Title: Explicit and Inexplicit higher form symmetries at quantum criticality

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Abstract: Recent years new concepts of symmetries have been developed such as higher form symmetries, and categorical symmetries. The higher form symmetries can be either explicit in a Hamiltonian, or inexplicit as a dual of an ordinary symmetry. The behavior of higher form symmetries are easy to evaluate in phases with gaps. But at quantum criticalities their behaviors are more nontrivial. We evaluate the behaviors of higher form symmetries (either explicit or inexplicit) at various quantum critical points, and demonstrate that for many quantum critical points a universal logarithmic contribution arises, which is analogous to the quantum entanglement entropy. This logarithmic contribution is related to the universal conductance at the quantum critical points, and in some cases can be computed exactly using duality between CFTs developed in last few years. We also evaluate the behavior of categorical symmetries for more exotic cases with subsystem symmetries.

Higher Form Symmetries at Quantum Criticality

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Higher Form Symmetries at Quantum Criticality

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Content:

1, Review of higher form symmetry and Categorical symmetry;

2, Universal features of 1-form ODO at a class of quantum critical points;

3, ODO with subsystem categorical symmetries;

4, quick comment on the relation between EE and ODO

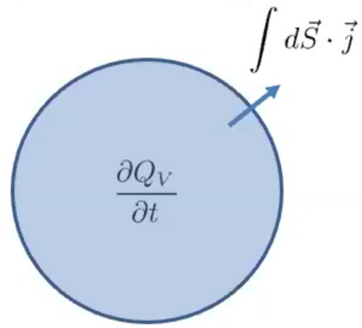
Reference: arXiv:2012.03976, + arXiv:2101:xxxxx

Generalized Symmetries:

1, 1-form symmetries (Nussinov et.al. 2009, Aharony, et.al. 2013 and many many others)

Symmetry is always associated with conservation. An ordinary global symmetry charge in d-dim space, is the total charge in the entire system, for example the total particle number in the space for a system with U(1) symmetry.

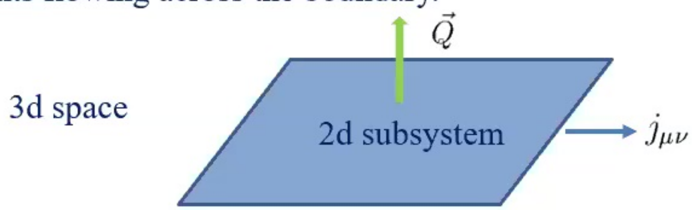
Consider U(1) symmetry charge localized in a d-dim subsystem V with boundary, the symmetry charges within V can only change through symmetry current flowing across boundary of V:



Generalized Symmetries:

1, 1-form symmetries (Nussinov et.al. 2009, Aharony, et.al. 2013 and many many others)

1-form symmetry charge is a vector (flux) penetrating a (d-1)-dim subsystem. For a closed (d-1)-dim subsystem without boundary, the flux is conserved; for a (d-1)-dim subsystem with (d-2)-dim boundary, the flux through the subsystem changes through 2-form currents flowing across the boundary.



Higher Form Symmetries at Quantum Criticality

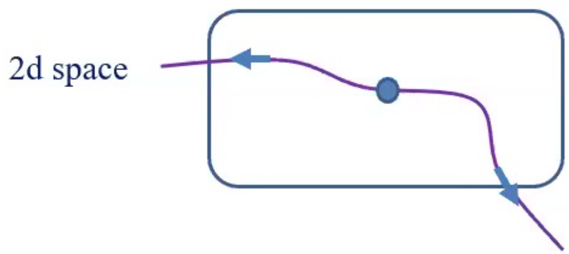
Generalized Symmetries:

1, 1-form symmetries (Nussinov et.al. 2009, Aharony, et.al. 2013 and many many others)

Example: U(1) gauge theory with only even electric charges

$$\vec{\nabla} \cdot \vec{e} = 2n$$

Either zero or even electric fluxes through a closed surface, flux conserved mod 2. Still a $Z_2^{(1)}$ 1-form symmetry:



Generalized Symmetries:

2, Categorical symmetry (Ji, Wen 2019): treat explicit symmetry and dual inexplicit symmetry on equal footing:

Well-known dualities, example 1:

$$H = \sum_j -K\sigma^3_j\sigma^3_{j+1} - h\sigma^1_j \quad \leftrightarrow \quad H_d = \sum_{\tilde{j}} -K\tau^1_{\tilde{j}} - h\tau^3_{\tilde{j}}\tau^3_{\tilde{j}+1}.$$

This inequity is remedied in the following ways:
1, we realize the two phases of the Ising model as the e-boundary and m-boundary of the 2d toric code, then neither phase has ground state degeneracy;
2, we consider a subset of the Hilbert space: only the symmetric subspace of the Z_2, for example we only consider the "cat state" in the Ising ordered phase

$$|\mathrm{Cat}\rangle \sim |\uparrow\uparrow\cdots\uparrow\rangle + |\downarrow\downarrow\cdots\downarrow\rangle$$

Generalized Symmetries:

Well-known dualities, example 2:

$$H = \sum_{<\mathbf{x},\mathbf{x}'>} -K\sigma^3_{\mathbf{x}}\sigma^3_{\mathbf{x}'} - \sum_{\mathbf{x}} h\sigma^1_{\mathbf{x}} \leftrightarrow$$

$$H_d = \sum_{\tilde{\mathbf{x}},\hat{\mu}} -K\tau^1_{\tilde{\mathbf{x}},\hat{\mu}} - \sum_{\tilde{\mathbf{x}}} h\tau^3_{\tilde{\mathbf{x}},\hat{x}}\tau^3_{\tilde{\mathbf{x}},\hat{y}}\tau^3_{\tilde{\mathbf{x}}+\hat{x},\hat{y}}\tau^3_{\tilde{\mathbf{x}}+\hat{y},\hat{x}}.$$

The 2d quantum Ising model is dual to a quantum Ising gauge theory.

The electric flux of the dual 1-form symmetry is the Ising domain wall of the original Ising model.

Again, the phase with $h >> K$, where the dual *inexplicit* $Z_2^{(1)}$ 1-form symmetry is supposed to be spontaneously broken, has no degeneracy. The Ising symmetry and dual 1-form symmetry are not on equal footing.

Higher Form Symmetries at Quantum Criticality

Q1: what is the correct language/quantity to characterize both the explicit symmetry and inexplict dual symmetry of these models?

Usual concepts and phenomena: order parameter, ground state degeneracy due to SSB, etc.

In order to describe physics of both the explicit and inexplict dual symmetry, we need to give up the ground state degeneracy, and the order parameter. But we can still use the generalized version of the concepts of "short range" or "long range" correlation.

Order diagnosis operator (ODO) for each symmetry: the expectation value of ODO associated with each symmetry characterizes whether the system preserves the symmetry, or have SSB.
(Patch operator in arXiv:1912.13492)

Higher Form Symmetries at Quantum Criticality

Basic example 1: ODO for Z_2 and dual Z_2 symmetries of the 1d quantum Ising model

$$H = \sum_j -K\sigma^3_j\sigma^3_{j+1} - h\sigma^1_j \quad \leftrightarrow \quad H_d = \sum_{\tilde{j}} -K\tau^1_{\tilde{j}} - h\tau^3_{\tilde{j}}\tau^3_{\tilde{j}+1}.$$

$$O_{i,j} = \sigma^3_i\sigma^3_j, \quad \tilde{O}_{\tilde{i},\tilde{j}} = \tau^3_{\tilde{i}}\tau^3_{\tilde{j}} = \prod_{i<k<j}\sigma^1_k.$$

The ODOs reduce to ordinary correlation functions in this simple example, and the expectation values of the ODOs characterize whether the state preserves the symmetry, or has SSB.

Q2: How do ODOs behave at quantum critical points?

At the critical point $K = h$, both ODOs have power-law expectation values, or else we can say both symmetries are preserved.

