford Joint Laboratory
for Quantum Information and Computation
³Department of Computer Science
The University of Hong Kong
⁴Peng Cheng Laboratory, Shenzhen
⁵SUSTech, Shenzhen
⁶Wolson College, University of Oxford

superposition of times of a correlated channel

$$M_{kl} := T_n(A_k^\dagger A_l) \quad \langle \alpha | \alpha \rangle = 1$$

$$\|T\|^2 = \sum_{k,l} M_{kl}$$

Theorem: α maximizes $\|T\|^2$
 $\Leftrightarrow \forall m \quad \|T\|^2 \alpha_m = \sum_k \alpha_k M_{km}$
i.e. $M|\alpha\rangle = \|T\|^2 |\alpha\rangle$

Take the special case $M_{kl} = \delta_{kl}$
 \Rightarrow eigenstate on the (t_0, t_1)

It is needed for $\alpha =$

$$\|T\|^2 = \text{Tr } T^\dagger T$$

$$= T$$

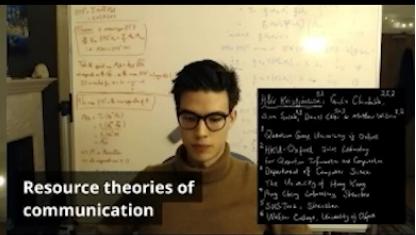
=

Constraint:

$$\text{MAX } \|T\|^2$$



H. K., W.-X. Mao, G. Chiribella,
arXiv:2004.06090 (2020)



superposition of the
direction of communication

$$M_{k\ell} := T_n(A_k^\dagger A_\ell) \quad \langle \alpha | \alpha \rangle = 1$$

$$\|T\|^2 = \sum_{k,\ell} M_{k\ell}$$

$$\begin{aligned} & \text{Theorem: } \alpha \text{ maximizes } \|T\| \\ & \text{if } \forall m \quad \|T\|^2 \alpha_m = \sum_k \alpha_k M_{km} \\ & \text{i.e. } M|\alpha\rangle = \|T\|^2 |\alpha\rangle \end{aligned}$$

Take the special case $M_{k\ell} = S_{k\ell}$
 \Rightarrow eigenstate on the PSL

It is needed for

$$\|T\|^2 = \text{Tr } T^\dagger T$$

$$= T$$

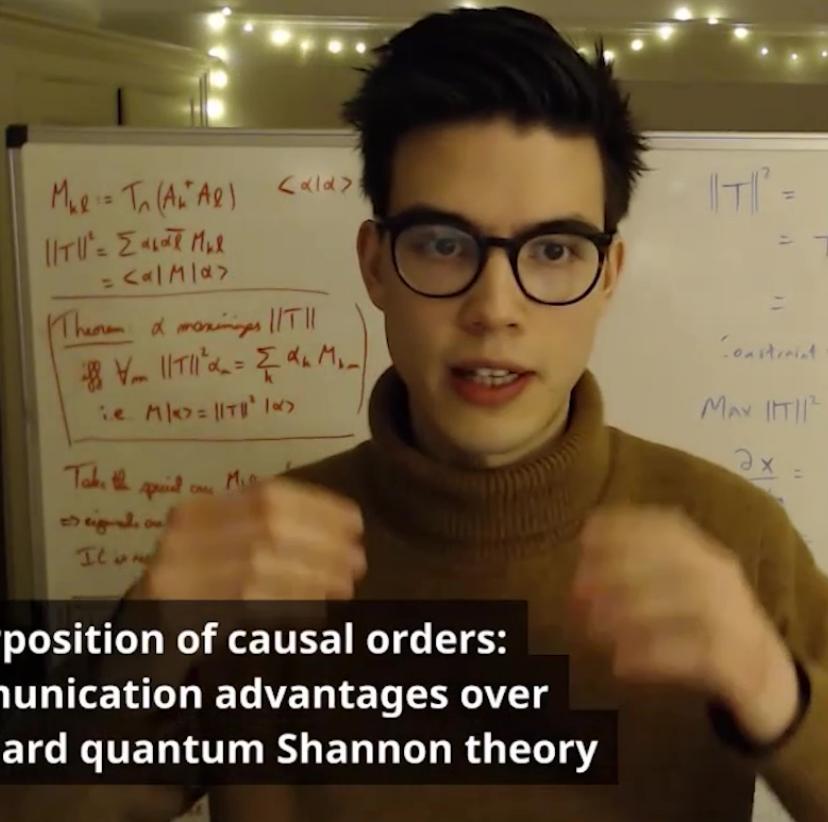
constraint:

$$\text{MAX } \|T\|^2$$

$$\partial_x =$$



F. del Santo & B. Dakic,
PRL (2018)



**superposition of causal orders:
communication advantages over
standard quantum Shannon theory**

PHYSICAL REVIEW LETTERS 120, 120502 (2018)

Enhanced Communication with the Assistance of Indefinite Causal Order

David Eller,^{1,*} Sina Sali¹, and Giulio Chiribella^{2,3,4}

¹Quantum Institute, University of Oxford, 1 Keble Road, OX1 3RH, United Kingdom

²Quantum communication in a superposition of causal orders

³Sina Sali,¹ David Eller,¹ and Giulio Chiribella^{2,3}

⁴Department of Computer Science, University of Hong Kong, Pokfulam Road, Hong Kong

Quantum tangledness is a measure of the non-locality of a quantum state. It is zero for classical states and maximal for Bell states. We show that the causal order of a channel can be manipulated to increase its capacity. In particular, we prove that the capacity of a channel with zero capacity in a causal order is strictly positive in another causal order. This result is obtained by showing that the capacity of a channel with zero capacity in a causal order is strictly positive in another causal order. This result is obtained by showing that the capacity of a channel with zero capacity in a causal order is strictly positive in another causal order.

PHYSICAL REVIEW A 101, 012346 (2020)

Sending classical information via three noisy channels in superposition of causal orders

Lorenzo M. Pironio,^{1,2} Francisco Delgado,^{1,2} Mario Enriquez,^{1,2} Nadia Beltrán,¹ and Juan ARIEL LEVISON¹

¹Centre for Nanoscience and Nanotechnology CSN, CNRS Université Paris-Saclay, Orsay, France

²Quantum Enhancement through Quantum Coherent Control of N Channels in an Indefinite Causal-Order Scenario

Communication Enhancement through Quantum Coherent Control of N Channels in an Indefinite Causal-Order Scenario

PHYSICAL REVIEW A 101, 012346 (2020)

Quantum and Classical Data Transmission Through Completely Depolarizing Channels in a Superposition of Cyclic Orders

Giulio Chiribella¹

¹QUT Quantum Information and Computation Institute, Department of Computer Science, The University of Hong Kong, Pokfulam Road, Hong Kong

²Department of Computer Science, University of Oxford, 1 Keble Road, OX1 3RH, United Kingdom

³HKU-Oxford Joint Laboratory for Quantum Information and Computation and Perimeter Institute for Theoretical Physics, 11 Caroline Street North, Waterloo, Ontario, Canada

Matthew Wilson¹

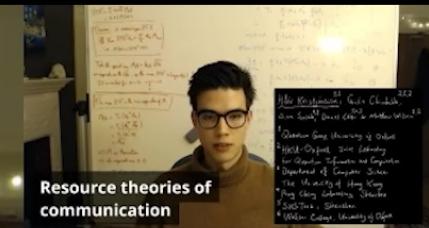
¹Department of Computer Science, University of Oxford, Parks Road, Oxford, United Kingdom and HKU-Oxford Joint Laboratory for Quantum Information and Computation

H. F. Chau²

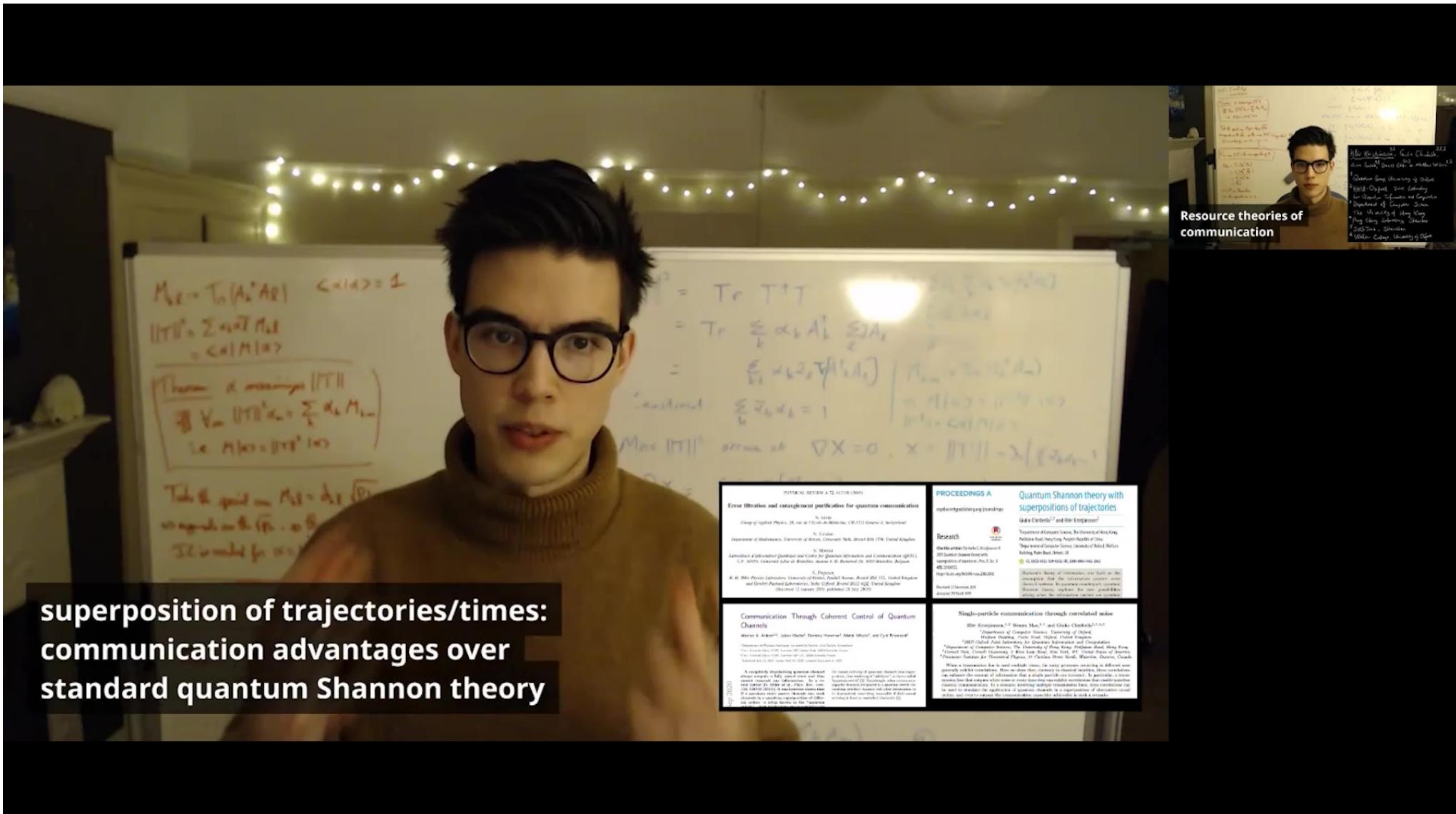
²Department of Physics, The University of Hong Kong, Pokfulam Road, Hong Kong

