Title: Physical implications of a fundamental period of time

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Abstract: If time is described by a fundamental process rather than a coordinate, it

interacts with any physical system that evolves in time. The resulting dynamics has recently been shown to be consistent provided the fundamental period of the time system is sufficiently small. A strong upper bound T_C < 10^{-33}s of the fundamental period of time, several orders of magnitude below any direct time measurement, can be obtained from bounds on dynamical variations of the period of a lab system evolving in time. Possible cosmological implications will be discussed.

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Physical implications of a fundamental period of time

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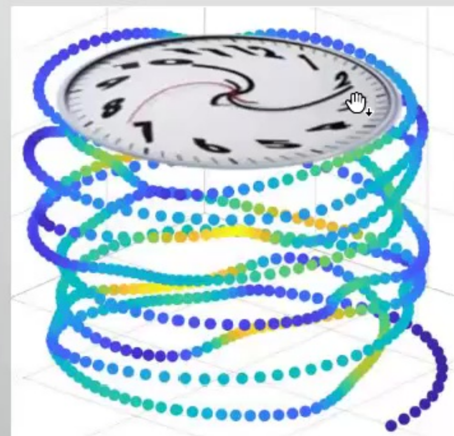

G. Wendel, L. Martínez, MB: PRL 124 (2020) 241301, M. Amaral, MB: Ann. Phys. 388C (2018) 241
MB, A. Tsobanjan, Commun. Math. Phys. to appear [arXiv:1906.04792]

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Time and clocks

Intuitive experience of time
complicates fundamental understanding.

→ Experience directed monotonic flow. “Time”

→ Measured by periodic processes. “Clock”

Conventional labeling of events by time (and date) reflects this dichotomy.

Time is a constructed concept, could be an effective parameter.

Is time based on a monotonic or periodic fundamental process?

First check if periodic clock can be consistent.

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Internal time

Position q with momentum p.

Clock ϕ with momentum $-E$, energy.

Energy equation $E = H(q, p, \phi)$ with Hamiltonian H imposes constraint

$$C = -E + H(q, p, \phi) = 0$$

Only one independent variable.

Hamilton's equation

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{\partial C}{\partial(-E)} = 1$$

implies identification of time t with clock variable ϕ.

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Quantization

Introduce operators $\hat{q}$, $\hat{p}$, $\hat{\phi}$ and $\hat{E}$ such that

$$[\hat{q},\hat{p}] = i\hbar \quad \text{and} \quad [\hat{\phi},\hat{E}] = -i\hbar$$

and impose quantum constraint

$$\hat{C}\psi(q,\phi) = -\hat{E}\psi(q,\phi) + \hat{H}\psi(q,\phi) = 0$$

on wave functions $\psi(q,\phi)$.

Equivalent to Schrödinger equation if $\hat{E} = i\hbar\partial/\partial\phi$ is used.

Position q and clock ϕ still different conceptually:

- → ϕ doesn't fluctuate, implication of constraint $\hat{C}$ [MB, Tsobanjan]
- → $\hat{C}$ linear in $\hat{E}$, quadratic in $\hat{p}$ for standard $\hat{H}$

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Relativity

Relativistic energy equation, modeled on $-E^2 + p^2 = -m^2$:

$$C' = -E^2 + H^2 = 0$$

Hamilton's equations for $(\phi, -E)$:

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{\partial C'}{\partial(-E)} = 2E$$

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\partial C'}{\partial \phi} = 2H\frac{\partial H}{\partial \phi}$$

$\mathrm{d}\phi/\mathrm{d}t$ not constant if H depends on ϕ.

- → $E = \pm H$ may be positive or negative as solution of $C' = 0$.
- → Oscillating $\phi(t)$ possible if Hamiltonian time-dependent.

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Cosmology

Friedmann equation $(\dot{a}/a)^2 = \frac{8\pi G}{3}\rho$ in canonical form:

$V = a^3$ with momentum $p_V = -\dot{a}/(4\pi G a)$

scalar field ϕ with momentum p_ϕ

$$\begin{aligned} C &= V\rho(\phi, p_\phi) - 6\pi G V p_V^2 \\ &= \frac{1}{2}\frac{p_\phi^2}{V} + \frac{1}{2} V m^2 \phi^2 - 6\pi G V p_V^2 \\ &= (-p_\phi + H(V, p_V, \phi))\,(p_\phi + H(V, p_V, \phi)) \end{aligned}$$

with

$$H(V, p_V, \phi) = \sqrt{12\pi G V^2 (P_V^2 - m^2\phi^2)}$$

Simplifies for $m = 0$ ("deparameterization"), but not generic as fundamental field: no mass or self-interactions.

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Quantization

1. Time-independent Hamiltonian, $[\hat{E},\hat{H}]=0$ in $\hat{C}'=-\hat{E}^2+\hat{H}^2$:

$$(-\hat{E}^2+\hat{H}^2)\psi=(-\hat{E}+\hat{H})(\hat{E}+\hat{H})\psi=(\hat{E}+\hat{H})(-\hat{E}+\hat{H})\psi=0$$

$\hat{E}\psi=i\hbar\partial\psi/\partial\phi=\pm\hat{H}\psi$, two Schrödinger equations.

