Title: The black hole spectrum in supergravity

Speakers: G. Joaquin Turiaci

Series: Quantum Fields and Strings

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Abstract: The talk will focus on the spectrum of near-extremal black holes in gravity and near-BPS black holes in supergravity. For concreteness, we will study cases in asymptotically four-dimensional flat space and three-dimensional Anti-de Sitter. This will be done by analyzing quantum effects near the horizon captured by an emergent Jackiw-Teitelboim mode at low temperatures. This will allow us to systematically study questions such as the extremal degeneracy and the size of the gap in the black hole spectrum, which can be compared to some string theory constructions.

The Black Hole Spectrum in (Super)gravity

Gustavo Joaquin Turiaci
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Perimeter Institute
Jan 2021

Plan of the talk

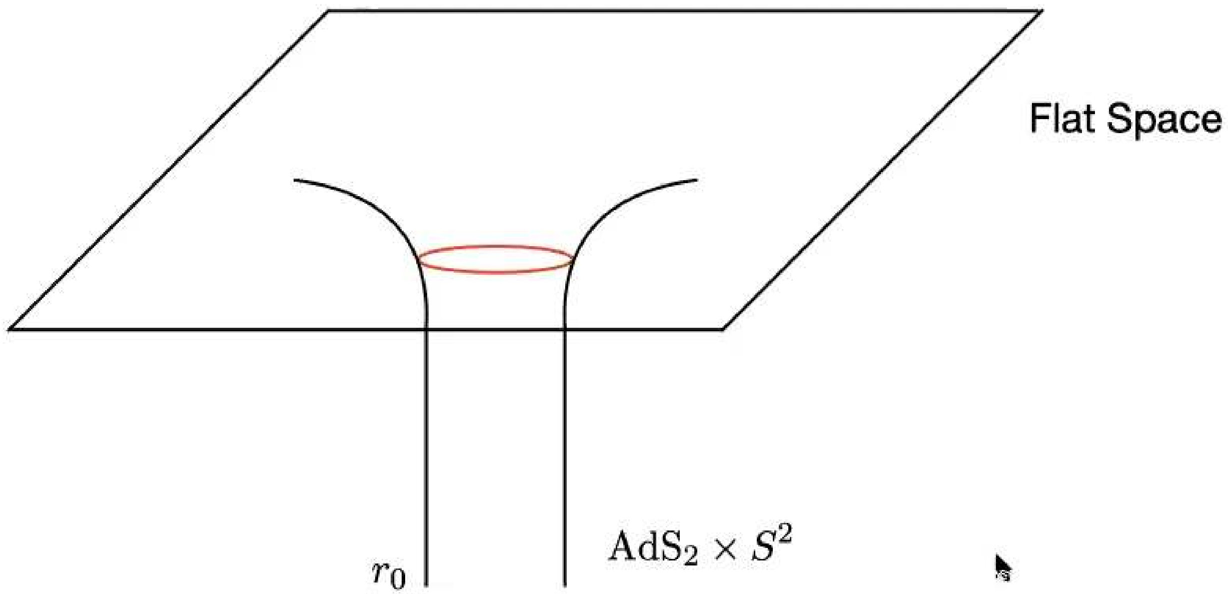
- Motivations: Spectrum of near-extremal black holes

- BTZ black hole in AdS_3
- 4D charged black hole

→ Einstein Gravity

- Same examples in Supergravity

Near-Extremal Black Hole



- Large Charge and Large Extremal Bekenstein-Hawking Entropy/ Large Area
- Low Temperature/ Near-Extremal Limit $E = M \to Q$

Black Hole Thermo: Issues

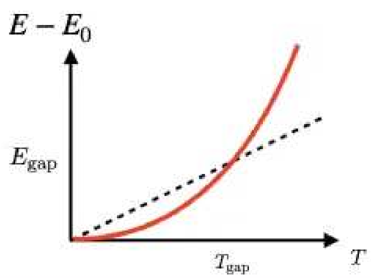
- Bekenstein-Hawking Entropy vs (low) T

$$S = \underbrace{\pi Q^2}_{S_0} + 4\pi^2 Q^3 T$$

- Energy vs (low) T

$$E = \underbrace{Q}_{E_0} + 2\pi^2 Q^3 T^2$$

- Gap-Scale: Thermodynamical description was thought to break down



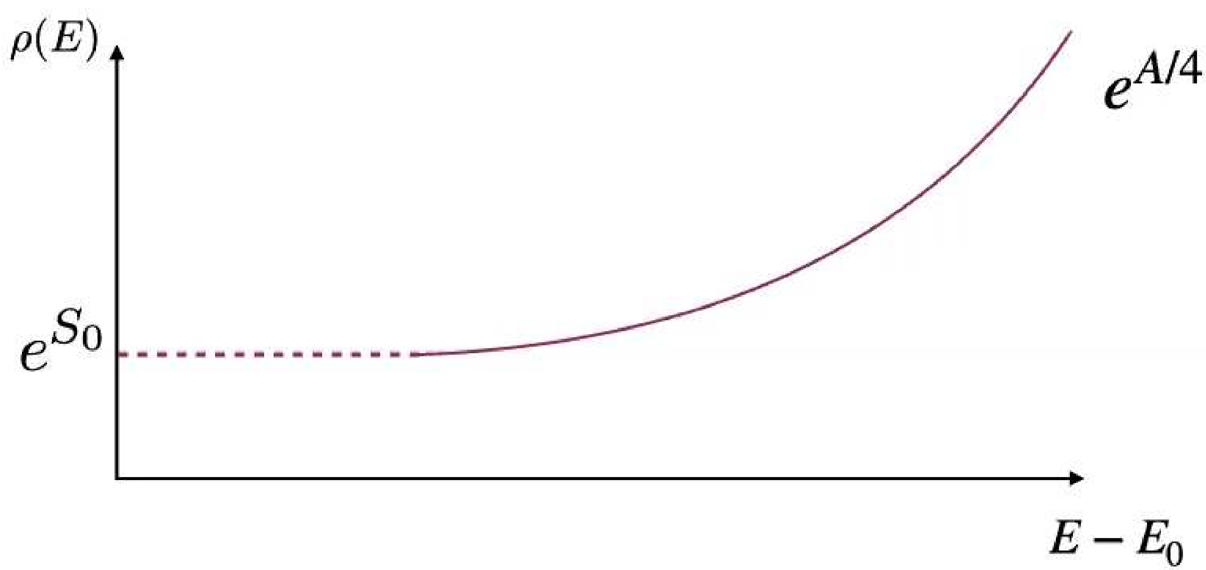
$$E_{\text{gap}} \sim T_{\text{gap}} \Rightarrow E_{\text{gap}} \sim \frac{1}{Q^3}$$

[Preskill, Schwarz, Shapere, Trivedi, Wilczek 91]

Spectrum of near-extremal black holes??

- Statistical Description breaks down at low energies?

[Preskill et al 91]



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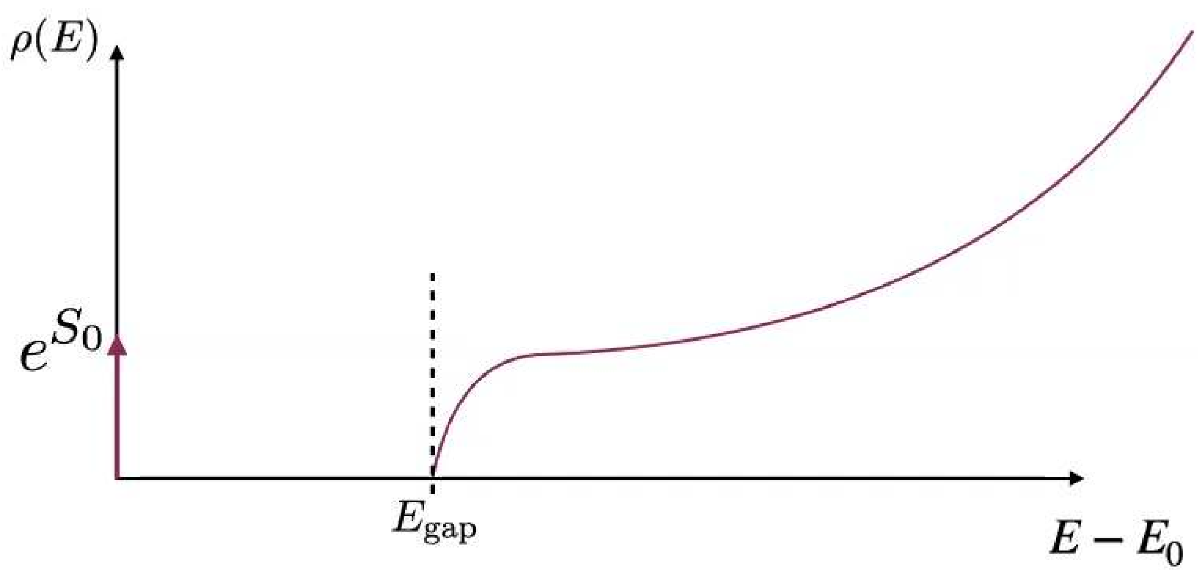
Spectrum of near-extremal black holes??