(Dated November 1, 2018)

Completely degrading channels, which transform every input state into white noise, are often regarded as the prototype of physical processes that are useless for communication. Here we show that the ability to combine N completely degrading channels in a superposition of N alternative



Resource theories of communication



debate on the nature and relation of these advantages

$$M_{kl} := T_n(A_k^* A_l) \quad \langle \alpha | \alpha \rangle = 1$$

$$\|T\|^2 = \sum_{k,l} M_{kl}$$

(Theorem: α maximizes $\|T\|^2$)
 $\Rightarrow \forall m \quad \|T\|^2 \alpha_m = \sum_k \alpha_k M_{km}$
 i.e. $M|\alpha\rangle = \|T\|^2 |\alpha\rangle$

Take the special case $M|\psi\rangle =$
 \Rightarrow signals on the $P(\psi)$
 It is needed for:

$$\|T\|^2 = \text{Tr } T^* T$$

=

constraint

$$\text{MAX } \|T\|^2$$

Communication Through Coherent Control of Quantum Channels

Alastair A. Abbott^{1,2}, Julian Wechs², Dominic Horman³, Mehdi Mhalla⁴, and Cyril Branciard²

¹Département de Physique Appliquée, Université de Genève, 1211 Geneva, Switzerland

²Vienne Center for Quantum Science and Technology (VCQ), Faculty of Physics, University of Vienna,
Boltzmanngasse 5, 1090 Vienna, Austria

³Umeå Graduate School, CMPS, Göteborg University, 412 90 Göteborg, Sweden

⁴Submitted July 15, 2019; revised April 14, 2020; accepted September 8, 2020

PHYSICAL REVIEW A **99**, 062317 (2019)

Communication through quantum-controlled noise

Philippe Allart Guérin,^{1,2} Giulio Ruffo,¹ and Caslav Brukner^{1,2}

¹Vienna Center for Quantum Science and Technology (VCQ), Faculty of Physics, University of Vienna,
Boltzmanngasse 5, 1090 Vienna, Austria

²Institute for Quantum Optics and Quantum Information (IQOQI), Austrian Academy of Sciences,
Boltzmanngasse 3, 1090 Vienna, Austria

(Received 21 January 2019; published 17 June 2019)

PHYSICAL REVIEW A **101**, 012340 (2020)

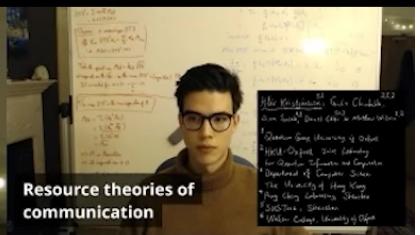
Channel capacity enhancement with indefinite causal order

Nicolas Löffenzig¹
Department of Physics, New York University, 726 Broadway, New York, New York 10003, USA

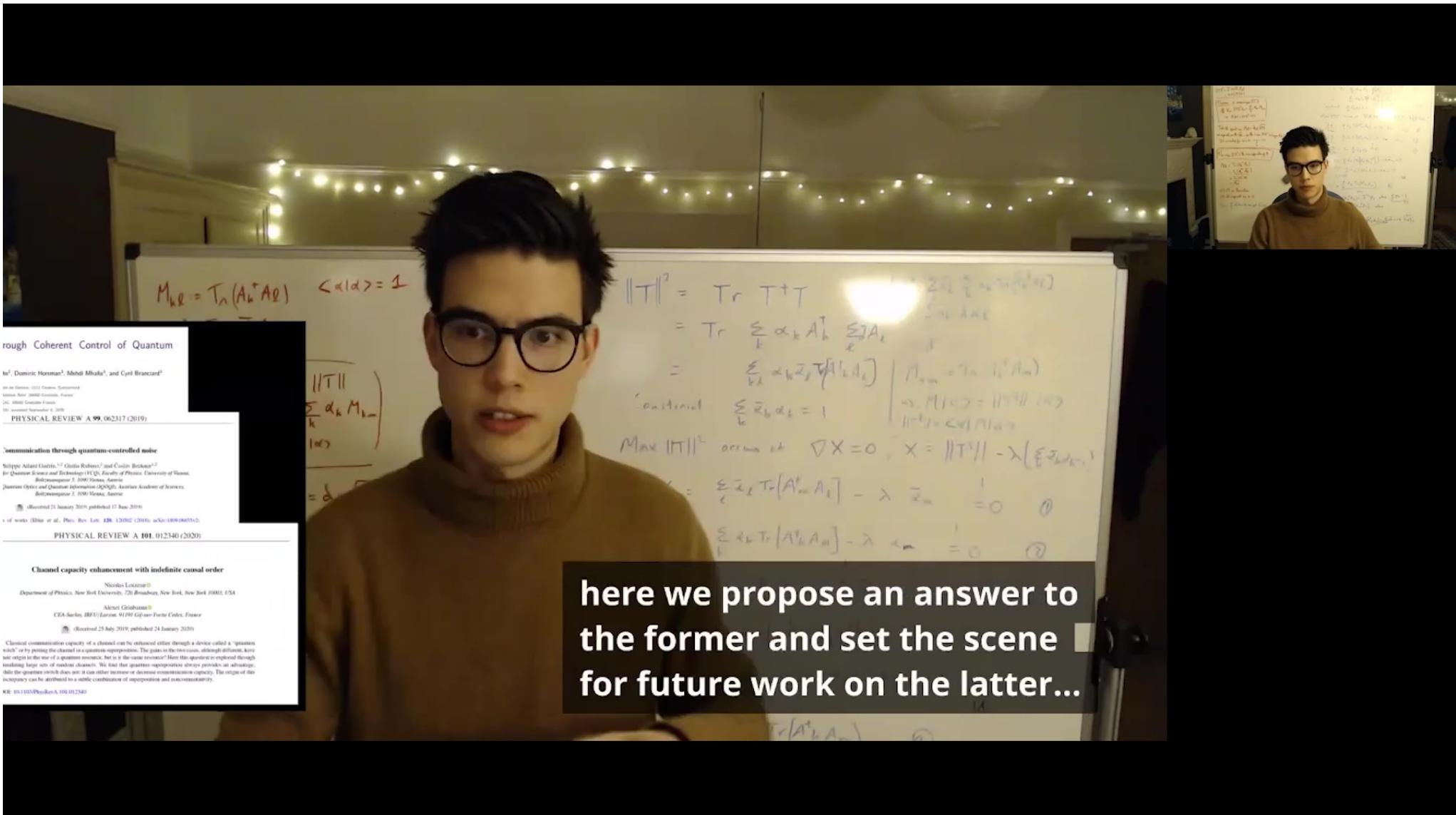
Alexei Grinbaum²
CEA-Saclay, IRFU/Laboratory, 91191 Gif-sur-Yvette Cedex, France

(Received 25 July 2019; published 24 January 2020)

DOI: 10.1103/PhysRevA.101.012340



Resource theories of communication



$$M_{kl} := T_n(A_k^* A_l) \quad \langle \alpha | \alpha \rangle = 1$$

$$\|T\|^2 = \sum_{k,l} \alpha_k \alpha_l M_{kl} = \langle \alpha | M | \alpha \rangle$$

Theorem: α maximizes $\|T\|^2$
 $\Leftrightarrow \forall m \quad \|T\|^2 \alpha_m = \sum_k \alpha_k M_{km}$
 i.e. $M|\alpha\rangle = \|T\|^2 |\alpha\rangle$

Take the special case $M_{kk} = \delta_{kk}$
 \Rightarrow signals on the form $|\alpha\rangle$
 It is needed for $\alpha =$

here we propose an answer to
 the former and set the scene
 for future work on the latter...

$$\begin{aligned} \|T\|^2 &= \text{Tr } T^* T \\ &= \text{Tr } \sum_k \alpha_k A_k^* \sum_l \alpha_l A_l \\ &= \sum_k \alpha_k \text{Tr}[A_k^* A_k] \quad | \quad M_{kk} = \text{Tr}[A_k^* A_k] \\ \text{constraint: } \sum_k \alpha_k \alpha_k &= 1 \quad \Rightarrow M|\alpha\rangle = \|T\|^2 |\alpha\rangle \\ \text{MAX } \|T\|^2 \text{ occurs at } \nabla X &= 0, \quad X = \|T\|^2 - \lambda \left(\sum_k \alpha_k \alpha_k \right) \\ \sum_k \alpha_k \text{Tr}[A_k^* A_k] - \lambda \sum_k \alpha_k &= 0 \end{aligned}$$

- How should potential advantages be quantified and compared?
- What is the origin of these advantages?

$$M_{kk} := T_n(A_k^* A_k) \quad \langle \alpha | \alpha \rangle = 1$$

$$\|T\|^2 = \sum_{k \in \mathcal{K}} M_{kk} = \langle \alpha | M | \alpha \rangle$$

Theorem: α maximizes $\|T\|^2$
iff $\forall m \quad \|T\|^2 \alpha_m = \sum_k \alpha_k M_{km}$
i.e. $M|\alpha\rangle = \|T\|^2 |\alpha\rangle$

Take the special case $M_{kk} = \delta_{kk}$
 \Rightarrow signals on the form $\alpha = \sum_k \alpha_k |k\rangle$
It is needed for α :

... by formulating a resource theory of communication

$$\begin{aligned} \|T\|^2 &= \text{Tr } T^* T \\ &= \text{Tr } \sum_k \alpha_k A_k^* \sum_l \alpha_l A_l \\ &= \sum_k \alpha_k \text{Tr}[A_k^* A_k] \quad | M_{km} = \text{Tr}[A_k^* A_m] \end{aligned}$$

constraint: $\sum_k \bar{\alpha}_k \alpha_k = 1 \quad \Rightarrow M|\alpha\rangle = \|T\|^2 |\alpha\rangle$

$\max \|T\|^2$ occurs at $\nabla X = 0 \quad X = \|T\|^2 \propto 1 - \alpha$

- Quantifies the differences between the different forms of coherent control
- Provides a general framework to formally define new paradigms of communication

- ④ Quantifies the differences between the different forms of coherent control
- ④ Provides a general framework to formally define new paradigms of communication

what is a resource theory?

$$M_{k\ell} = T_n(A_k^\dagger A_\ell) \quad \langle \alpha | \alpha \rangle = 1$$

$$\|T\|^2 = \sum \alpha_k \alpha_\ell M_{k\ell}$$

$$\begin{aligned} & \text{Theorem: } \alpha \text{ maximizes } \|T\| \\ & \text{if } \sqrt{\alpha_m} \|T\|^2 \alpha_m = \sum_k \alpha_k M_{k\ell} \\ & \text{i.e. } M|\alpha\rangle = \|T\|^2 |\alpha\rangle \end{aligned}$$

$$\text{Take the special case } M_{k\ell} = \delta_{k\ell} \sqrt{p_k}$$

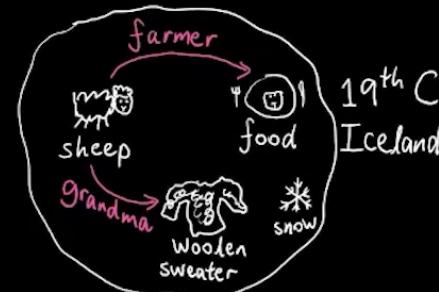
\Rightarrow eigenstate on the (p_k) , so the max.