Higher Form Symmetries at Quantum Criticality

Basic example 2: ODO for Z_2 and dual $Z_2^{(1)}$ 1-form symmetries of the 2d quantum Ising model

$$
\begin{aligned}
H &= \sum_{<\mathbf{x},\mathbf{x}'>} -K\sigma^3_{\mathbf{x}}\sigma^3_{\mathbf{x}'} - \sum_{\mathbf{x}} h\sigma^1_{\mathbf{x}} \;\; \leftrightarrow \\
H_d &= \sum_{\tilde{\mathbf{x}},\hat{\mu}} -K\tau^1_{\tilde{\mathbf{x}},\hat{\mu}} - \sum_{\tilde{\mathbf{x}}} h\tau^3_{\tilde{\mathbf{x}},\hat{x}}\tau^3_{\tilde{\mathbf{x}},\hat{y}}\tau^3_{\tilde{\mathbf{x}}+\hat{x},\hat{y}}\tau^3_{\tilde{\mathbf{x}}+\hat{y},\hat{x}}.
\end{aligned}
$$

In the two fully gapped phases, ODO can be computed perturbatively, perturbation protected by the gap. Q2: But what about at the quantum critical point? Are both symmetries preserved, or spontaneously broken? We need to evaluate the ODOs at the critical points.

Analytical evaluation of ODOs at QCPs can be difficult;
Numerics shows a perimeter law, plus a corner-induced logarithmic term (arXiv:2011.12543, Zhao,et.al.), reminiscent of entanglement entropy (Swingle 2009, Bueno, et.al.2015, Faulkner, et.al. 2016).

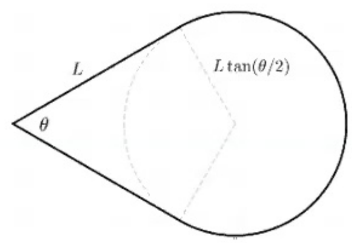
Higher Form Symmetries at Quantum Criticality

Universal features of 1-form ODO at a class of QCPs;

One can show that for a class of a quantum critical points with 1-form symmetries (either explicit or inexplict), the 1-form ODO receives a universal (UV-independent) subleading logarithmic contribution from sharp corners.

$$-\langle(\log \tilde{O}_C)^2\rangle = \frac{\sigma\pi^2}{N^2}\left(\frac{\pi P}{\epsilon} - f(\theta)\log P\right) + \mathcal{O}(1)$$
$$f(\theta) = 2(1 + (\pi - \theta)\cot(\theta))$$

Here the 1-form symmetry is the dual inexplicit $Z_N^{(1)}$ 1-form symmetry of a Z_N symmetry; for a class of quantum critical points, Z_N will enlarge to U(1) in the IR; σ is proportional to the universal conductivity.



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Example 1: 0-form Z_N order-disorder transition: when N >=4, it is just a (2+1)d XY transition with irrelevant anisotropy.

$$\mathcal{S} = \int d^2x d\tau \ |\partial \Phi|^2 + r|\Phi|^2 + g|\Phi|^4 + u(\Phi^N + h.c.) \leftrightarrow$$
$$\mathcal{S}_d = \int d^2x d\tau \ |(\partial - \mathrm{i}a)\phi|^2 + \tilde{r}|\phi|^2 + \tilde{g}|\phi|^4 \ + \ u(\mathrm{M}^N + h.c.).$$

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$$\tilde{O}_C = \exp\left(\mathrm{i}\frac{2\pi}{N}\sum_{j\in\mathcal{A}}\hat{n}_j\right) \qquad J = \frac{\mathrm{i}}{2\pi} * da$$

$$\langle(\log \tilde{O}_C)^2\rangle = -\frac{1}{N^2}\int_C dl^\mu \int_{C'} dl'^\nu \langle a_\mu(\mathbf{x})a_\nu(\mathbf{x}')\rangle.$$

The gauge field correlation is completely dictated by the current-current correlation:

Current-current correlation is proportional to the universal conductivity of the XY transition (studied extensively)

$$\langle J_\mu(0)J_\nu(\mathbf{x})\rangle = \sigma\frac{I_{\mu\nu}(\mathbf{x})}{|\mathbf{x}|^4}$$

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Comment 1: current has a fixed scaling dimension at any CFT.