- Proposal from microscopic models of BHs: gap

(No precise derivation of shape)

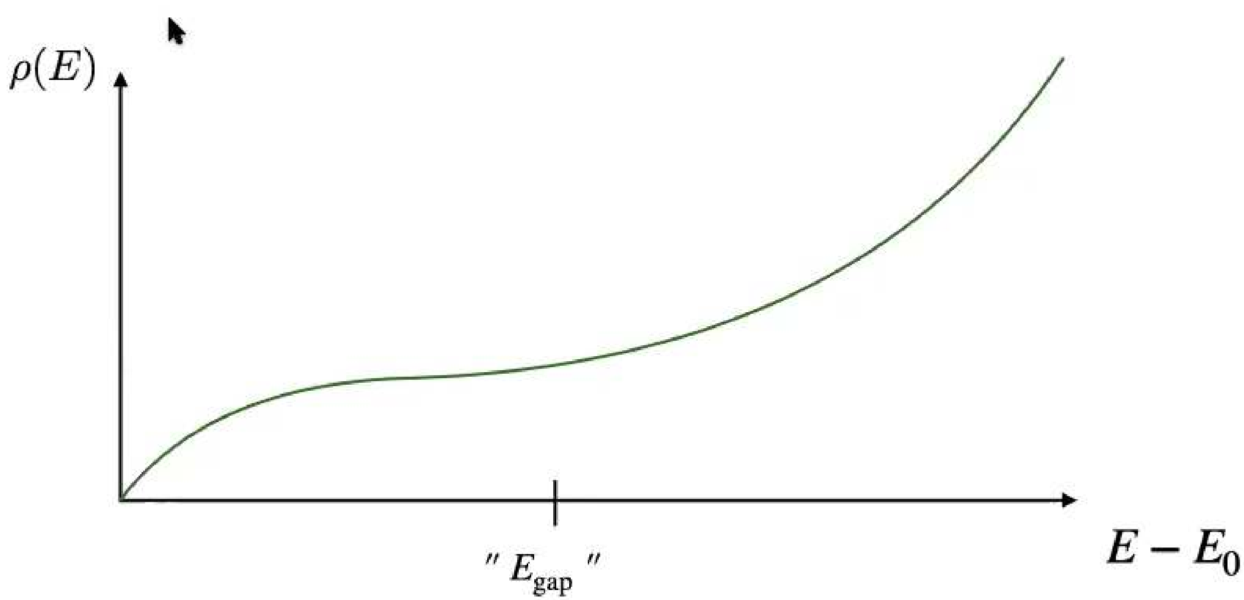
[Maldacena, Susskind 96]
[Maldacena, Strominger 97]



Answer 1: Einstein Gravity

[Iliesiu GJT 20]
[Ghosh Maxfield GJT 19]

- No Gap, quantum effects near the horizon become large and modify answer



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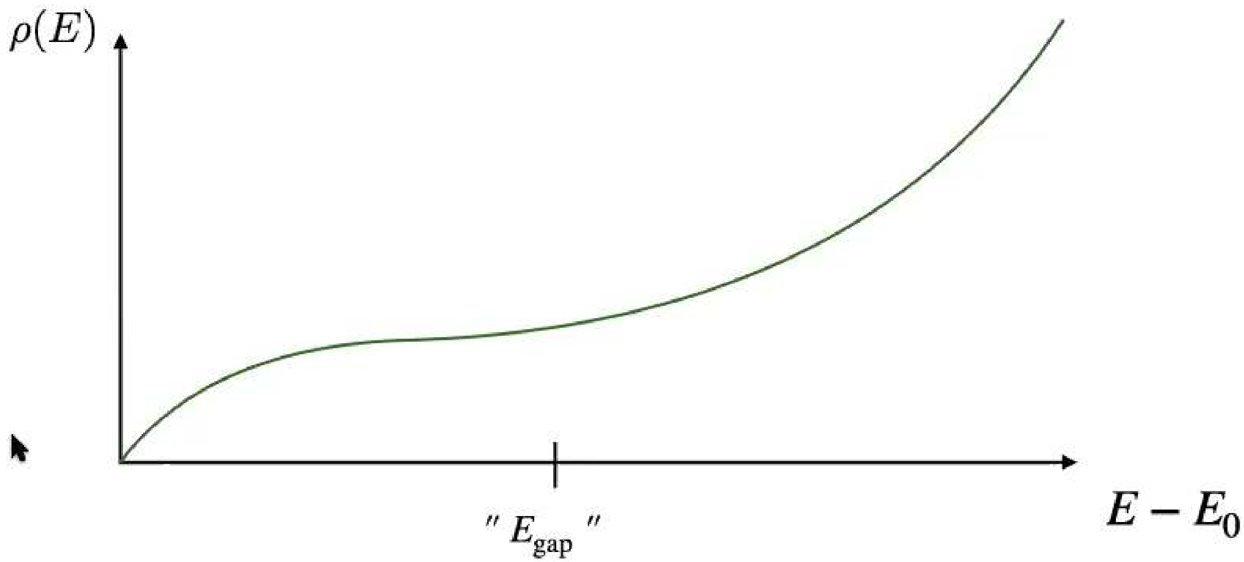
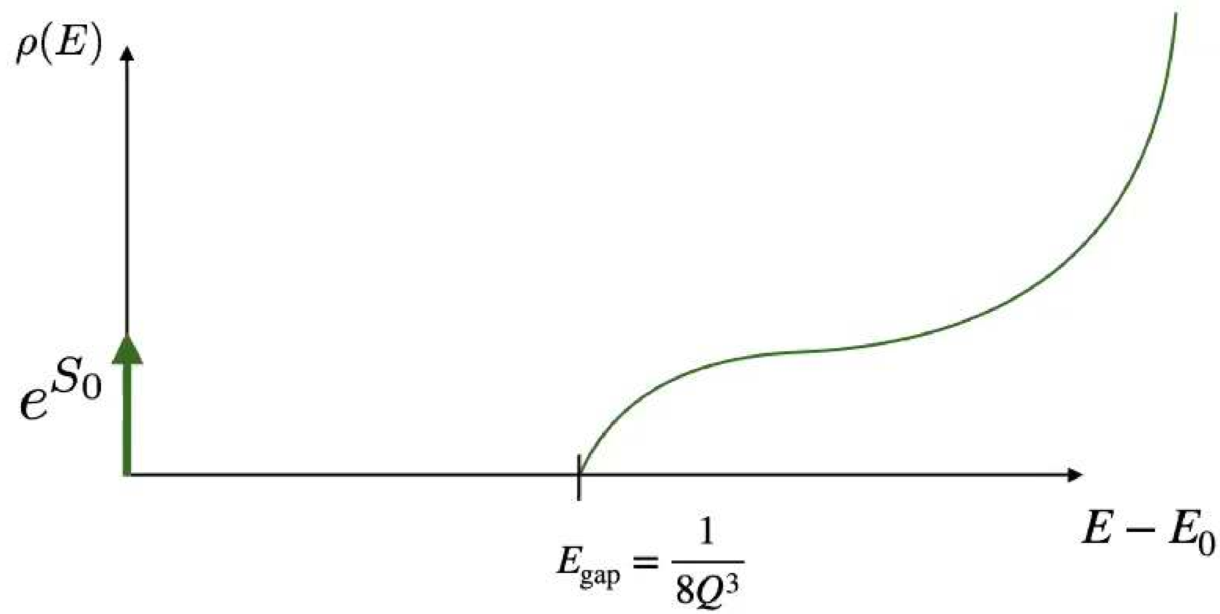


Answer 2: Supergravity

[Heydeman, Iliesiu, Zhao, GJT 20]

- Emergent, but **broken**, superconformal symmetry $PSU(1,1|2)$. Precise derivation of the shape



3D gravity and BTZ

- Consider 3D Einstein gravity (possibly coupled to matter)

