

Title: Emergent fractons in Elusive Bose Metal --- When IR theory blends with UV physics

Speakers: Yizhi You

Series: Quantum Matter

Date: January 18, 2021 - 2:00 PM

URL: <http://pirsa.org/21010005>

Abstract: The entanglement pattern of a quantum many-body system can be characterized by quasiparticles and emergent gauge fields, much like those found in Maxwell's theory. My talk begins with the basic aspects of symmetry fractionalization and emergent gauge fields in strongly correlated systems. I will further extend this paradigm into a new type of quantum many-body state, dubbed "fracton phase," from a quantum melting transition of plaquette paramagnetic crystals. These exotic states contain fractionalized sub-dimensional quasiparticles with constraint motion and emergent higher-rank gauge fields. Such constraint dynamics of the quasiparticles bring about an intriguing Bose metal phase with quasi-long range order and yields non-local quantum entanglement. In particular, the key peculiarities of this phase is the UV/IR mixing, where the short wavelength physics controls the low energy theory and hence challenges the standard notion of the renormalization group perspective.

# Emergent Fractons in Elusive Bose Metals

Yizhi You

Princeton University



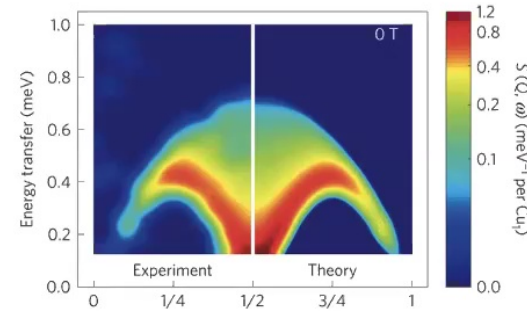
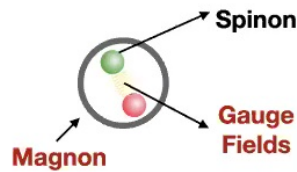
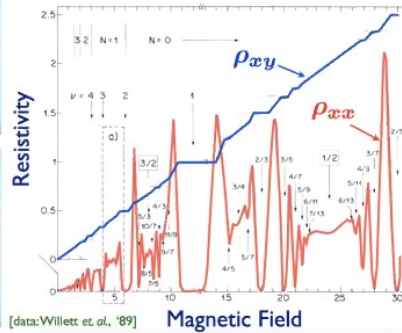
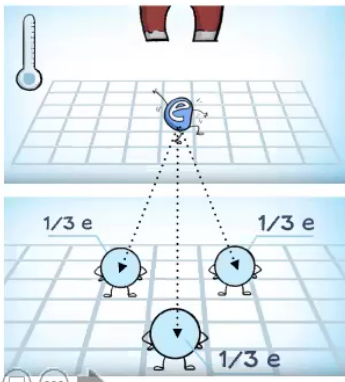
(UV) Hamiltonian  $\rightarrow \rightarrow \rightarrow$  (IR) effective field theory

Emergence!

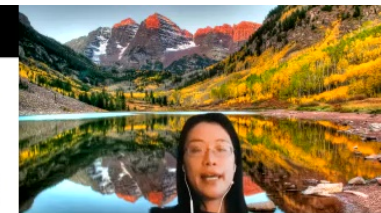
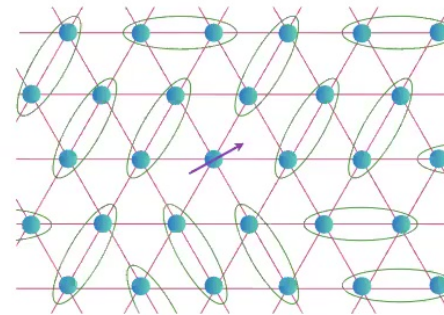
Emergent phenomena  $\rightarrow$  richer context

✓ Fractionalization

✓ Emergent gauge field



$$\Psi = \sum \text{[Diagram of a lattice of red ellipsoids representing a spin configuration]}$$



# Outline

- ◆ Emergence of fractionalization?
- ◆ Effective theory to describe them?