Comment 2: careful integral requires UV regularization when x and x' are close. To keep gauge invariance, we keep C and C' separated slightly along the temporal direction;

Comment 3: $-\langle (\log \tilde{O}_{\mathcal{C}})^2 \rangle$ is the second order expansion of $2\langle \tilde{O}_{\mathcal{C}} \rangle$

Higher Form Symmetries at Quantum Criticality

Example 2: "QED$_{(Nf,\,N,\,k)}$ theory", with various applications.

$$\mathcal{S} = \int d^2x d\tau \sum_{\alpha=1}^{N_f} \bar{\psi}_\alpha \gamma \cdot (\partial - iNa)\psi_\alpha + m\bar{\psi}\psi + \frac{\mathrm{i}k}{4\pi} a da + \cdots$$

Application 1: QED$_{(2,\,1,\,0)}$, the 0-form SPT-trivial transition; the result in previous example applies with a different σ.

Application 2: QED$_{(1,\,2N,\,0)}$, can be realized at the 2d boundary of a 3d SPT phase with $Z_N^{(1)}$ and U(1) symmetry (conservation of gauge flux). Using the fermion-vortex duality (Son 2015, and others)

$$\mathrm{QED}_{(1,2N,0)} \leftrightarrow$$

$$\bar{\chi}\gamma \cdot \partial\chi \text{ coupled to } Z_N \text{ gauge theory} + \cdots \qquad J_\chi = \mathrm{i}\frac{2N}{4\pi} * da,$$

$$\langle J_{\chi,\mu}(0) J_{\chi,\nu}(\mathbf{x})\rangle = \frac{1}{8\pi^2}\frac{I_{\mu\nu}(\mathbf{x})}{|\mathbf{x}|^4}. \qquad -\langle(\log O_C)^2\rangle = \frac{1}{8N^2}\left(\frac{\pi P}{\epsilon} - f(\theta)\log P\right)$$

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Application 3: QED$_{(Nf,\ N,\ qN\wedge 2)}$, can be realized in 2d, and it is a topological transition between topological orders.

q is at order of N_f, Then with large N_f, the gauge field correlation is

$$\langle a_\mu(0) a_\nu(\mathbf{x}) \rangle = \frac{8}{N_f N^2} \frac{1}{\pi^2 |\mathbf{x}|^2} \times \left(\frac{\cos \hat{\boldsymbol{K}}}{|\boldsymbol{K}|} \frac{\delta_{\mu\nu} - \zeta I_{\mu\nu}(\mathbf{x})}{|\mathbf{x}|^2} + \frac{\sin \hat{\boldsymbol{K}}}{|\boldsymbol{K}|} \frac{\mathrm{i}\pi}{2} \frac{\varepsilon_{\mu\nu\sigma} x_\sigma}{|\mathbf{x}|} \right)$$

$$-\langle (\log O_C)^2 \rangle = \frac{8N^2 N_f}{64k^2 + \pi^2 N^4 N_f^2} \left(\frac{\pi P}{\epsilon} - f(\theta) \log P \right)$$

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ODO with subsystem categorical symmetries

In previous examples, ODOs reduce to either ordinary correlation functions or Wilson loops. But for systems with "special" symmetries, ODOs and their behavior can be much richer.

Example: a 2d quantum Z_2 gauge theory with extra subsystem symmetries (Xu, Fu, 2009):

$$H = \sum_{\mathbf{x}} -K\sigma^3_{\mathbf{x},\hat{x}}\sigma^3_{\mathbf{x},\hat{y}}\sigma^3_{\mathbf{x}+\hat{x},\hat{y}}\sigma^3_{\mathbf{x}+\hat{y},\hat{x}} - J\sigma^1_{\mathbf{x},\hat{x}}\sigma^1_{\mathbf{x}+\hat{x},\hat{x}} - J\sigma^1_{\mathbf{x},\hat{y}}\sigma^1_{\mathbf{x}+\hat{y},\hat{y}}.$$

This model has a series of subsystem conserved quantities such as

$$\Sigma_{\hat{x},y} = \prod_{y=\mathrm{Const}} \sigma^3_{\mathbf{x},\hat{x}}, \quad \Sigma_{\hat{y},x} = \prod_{x=\mathrm{Const}} \sigma^3_{\mathbf{x},\hat{y}}.$$