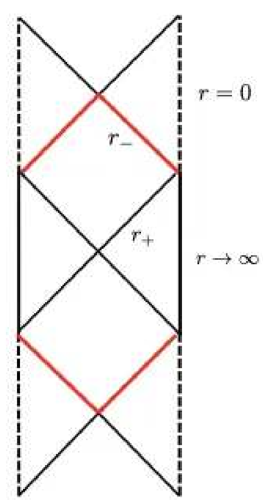
$$I_{EH} = -\frac{1}{16\pi G_N}\int d^3x\sqrt{g_3}\left(R_3 + \frac{2}{\ell_3^2}\right)$$

- We will be interested in the rotating BTZ black hole solution near extremality with mass E and angular momentum J

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\varphi - \frac{r_- r_+}{r^2}dt\right)^2$$

$$f(r) = \frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^2}$$

- Extremality bound: $E \geq |J|$



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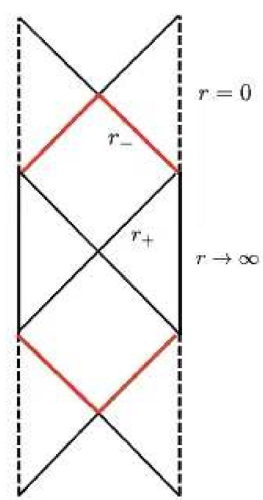


- Extremality bound: $E \geq |J|$

The near extremal limit

- We will focus on states with angular momentum, at very low T. This implies that

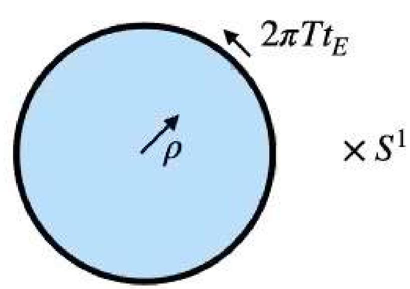
$$T \sim \frac{r_+ - r_-}{\pi \ell_3} \to 0 \qquad E - |J| \to 0 \qquad r_- \sim r_+ \sim r_0$$

- The metric in the throat is approximately $AdS_2 \times S^1$ in terms of

$$ds^2 = \left(\frac{\ell_3}{2}\right)^2 \left[-(\rho^2 - 1)(2\pi T dt)^2 + \frac{d\rho^2}{\rho^2 - 1}\right] + r_0^2 \left(d\varphi - dt + \frac{\ell_3/2}{r_0}\rho 2\pi T dt\right)^2$$

AdS_2 with radius $\ell_2 = \ell_3/2$

Background U(1) gauge field



- In terms of

$$r = \frac{r_+ + r_-}{2} + \frac{r_+ - r_-}{2}\rho$$

Semiclassical thermodynamics

- The extremal energy and Bekenstein-Hawking entropy are

$$E_0 = |J|, \quad S_0 = 2\pi\sqrt{\frac{c\,|J|}{6}}$$

- Thermodynamics at low temperatures

$$E = E_0 + 2\pi^2\Phi_r T^2, \quad S = S_0 + 4\pi^2\Phi_r T$$

- This will be the universal behavior, with a model dependent parameter Φ_r, that in this case is

$$\Phi_r = \frac{c}{24}$$

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Dimensional reduction in throat

- In the gravitational path integral only some modes become relevant at low T

Dilaton

$$ds^2 = ds^2_{2D} + \Phi^2(d\varphi + A)^2$$

2D metric in (t, r)

U(1) field

- Dimensional reduction of 3D Einstein action gives the Achucarro-Ortiz action

$$I = -\frac{1}{8G_N}\int \sqrt{g_2}\,\Phi\left(R_2 - \frac{1}{4}\Phi^2 F^2 + \frac{2}{\ell_3^2}\right)$$

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Search

NHR: Two simplifications

- After integrating out the gauge field (with **fixed charge** boundary condition) we get a 2D dilaton-gravity theory

$$I_J = -\frac{1}{8G_N}\int\sqrt{g}(\Phi R - U_J(\Phi)), \qquad U_J(\Phi) = \frac{1}{2}\frac{(8G_N J)^2}{\Phi^3} - \frac{2}{\ell_3^2}\Phi$$

- In the near horizon region $\Phi \approx \Phi_0$ and the EOM is $U(\Phi_0) = 0$, determines extremal size. Expand for small fluctuations $\Phi = \Phi_0 + 4G_N\phi$

$$\boxed{I_J = -S_0\chi - \frac{1}{2}\int\sqrt{g}\phi\left(R + \frac{2}{\ell_2^2}\right)}$$

$$S_0 = \frac{2\pi\Phi_0}{4G_N} = 2\pi\sqrt{\frac{c\,|J|}{6}}$$

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Near-extremal limit

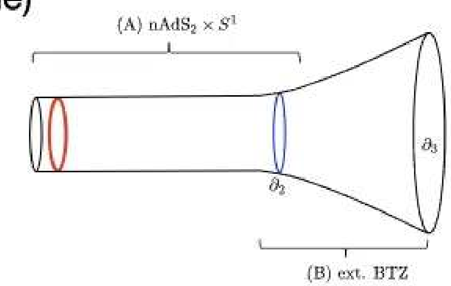
- Final answer for gravitational path integral near extremality

$$Z_{BTZ}[\beta, J] = e^{-\beta E_0} e^{S_0} \int \mathcal{D}g \mathcal{D}\phi \; e^{-I_{JT}[g,\phi]}$$

- Boundary conditions at the throat (from gluing with outside)

$$\phi|_\partial = \frac{1}{\varepsilon}\frac{c}{24}, \qquad L|_\partial = \frac{\ell_2 \beta}{\varepsilon}, \qquad \varepsilon \to 0$$

Φ_r : Renormalized dilaton



- Contributions from KK modes are easy: interactions are suppressed and they give temperature independent corrections (shift S_0 and E_0)

[Sen]

$$I \to \Phi_0 I_{\text{2D matter, KK modes}}$$

Jackiw-Teitelboim (JT) gravity

- Integrating out the (linear) dilaton first, the theory reduces to a boundary mode on rigid AdS2 [Almheiri, Polchinski] [Jensen] [Maldacena,Stanford,Yang] [Englesoy, Mertens Verlinde]...

$$I_{JT} = \Phi_r \int_0^\beta d\tau \, \{f, \tau\} \qquad f(\tau+\beta) = f(\tau)$$

  - This mode controls finite-temperature effects, breaks the emergent $SL(2,\mathbb{R})$ symmetry

- This theory can be quantized exactly to obtain the disk partition function [Altland, Bagrets, Kamenev] [Stanford Witten] [Mertens GJT Verlinde]

$$Z_{\rm JT}(\beta, J) \sim \frac{\Phi_r^{1/2}}{\sqrt{2\pi}\beta^{3/2}} e^{\frac{2\pi^2\Phi_r}{\beta}}$$

A check: Pure 3D gravity

- The exact result in 3D pure gravity including perturbative quantum effects was computed by Maloney and Witten:

$$Z_{\rm BTZ} = \chi_1(-1/\tau)\chi_1(1/\bar{\tau}) \qquad \chi_1(\tau) = \frac{(1-q)q^{-\frac{c-1}{24}}}{\eta(\tau)},$$

- We can take a near extremal approximation of this formula, gives

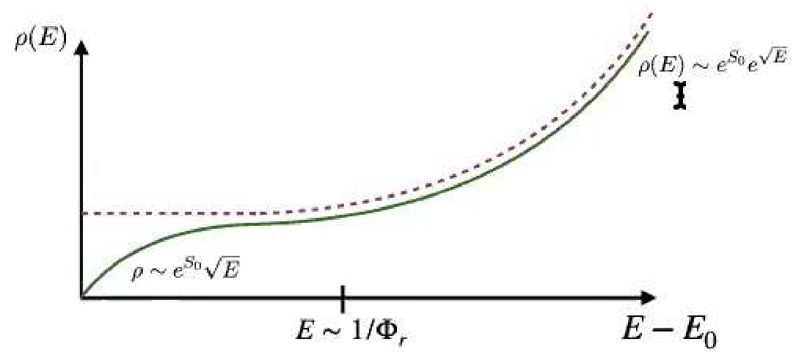
$$\boxed{Z_{\rm BTZ}(\beta, J) \sim \frac{\Phi_r^{1/2}}{\sqrt{2\pi}\beta^{3/2}} e^{S_0 - \beta E_0 + \frac{2\pi^2 \Phi_r}{\beta}}} \qquad \Phi_r = \frac{c}{24}$$

- This is precisely the answer we expected from the reduction to JT gravity!