1) Melting transition of Valence Bond Solid

Critical theory = spinon + emergent gauge field

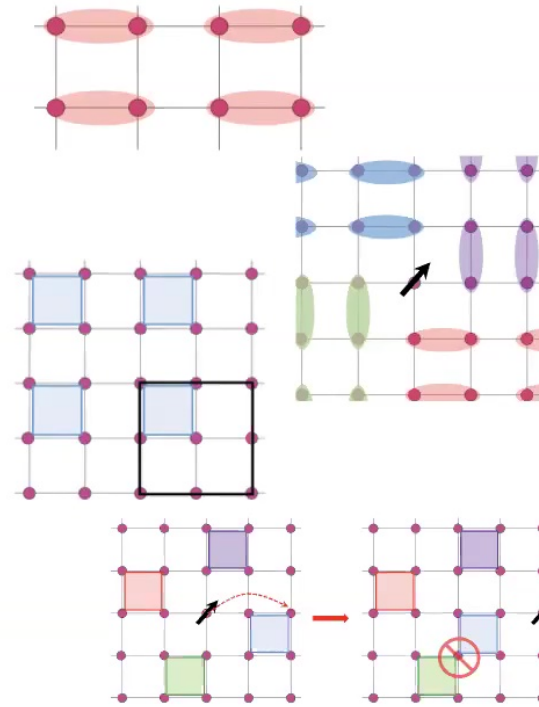
2) Phase transition: Valence Plaquette Solid  $\rightarrow$  Bose metal

✓ Spinon with **constrain dynamics**

✓ Emergent gauge field: **Higher-rank gauge theory**

◆ UV-IR mixing!

◆ Short wavelength physics control IR





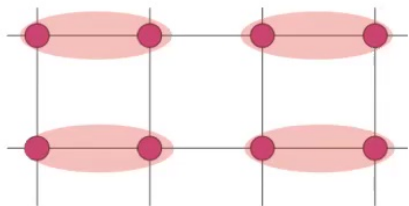
## Emergence of fractionalization & Gauge fields?    Where to find them?

### Valence Bond Solid

- ✓ Paramagnetic crystal
- ✓ Breaks  $T_x$  and  $C_4$

*The Quantum melting of VBS?*

**= Disorder the VBS pattern ?**



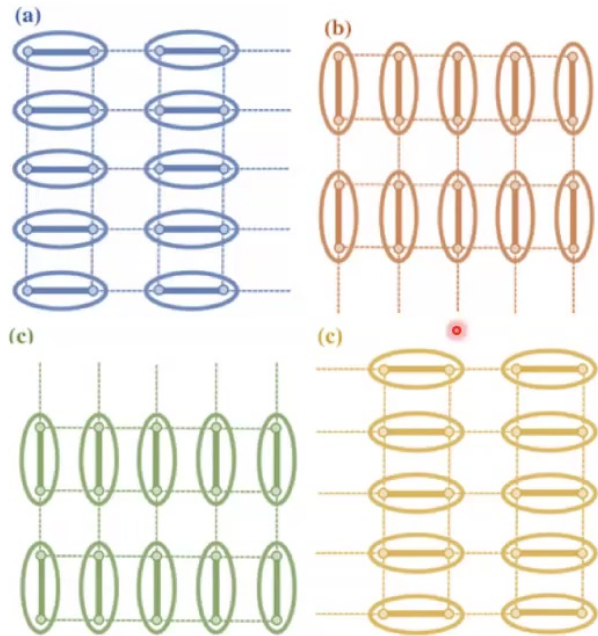
$$\text{oval} = \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow)$$

