Title: Universal thermal transport in TTbar-deformed conformal field theories

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Abstract: In this talk I will discuss the universal properties of thermal transport in conformal field theories that are perturbed by a TTbar operator. TTbar-deformation is known to be an exactly solvable deformation in that the spectrum of the undeformed theory alone suffices to predict that of the deformed theory. Unique properties of TTbar deformation allow us to study the TTbar-deformed CFTs using two disparate methods: integrability and holography. I will apply these two approaches to study the non-equilibrium steady states and Drude weights, finding perfect agreement. I will also explain how the integrability-based approach yields the exact momentum diffusion constant, which, to our surprise, also happens to satisfy a universal formula. Finally I will briefly touch upon a curious connection between TTbar-deformed CFTs and an integrable cellular automaton model called the Rule 54 chain.

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Universal thermal transport

in

$T\bar{T}$-deformed conformal field theories

Takato Yoshimura (Tokyo Tech.)

w/ M. Medenjak and G. Policastro

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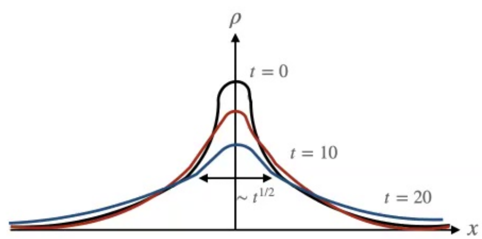
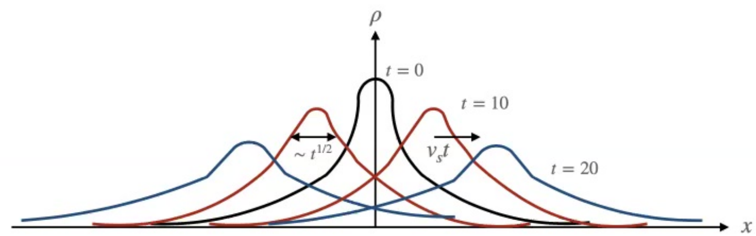
Motivation

- The late time dynamics of many-body systems typically exhibits diffusion

purely diffusive



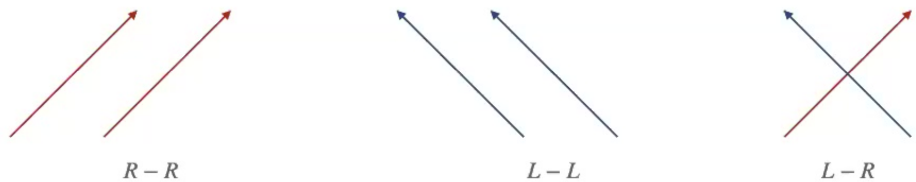
ballistic + diffusive



- Diffusion is also known to be related to chaos in strongly-interacting systems (e.g. SYK) Blake; Gu, Xi, Stanford; Blake, Lee, Liu
- Recently a part of it was uncovered Medenjak, De Nardis, TY; Doyon

Can we understand the mechanism of diffusion in (quantum) many-body systems in a controlled way?

- To do so, it would be nice to start with a simple system: 2d conformal field theory (CFT)
- 2d CFTs are quantum field theories that characterise the fixed points of renormalisation group flows
- No characteristic scale in CFTs $\longrightarrow$ no asymptotic particles
- Elementary excitations are either right or left movers, each of which does not scatter



- No phase shift for $L-R$ scatterings, hence no diffusion
- How can we introduce $L-R$ scatterings? We can deform CFTs by some operators
- $T\bar{T}$-deformation

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Plan

- Hydrodynamics and diffusion
- What is $T\bar{T}$-deformation?
- Thermodynamics and hydrodynamics of $T\bar{T}$-deformed conformal field theories (CFTs)
- Transport quantities in $T\bar{T}$-deformed CFTs
- Conclusion and Outlook

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Hydro and diffusion

- Diffusion is conveniently captured by hydrodynamics

- Hydrodynamics is an effective theory that describes the long wave-length dynamics of many-body systems

$$\partial_t \langle q_i \rangle + A_i^{\ j} \partial_x \langle q_j \rangle = \mathfrak{D}_i^{\ j} \partial_x^2 \langle q_j \rangle$$ Navier-Stokes equation