Near-Extremal Spectrum

- Since the reduction to JT is valid beyond pure gravity, we have a universal spectrum near extremality

$$\rho_J(E) = \frac{e^{S_0}}{2\pi^2} \sinh\left(2\pi\sqrt{2\Phi_r(E-E_0)}\right)$$

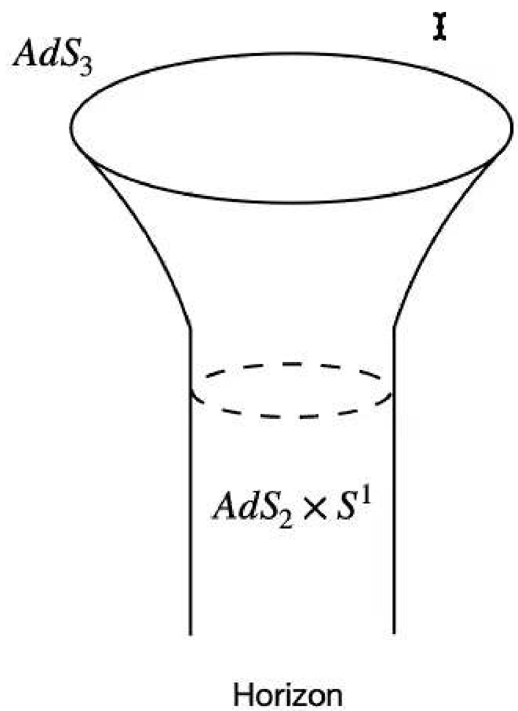


- According to [Preskill Schwarz Shapere Trivedi Wilczek 91] the statistical mechanical description of NEBH was supposed to break down at $E \sim 1/\Phi_r$, and it was believed to be a gap in their spectrum

- Instead, there is no gap. A gravitational mode becomes strongly coupled and the density of states goes smoothly to zero

Universal sector in 2D CFT

- Universal gravitational sector in AdS_3 when looking at near extremal states. Is there a universal sector of 2D CFTs?



Universal sector in 2D CFT

- Universal gravitational sector in AdS_3 when looking at near extremal states. Is there a universal sector of 2D CFTs?

✓ Yes! Only assumptions: twist gap, large c and modular invariance

[Ghosh, Maxfield, GJT 19]

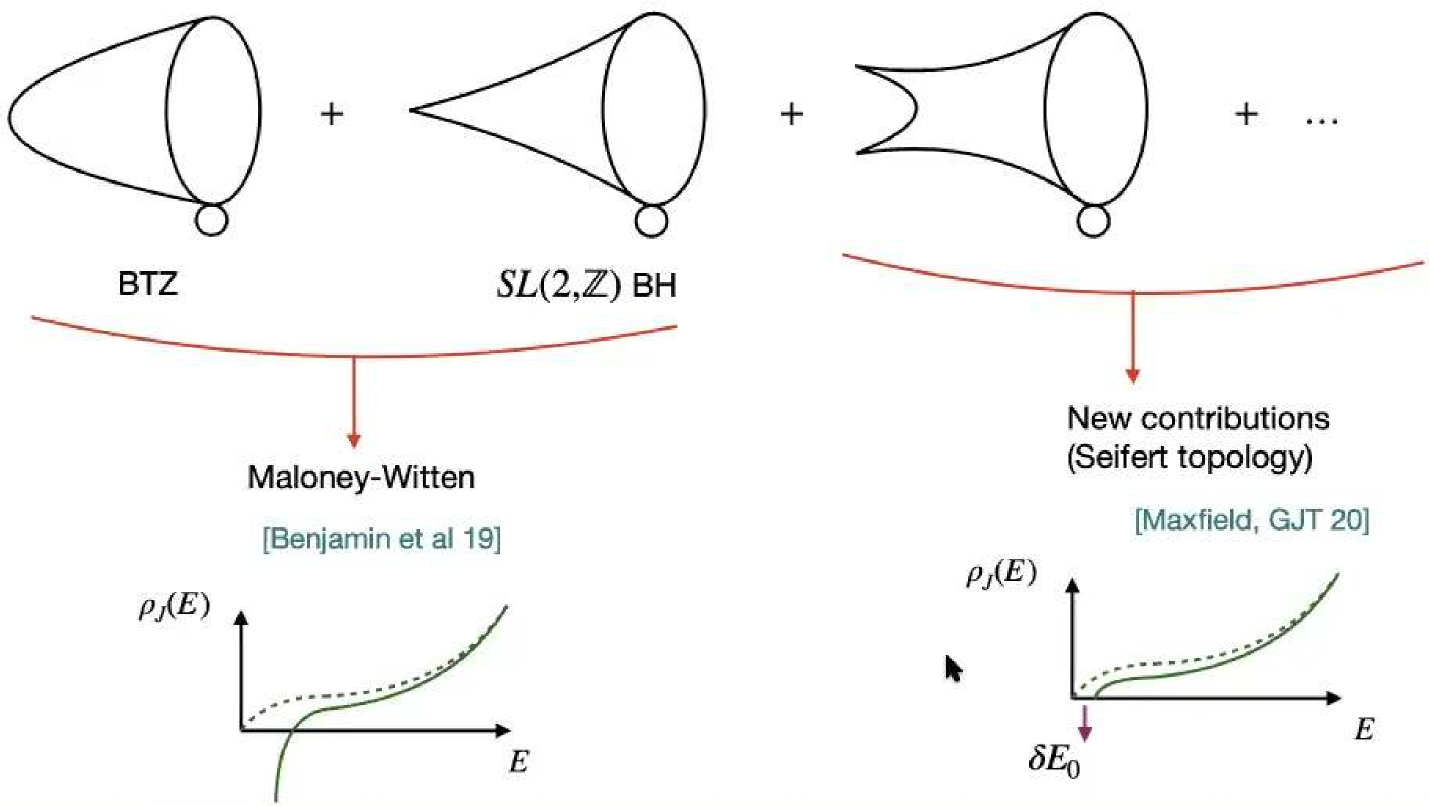
- 2D CFT description of the states:
  - If we fix angular velocity: $\beta_L \sim 2\beta \qquad \beta_R \sim 2\pi\sqrt{\frac{c}{24J}}$

$$Z \approx \chi_1(-1/\tau)\chi_1(-1/\bar{\tau}) + \ldots$$

  - Similar phenomena with correlators

Pure 3D gravity

- Including only BTZ and the $SL(2,\mathbb{Z})$ black hole gives non-unitary partition function. Other interesting configurations in near-extremal limit are:



Einstein Gravity

Case 2: 4D Charged BH

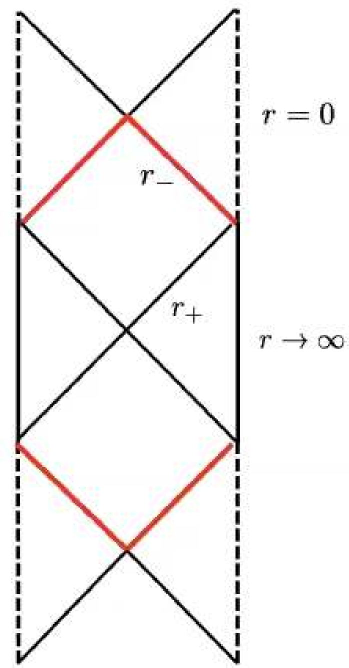
Charged Black Hole in AdS_4

- 4D action: $I = -\int (R + \frac{6}{L^2}) - \frac{1}{e^2}\int F^2 + I_{\text{Bdy}}$

- AdS RNBH

(metric) $ds^2 = f d\tau^2 + \frac{dr^2}{f} + r^2 d\Omega^2, \quad f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{L^2}$

(gauge field) $A = i\mu\left(1 - \frac{r_h}{r}\right) d\tau \qquad \mu = \frac{e}{4\pi}\frac{Q}{r_h}$