2. Time-dependent Hamiltonian:

$$(-\hat{E}+\hat{H})(\hat{E}+\hat{H})=-\hat{E}^2-[\hat{E},\hat{H}]+\hat{H}^2=-\hat{E}^2+\hat{H}^2+i\hbar\widehat{\partial H/\partial\phi}$$

$$\neq\ (\hat{E}+\hat{H})(-\hat{E}+\hat{H})=-\hat{E}^2+[\hat{E},\hat{H}]+\hat{H}^2=-\hat{E}^2+\hat{H}^2-i\hbar\widehat{\partial H/\partial\phi}$$

Cannot impose both $\hat{E}\psi=\hat{H}\psi$ and $\hat{E}\psi=-\hat{H}\psi$ based on a single $\hat{C}'$. Periodic clock impossible?

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The problem of time as a Gribov problem

[Amaral, MB]

Choosing ϕ as time, such that $\phi = t$ with a real parameter t, is gauge fixing:

$$\{\phi - t, C'\} = 2E \neq 0$$

as long as $E \neq 0$. ($C' = -E^2 + H^2$)

→ If H is ϕ-independent, $\{E, C'\} = 0$ and E is constant. Stays non-zero if initially non-zero.

→ If H depends on ϕ, E is not constant, goes through zero as ϕ oscillates.
$\phi - t$ no longer fixes gauge when $E = 0$: Gribov horizon.

Path integral quantization of gauge theories:
Do not cross Gribov horizons in order to avoid overcounting degrees of freedom. $\mathrm{sgn} E$ fixed.

How can ϕ oscillate?

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Global evolution

$\mathrm{d}\phi/\mathrm{d}t = 1$ for $C = -E + H = 0$ presupposes direction of ϕ.

More general: $\phi = \tau + A_+$ or $\phi = -\tau + A_-$ at different times.

Periodic clock ϕ, monotonic time-and-date τ.

Schrödinger equation with respect to τ:

$$i\hbar\frac{\partial\psi}{\partial\tau} = i\hbar\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\frac{\partial\psi}{\partial\phi} = \frac{\mathrm{d}\phi}{\mathrm{d}\tau}\hat{E}\psi$$

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Constrained path integral

Transition amplitude

$$(q_{\rm b}|\hat{T}(\tau_{\rm a},\tau_{\rm b})|q_{\rm a}) = \int \mathcal{D}q\mathcal{D}p\mathcal{D}\phi\mathcal{D}E\mathcal{D}N|\{C,G_\tau\}|\delta(G_\tau)\theta(\pm E)$$
$$\times \exp\left(\frac{i}{\hbar}\int_{\tau_{\rm a}}^{\tau_{\rm b}} d\tau\left(p\dot{q}-E\dot{\phi}-NC\right)\right)$$

Constraint C gauge-fixed by $G_\tau = 0$, $\theta(\pm E)$ restricts to single Gribov region.

After N-integration:

$\longrightarrow$ $E = \pm H$, (q,p)-Hamiltonian given by $E\mathrm{d}\phi/\mathrm{d}\tau$.

$\longrightarrow$ ϕ can oscillate even with positive Hamiltonian, $E\mathrm{d}\phi/\mathrm{d}\tau > 0$.

$\longrightarrow$ E changes sign, but one Gribov region at a time.

$\longrightarrow$ Combine transition amplitudes for different branches.

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Main example

$C = -E^2 - \lambda^2\phi^2 + H(q,p)^2 = 0$

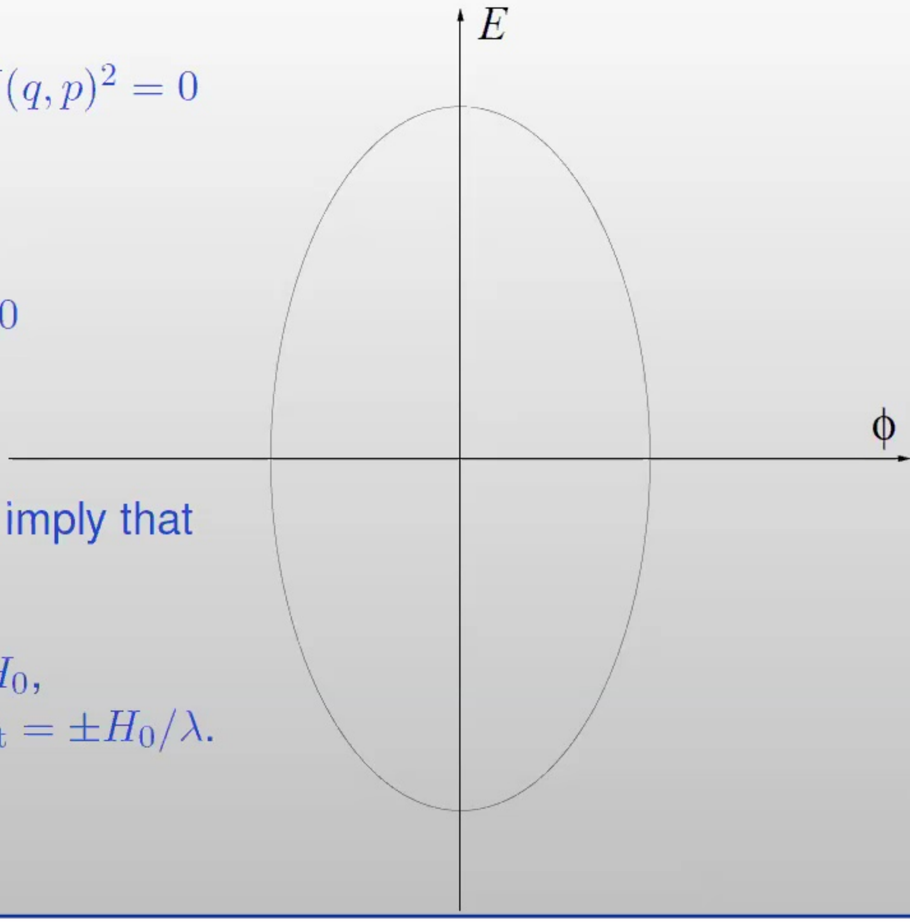
Clock ϕ, system q coupled minimally:

Energy balance $C = 0$ instead of force.

Hamilton's equations imply that $H(q,p)$ constant.

For given $H(q,p) = H_0$, ϕ turns around at $\pm\phi_{\rm t} = \pm H_0/\lambda$.

Clock "frequency" λ will be large.



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Stationary states

$C=0$ implies $E=\sqrt{H^2-\lambda^2\phi^2}$, choosing $E>0$ (for now).

Stationary states ψ_k of $\hat{H}$:

$$i\hbar\frac{\mathrm{d}\psi_k(\phi)}{\mathrm{d}\phi}=\sqrt{E_k^2-\lambda^2\phi^2}\;\psi_k(\phi)$$

Solution: $\psi_k(\phi)=\psi_k(0)\exp(i\Theta_k(\phi))$ with

$$\Theta_k(\phi)=-\frac{1}{2\hbar}\left(\phi\sqrt{E_k^2-\lambda^2\phi^2}+\frac{E_k^2}{\lambda}\arcsin\left(\frac{\lambda\phi}{E_k}\right)\right)$$

Non-linear phase in exponential function replaces standard $\exp(-iE_kt/\hbar)$.

Phase $\Theta_k(\phi)$ real only if $|\phi|\leq\phi_{\mathrm{t}}=E_k/\lambda$: ϕ-evolution not unitary.

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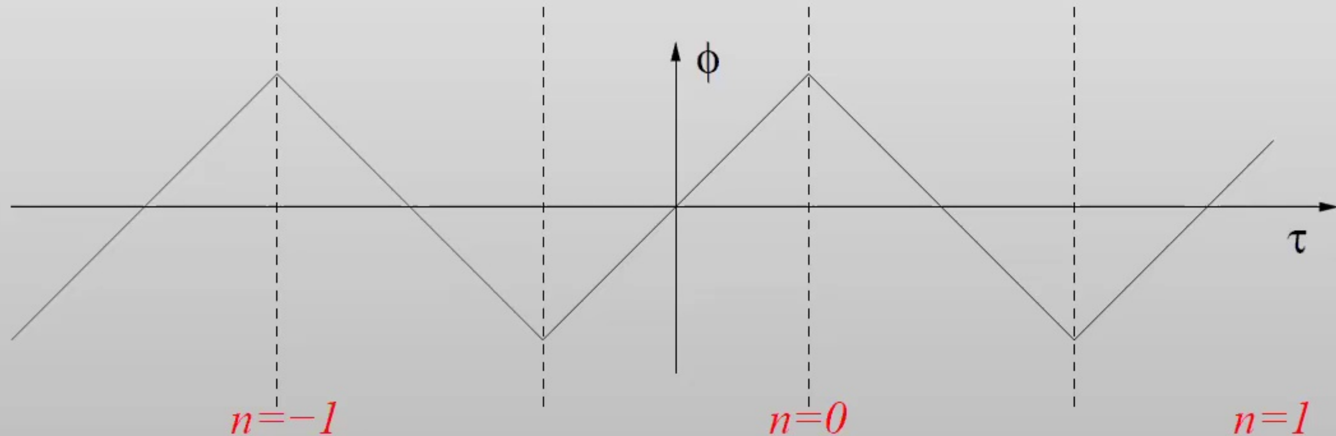
Unwinding time

For fixed k, define

$$\phi(\tau) = \begin{cases} \tau - 4n\phi_{\rm t} & \text{if } 4n-1 \le \tau/\phi_{\rm t} \le 4n+1 \\ (4n+2)\phi_{\rm t} - \tau & \text{if } 4n+1 \le \tau/\phi_{\rm t} \le 4n+3 \end{cases}$$