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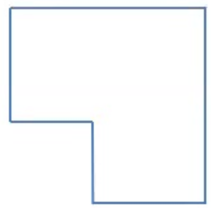
Because of these subsystem conserved quantities, in the large K phase where the 1-form symmetry is "SSB", the ODO (Wilson loop) decays as a "corner law"



$$O^{(1)}_{\mathcal{C}} = \prod_{l\in\mathcal{C}} \sigma^3_l, \quad \langle O^{(1)}_{\mathcal{C}}\rangle \sim e^{-\alpha_3 (J/K)^2 N_{\mathcal{C}}}.$$

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This model is dual to a Ising plaquette model, which also has subsystem symmetries.

$$H_d = \sum_{\tilde{\mathbf{x}}} -K\tau^1_{\tilde{\mathbf{x}}} - 2J\tau^3_{\tilde{\mathbf{x}}}\tau^3_{\tilde{\mathbf{x}}+\hat{x}}\tau^3_{\tilde{\mathbf{x}}+\hat{y}}\tau^3_{\tilde{\mathbf{x}}+\hat{x}+\hat{y}}.$$

$$\tilde{O}^{(\mathrm{sub})}_{x,y} = \tau^3_{0,0}\tau^3_{x,0}\tau^3_{0,y}\tau^3_{x,y}.$$

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ODO with subsystem categorical symmetries

Both the special Z2 gauge theory, and the dual plaquette Ising model can be embedded into a special U(1) gauge theory and a dual U(1) model with subsystem symmetries.

$$H = \int d^2x \ \frac{U}{2}(\vec{\nabla} \times \hat{\vec{a}})^2 + \frac{t}{4}\left((\nabla_x \hat{e}_x)^2 + (\nabla_y \hat{e}_y)^2\right)$$

$$\mathcal{S}_d = \int d\tau d^2\tilde{x} \ \frac{1}{2U}(\partial_\tau \theta)^2 + \frac{t}{2}(\nabla_x \nabla_y \theta)^2,$$

The dual U(1) theory is analogous to the bose-metal model proposed before (Paramekanti, et.al. 2002), with subsystem symmetry.

ODO with subsystem categorical symmetries

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$$\mathcal{S}_d = \int d\tau d^2\tilde{x} \, \frac{1}{2U}(\partial_\tau \theta)^2 + \frac{t}{2}(\nabla_x \nabla_y \theta)^2,$$

The ODOs can be evaluated at the Gaussian phase of both theories.
The leading order of the ODO is different from the gapped phases.

Special 2d Z_2 Gauge theory Eq. 17	$K \gg J$	$K \ll J$	Gapless Phase
$O_{\mathcal{C}}^{(1)}$	Corner law	Area law	$\exp\left(-c\pi^2\sqrt{t/U}\ \log\lvert x\rvert \log\lvert y\rvert\right)$ for rect. $\mathcal{C}$
$\tilde{O}_{x,y}^{\text{sub}}$	Area law	Long range	$\exp\left(-\tilde{c}\pi^2\sqrt{U/t}\ \log\lvert x\rvert \log\lvert y\rvert\right)$

Higher Form Symmetries at Quantum Criticality

3d systems with 1-form and sub-system symmetries:

$$H = \sum_{\mathbf{x},\hat{\mu},\hat{\nu}} -K\sigma^3_{\mathbf{x},\hat{\mu}}\sigma^3_{\mathbf{x},\hat{\nu}}\sigma^3_{\mathbf{x}+\hat{\mu},\hat{\nu}}\sigma^3_{\mathbf{x}+\hat{\nu},\hat{\mu}} - J\sigma^1_{\mathbf{x},\hat{\mu}}\sigma^1_{\mathbf{x}+\hat{\mu},\hat{\mu}}$$

$$\leftrightarrow \quad H_d = \sum_{\tilde{\mathbf{x}},\hat{\mu},\hat{\nu}} -K\tau^1_{\tilde{\mathbf{x}},\hat{\mu}} - \sum_{\hat{\rho}\perp\hat{\mu},\hat{\nu}} J\hat{\mathcal{B}}_{\tilde{\mathbf{x}},\hat{\mu}\hat{\nu}}\hat{\mathcal{B}}_{\tilde{\mathbf{x}}+\hat{\rho},\hat{\mu}\hat{\nu}}.$$