$A_i^{\ j} = \partial \langle j_j \rangle_{\mathrm{sta}} / \partial \langle q_i \rangle_{\mathrm{sta}}$: linearisation matrix evaluated wrt the back ground state

$\mathfrak{D}_i^{\ j}$: diffusion matrix that satisfies $\mathfrak{L} = \mathfrak{D} C$, where $C_{ij} = \int \mathrm{d}x\, S_{ij}(x,t)$ with $S_{ij}(x,t) = \langle q_i(x,t) q_j(0,0) \rangle^c_{\mathrm{sta}}$

- The Onsager matrix $\mathfrak{L}$ controls the diffusive broadening of fluid packets

$$\int \mathrm{d}x\, x^2 (S_{ij}(x,t) + S_{ij}(x,-t)) = D_{ij} t^2 + \mathfrak{L}_{ij} t + \mathcal{O}(1) \qquad \text{as } t \to \infty$$

Drude weights

$$D_{ij} = \lim_{t\to\infty} \frac{1}{2t} \int_{-t}^{t} \mathrm{d}s \int \mathrm{d}x \langle j_i(x,s) j_j(0,0) \rangle^c$$

$$\mathfrak{L}_{ij} = \int \mathrm{d}t \left(\int \mathrm{d}x \langle j_i(x,s) j_j(0,0) \rangle^c - D_{ij} \right).$$

- The computation of the Onsager matrix is a rather formidable task. Possible in integrable systems De Nardis, Bernard, Doyon

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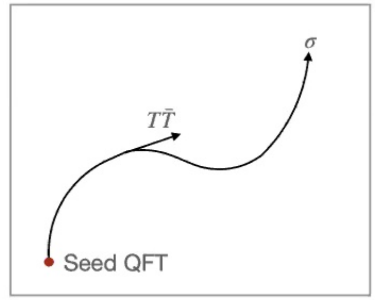
$T\bar{T}$-deformation



- $T\bar{T}$-deformation is defined via an infinitesimal deformation of the Lagrangian

$$\mathscr{L}^{(\sigma+\delta\sigma)} = \mathscr{L}^{(\sigma)} + \frac{\delta\sigma}{2}\det T_{\mu\nu}$$

- Irrelevant deformation (dim=(2,2)) but solvable. UV complete?

  Finite size spectrum, thermodynamics, etc Smirnov and Zamolodchikov; Cavaglia et al; Cardy

- The finite volume energy eigenvalue $E_n(R,\sigma)$ satisfies the Burgers equation

$$\partial_\sigma E_n(R,\sigma) = E_n(R,\sigma)\partial_R E_n(R,\sigma) + \frac{1}{R}P_n^2(R)$$

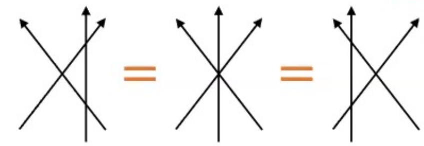
The spectrum of the seed theory ⟶ that of the deformed theory!

- Finite temperature free energy in the thermodynamic limit ⟷ the g.s. energy in the finite volume: $f(\beta,\sigma) = E_0(\beta,\sigma)/\beta$

$$f(\beta,\sigma) = -\frac{1}{\sigma}\left(1-\sqrt{1-\frac{\pi c\sigma}{3\beta^2}}\right)$$

$\sigma > 0$: complex free energy
$\sigma < 0$: Hagedorn growth

CDD factor and integrability



- $T\bar{T}$-deformed integrable systems are still integrable

- An infinite number of conserved charges $\longrightarrow$ factorisation of a S-matrix into the two-body S-matrices $S_{2\to2}$