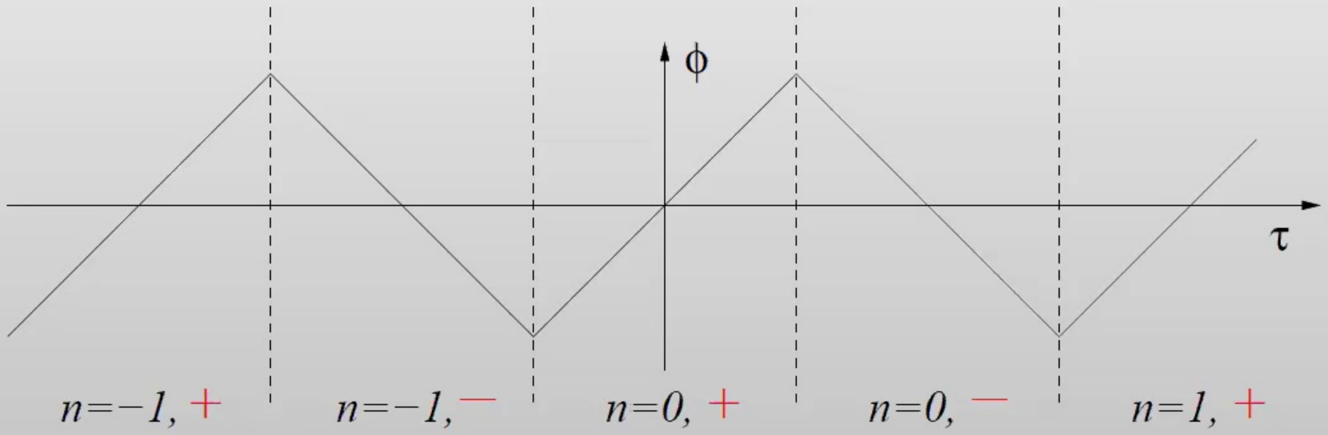
such that $|\phi(\tau)| \le \phi_{\rm t} = E_k/\lambda$ for all τ, $n = \left\lfloor \frac{1}{4}(1 + \tau/\phi_{\rm t}) \right\rfloor$ cycles.



Global evolution

$$i\hbar\frac{\mathrm{d}\psi_k(\tau)}{\mathrm{d}\tau} = \frac{\mathrm{d}\phi}{\mathrm{d}\tau}\hat{E}\psi_k(\tau) = \sqrt{E_k^2 - \lambda^2\phi(\tau)^2}\;\psi_k(\tau)$$

Replace $\Theta_k(\phi)$ with $\mathrm{sgn}(\mathrm{d}\phi/\mathrm{d}\tau)\Theta_k(\phi(\tau))$ in stationary solutions.



Superposition of stationary states $\psi_k(\tau)$ according to initial state.

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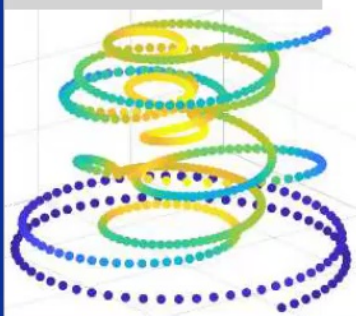
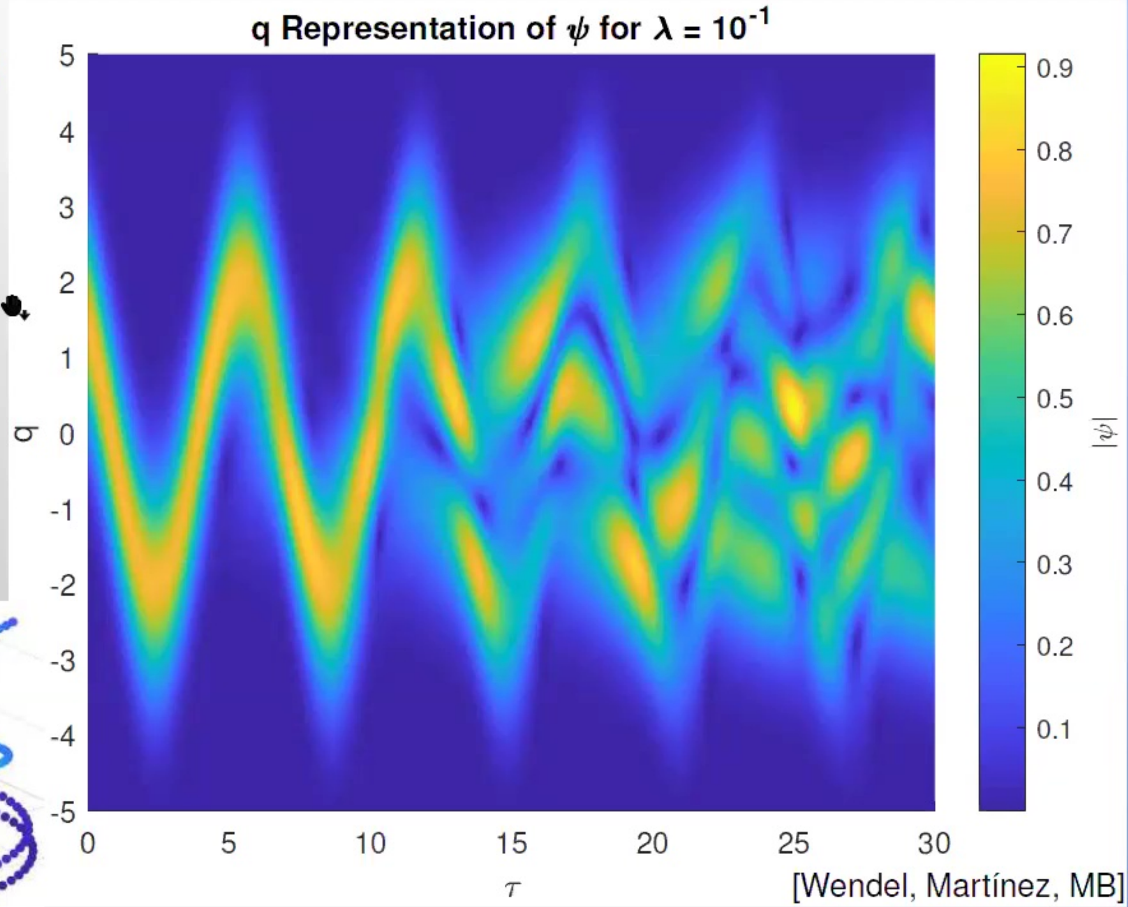
Lost coherence: Harmonic oscillator