→ develop topological defects

→ fluctuation between distinct patterns

**Magic appears!**

## Valence Bond Solid

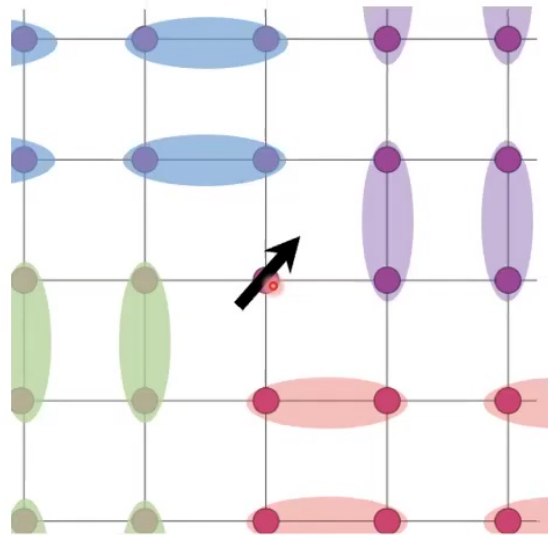


4 patterns, related by  $C_4$  rotation





## VBS Pattern Melting



$Z_4$  Vortex defect of VBS

Destroy the VBS (*Senthil-2004*)

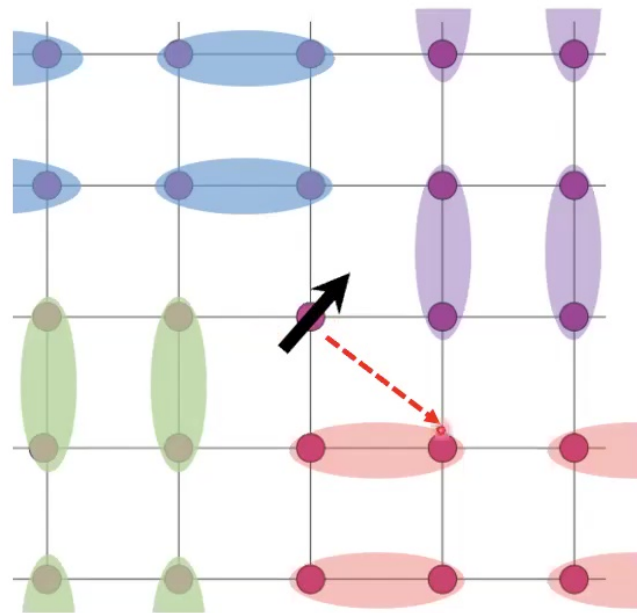
→ Proliferate the defect (vortex)

Defect of the VBS order:  
Carry a **spinon** !

Spinon: Carry  $\frac{1}{2} S_z$  charge  
→ fractionalization at QCP!

Compare to Magnon excitation:  $\uparrow \dots \rightarrow \downarrow$   
Change  $S_z$  number by integer!