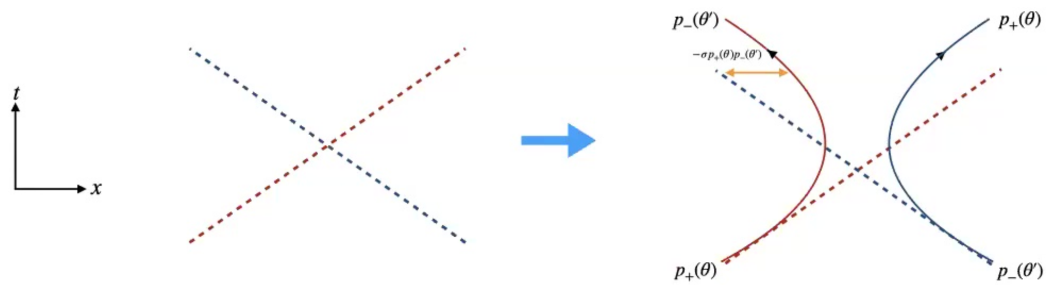
- $T\bar{T}$-deformation gives the CDD factor to $S_{2\to2}$ (gravity dressing)   Dubovski et al

$$S^{(\sigma)}_{2\to2}(\theta) = S^{(0)}_{2\to2}(\theta) \times e^{\mathrm{i}\Sigma(\theta)}$$

massive: $\Sigma(\theta) = \sigma m^2 \sinh\theta$
massless: $\Sigma(\theta) = -\sigma p_+(\theta_1)p_-(\theta_2)$

$$p_\pm(\theta) = \pm\frac{Me^{\pm\theta}}{2}$$

- We can formally apply the logic of Thermodynamic Bethe ansatz to CFTs   Bazhanov, Lukyanov, Zamolodchikov



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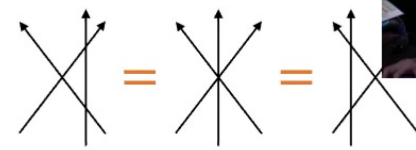
CDD factor and integrability



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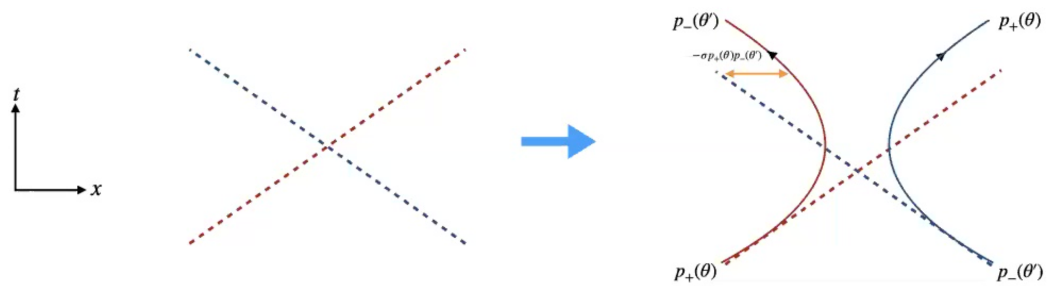

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Fundamental width of particles

- Intuitively speaking, $T\bar{T}$-deformation induces the "width" of particles in the original theory   Cardy and Doyon

e.g. free boson gas $\longrightarrow$ hard-rod gas   Jiang
CFT $\longrightarrow$ cellular automaton-like system   Medenjak, Policastro, TY

- Consistent with "$T\bar{T}$-deformation $\longleftrightarrow$ state-dependent coordinate change"   Conti, Negro, Tateo; Cardy

$$\delta x^1 = \delta\sigma \times 2\pi \int_{x^1+\epsilon}^{\infty} \mathrm{d}y^1\, T_{00}^{(\sigma)}(0, y^1) = \delta\sigma(E_> - E_<)$$

$$\delta x^0 = -\,\delta\sigma \times 2\pi \int_{x^1+\epsilon}^{\infty} \mathrm{d}y^1\, T_{01}^{(\sigma)}(0, y^1) = -\,\delta\sigma(P_> - P_<)$$

- In the new coordinate (or metric), the free space in which particles can explore reduces/increases

Emergence of scattering $\longleftrightarrow$ change of the free space

- The same idea (!) was employed in the context of hard rods and generalised hydrodynamics (GHD)   Spohn; Doyon and Spohn; Doyon, Spohn, TY