It is needed for $\alpha = 1$.

Resource theory:

- objects \leftrightarrow resources
- operations \leftrightarrow free transformations

Set of operations on the objects is closed under parallel + sequential composition



[B. Coecke, T. Fritz, R. Spekkens,
Inf. and Comp. 250 (2016)]



resource theory of communication: allowed ways to combine channels

$$M_{k\ell} := T_n(A_k^\dagger A_\ell) \quad \langle \alpha | \alpha \rangle = 1$$

$$\|T\|^2 = \sum \alpha_k \alpha_\ell M_{k\ell}$$

$$\begin{aligned} & \text{Theorem: } \alpha \text{ maximizes } \|T\| \\ & \text{if } \forall m \quad \|T\|^2 \alpha_m = \sum_k \alpha_k M_{km} \\ & \text{i.e. } M|\alpha\rangle = \|T\|^2 |\alpha\rangle \end{aligned}$$

Take the special case $M_{k\ell} = \delta_{k\ell} \sqrt{P_k}$

\Rightarrow signals on the (P_k) go through

It is needed for $\alpha = 1$

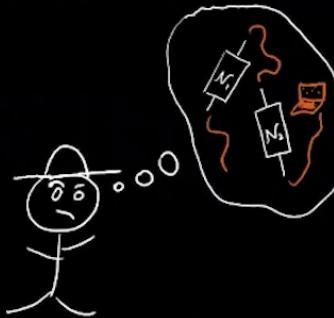
$$\|T\|^2 =$$

$$=$$

$$\text{Constraints}$$

$$\text{Max } \|T\|$$

$$\alpha =$$



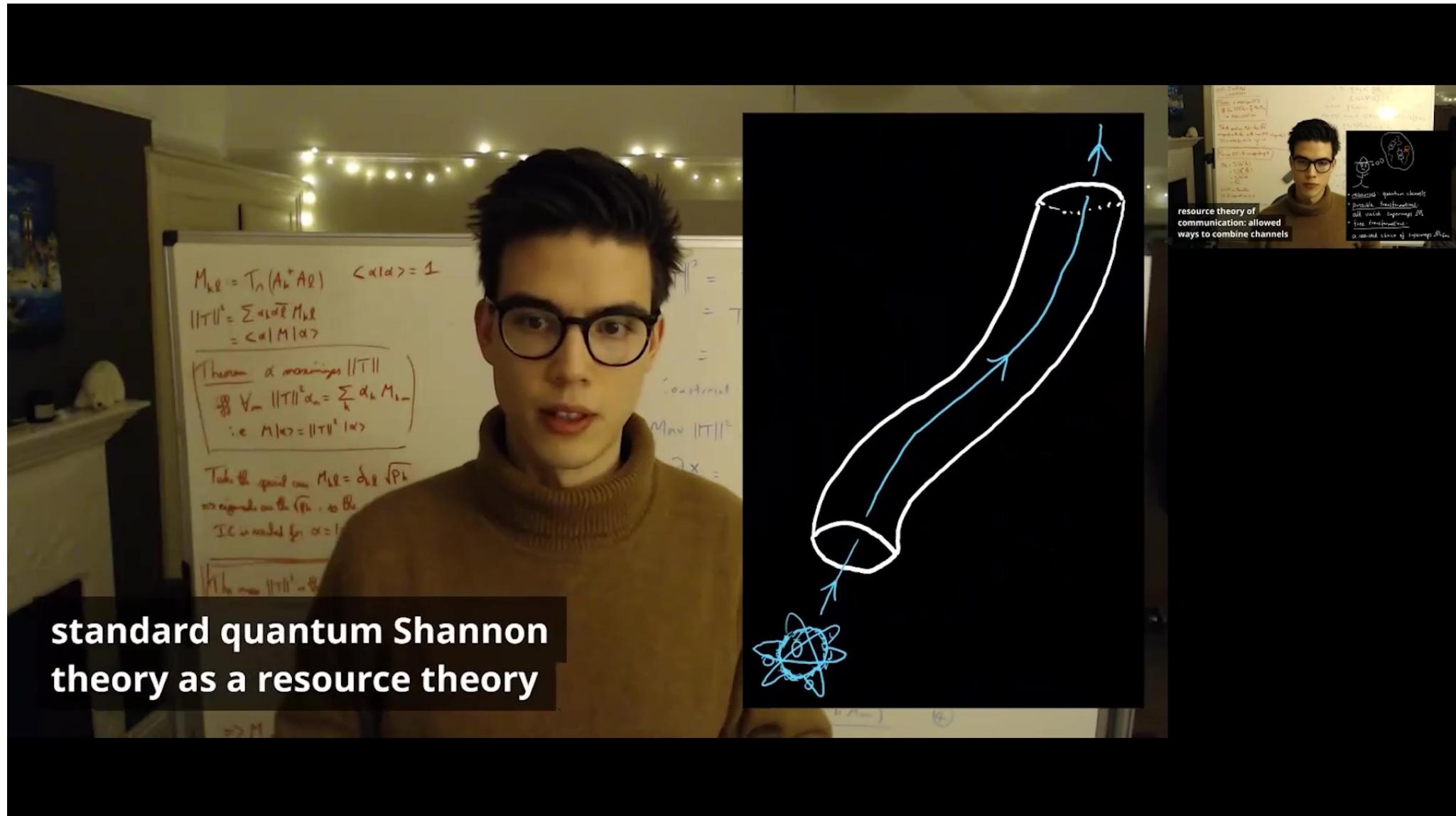
- resources: quantum channels
- possible transformations:
all valid supermaps M
- free transformations:
a restricted choice of supermaps M_{free}

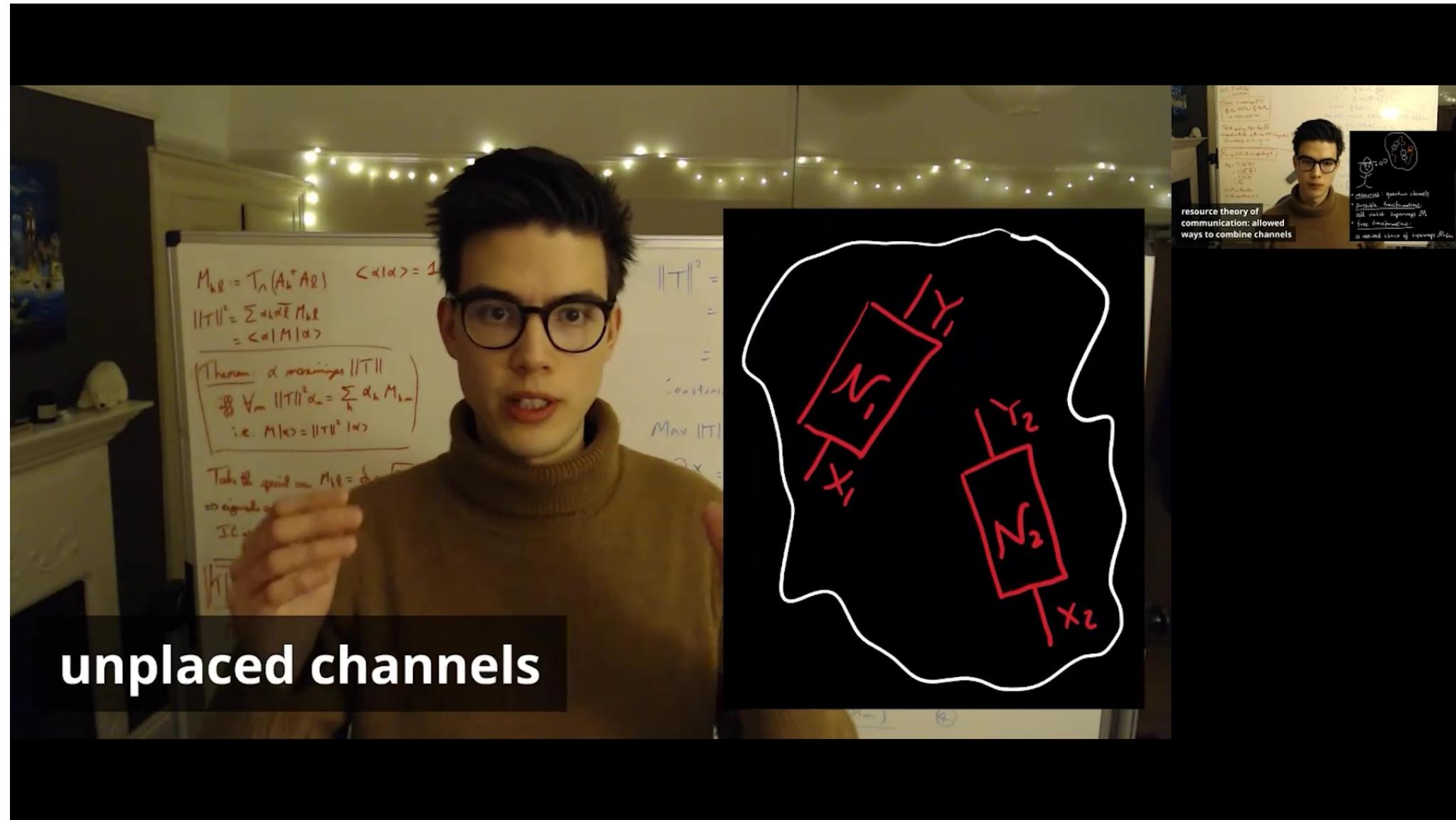
Resource theory:
• Objects as resources
• operations \Leftrightarrow the transformations
Set of operations on the objects is closed under parallel + sequential compositions