**Where is the emergent gauge field?**



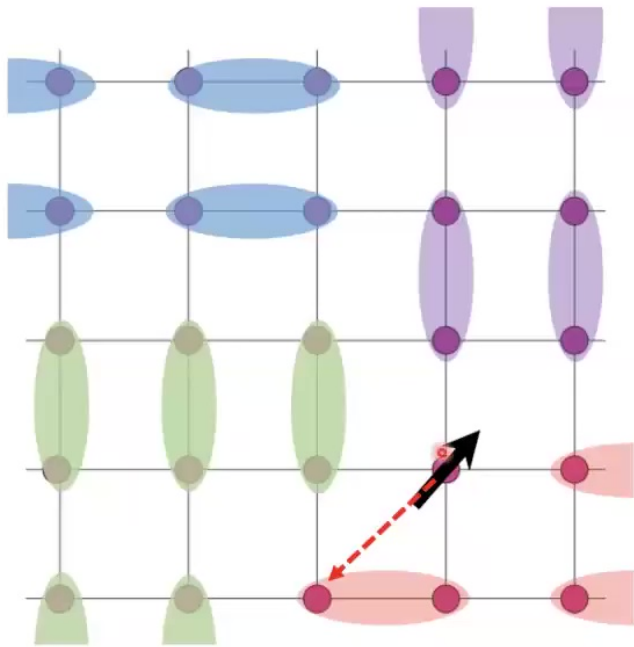
## Spinon's kinetics

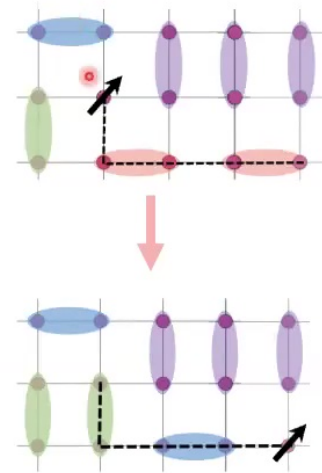
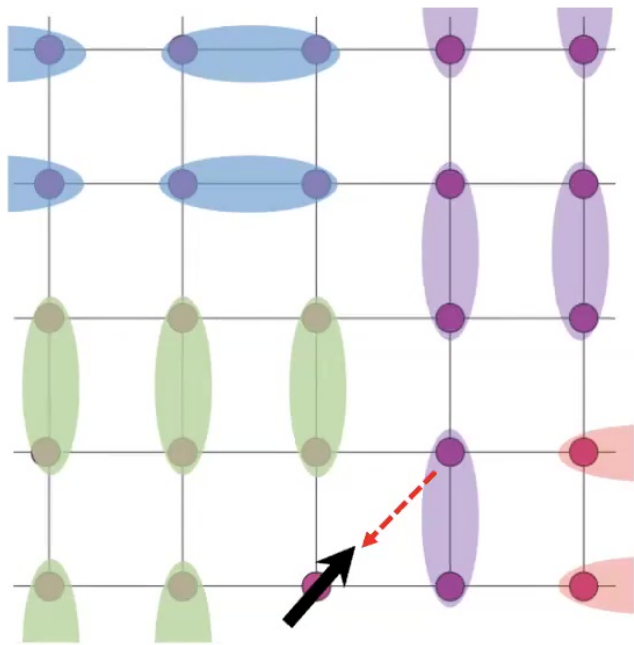
If spinon is energetically deconfined,

It can fluctuate in the dimer background!









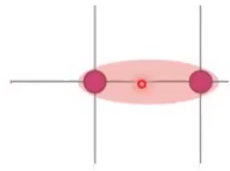
Moving spinon requires changing VBS pattern !

→ Emergent gauge field



### Mapping (Moessner, Kivelson)

Dimer  $\leftrightarrow$  U(1) gauge theory



VB coverage =  $\mathbf{E}$  field

$$E_i(\mathbf{r}) = (-1)^{i_r} D_i(\mathbf{r})$$

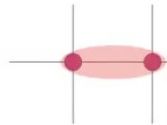
Gauss Law  $\rightarrow$  charge conservation

$$\partial_i E_i(\mathbf{r}) = (-1)^{i_r} (1 - q(\mathbf{r}))$$

No dimer adjacent to site  $\rightarrow$  spinon

1 dimer adjacent to site  $\rightarrow$  no spinon

### Emergent gauge field $\leftrightarrow$ constraint !!!

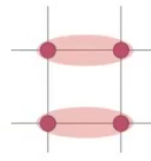


$q=0$

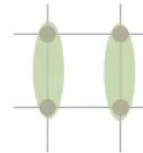


$q=1$

$$[A_i(\mathbf{x}), E_j(\mathbf{y})] = \frac{i}{2\pi} \delta_{ij} \delta_{\mathbf{x}\mathbf{y}} \quad \text{Create/annihilate VB}$$