Developments

- Novel solvable deformation of QFT in (1+1)-demension — Zamoldchikov; Smirnov and Zamolodchikov
- $T\bar{T}$-deformation = coupling to Jackiw-Teitelboim gravity — Dubovsky, Gorbenko, Mirbabayi
- **A $T\bar{T}$-deformed holographic CFT still has a gravity dual** — McGough, Mezei, Verlinde
- $T\bar{T}$-deformation = coupling to a random geometry — Cardy
- Correlation functions — Cardy
- Similar deformations to lattice models and non-relativistic systems — Cardy; Pozsgay, Jiang, Tacaks; Jiang
- Other solvable deformations, e.g. $J\bar{T}$-deformation — Guica
- Out-of-equilibrium dynamics of $T\bar{T}$-deformed theories — Bernard and Doyon; Medenjak, Policastro,TY
- For more info, see a nice review by Jiang — Jiang

Disclaimer: by no means meant to be exhaustive!

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Generalised hydrodynamics for $T\bar{T}$-deformed CFTs

Generalised hydrodynamics

- GHD is a hydrodynamic theory for integrable systems — Castro-Alvaredo, Doyon, TY; Bertini, Collura, De Nardis, Fagotti
- An infinite # of conserved charges ⟶ An infinite # of hydrodynamic equations

$$\partial_t \langle q_i \rangle + \partial_x \langle j_i \rangle = 0, \quad i = 1, \cdots$$

- Integrability provides explicit expressions of $\langle q_i \rangle$ and $\langle j_i \rangle$ in terms of quasi-particle momentum

$$\langle q_i \rangle = \int \mathrm{d}\theta\, \rho(\theta) h_i(\theta)$$

Zamolodchikov; Saleur; Kostov, Serban, Vu

$$\langle j_i \rangle = \int \mathrm{d}\theta\, \rho(\theta) v^{\mathrm{eff}}(\theta) h_i(\theta)$$

Castro-Alvaredo, Doyon, TY;
Bertini, Collura, De Nardis, Fagotti

- These ingredients allow us to rewrite the hydro equations as a single equation

$$\partial_t \rho(\theta) + \partial_x (\rho(\theta) v^{\mathrm{eff}}(\theta)) = 0$$

- Vast developments with experimental confirmations — 100+ papers; Schemmer et al; Malvania et al

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GHD for $T\bar{T}$-deformed CFTs

- BLZ allows us to write down TBA-like non-linear integral equations (NLIE) for CFTs

  e.g. minimal models, Liouville CFT. Not known if it's possible for a generic CFT

  Bazhanov, Lukyanov, A. B. Zamolodchikov; Al. Zamolodchikov

- NLIEs for the simplest case

$$\varepsilon_\pm(\theta) = (\beta \mp \nu)E_\pm(\theta) - \int_{\mathbb{R}} \mathrm{d}\theta' T(\theta,\theta')\log(1+e^{-\varepsilon_\pm(\theta')}) - \int_{\mathbb{R}} \mathrm{d}\theta' T_{\pm\mp}(\theta,\theta')\log(1+e^{-\varepsilon_\mp(\theta')})$$

energy $E_\pm(\theta) = \pm p_\pm(\theta)$ phase shift of CFT $R-L$ phase shift $T_{\pm\mp}(\theta,\theta') = -\frac{\sigma p_\pm(\theta)p_\mp(\theta')}{2\pi}$

- Scaling $M \mapsto sM$ merely shifts the rapidity $\theta \mapsto \theta + s$ no characteristic scale

- The pseudo-energy $\varepsilon_\pm(\theta)$ determines the free energy of the system

$$f = -T\sum_{a=\pm}\int_{\mathbb{R}} \frac{\mathrm{d}p_a(\theta)}{2\pi}\log(1+e^{-\varepsilon_a(\theta')}) \xrightarrow[\sigma\to 0]{} -\frac{\pi c}{6\beta^2}$$

CFT result ✓

Energy hydro equation in $T\bar{T}$-deformed CFTs

- The effective velocity $v_\pm^{\mathrm{eff}}(\theta)$ in $T\bar{T}$-deformed CFTs are θ-independent

$$v_\pm^{\mathrm{eff}}(\theta) = \frac{\pm 1 + \sigma(\rho_+ - \rho_-)}{1 + \sigma(\rho_+ + \rho_-)}$$

In thermal state $v_\pm^{\mathrm{eff}} = \pm\sqrt{1 - \frac{\pi c \sigma}{3\beta^2}}$