$$\hat{\mathcal{B}}_{\tilde{\mathbf{x}},\hat{\mu}\hat{\nu}} = \tau^3_{\tilde{\mathbf{x}},\hat{\mu}}\tau^3_{\tilde{\mathbf{x}},\hat{\nu}}\tau^3_{\tilde{\mathbf{x}}+\hat{\mu},\hat{\nu}}\tau^3_{\tilde{\mathbf{x}}+\hat{\nu},\hat{\mu}}.$$

With a modified Hamiltonian of the Z_2 gauge theory, many subsystem conserved quantities:

$$\Sigma_{\hat{x};(y,z)} = \prod_{y,z=\mathrm{Const}} \sigma^3_{\mathbf{x},\hat{x}}, \quad \Sigma_{\hat{y};(x,z)} = \prod_{x,z=\mathrm{Const}} \sigma^3_{\mathbf{x},\hat{y}}, \quad \Sigma_{\hat{z};(x,y)} = \prod_{x,y=\mathrm{Const}} \sigma^3_{\mathbf{x},\hat{z}}$$

Because of the all the subsystem symmetries, the dual 1-form symmetry ODO is defined with a pair of separate and parallel loops:

$$\tilde{O}^{(1)}_{\mathcal{C},\mathcal{C}'} = \prod_{\tilde{l}\in\mathcal{C}} \tau^3_{\tilde{l}} \prod_{\tilde{l}\in\mathcal{C}'} \tau^3_{\tilde{l}}.$$

Higher Form Symmetries at Quantum Criticality

Again this theory can be imbedded into a U(1) gauge theory with subsystem symmetries and a Gaussian gapless phase. For example, the dual action of the gapless phase reads:

$$\mathcal{S}_d = \int d^3\tilde{x} d\tau \ \frac{1}{2U}(\partial_\tau \vec{a})^2 + \frac{t}{2}\left(\nabla_z(\nabla_x a_y - \nabla_y a_x)\right)^2 + (\text{permute } x, y, z).$$

$$\tilde{O}^{(1)}_{\mathcal{C},\mathcal{C}'} = \prod_{\tilde{i}\in\mathcal{C}} \tau^3_{\tilde{i}} \prod_{\tilde{i}\in\mathcal{C}'} \tau^3_{\tilde{i}} \sim \exp\left(\mathrm{i}\oint_{\mathcal{C}} \hat{a}_\mu dx^\mu\right) \exp\left(-\mathrm{i}\oint_{\mathcal{C}'} \hat{a}_\nu dx^\nu\right)$$

Special $3d$ Z_2 Gauge theory Eq. 32	$K \gg J$	$K \ll J$	Gapless Phase
$O^{(1)}_{\mathcal{C}}$ with rect. $\mathcal{C}$ in XY	Corner law	Area law	$\frac{1}{\lvert x\rvert^{\Delta_{\mathcal{C}}}}$, with $y = 1$ and $x \gg 1$.
$\tilde{O}^{(1)}_{\mathcal{C},\mathcal{C}'}$ parallel $\mathcal{C}, \mathcal{C}'$ in XY; separated along $\hat{z}$	Area law of $\mathcal{C}, \mathcal{C}'$; exponential decay with Z	Perimeter law of $\mathcal{C}$; long range with Z	$\frac{1}{\lvert Z\rvert^{2\Delta_{\mathcal{C},\mathcal{C}'}}}$, for unit square $\mathcal{C}, \mathcal{C}'$ separated along z

Higher Form Symmetries at Quantum Criticality

Comment on general connection between Renyi entropy and ODO:

To compute the Renyi entropy, one makes *n*-copies of the system, the symmetry is granted with an extra swapping symmetry between the different copies. The Renyi entropy can be viewed as the <ODO> for the swapping symmetry.

One can start with the *n*-copy system, and compute both the ODO for the dual of the intrinsic symmetry, and the extra swapping symmetry, to extract the EE and information of dual symmetry simultaneously.