[B. Coecke, T. Selby, R. Spekkens,
T-01-040, 2009 (2011)]

standard quantum Shannon theory as a resource theory





unplaced channels

placed channels

$$M_{kL} = T_k (A_k^\dagger A_L) \quad \langle \alpha | \alpha \rangle = 1$$

$$\|T\|^2 = \sum_{k,m} M_{km}$$

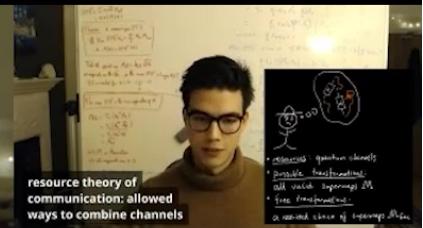
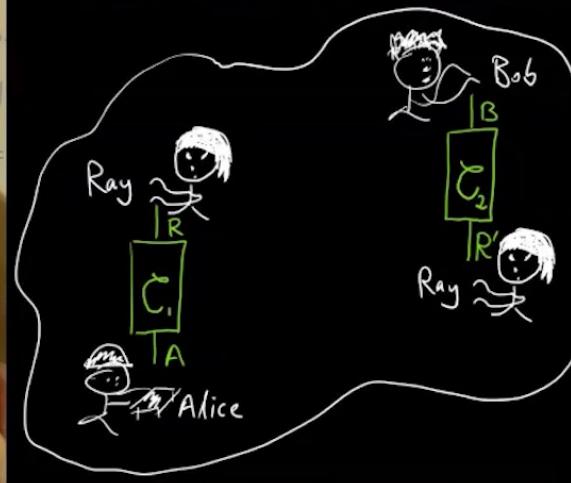
Theorem: α maximizes $\|T\|^2$
 $\forall m \quad \|T\|^2 \alpha_m = \sum_k \alpha_k M_{km}$
i.e. $M|\alpha\rangle = \|T\|^2 |\alpha\rangle$

Take the special case $M_{kL} = \delta_{kL} \sqrt{P_L}$
 \Rightarrow signals on the (P_1, \dots, P_n)
It is needed for α

$$\begin{aligned} \|T\|^2 &= \text{Tr } T^\dagger T \\ &= \text{Tr } \sum_k \alpha_k A_k^\dagger \sum_l \alpha_l A_l \\ &= \sum_{k,l} \alpha_k \alpha_l \text{Tr}(A_k^\dagger A_l) \end{aligned}$$

constraint
Max $\|T\|^2$

$\partial x =$



**basic placement supermap:
performed by
communication provider**

Whiteboard content:

- $M_{kk\ell} := T_\alpha (A_k^\dagger A_\ell)$ $\langle \alpha | \alpha \rangle = 1$
- $\|T\|^2 = \sum \alpha_k \alpha_\ell M_{kk\ell}$ $= \langle \alpha | M | \alpha \rangle$
- (Theorem: α maximizes $\|T\|$)
 $\forall m \quad \|T\|^2 \alpha_m = \sum_k \alpha_k M_{km}$
 i.e. $M|\alpha\rangle = \|T\|^2 |\alpha\rangle$
- Take the special case M
 \Rightarrow signals on the \mathbb{R}^n
 IC is needed for
- $\|T\|^2 = \text{Tr } T^\dagger T$
 $= \text{Tr } \sum_k \alpha_k A_k^\dagger \sum_\ell A_\ell$
 $= \sum_k \alpha_k \text{Tr} [A_k^\dagger A_k]$ $|M_{km} = \text{Tr } M_k^\dagger M_m\rangle$
 constraint: $\sum_k \alpha_k \alpha_k = 1$ $\Rightarrow M|\alpha\rangle = \|T\|^2 |\alpha\rangle$
 $\|T\|^2 = \langle \alpha | M | \alpha \rangle$
- $M_{km} = \text{occurs at } \nabla X = 0, X = \|T\|^2 - \lambda \left(\sum_k \alpha_k \alpha_k \right)$
- $T = T^\dagger$

Diagram on the right:

parallel placement

$$M_{kk} = T_n(A_k^* A_k) \quad \langle \alpha | \alpha \rangle = 1$$

$$\|T\|^2 = \sum_{k=1}^n M_{kk} = \langle \alpha | M | \alpha \rangle$$

Theorem: α maximizes $\|T\|^2$
 $\forall \alpha \quad \|T\|^2 \alpha = \sum_k \alpha_k M_{1-k}$
 i.e. $M|\alpha\rangle = \|T\|^2 |\alpha\rangle$

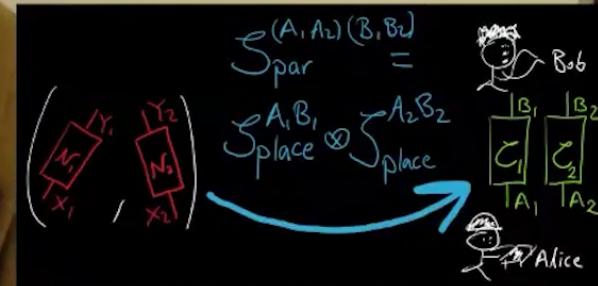
Take the special case $M_{kk} = \delta_{kk} \sqrt{p_k}$
 or eigenvalues on the \mathbb{R}_+ .

It's needed for

$\|T\| = \sqrt{\sum_k p_k}$

$$\begin{aligned} \|T\|^2 &= \text{Tr } T^* T \\ &= \text{Tr } \sum_k \alpha_k A_k^* \sum_l \beta_l A_l \\ &= \sum_{k,l} \alpha_k \beta_l \text{Tr}(A_k^* A_l) \quad | \quad M_{1-m} = \text{Tr}(A_m^* A_m) \\ \text{constraint: } \sum_k \beta_k \alpha_k &= 1 \quad | \quad M|0\rangle = \|T\|^2 |0\rangle \\ \text{Max } \|T\|^2 \text{ occurs at } \nabla X &= 0, X = \|T\|^2 - \lambda \left(\sum_k \alpha_k \delta_{km} \right) \end{aligned}$$

$$\frac{\partial X}{\partial \alpha_m} = \sum_k \beta_k \text{Tr}(A_m^* A_k) - \lambda \sum_k \delta_{km} = 0$$



sequential placement

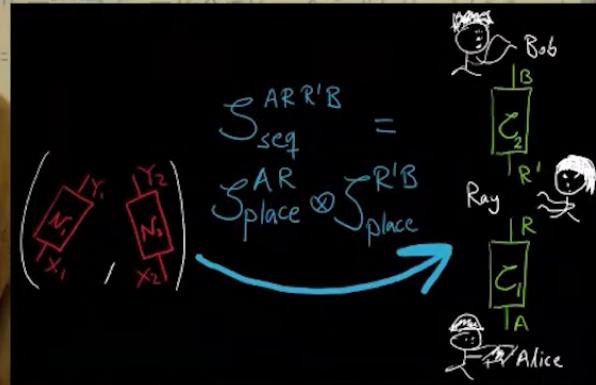
$$M_{kk} = T_n(A_k^* A_k) \quad \langle \alpha | \alpha \rangle = 1$$

$$\|T\|^2 = \sum \alpha_k M_{kk}$$

Theorem: α maximizes $\|T\|^2$
 $\forall m \quad \|T\|^2 \alpha_m = \sum_k \alpha_k M_{km}$
i.e. $M|\alpha\rangle = \|T\|^2 |\alpha\rangle$

Take the special case $M_{kk} = \delta_{kk} \sqrt{p_k}$
 \Rightarrow signals on the \mathbb{R}_{+} ,
It is needed for

$$\begin{aligned} \|T\|^2 &= \text{Tr } T^\dagger T \\ &= \text{Tr } \sum_k \alpha_k A_k^\dagger \sum_\ell \alpha_\ell A_\ell \\ &= \sum_k \alpha_k^2 T(A_k^\dagger A_k) \quad | \quad A_{nm} = T_n H_m^* A_m \\ \text{constraint: } \sum_k \alpha_k &= 1 \quad | \quad \Rightarrow \|T\|^2 = \text{Tr } T^2 \quad \text{or} \\ \text{Max } \|T\|^2 \text{ occurs at } \nabla X &= 0 \quad X = \|T\|^2 \geq 0 \end{aligned}$$



party supermap: encoding-repeater- decoding

$$M_{BL} = T_n(A_k^* A_k) \quad \langle \alpha | \alpha \rangle$$

$$\|T\|^2 = \sum_{k=0}^n M_{kk}$$

Theorem: α maximizes $\|T\|^2$
 $\forall m \quad \|T\|^2 \alpha_m = \sum_k \alpha_k M_{km}$
i.e. $M|\alpha\rangle = \|T\|^2 |\alpha\rangle$

Take the special case $M_{BL} = \delta_{jk}$
or eigenvalues on the $(T^*)^*$
 T is unitary

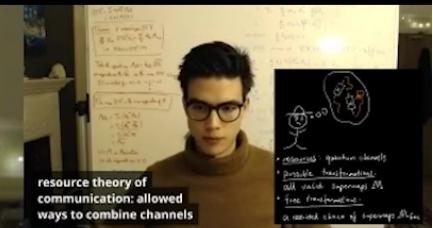
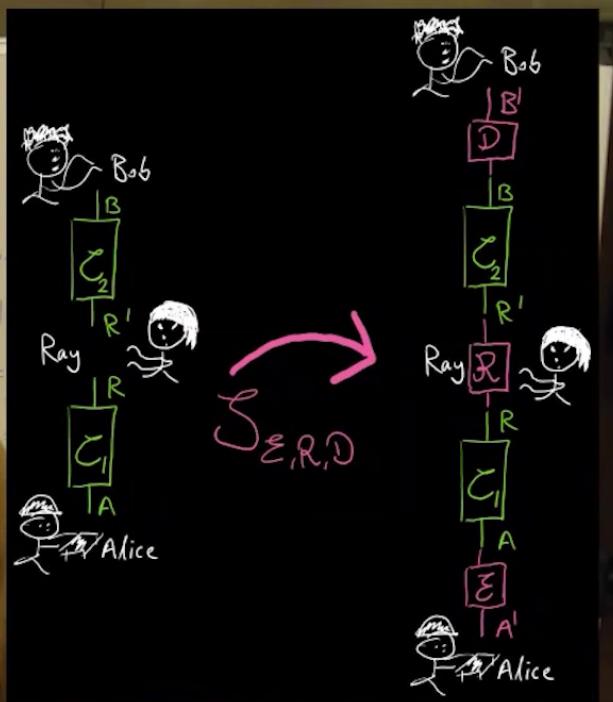
$$\|T\|^2 =$$