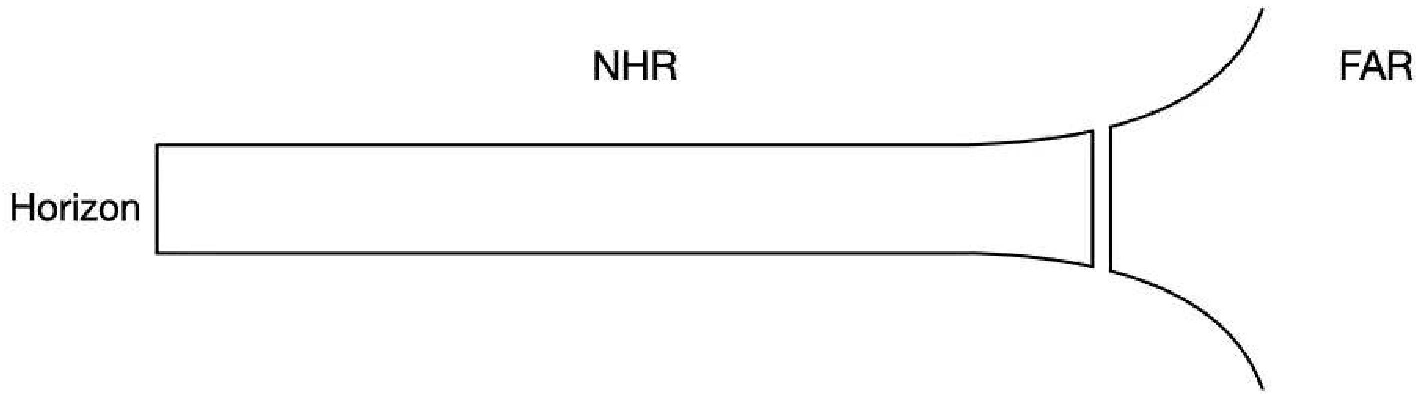
- Large BH in AdS

$Q^2 \sim \frac{r_0^4}{L^2}, \quad E_0 \sim \frac{r_0^3}{L^2} \sim Q^{3/2}, \quad S_0 \sim Q \gg 1$

- "Gap" scale

$\Phi_r = r_0 L$

Charged Black Hole in AdS_4



- Throat (NHR)

$$\mathrm{AdS}_2 \times S^2$$

$$L_2 \sim L_1 \qquad R_{S^2} = r_0$$

- Matching surface:

$$r_c - r_0 \sim \frac{1}{\varepsilon} \gg L_2$$

Massless Sector at low T

- Reduction to 2D: we only want to keep massless modes [Michelson, Spradlin 99]

(metric) $$ds^2_{4D} = \frac{r_0}{\Phi^{1/2}} ds^2_{2D} + \Phi \, h^{(S^2)}_{mn} (dy^m + \mathbf{B}^a \xi^m_a)(dy^n + \mathbf{B}^b \xi^n_b)$$

SO(3) gauge field → $\mathbf{B}^a$; Killing vectors of sphere → ξ^m_a

(gauge field) $$A_\mu(x,y) \;=\; a_\mu(x)$$ ← U(1) gauge field

- Action:

$$I_{2D} = \int (\Phi R - 2U(\Phi)) - \frac{1}{r_0}\int \Phi^{5/2} H^2 - \frac{1}{e^2 r_0}\int \Phi^{3/2} f^2$$

$$U(\chi) = r_0 \left[-\frac{3}{L^2}\Phi^{1/2} - \frac{1}{\Phi^{1/2}} \right]$$

Effective 2D theory

- 2D Yang Mills: easy to integrate out fields (all Dirichlet)

$$Z = \sum_{j,Q} (2j+1)^2 e^{\beta\mu \frac{Q}{\epsilon}} \int \mathcal{D}\Phi \mathcal{D}g \; e^{\int (\Phi R - 2U_{Q,j}(\Phi))}$$

- Charge dependent dilaton potential: $U_{Q,j}(\chi) = r_0 \left[\frac{Q^2}{\Phi^{3/2}} + \frac{3j(j+1)}{\Phi^{5/2}} - \frac{3}{L^2}\Phi^{1/2} - \frac{1}{\Phi^{1/2}} \right]$

- NHR: Linear dilaton approximation $\Phi = \Phi_0 + \phi$

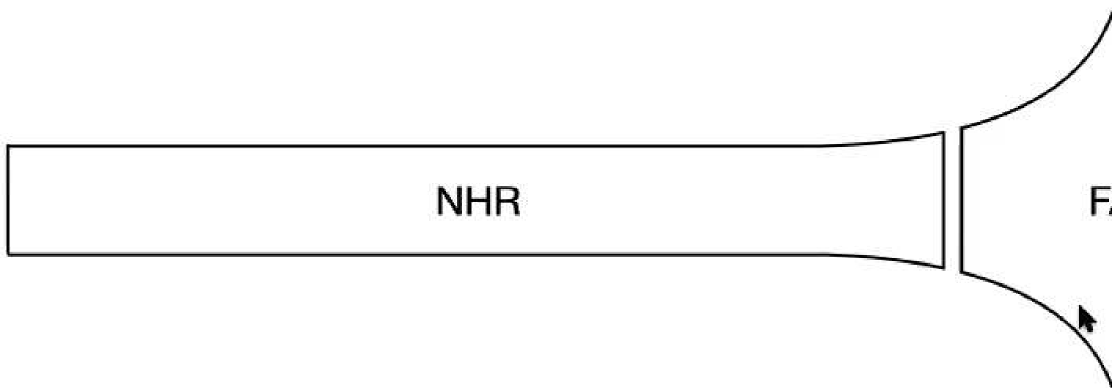
$$U_{Q,j}(\Phi_0 = r_0) = 0, \quad \Rightarrow \quad U(\Phi_0 + \phi) \approx -\frac{1}{L_2^2}\phi$$

$$Z_{\rm JT} = \int \mathcal{D}\phi \mathcal{D}g e^{\int_{\rm NHR} \phi (R + \frac{2}{L_2^2}) + I_{\rm Bdy}}$$

$$\phi|_{\rm Bdy} = \frac{\Phi_r}{\varepsilon}$$

$$L|_{\rm Bdy} = \beta \frac{L_2}{\varepsilon}$$

Partition Function



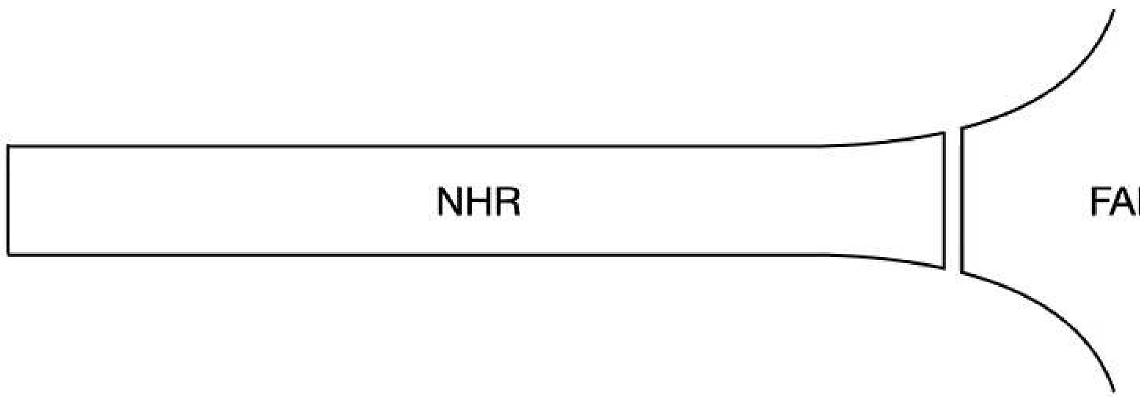
NHR: Constant Dilaton

$$Z = \sum_{j,Q} (2j+1)^2 \, e^{\beta\mu\frac{Q}{e} - \beta E_0} \, e^{S_0} \, Z_{\text{JT}}[\Phi_r, \beta]$$

FAR: Classical fluctuations

NHR: Linear Dilaton / JT

Partition Function: Corrections



- Uncharged matter and KK modes

$$\delta \log Z = \beta \delta E_0 + \delta S_0 + \mathcal{O}(\varepsilon)$$

- Interactions: suppressed in r_0 and ε (Log corrections $\delta S_0 \sim \log L_2$)

- Non-linear dilaton corrections: supp. in r_0 and T

- Non-perturbative: exponentially suppressed in S_0

Statistical Mechanics

- The 2D gauge modes are frozen when we fix charges. In a sector of fixed Q and J the spectrum is again:

$$\rho_{Q,J}(E) = \frac{e^{S_0}}{2\pi^2} \sinh\left(2\pi\sqrt{2\Phi_r(E-E_0)}\right)$$

- Fixed chemical potential and j=0

$$Z = e^{S_0 - \beta E_0} Z_{\rm JT}(Q_0) \sum_{Q=Q_0+q} e^{2\pi \mathcal{E} q - \beta \frac{q^2}{2K}}$$

$$K = \frac{\partial Q}{\partial \mu}\Big|_{T=0}$$ (compressibility)

$$\mathcal{E} = \frac{\partial S_0}{\partial Q}\Big|_{T=0}$$ (electric field)