- Hydro equation for the chiral energies $\rho_\pm = \int_{\mathrm{R}} \mathrm{d}\theta \rho_\pm(\theta) E_\pm(\theta)$ is closed

$$\boxed{\partial_t \rho_\pm + \partial_x (v_\pm^{\mathrm{eff}} \rho_\pm) = 0}$$

Medenjak, Policastro, TY

meaning $\partial_t \langle q_E \rangle + \partial_x \langle j_E \rangle = 0$ and $\partial_t \langle q_P \rangle + \partial_x \langle j_P \rangle = 0$ are decoupled from other hydro equations

- The equation is nothing but the hydro equation for soliton densities in the reversible cellular automaton rule 54 (RCA54)

Friedman, Gopalakrishnan, Vasseur

energy quanta of $T\bar{T}$-deformed CFTs $\xleftrightarrow{\text{hydro}}$ solitons in the RCA54 — But not thermodynamics!

- A Riemann problem of it was solved before

El and Kamchatnov

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RCA54 hydro from the trace relation

- The hydro equation for the chiral energies in $T\bar{T}$-deformed CFTs is equivalent to

$$\partial_t \langle T_{00}\rangle + \partial_x \langle T_{10}\rangle = 0, \quad \partial_t \langle T_{01}\rangle + \partial_x \langle T_{11}\rangle = 0, \quad \langle T_{11}\rangle = \frac{\langle T_{00}\rangle + \sigma \langle T_{01}\rangle^2}{1 + \sigma \langle T_{00}\rangle}.$$

- The equations of state can be obtained by the trace relation, which is valid inside correlation functions

Jiang

$$T^{\mu}_{\ \mu} = -\,\sigma \det T_{\mu\nu}$$

- To obtain it, note by definition that, on a curved space endowed with a metric g

$$\frac{\mathrm{d}S}{\mathrm{d}\sigma} = \int \mathrm{d}^2x \sqrt{g}\, \det T_{\mu\nu}$$

- On the other hand when the theory has a single mass scale μ, an infinitesimal change of the mass scale is induced by the coordinate transformation $x^\nu \mapsto (1 + \delta \log \mu) x^\nu$, under which the action changes as

$$\frac{\mathrm{d}S}{\mathrm{d}\log \mu} = \int \mathrm{d}^2x \sqrt{g}\, T^{\mu}_{\mu}\,.$$

- Since $\sigma = 1/(2\mu^2)$, equating them gives the trace relation

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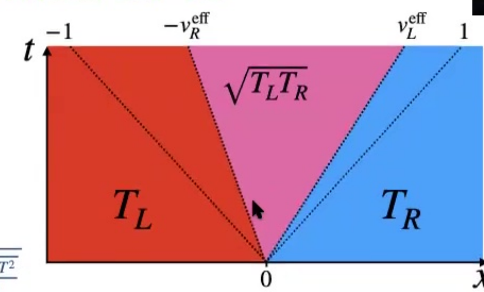
NESS and Drude weights in $T\bar{T}$-deformed CFTs



- Non-equilibrium steady state in the partitioning protocol turns out to be simple

- Energy and momentum currents are given by

$$\langle j_E \rangle_{\text{NESS}} = \frac{\pi c}{12} e_{RL} (\tilde{T}_L^2 - \tilde{T}_R^2)$$

effect of scattering — change of thermodynamics

$$\tilde{T}(T) = \frac{2T}{1+\sqrt{1-\frac{\pi\sigma c T^2}{3}}}$$

$$e_{RL} = \frac{1}{1-\left(\frac{\pi\sigma c}{12}\right)^2 \tilde{T}_L^2 \tilde{T}_R^2}$$

- Reproduces the CFT result when $\sigma \to 0$ — Bernard and Doyon

- We can also compute the energy and momentum Drude weights

$$D_{EE} = \frac{\langle q_E \rangle + \langle j_P \rangle}{\beta} = \frac{\pi c}{3 v^{\text{eff}}} T^3, \quad D_{PP} = \left(\frac{\langle j_P \rangle}{\langle q_E \rangle}\right)^2 D_{EE} = \frac{\pi c v^{\text{eff}}}{3} T^3$$

energy conservation — massless-ness

In CFT

$$D_{EE} = \frac{\pi c}{3 v} T^3, \quad D_{PP} = \frac{\pi c v}{3} T^3$$

v: sound velocity

Bernard and Doyon

- Perfect agreement with holographic computations! — Medenjak, Policastro, TY

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NESS in $T\bar{T}$-deformed holographic CFTs