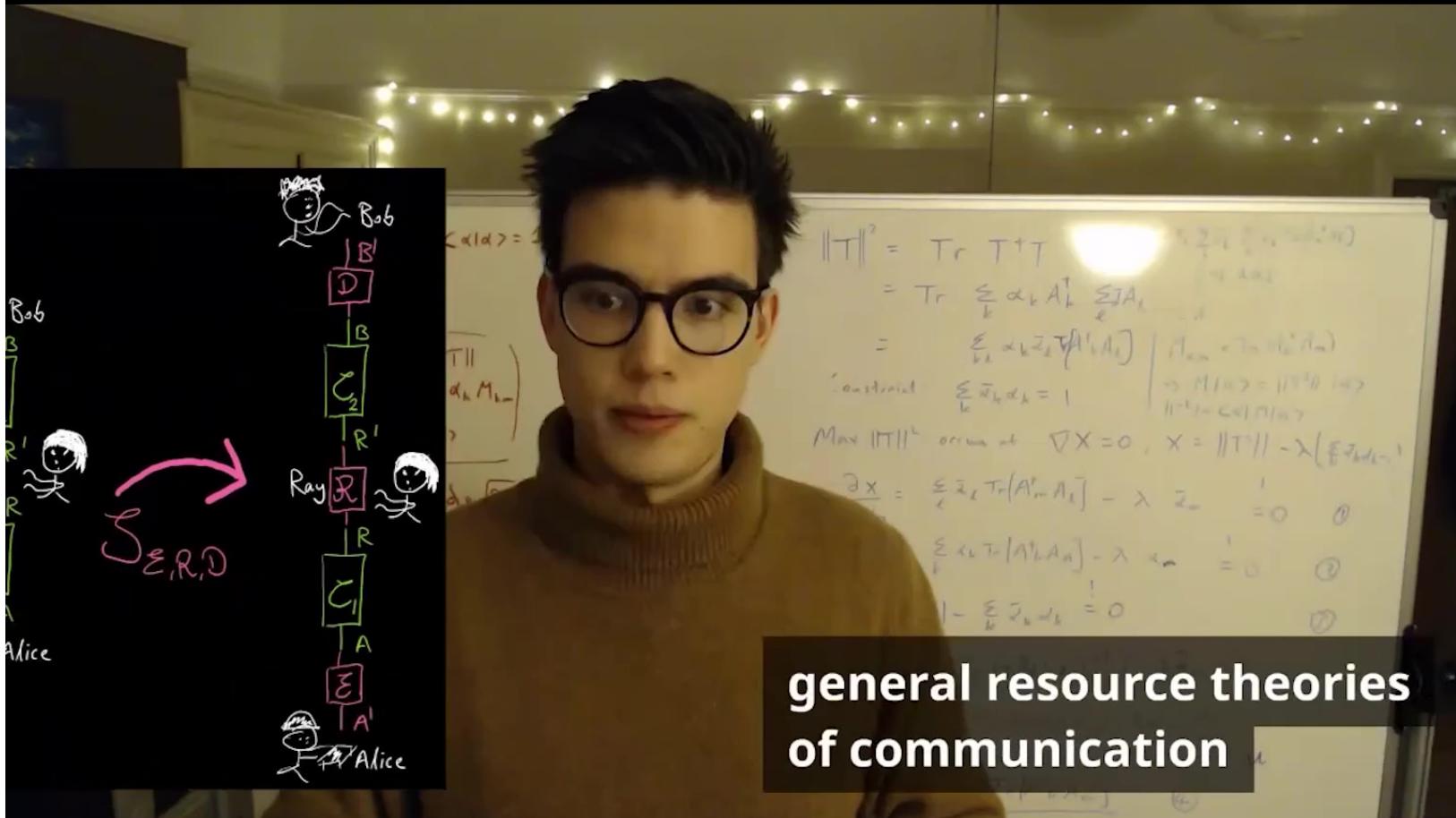
=

constraint

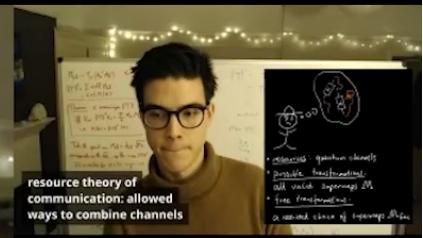
$$\text{Max } \|T\|$$

$$\frac{\partial \chi}{\partial \alpha_m}$$





general resource theories of communication



general resource theories of communication

$$M_{k\ell} = T_k (A_k^\dagger A_\ell)$$

$$\|T\|^2 = \sum_{k,\ell} M_{k\ell}$$

Theorem of majorization:
 $\nabla \text{Max } \|T\|^2$
i.e. $M_{\text{Max}} = \|\text{Max}\|$

Take the ap...
original
It is...

$$\begin{aligned} \|T\|^2 &= \text{Tr } T^\dagger T \\ &= \text{Tr } \sum_k \alpha_k A_k^\dagger \sum_\ell \beta_\ell A_\ell \\ &= \sum_{k,\ell} \alpha_k \beta_\ell T^\dagger [A_k^\dagger A_\ell] \quad | \quad M_{\text{Max}} = \text{Tr } (A_k^\dagger A_m) \\ \text{constraint: } \sum_k \alpha_k &= 1 \quad | \quad \Rightarrow M_{\text{Max}} = \|T\|^2 = \text{Max } \|T\|^2 \\ \text{Max } \|T\|^2 \text{ occurs at } \nabla X &= 0, X = \|T\|^2 - \lambda \left(\sum_k \alpha_k \right) \end{aligned}$$

Definition 1

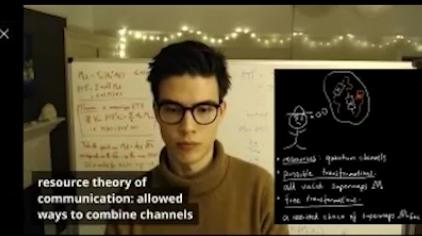
a resource theory of communication:
specified by a set of free
supermaps M_{free} containing
(a) placement supermaps
(b) party supermaps



Definition 1

a resource theory of communication:
specified by a set of free
supermaps M_{free} containing

- (a) placement supermaps
- (b) party supermaps



minimal requirement: no side-channel generation

$$M_{kL} = T_n(A_k^T A_L) \quad \langle \alpha | \alpha \rangle = 1$$

$$\|T\|^2 = \sum_{k \in L} M_{kk}$$

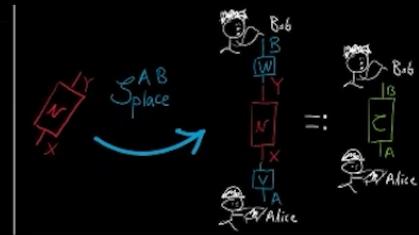
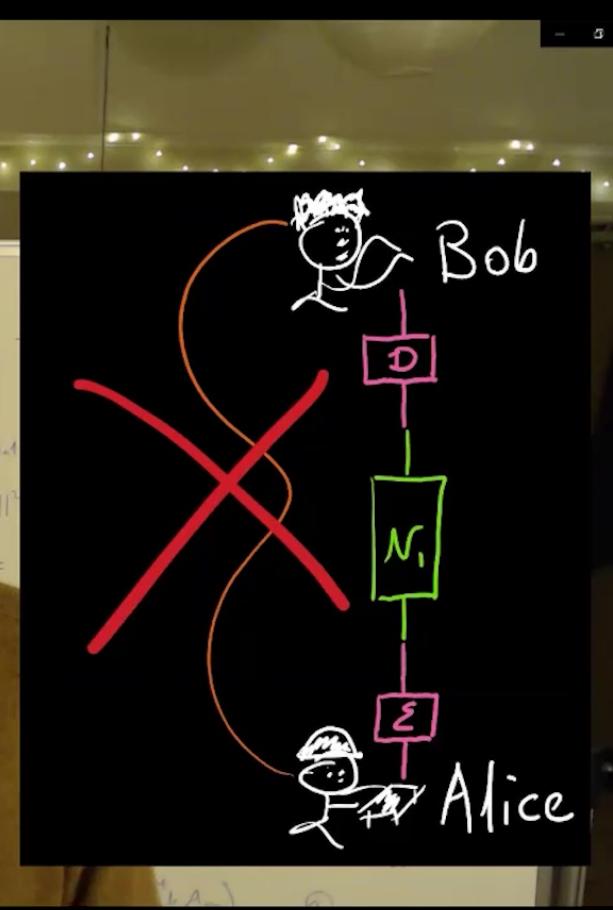
$$= \langle \alpha | M | \alpha \rangle$$

Theorem: α maximizes $\|T\|^2$

$$\forall m \quad \|T\|^2 \alpha_m = \sum_k \alpha_k M_{km}$$

i.e. $M|\alpha\rangle = \|T\|^2 |\alpha\rangle$

Take the special case $M_{kL} = \alpha_{kL} \sqrt{P_k}$
⇒ signals on the $\sqrt{P_k}$ is the max $\|T\|^2$
IC is needed for $\alpha = 10^{-10}$



formally, the set of free supermaps must not contain side-channel generating operations

$$M_{kL} = T_L(A_k^* A_L) \quad \langle \alpha | \alpha \rangle = 1$$

$$\|T\|^2 = \sum \alpha_k M_{kk} = \langle \alpha | M | \alpha \rangle$$

Theorem α maximizes $\|T\|^2$
 $\Leftrightarrow \forall m \quad \|T\|^2 \alpha_m = \sum_k \alpha_k M_{km}$
 i.e. $M|\alpha\rangle = \|T\|^2 |\alpha\rangle$

Take the special case $M_{kL} = \alpha_{kL} \sqrt{P_L}$
 \Rightarrow signals on the P_L 's to the max.
 IC is needed for $\alpha = 10$

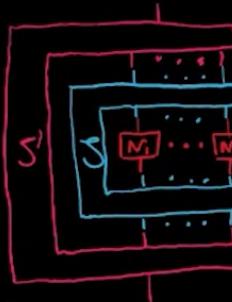
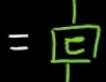
Definition 2

(side channel-generating operation);

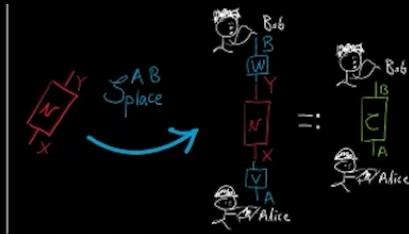
A supermap  generates

side-channels if \exists a free supermap

$$\begin{matrix} \text{[S]} \\ \text{---} \\ \text{[S']} \end{matrix} \text{ s.t. } \forall \left(\begin{matrix} \text{[N}_1\text{]} \\ \vdots \\ \text{[N}_k\text{]} \end{matrix} \right),$$

We have  = 

where  has capacity > 0 .