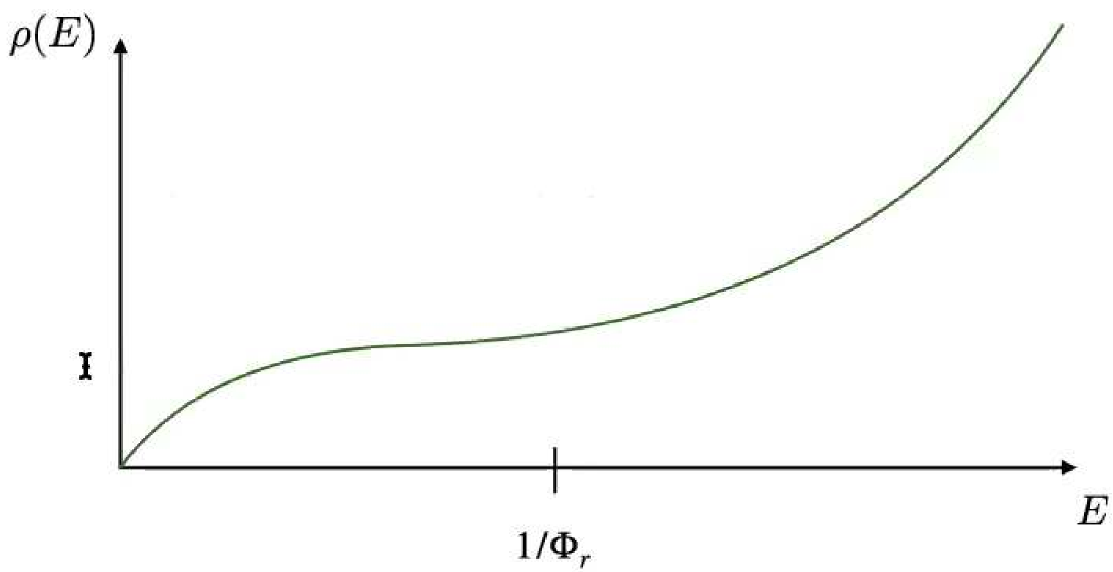
[Sen]
[Sachdev 19]

- Fixed charge and boundary metric (zero angular velocity)

$$Z = e^{S_0 - \beta E_0} Z_{\rm JT}(Q_0) \sum_j (2j+1)^2 e^{-\beta \frac{j(j+1)}{r_0^3}}$$

Black Hole Spectrum for Fixed Charges

- No Gap, quantum effects become large. Answer for **non-SUSY** theories



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4D $\mathcal{N} = 2$ Supergravity ($\Lambda = 0$)

- Fields: Metric G_{MN}, doublet of gravitinos Ψ^I_M and U(1) gauge field A_M

- Lagrangian: $$E^{-1}\mathcal{L} = \kappa^{-2}\Big(\frac{1}{2}R - \overline{\Psi}_{IM}\Gamma^{MNP}D_N\Psi^I_P - \frac{1}{4}F_{MN}F^{MN} + \frac{\varepsilon^{IJ}}{2\sqrt{2}}\overline{\Psi}^M_I(F_{MN} + i\star F_{MN}\Gamma_5)\Psi^N_J + 4\text{ gravitino}\Big),$$
  $8\pi G_N = \kappa^2$

- Local SUSY transformations: $$\delta_\epsilon E^A_M = \frac{1}{2}\bar{\epsilon}^I\Gamma^A\Psi_{MI} + \text{h.c.},$$ $$\delta_\epsilon A_M = \frac{1}{\sqrt{2}}\varepsilon^{IJ}\bar{\epsilon}_I\Psi_{MJ} + \text{h.c.},$$ $$\delta_\epsilon\Psi^I_M = (\partial_M + \frac{1}{4}\omega^{AB}_M\Gamma_{AB})\epsilon^I - \frac{1}{4\sqrt{2}}\Gamma^{AB}F_{AB}\Gamma_M\varepsilon^{IJ}\epsilon_J.$$

- In this theory, the Reissner-Nordstrom black hole is still a solution

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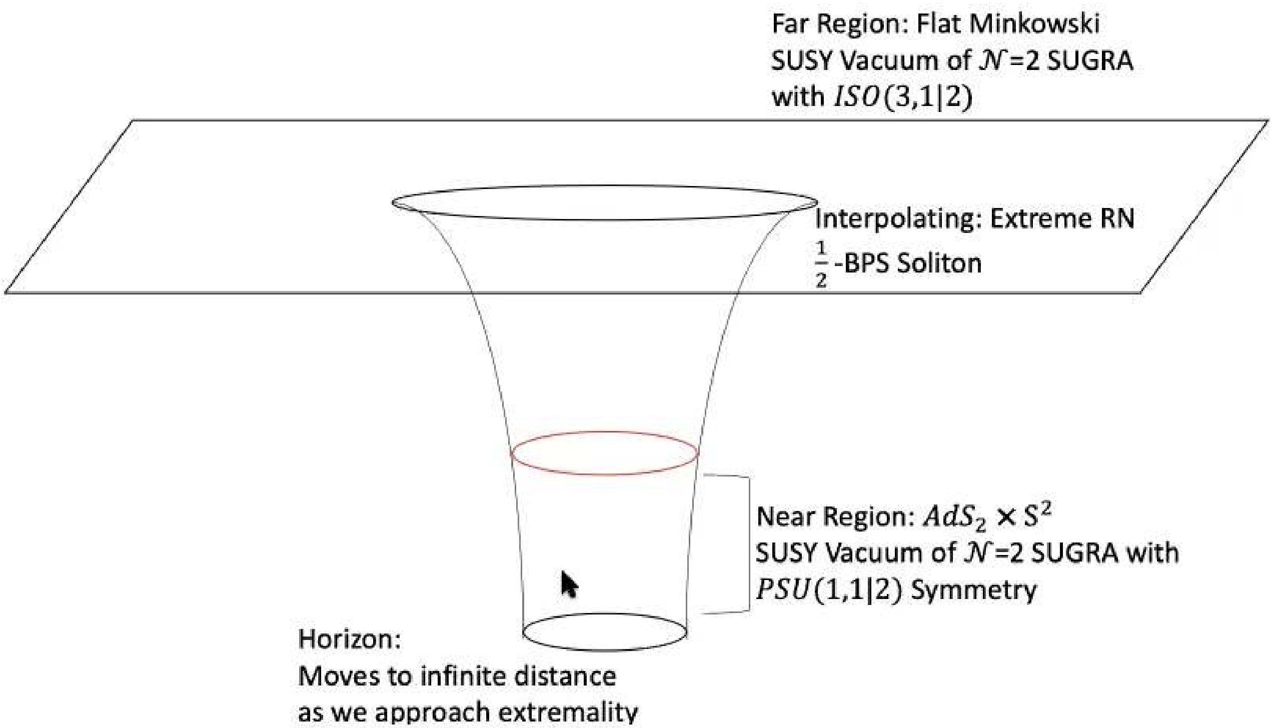
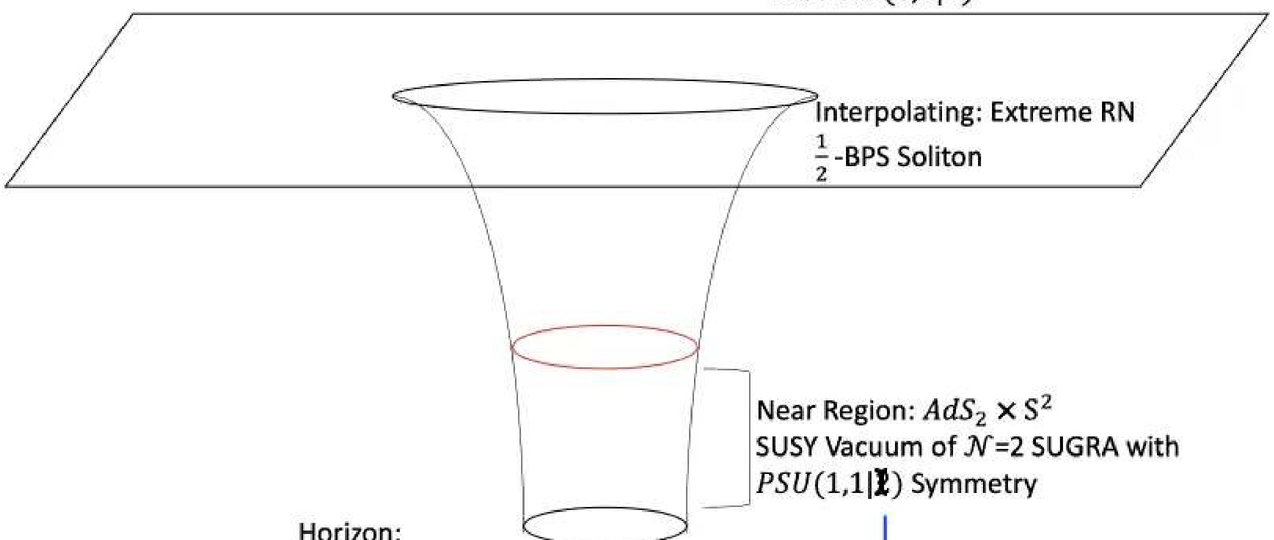
4D $\mathcal{N} = 2$ Supergravity



$$PSU(1,1\,|\,2) \supset SL(2,\mathbb{R}) \times SU(2)$$

4D $\mathcal{N} = 2$ Supergravity



This symmetry comes from the S^2 (NOT the outer $SU(2)$ which is usually broken)