$e^{iB}$



VB fluctuation =  $\mathbf{B}$  field

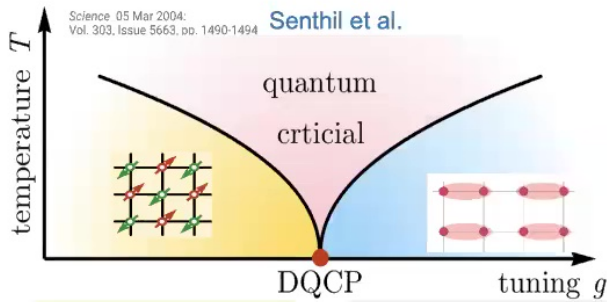
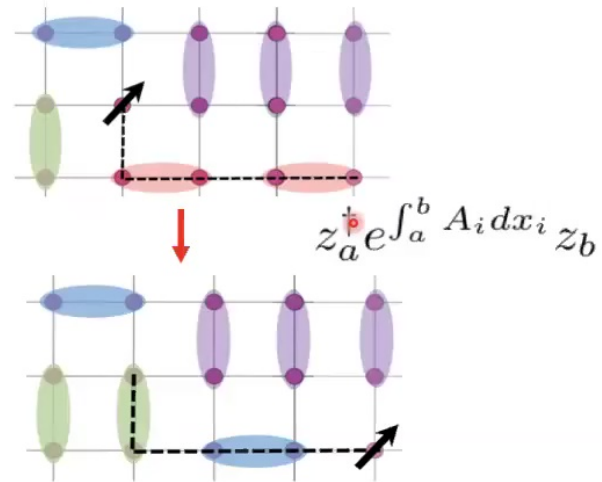




## Emergence of fractionalization & Gauge fields?

- ✓ Spinon couple with the **emergent gauge field**
- ✓ Moving spinon requires changing VBS pattern

VBS melting → VBS defect + spinon condensate  
 condensate → Magnetic order

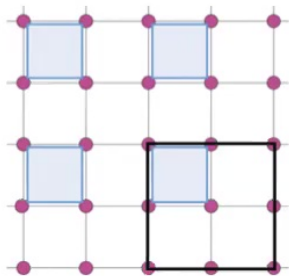


Deconfined  
Quantum  
Criticality



- ✓ Spinon ~ half-Magnon
- ✓ Emergent U(1) gauge theory

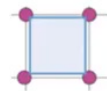
## Beyond fractionalization & Gauge fields?



### Valence Plaquette Solid

✓ Breaks  $T_x$ ,  $T_y$  and  $C_4$

*(SrCu<sub>2</sub>(BO<sub>3</sub>)<sub>2</sub>)*



$|1010\rangle + |0101\rangle$

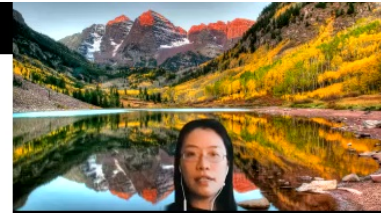
## The Quantum melting of VPS?