$M_{b2} = T_A (A_b^\dagger A_2)$ $\langle \alpha | \alpha \rangle =$
 $\|T\|^2 = \sum_{\alpha \in \mathcal{A}} M_{\alpha\alpha}$
 $= \langle \alpha | M | \alpha \rangle$
 (Theorem: α maximizes $\|T\|^2$)
 $\Leftrightarrow \forall m \quad \|T\|^2 \alpha_m = \sum_k \alpha_k M_{k\alpha_m}$
 i.e. $M|\alpha\rangle = \|T\|^2 |\alpha\rangle$
 Take the special case $M_{b2} = \alpha_{b2} \sqrt{P_A}$
 \Rightarrow signals on the (P_A, α_{b2})
 IC is needed for ...

superposition of encoding operations:
generates side-channels

$\|T\|^2 =$
 $=$
 $=$
 constraint
 $\text{MAX } \|T\|^2$
 $\frac{\partial}{\partial \alpha_n} =$

P.A. Guérin et al.
 Phys. Rev. A
 2019

00:24:11 00:17:44

$$M_{kL} = T_n(A_k^* A_L) \quad \langle \alpha | \alpha \rangle = 1$$

$$\|T\|^2 = \sum_{k,L} M_{kL}$$

Theorem: α maximizes $\|T\|^2$

$$\forall m \quad \|T\|^2 \alpha_m = \sum_k \alpha_k M_{km}$$

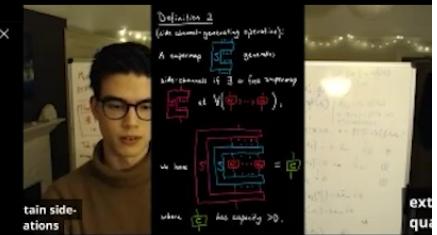
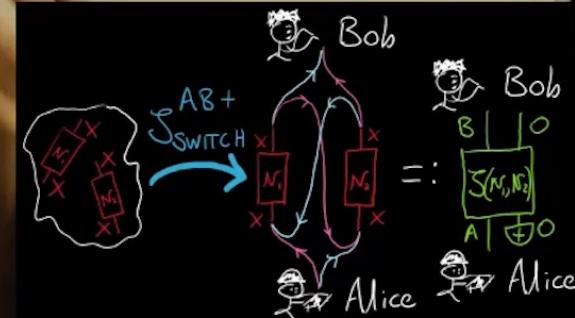
i.e. $M|\alpha\rangle = \|T\|^2 |\alpha\rangle$

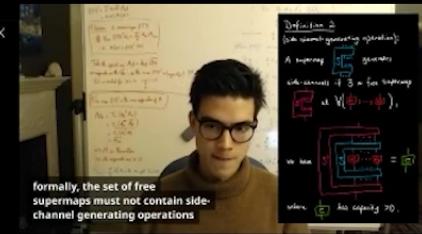
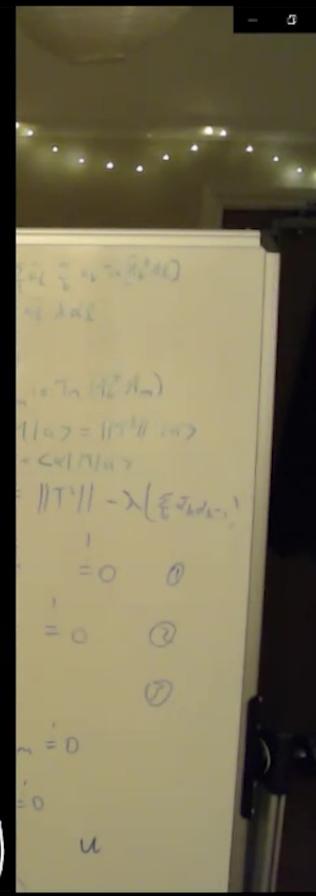
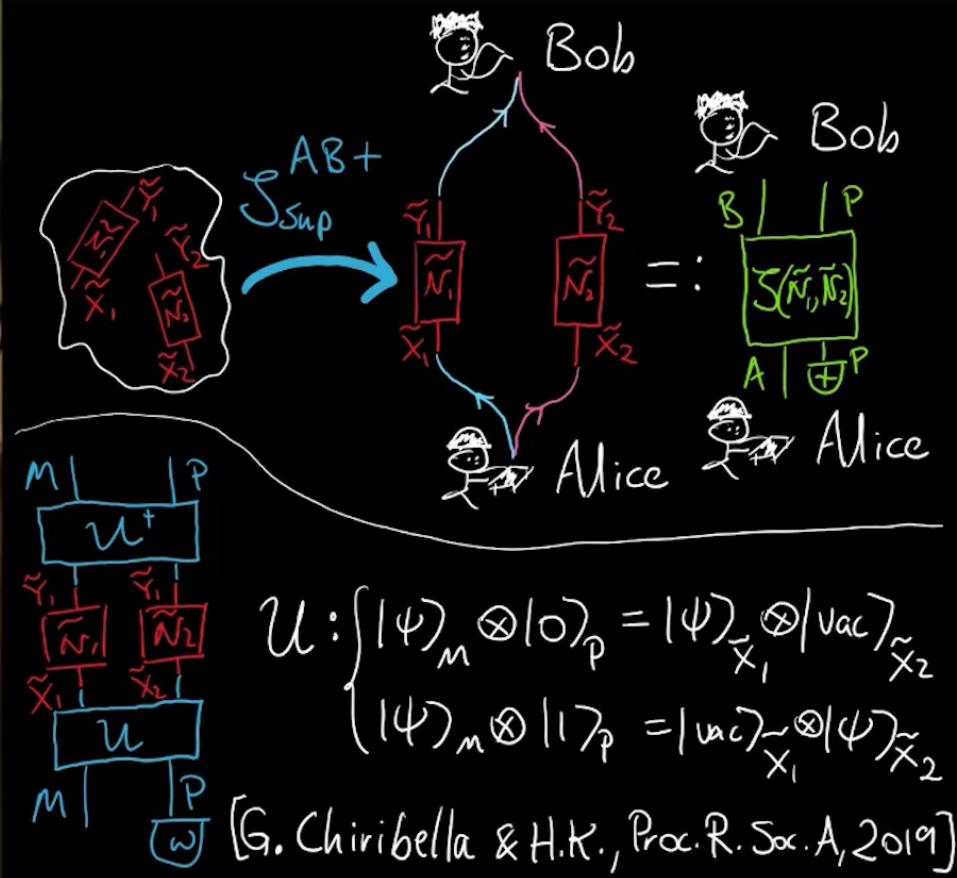
Take the special case $M_{kL} = \delta_{kL}$
 \Rightarrow signals on the (p_k, α_k)
 IC is needed for α

SWITCH placement

Proposition 1:
does not generate side-channels

$$\begin{aligned} \|T\|^2 &= \text{Tr } T^\dagger T \\ &= \text{Tr } \sum_k \alpha_k A_k^\dagger A_k \\ &= \sum_k \alpha_k \text{Tr } A_k^\dagger A_k \quad | \quad \text{Tr } A_k^\dagger A_k = M_{kk} \\ &\text{Constraint: } \sum_k \alpha_k \alpha_k = 1 \quad | \quad \Rightarrow M|\alpha\rangle = \|T\|^2 |\alpha\rangle \\ \text{Max } \|T\|^2 \text{ s.t. } \nabla X = 0, \quad X = \|T\|^2 - \lambda \left(\sum_k \alpha_k \alpha_k - 1 \right) \end{aligned}$$





SWITCH and superposition placements take different initial resources

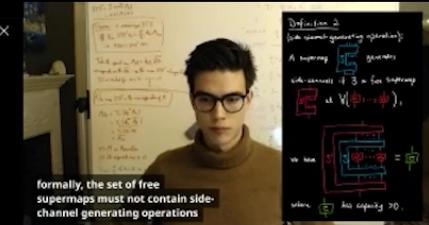
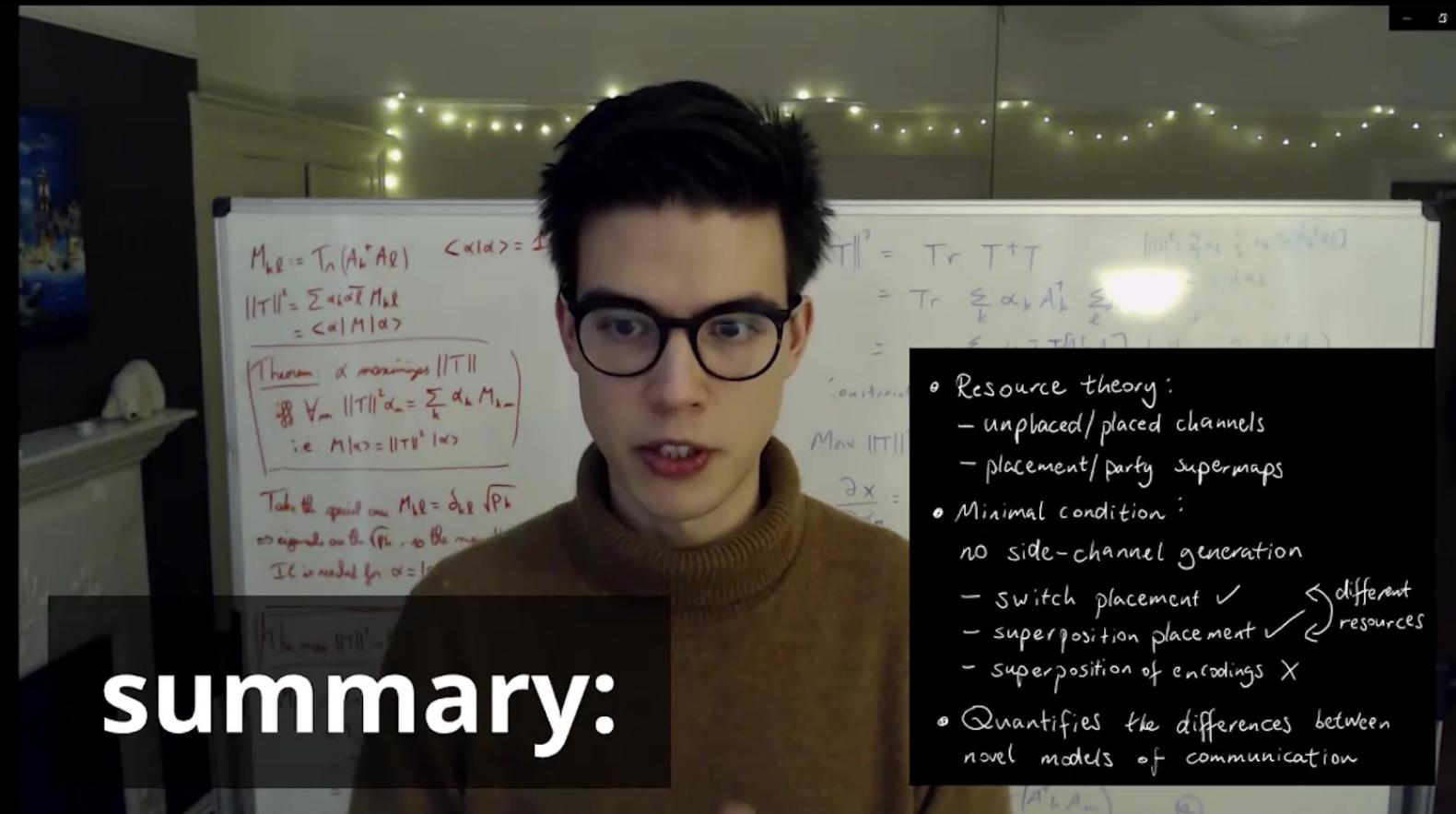
$$\boxed{X} \neq \boxed{\tilde{X}} := X \oplus \text{Vac}$$