Dimensional Reduction

- The emergent, broken, symmetry in the throat is now PSU(1,1|2), new fermionic modes from gravitino become relevant at low T [Michelson, Spradlin 99]

- The reduction of the higher dimensional supergravity theory in the throat is $\mathcal{N}=4$ supersymmetric JT gravity. This can be rewritten as a $\mathfrak{psu}(1,1\,|\,2)$ BF theory:

$$I_{BF} = -i\int \mathrm{Str}\,\phi F\,, \qquad F = dA - A\wedge A\,,$$

- Where A is a $\mathfrak{psu}(1,1\,|\,2)$ gauge field and ϕ is a zero-form in the adjoint of $\mathfrak{psu}(1,1\,|\,2)$

- Boundary conditions to glue to far-away region: $\delta(2i\Phi_r A_\tau(\tau)+\phi(\tau))|_{\partial\mathcal{M}}=0$

$$I_{BF,\,\mathrm{bdy.}} = \frac{i}{2}\int_{\partial\mathcal{M}} \mathrm{Str}\,\phi A = \Phi_r \int_{\partial\mathcal{M}} d\tau\,\mathrm{Str}\,A_\tau^2$$

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- Where A is a $\mathfrak{psu}(1,1\,|\,2)$ gauge field and ϕ is a zero-form in the adjoint of $\mathfrak{psu}(1,1\,|\,2)$

- Boundary conditions to glue to far-away region: $\delta(2i\Phi_r A_\tau(\tau)+\phi(\tau))|_{\partial\mathcal{M}}=0$

$$I_{BF,\,\mathrm{bdy.}} = \frac{i}{2}\int_{\partial\mathcal{M}} \mathrm{Str}\,\phi A = \Phi_r\int_{\partial\mathcal{M}} d\tau\,\mathrm{Str}\,A_\tau^2$$

↓ $\mathcal{N}=4$ Schwarzian Theory

$\mathcal{N} = 4$ Schwarzian Theory

- Parametrization of the $\mathcal{N} = 4$ superline $Z = (\tau, \theta^a, \bar{\theta}_b)$. The supercovariant derivatives are

$$D_a = \frac{\partial}{\partial \theta^a} + \frac{1}{2}\bar{\theta}_a \partial_\tau, \qquad \bar{D}^a = \frac{\partial}{\partial \bar{\theta}_a} + \frac{1}{2}\theta^a \partial_\tau.$$

- Super-reparametrization: $\tau \to \tau'(\tau, \theta, \bar{\theta}), \quad \theta^a \to \theta'^a(\tau, \theta, \bar{\theta}), \quad \bar{\theta}_b \to \bar{\theta}'_b(\tau, \theta, \bar{\theta}),$

Satisfy the constrains [Matsuda Uematsu] [Schoutens]

$$D_a \bar{\theta}'_b = 0 \quad , \quad \bar{D}^a \theta'^b = 0,$$
$$D_a \tau' - \frac{1}{2}(D_a \theta'^b)\bar{\theta}'_b = 0 \quad , \quad \bar{D}^a \tau' - \frac{1}{2}(\bar{D}^a \bar{\theta}'_b)\theta'^b = 0$$

$\mathcal{N} = 4$ Schwarzian Theory

- We can parametrize the super-reparametrizations $\text{Diff}\left(S^{1|4}\right)$ by the following functions:

$$f(\tau) \in \text{Diff}(S^1), \quad g(\tau) \in SU(2), \quad \eta^a(\tau), \quad \bar{\eta}_a(\tau).$$

- A finite reparam with all parameters turned on looks very complicated. A special subset are $PSU(1,1\,|\,2)$ transformations. For example the bosonic subgroup $SL(2,\mathbb{R}) \times SU(2)$ is

$$\begin{aligned}
\tau &\to \frac{a\tau + b}{c\tau + d} - \frac{c}{4(c\tau + d)^3}(\bar{\theta}\theta)^2, \\
\theta^a &\to [e^{i\vec{t}\cdot\vec{\sigma}}]^a_b \theta^b \frac{1}{c(\tau - \frac{1}{2}\bar{\theta}\theta) + d}, \\
\bar{\theta}_a &\to \bar{\theta}_b [e^{i\vec{t}\cdot\vec{\sigma}}]^b_a \frac{1}{c(\tau + \frac{1}{2}\bar{\theta}\theta) + d},
\end{aligned}$$

$\mathcal{N} = 4$ Schwarzian Theory

- The N=4 Schwarzian derivative was defined by Matsuda and Uematsu

$$S^i(Z;Z') = -\frac{1}{6} D\sigma^i \bar{D} \log\left(\frac{1}{2}(D_a\theta'^b)(\bar{D}^a\bar{\theta}'_b)\right).$$

- The Schwarzian action corresponds to one component of this field

$$I_{\mathcal{N}=4} = -\Phi_r \int d\tau\, S_b[f(\tau), g(\tau), \eta(\tau)]$$

- More Explicitly:

$$I_{\mathcal{N}=4} = -\Phi_r \int_0^\beta d\tau \left[\mathrm{Sch}(f,\tau) + \mathrm{Tr}(g^{-1}\partial_\tau g)^2 + (\text{fermions})\right]$$

Bosonic Schwarzian

Particle moving on $SU(2)$

Summary of Steps

4D $\mathcal{N} = 2$ supergravity

↓ Fixed U(1) charge, Look at throat

2D $\mathcal{N} = 4$ Super-JT ⟷ $PSU(1,1|2)$ BF theory

↓ Integrate out dilaton

1D Boundary mode ⟷ $\mathcal{N} = 4$ Super-Schwarzian

↓

Spectrum as function of energy and spin

$\mathcal{N} = 4$ Schwarzian Theory

- The spectrum of this two-dimensional black hole is extracted from the partition function:

$$Z(\beta, \alpha) = \int \frac{\mathcal{D}f \mathcal{D}g \mathcal{D}\eta \mathcal{D}\overline{\eta}}{PSU(1,1|2)} \exp\left(\Phi_r \int d\tau S_b[f, g, \eta, \overline{\eta}] \right) ,$$

- Boundary conditions:

$$f(\tau + \beta) = f(\tau), \quad g(\tau + \beta) = e^{2\pi i \alpha \sigma^3} g(\tau), \quad \eta(\tau + \beta) = -e^{2\pi i \alpha \sigma^3} \eta(\tau),$$

Inverse Temperature

$SU(2)$ chemical potential (Angular Velocity)

$$2\pi\alpha = i\beta\Omega$$

$\mathcal{N} = 4$ Schwarzian Theory

- The partition function can be computed exactly using localization [Stanford Witten] or canonical methods [Mertens GJT Verlinde]. The answer is:

$SU(2)$ mode 1-loop

Classical Action

$$Z = e^{S_0} \sum_{n\in\mathbb{Z}} \left(\frac{\Phi_r^{3/2}}{\beta^{3/2}}\right)\left(\frac{\Phi_r^{3/2}}{\beta^{3/2}}\frac{n+\alpha}{\sin(2\pi\alpha)}\right)\left(\frac{\beta^4}{\Phi_r^4}\frac{\cos^2(\pi\alpha)}{(1-4(n+\alpha)^2)^2}\right) e^{\frac{2\pi^2\Phi_r}{\beta}(1-4(n+\alpha)^2)}$$

Schwarzian 1-loop

Fermion 1-loop

$\mathcal{N} = 4$ Spectrum

- The $PSU(1,1\,|\,2)$ symmetry is broken, as much as the conformal symmetry in the bosonic case. There is a global super-translation group which survives with four supercharges. This organizes spectrum is supermultiplets

$$
\begin{aligned}
Z(\beta,\alpha) \;&=\; \sum_J \chi_J(\alpha)\rho_{\rm ext}(J) + \int dE\; e^{-\beta E}\left(\chi_{1/2}(\alpha) + 2\chi_0(\alpha)\right)\rho_{\rm cont}(1/2,E) \\
&\quad + \sum_{J\geq 1}\int dE\; e^{-\beta E}\left(\chi_J(\alpha) + 2\chi_{J-\frac{1}{2}}(\alpha) + \chi_{J-1}(\alpha)\right)\rho_{\rm cont}(J,E),
\end{aligned}
$$