$$\hat{H} = \frac{1}{2}(\hat{p}^2 + \hat{q}^2)$$

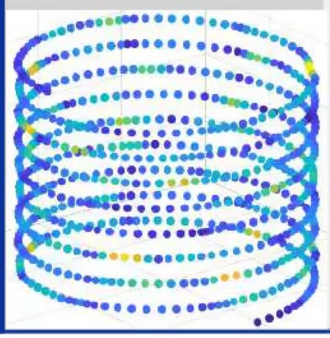


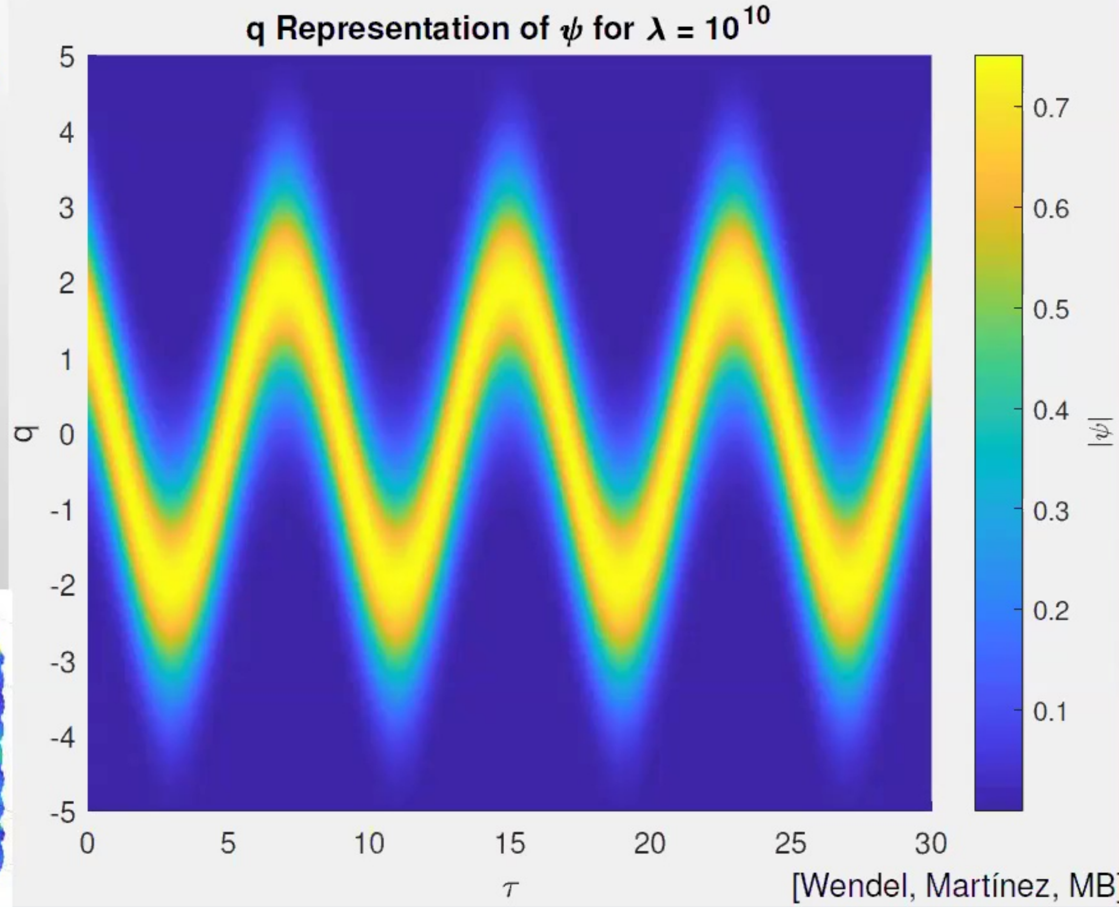


[Wendel, Martínez, MB]

Regained coherence: short clock period

$$\hat{H} = \frac{1}{2}(\hat{p}^2 + \hat{q}^2)$$





[Wendel, Martínez, MB]

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Rescaled system period

Large λ, short clock period.