Future work:

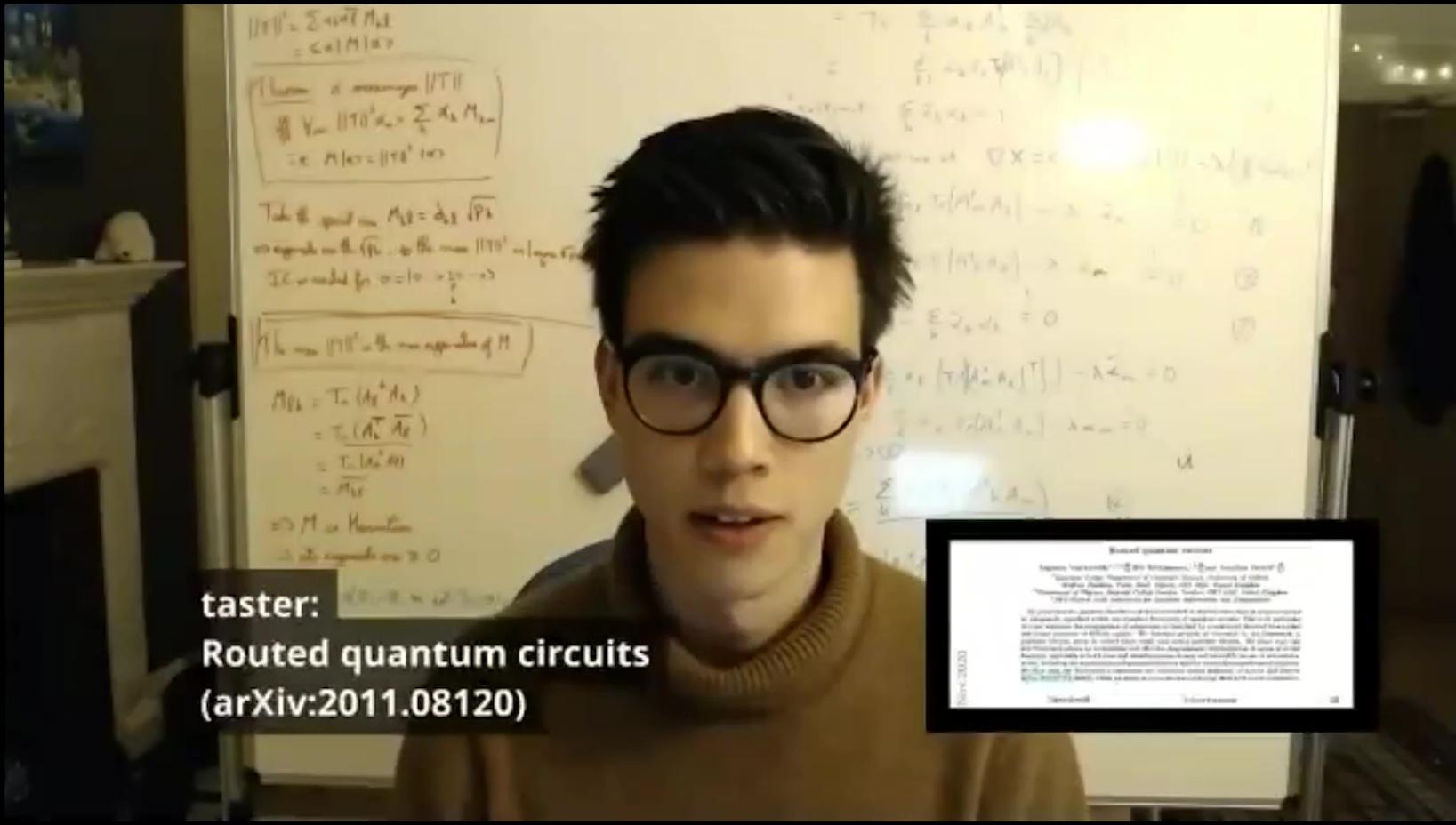
Investigate the extent to which indefinite causal order gives advantages, when the comparison is restricted to

- (a) non-side-channel generating operations
- (b) operations acting on the same initial resources

summary:



- Resource theory:
 - unplaced/placed channels
 - placement/partly supermaps
- Minimal condition:
 - no side-channel generation
 - switch placement ✓ ↗ different resources
 - superposition placement ✓ ↗ resources
 - superposition of encodings X
- Quantifies the differences between novel models of communication



- Resource theory:
 - unplaced/placed channels
 - placement/partly supermaps
- Minimal condition:
 - no side-channel generation
 - switch placement ✓ ↗ different resources
 - superposition placement ✓ ↗ different resources
 - superposition of encodings X
- Quantifies the differences between novel models of communication

resourcesPl_talk

00:32:18



00:09:37



recap of research aims:

- How should potential advantages be quantified and compared? ✓
- What is the origin of these advantages? ←

$M_{kk} := T_n(A_k^T A_k) \quad \langle \alpha | \alpha \rangle = 1$

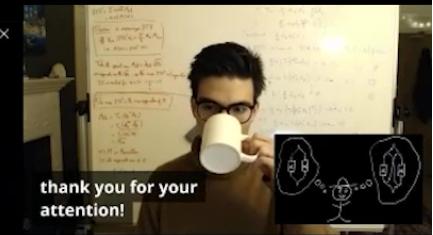
$\|T\|^2 = \sum \alpha_k \overline{\alpha_k} M_{kk}$
= $\langle \alpha | M | \alpha \rangle$

Theorem: α maximizes $\|T\|^2$
 $\Leftrightarrow \forall m \quad \|T\|^2 \alpha_m = \sum_k \alpha_k M_{km}$
i.e. $M|\alpha\rangle = \|T\|^2 |\alpha\rangle$

Take the special case $M_{kk} = \delta_{kk} \sqrt{P_k}$
⇒ signals on the $\sqrt{P_k}$, so the max $\|T\|^2$ is
It is needed for $\alpha = 10^{-25}$

$= \text{Tr } T^T T$
 $= \text{Tr } \sum_k \alpha_k A_k^T \sum_l \alpha_l$
 $= \sum_k \alpha_k \overline{\alpha_k} T^T(A_k^T A_k)$ | $M_{kk} = \text{Tr}(A_k^T A_k)$
constraint: $\sum_k \alpha_k \overline{\alpha_k} = 1 \Rightarrow M|\alpha\rangle = \|T\|^2 |\alpha\rangle$
 $\|T\|^2$ occurs at $\nabla X = 0$, $X = \|T\|^2 - \lambda \left(\sum_k \alpha_k \overline{\alpha_k} \right)$

$\partial X = \sum_k \text{Tr}[A_k^T A_k] - \lambda \frac{1}{\sum_k} = 0 \quad \square$



Nov 2020

←



Routed quantum circuits

Augustin Vanrietvelde,^{1,2,3,*} Hlér Kristjánsson,^{1,3,†} and Jonathan Barrett^{1,‡}

¹*Quantum Group, Department of Computer Science, University of Oxford,
Wolfson Building, Parks Road, Oxford, OX1 3QD, United Kingdom*

²*Department of Physics, Imperial College London, London, SW7 2AZ, United Kingdom*

³*HKU-Oxford Joint Laboratory for Quantum Information and Computation*

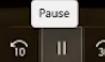
We argue that the quantum-theoretical structures studied in several recent lines of research cannot be adequately described within the standard framework of quantum circuits. This is in particular the case whenever the combination of subsystems is described by a nontrivial blend of direct sums and tensor products of Hilbert spaces. We therefore propose an extension to the framework of quantum circuits, given by *routed linear maps* and *routed quantum circuits*. We prove that this new framework allows for a consistent and intuitive diagrammatic representation in terms of circuit diagrams, applicable to both pure and mixed quantum theory, and exemplify its use in several situations, including the superposition of quantum channels and the causal decompositions of unitaries. We show that our framework encompasses the ‘extended circuit diagrams’ of Lorenz and Barrett [arXiv:2001.07774 (2020)], which we derive as a special case, endowing them with a sound semantics.