$SU(2)$ characters

$$\chi_J(\alpha) \equiv \sum_{m=-J}^{J} e^{4\pi i\alpha m} = \frac{\sin(2J+1)2\pi\alpha}{\sin 2\pi\alpha}$$

$\mathcal{N} = 4$ Spectrum

- The $PSU(1,1\,|\,2)$ symmetry is broken, as much as the conformal symmetry in the bosonic case. There is a global super-translation group which survives with four supercharges. This organizes spectrum is supermultiplets

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\end{aligned}
$$

Non-zero index: - $E = 0$: Supermultiplet (J)

Zero index: - $E \neq 0$: Supermultiplet $\mathbf{1/2} = (1/2) \oplus 2(0)$

Supermultiplet $\mathbf{J} = (J) \oplus 2(J - 1/2) \oplus (J - 1)$

$\mathcal{N} = 4$ Spectrum

- Density of states at fixed $SU(2)$ charge J, where $E_0(J) = J^2/2\Phi_r$

$$\rho_{\text{ext}}(J) = e^{S_0}\delta_{J,0}.$$
$$\rho_{\text{cont}}(J,E) = \frac{e^{S_0}J}{4\pi^2\Phi_r E^2}\sinh\left(2\pi\sqrt{2\Phi_r(E-E_0(J))}\right)\,\Theta\Big(E-E_0(J)\Big),\ \text{for } J \geq \frac{1}{2},$$

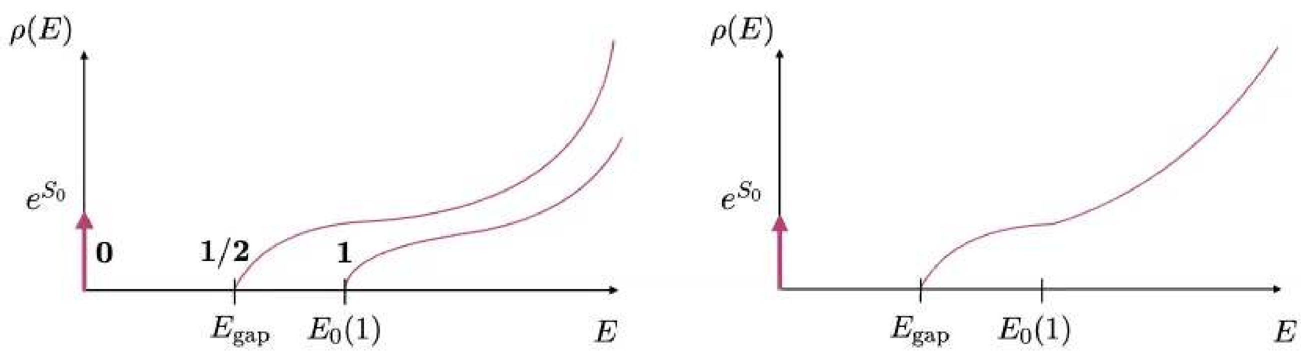
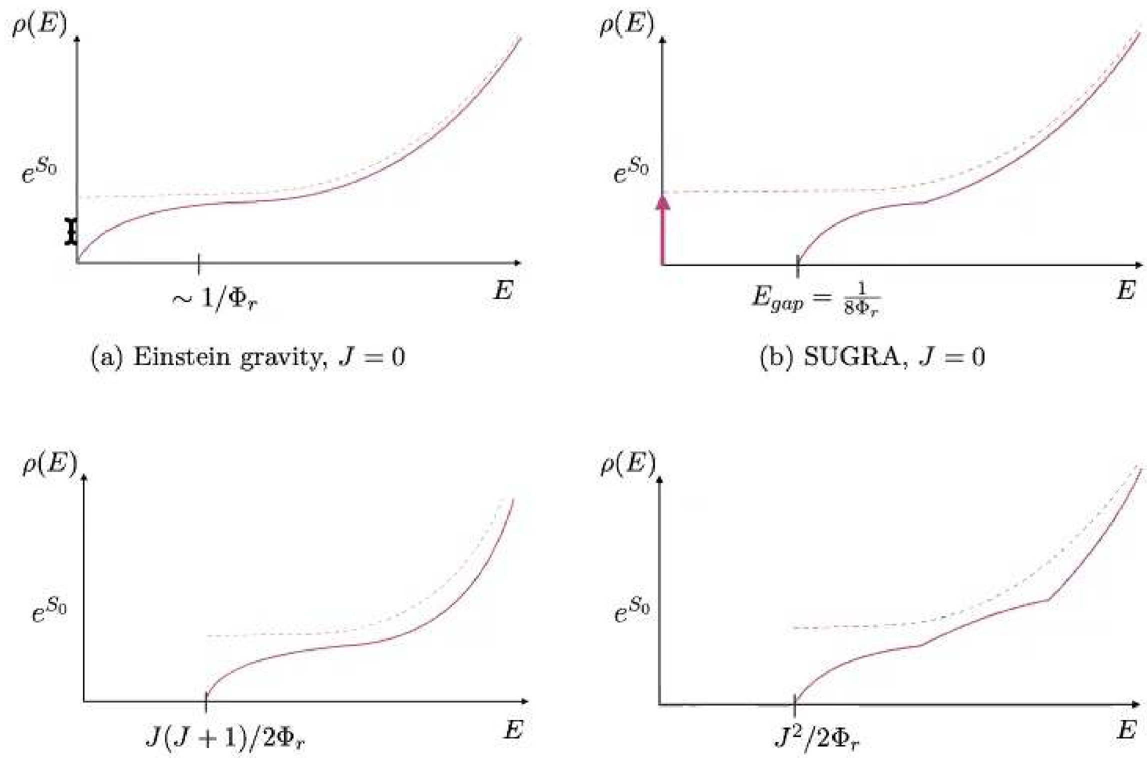


Figure 3: Left: Density of supermultiplets labeled by the highest spin **J**. We show **0**, which is simply a delta function at $E = 0$; **1/2** which is continuous but starts at $E_{\text{gap}} \equiv E_0(1/2)$; and **1** which is also continuous starting at $E_0(1)$. **Right:** Degeneracy for all states with $J = 0$. These come from **0**, the delta function at $E = 0$, **1/2**,starting at E_{gap}, and **1**, starting at $E_0(1)$. All other supermultiplets do not have a $J = 0$ component.



(a) Einstein gravity, $J = 0$

(b) SUGRA, $J = 0$

(c) Einstein gravity, $J \neq 0$

(d) SUGRA, $J \neq 0$

Figure 2: Schematic shape of the black hole spectrum at fixed $SU(2)$ charge as a function of energy above extremality E. We show the semiclassical answer (red dahsed) and the solution including quantum effects (purple). (a) Answer for Einstein gravity. We see there is no gap at scale $E \sim 1/\Phi_r$ and the extremal entropy goes to zero. (b) Answer for supergravity (either $\mathcal{N} = 2$ in 4D or $\mathcal{N} = (4,4)$ in 3D). We find a gap at the scale $E_{gap} = \frac{1}{8\Phi_r}$ and a number e^{S_0} of extremal states, consistent with string theory expectations. (c) Einstein gravity spectrum for $J \neq 0$. (d) Supergravity spectrum for $J \neq 0$, the jumps indicate contributions from different supermultiplets $\mathbf{J}, \mathbf{J+1/2}$ and $\mathbf{J+1}$.

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(4,4) Supergravity in AdS_3

- The gravity sector is described by two Chern-Simons theories with super-group $PSU(1,1|2)_L \otimes PSU(1,1|2)_R$ at level k
  - The bosonic sector is 3D Einstein gravity coupled to $SU(2)_L \otimes SU(2)_R$ Chern-Simons at level k
- In the near-extremal black hole appearing in the D1-D5 system, we have the same spectrum with

$$S_0 = 2\pi\sqrt{Q_1 Q_5 P}, \qquad \Phi_r = \frac{Q_1 Q_5}{4}$$

- This predicts correct index and a gap

$$E_{gap} = \frac{1}{2Q_1 Q_5}.$$

Conclusions

- Gravitational path integral near-extremality:

Extremal Physics + Near-extremal physics

↓ e.g. Sen's quantum entropy function
Localization

↓ JT mode, T dependence

- Universal Schwarzian sector in 2D CFTs. Controls near-extremal shape of d.o.s. and correlators

- Universal large-charge low-temperature limit of higher dimensional CFTs. Bootstrap?

- Another application: Hartle-Hawking wavefunction of a $S^1 \times S^2$ universe

[Maldacena Yang GJT 19]