Clock cycles:

$$\frac{n}{\lambda} = \frac{\lfloor 1/4(1+\lambda\tau/E_k)\rfloor}{\lambda} \sim \frac{\tau}{4E_k}$$

Accumulated phase at time τ:

$$\Delta\Theta_k = 2n(\Theta_k(\phi_t) - \Theta_k(-\phi_t)) = -\frac{\pi E_k^2}{\hbar}\frac{n}{\lambda} \sim -\frac{\pi}{4}\frac{E_k\tau}{\hbar}$$

System period $2\pi/\omega_k = 2\pi\hbar/E_k$ rescaled to $8/\omega_k$.

Not observable: Can be absorbed in “bare” system frequency.

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Phase variations

Non-linear $\Theta_k(\phi(\tau))$ at finite $\lambda \neq 0$:

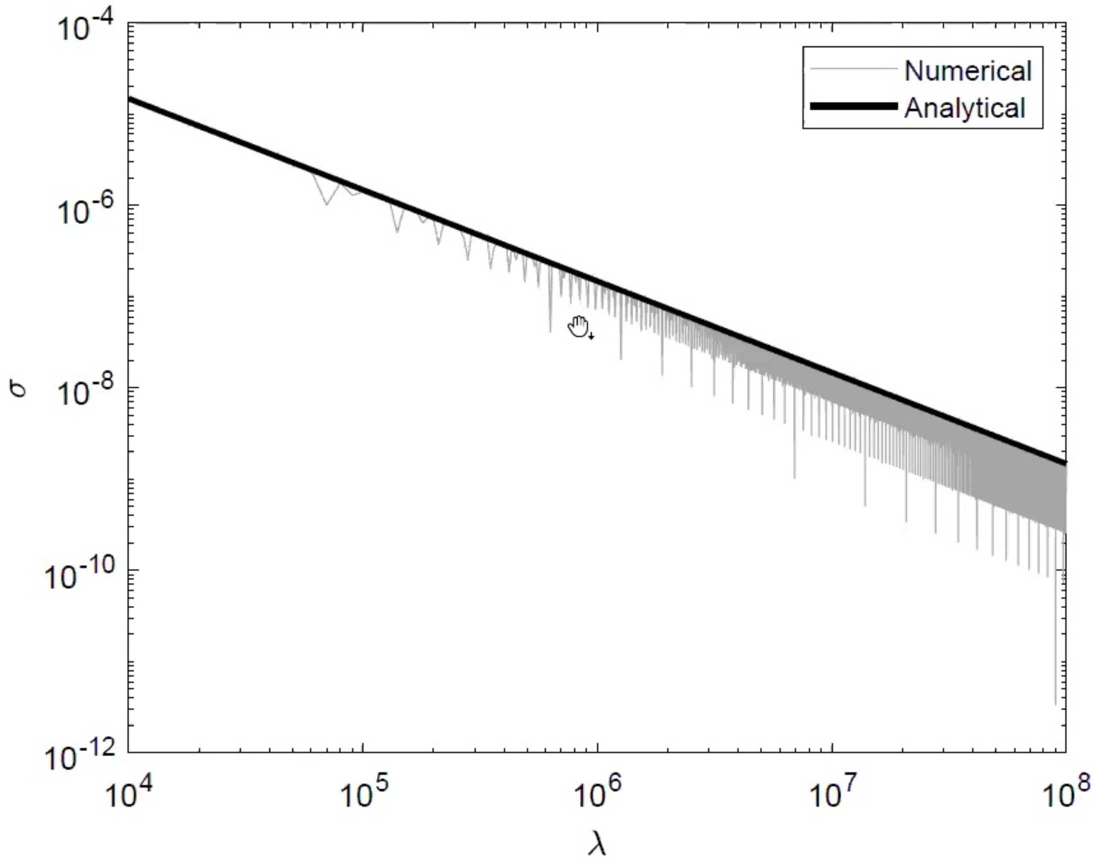
$$\begin{aligned}\sigma^2 &= \frac{1}{\phi_{\rm t}}\int_0^{\phi_{\rm t}} (\Theta_k(\phi(\tau)) - (\text{accumulated phase}))^2\, d\tau \\ &= \frac{E_k^4(21\pi^2 - 1024/5)}{24^2\lambda^2\hbar^2}\end{aligned}$$

Clock period $T_{\rm C} = 4\phi_{\rm t} = 4E_k/\lambda$ and system period $T_{\rm S} = 2\pi\hbar/E_k$ related by

$$T_{\rm C} = \frac{48\sigma T_{\rm S}}{\pi\sqrt{21\pi^2 - 1024/5}} \approx 9.7\sigma T_{\rm S} \qquad \text{for all } k$$

New dephasing effect largely model-independent. Requires non-linear $\Theta_k(\tau)$, always realized for ϕ-dependent Hamiltonian.

Standard deviation



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Upper bound

$T_\mathrm{C} \approx 9.7\sigma T_\mathrm{S}$ based on theory of fundamental clock.

If T_S can be measured with accuracy σ, then T_C cannot be greater than $9.7\sigma T_\mathrm{S}$.