CONTENTS

Acknowledgments



00:33:32



00:08:23



$$M_{k\ell} := T_n(A_k^\dagger A_\ell) \quad \langle \alpha | \alpha \rangle = 1$$

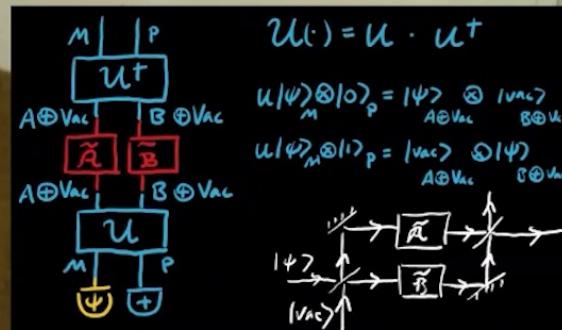
$$\|T\|^2 = \sum_{k,\ell} \alpha_k \alpha_\ell M_{k\ell}$$

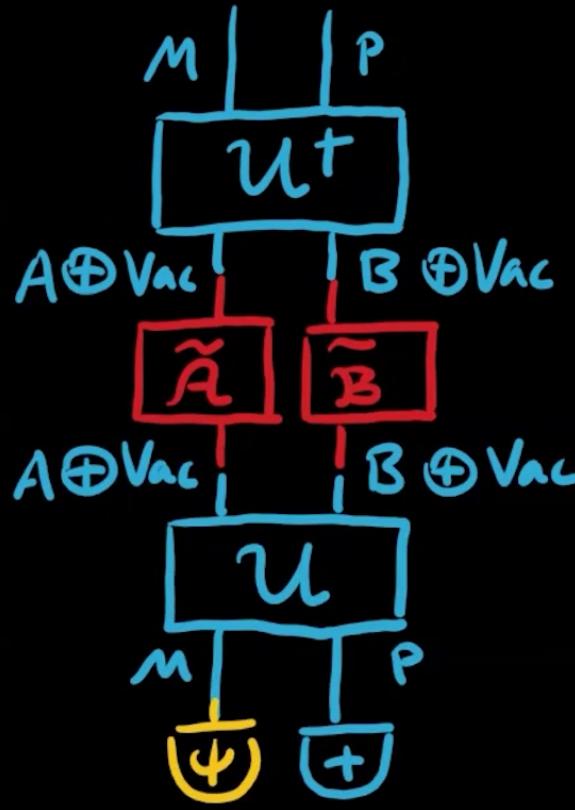
Theorem: α maximizes $\|T\|$
 $\Leftrightarrow \forall m \quad \|T\|^2 \alpha_m = \sum_k \alpha_k M_{km}$
 i.e. $M|\alpha\rangle = \|T\|^2 |\alpha\rangle$

Take the special case $M_{k\ell} = \delta_{k\ell} \sqrt{p_k}$
 \Rightarrow eigenstate on the $\sqrt{p_k}$, so the max $\|T\|^2$.
 It is needed for $\alpha = 1$.

circuit for superposition of channels - doesn't capture true composition

$$\begin{aligned}
 &= \text{Tr } T^\dagger T \\
 &= \text{Tr } \sum_k \alpha_k A_k^\dagger \sum_\ell \alpha_\ell \\
 &= \sum_k \alpha_k \bar{\alpha}_k T^\dagger (A_k A_k) \quad | M_{kk} = \text{Tr } (A_k^\dagger A_k) \\
 &\text{constraint: } \sum_k \bar{\alpha}_k \alpha_k = 1 \quad | M|\alpha\rangle = \|T\|^2 |\alpha\rangle \\
 &\text{MAX } \|T\|^2 \text{ occurs at } \nabla X = 0, \quad X = \|T\|^2 - \lambda \left(\sum_k \bar{\alpha}_k \alpha_k \right)
 \end{aligned}$$

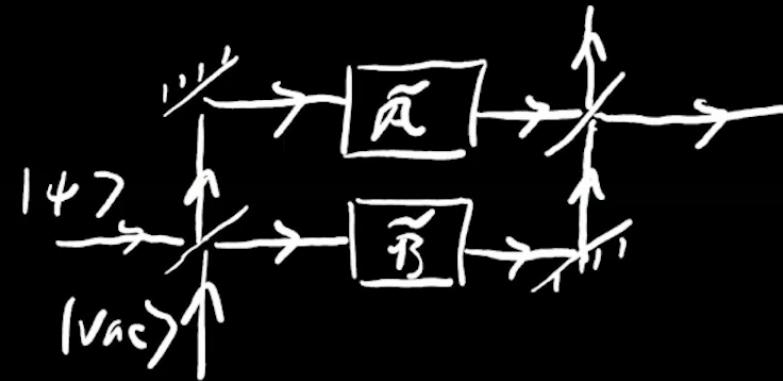




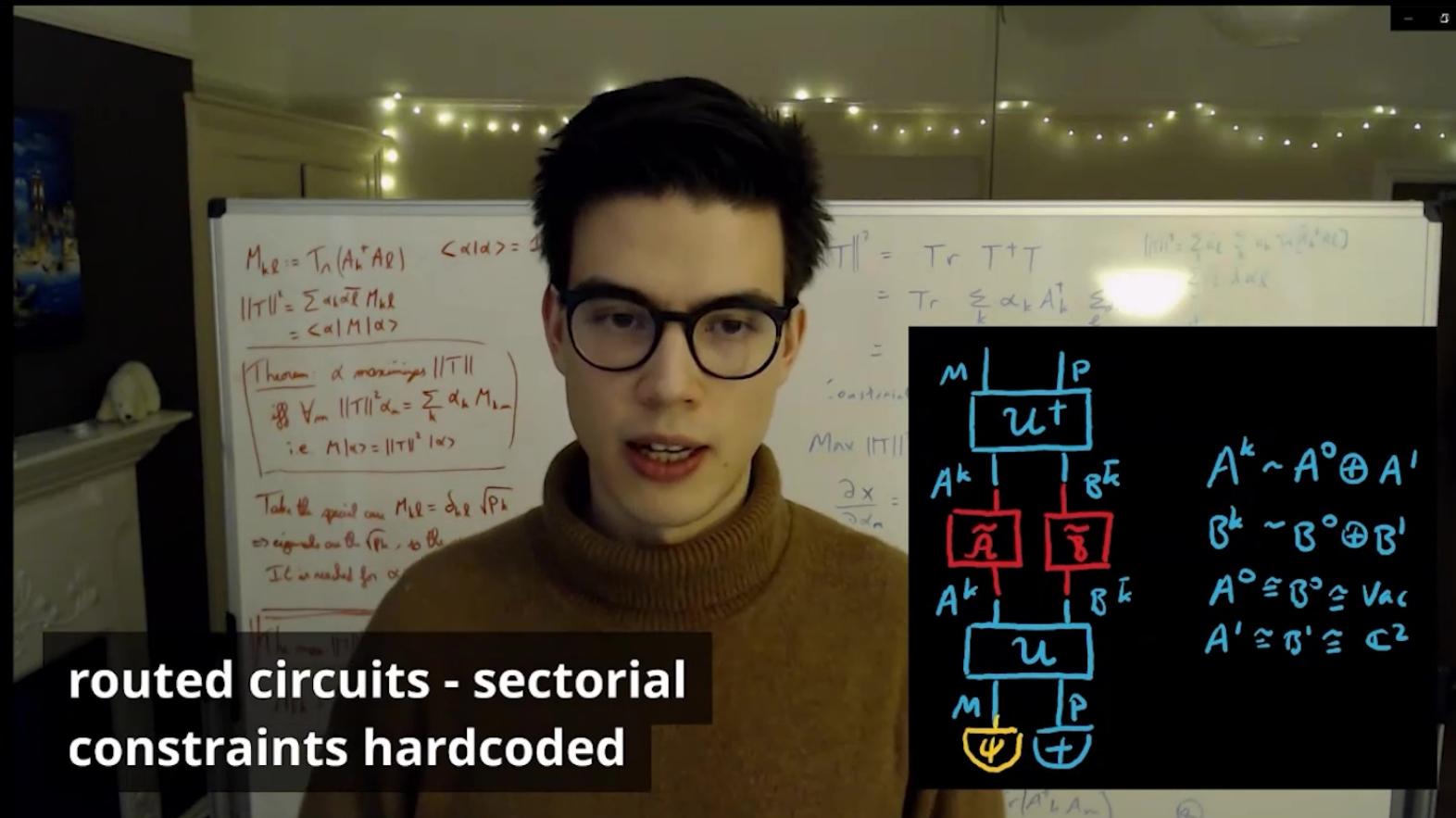
$$U(\cdot) = U \cdot U^\dagger$$

$$U|\psi\rangle_M \otimes |0\rangle_P = |\psi\rangle_{A \oplus \text{Vac}} \otimes |\text{vac}\rangle_{B \oplus \text{Vac}}$$

$$U|\psi\rangle_M \otimes |1\rangle_P = |\text{vac}\rangle_{A \oplus \text{Vac}} \otimes |\psi\rangle_{B \oplus \text{Vac}}$$



routed circuits - sectorial constraints hardcoded



more general routes: loss of particles

$$\begin{aligned}
 \|T\|^2 &= \text{Tr } T^\dagger T \\
 &= \text{Tr } \sum_k \alpha_k A_k^\dagger \sum_\ell \alpha_\ell \\
 &= \sum_{k,\ell} \alpha_k \bar{\alpha}_\ell T^\dagger(A_k A_\ell) \quad | \quad M_{kk} = \text{Tr}(A_k^\dagger A_k) \\
 &\text{constraint: } \leq T
 \end{aligned}$$

$\Rightarrow M_{kk} = \|T\|^2 / \alpha_k^2$

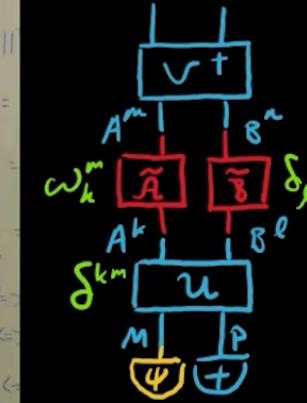
MAX $|T|$

$$\frac{\partial X}{\partial \alpha_m}$$

$$\frac{\partial X}{\partial \beta_n}$$

$$\frac{\partial X}{\partial \lambda}$$

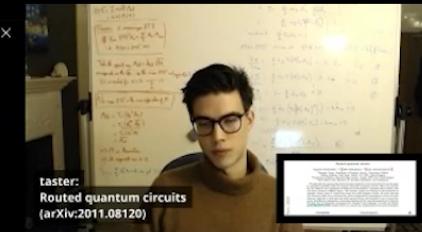
$$\textcircled{1} \Leftrightarrow$$



$$\begin{aligned}
 A^k &\sim A^0 \oplus A' \\
 B^k &\sim B^0 \oplus B' \\
 A^0 &\cong B^0 \cong \text{Vac} \\
 A' &\cong B' \cong \mathbb{C}^2
 \end{aligned}$$

$$\omega_k^m = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$M_{kk} = \sum_k \alpha_k \text{Tr}(A_k^\dagger A_k)$$



future work:

- applications to indefinite causal order?
- quantum switch in this framework
→ see already: J. Barrett, R. Lorenz, O. Oreshkov, arXiv:2002.12157v2 (2020)