✓ Fractionalization

Spinon with **constraint dynamics**

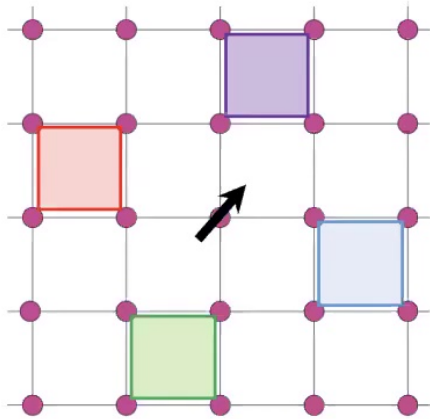
✓ Emergent gauge field

**Higher-rank gauge theory**

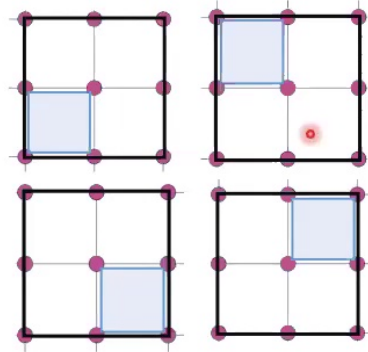


# VPS melting !

YY-2019



$Z_4$  Vortex defect of VPS



Four Valence Plaquette patterns

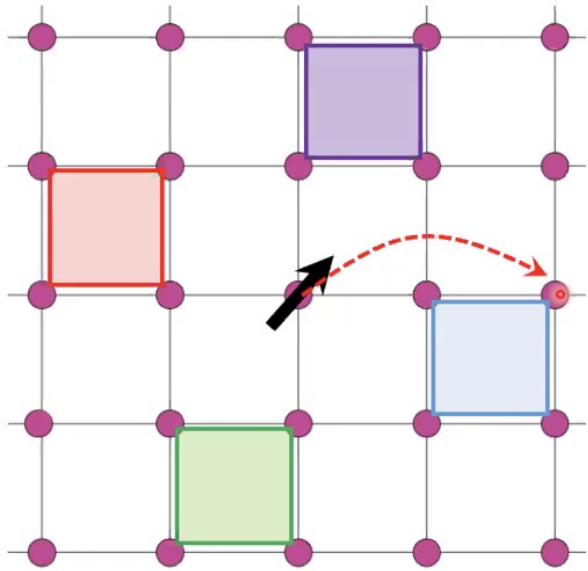
Defect of the VPS order: Carry a **spinon** !

**Spinon: Carry  $\frac{1}{2} S_z$  charge  $\rightarrow$  fractionalization!**

**Where is the emergent gauge field?**

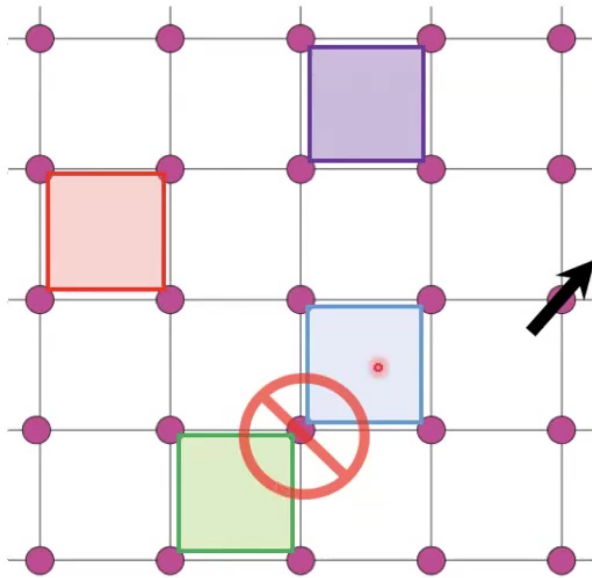
*Look for the spinon's kinetics and VPS pattern change!*





Spinon's kinetics

How to move a spinon???

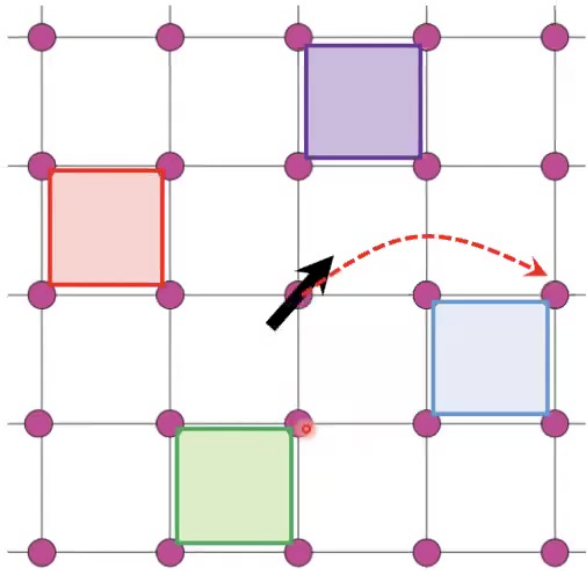


How to move a spinon???

Spinon is stuck?