Latest atomic clocks: $\sigma \approx 10^{-19}$ at system period of $T_\mathrm{S} \approx 2\,\mathrm{fs}$.

[Campbell et al. *Science* 358 (2017) 98]

Therefore,

$$T_\mathrm{C} < 2 \cdot 10^{-33}\,\mathrm{s} \approx 0.5 \cdot 10^{11} t_\mathrm{P}$$

Smallest direct measurement: Photon travel time across hydrogen molecule, $247 \cdot 10^{-21}\mathrm{s}$. [Grundmann *et al. Science* 370 (2020) 339]

Particle accelerators probe spatial distances of $10^{-20}\mathrm{m} \approx 10^{16}\ell_\mathrm{P}$. Corresponds to $\approx 10^{-28}\,\mathrm{s}$.

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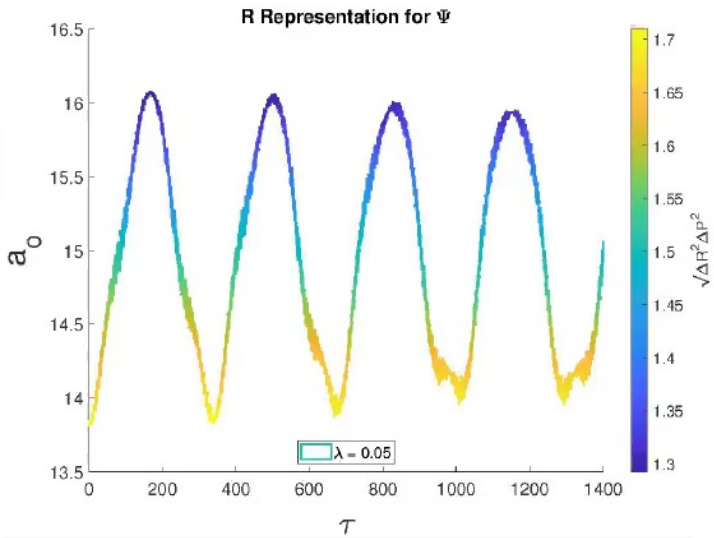
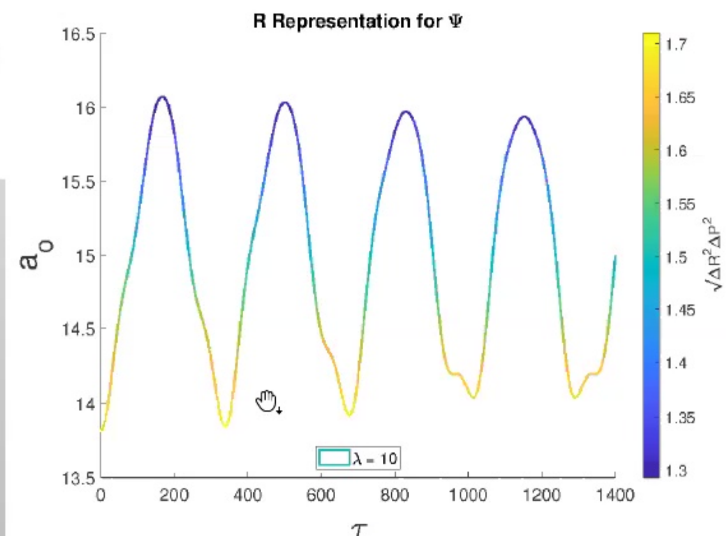
Anharmonic system or clock





Cosmology

Planck-mass clock field ϕ (not inflaton): [with Ding, Martínez]

$$C = -\frac{1}{2}\frac{p_\phi^2}{V} - \frac{1}{2}Vm^2\phi^2 + 6\pi G V p_V^2 + \cdots$$

→ Hubble friction: ϕ does not oscillate if $\dot{V}/V$ large ("small" V).

- Monotonic clock at Planck density: deparameterized.
- As V increases, first turning points of ϕ appear.
  Enhanced decoherence of V-state or matter state.

  Can this wash out trans-Planckian fatures?
- Stabilizes when mass term dominates.

→ Possible effective description at large λ: varying period.
New stochastic term of the form $(1+\xi)\mathrm{d}/\mathrm{d}t$, noise ξ.

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Quantum cosmology with periodic time

$$C = -\frac{1}{2}\frac{p_\phi^2}{V} - \frac{1}{2}Vm^2\phi^2 + 6\pi G V p_V^2 + \cdots$$

Quantization requires two non-commuting operators

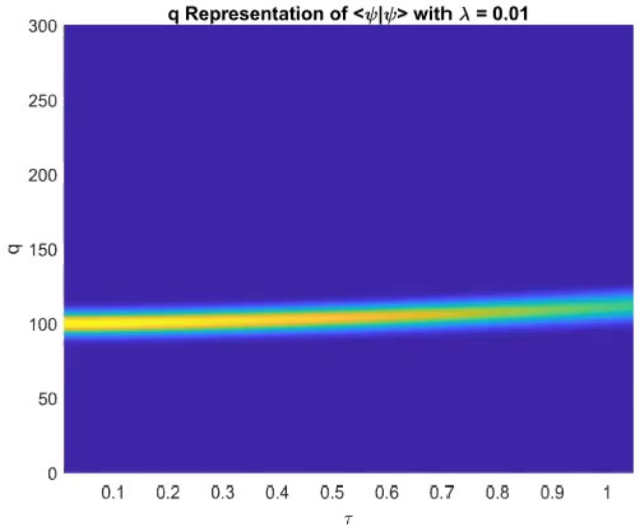
$$\hat{H} = \sqrt{12\pi G}\widehat{|V p_V|} \quad \text{and} \quad \hat{V}^2$$