- We work in pure gravity. Assuming that the space the $T\bar{T}$-deformed CFT lives is the Lorentzian cylinder, the most generic geometry is given by the Bañados geometry McGough, Mezei, Verlinde; Guica and Monten

$$ds^2 = \frac{\ell^2 d\rho^2}{4\rho^2} + \frac{du dv}{\rho} + \mathcal{L}(u)du^2 + \overline{\mathcal{L}}(v)dv^2 + \mathcal{L}(u)\overline{\mathcal{L}}(v)du dv$$

$$\mathcal{L}(u) = 8\pi G\ell \langle T_{uu}\rangle$$
$$\overline{\mathcal{L}}(v) = 8\pi G\ell \langle T_{vv}\rangle$$

- The $T\bar{T}$-deformed CFT lives on the $\rho = \mu$ surface. The following state dependent coordinate change $(u, v) \mapsto (U, V)$ yields the induced metric $dUdV$ at $\rho = \mu$

$$U = u + \mu \int^v dv' \overline{\mathcal{L}}(v'), \quad V = v + \mu \int^u du' \mathcal{L}(u')$$

- (u, v) are the light-cone coordinates for the original CFT, while (U, V) are those for the deformed CFT. In terms of (u, v),

$$\mathcal{L}(u) = \mathcal{L}_L \vartheta(-u) + \mathcal{L}_R \vartheta(u), \quad \overline{\mathcal{L}}(v) = \overline{\mathcal{L}}_L \vartheta(-v) + \overline{\mathcal{L}}_R \vartheta(v)$$

Bernard and Doyon

- In the case of partitioning protocol, we can carry out coordinate transformation and get the deformed light cone

$$x = -\frac{1 + \mu\overline{\mathcal{L}}_L}{1 - \mu\overline{\mathcal{L}}_L} t, \quad x = \frac{1 + \mu\mathcal{L}_R}{1 - \mu\mathcal{L}_R} t \qquad U = x + t, \quad V = x - t$$

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Momentum diffusion in $T\bar{T}$-deformed CFTs

- $R-L$ scatterings induce diffusion in $T\bar{T}$-deformed CFTs
- Lorentz invariance (i.e. energy conservation) $\longrightarrow$ no energy diffusion $\mathfrak{L}_{EE} = 0$
- A formula from GHD gives the momentum Onsager matrix $\mathfrak{L}_{PP}$

De Nardis, Bernard, Doyon; Gopalakrishnan, Huse, Khemani, Vasseur

$$\mathfrak{L}_{PP} = \frac{\sigma^2}{2} v^{\rm eff} D_{EE}^2 = \frac{\pi c}{6\beta^5}\sigma^2 s, \quad s = \frac{\pi c}{3 v^{\rm eff}\beta}$$

quadratic in σ

Medenjak, Policastro, TY

- Conformal perturbation confirms it up to the second order in σ. Universal for any CFT?
- Holographically challenging to compute

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Conclusion and Outlook

- $T\bar{T}$-deformation has a number of remarkable properties, and their full implications have not been fully understood yet
- $T\bar{T}$-deformed CFTs admit a variety of unrelated approaches, allowing us to understand the physics from different points of view
- Energy and momentum transport in $T\bar{T}$-deformed CFTs are rather universal, and natural generalisation of the pure CFTs case
- Momentum diffusion does not seem to distinguish chaotic and non-chaotic CFTs. What about other diffusion constants? Can we cook up another deformation that does distinguish them? $J\bar{T}$-deformation?
- Chaos and diffusion are known to be intimately related, at least in large-N theories. Will the $T\bar{T}$-deformed holographic CFTs be still maximally chaotic? Effective field theory for $T\bar{T}$-deformed chaotic CFTs? Haehl and Rozali
- Other universality classes from $T\bar{T}$-like deformations?