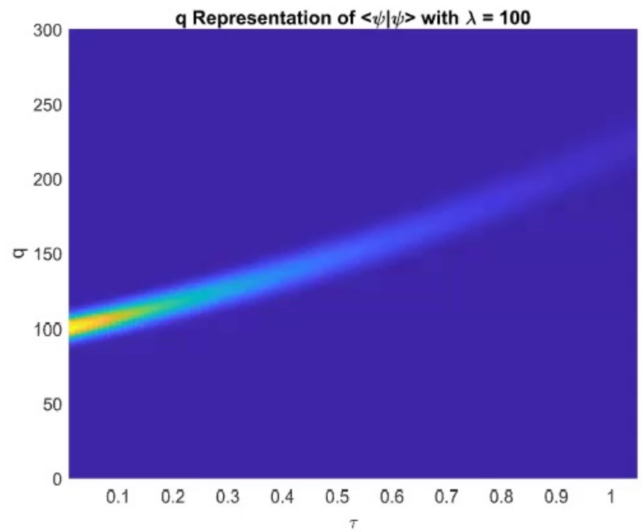
(or $\hat{V}$-dependent λ).

Stationary states of $\hat{H}$ do not decouple modes, ϕ-dependent evolution operator:

Numerics requires diagonalization at every time step, combined with implementation of turning-point condition for phase.

Small and large mass





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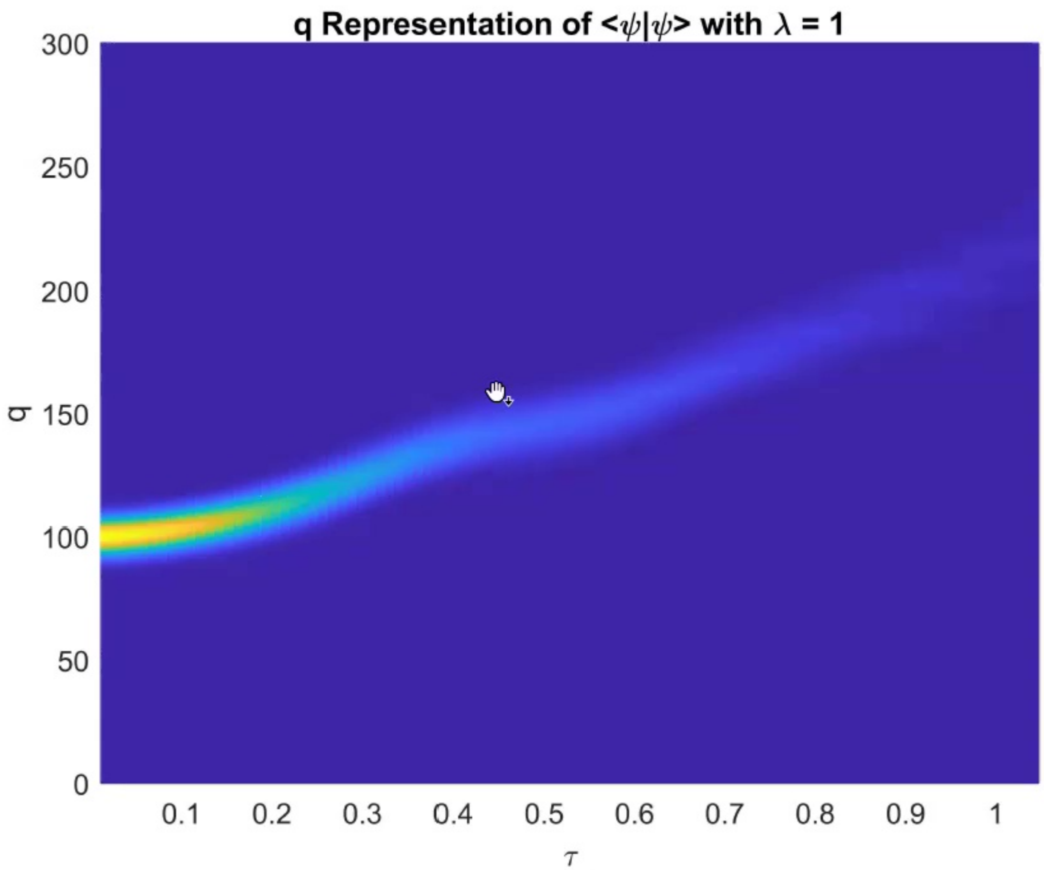
Intermediate mass



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Some open questions

- → Detailed cosmological analysis, inclusion of perturbative inhomogeneity.
- → Good effective theory to express discrete process of phase reflections in terms of observables for large λ.
- → Transformations between different clocks.