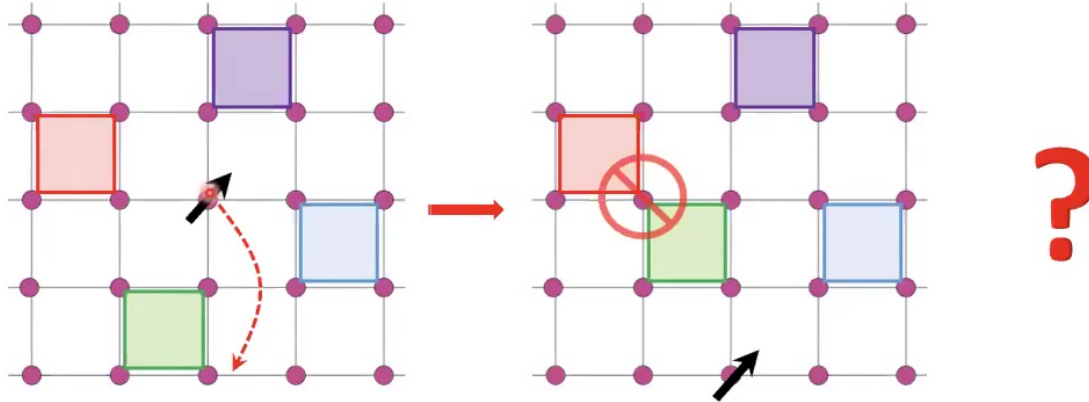






Spinon's kinetics

How to move a spinon???

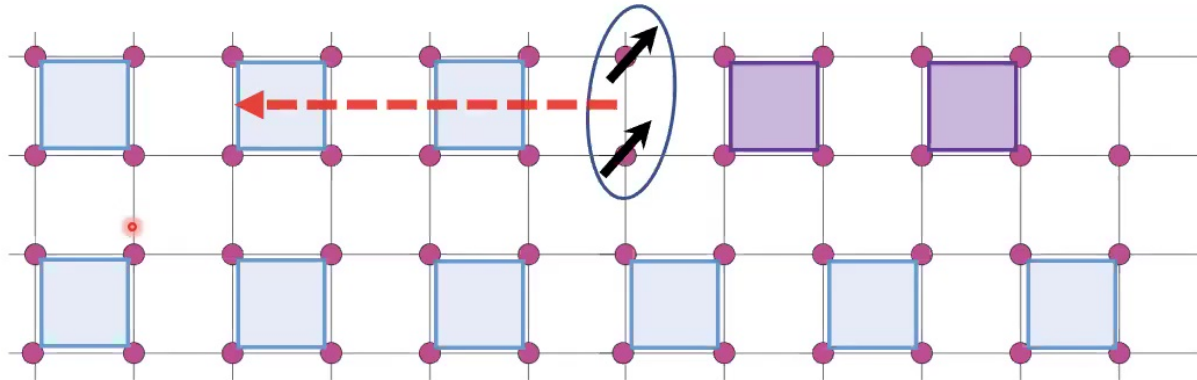


Spinon is stuck! Cannot move!  
**Immobile particle → Fracton**

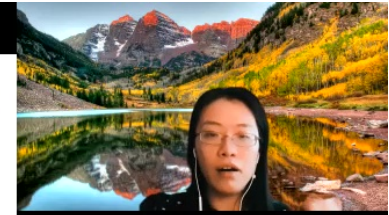
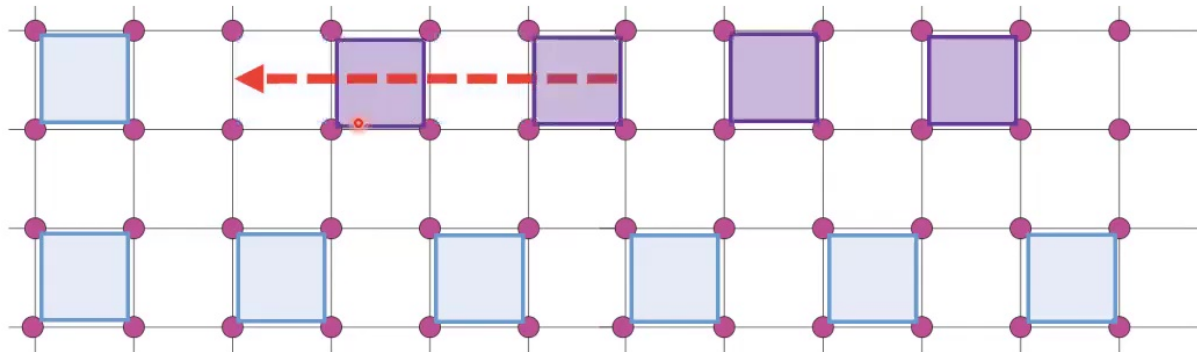
## Fracton Phases of Matter

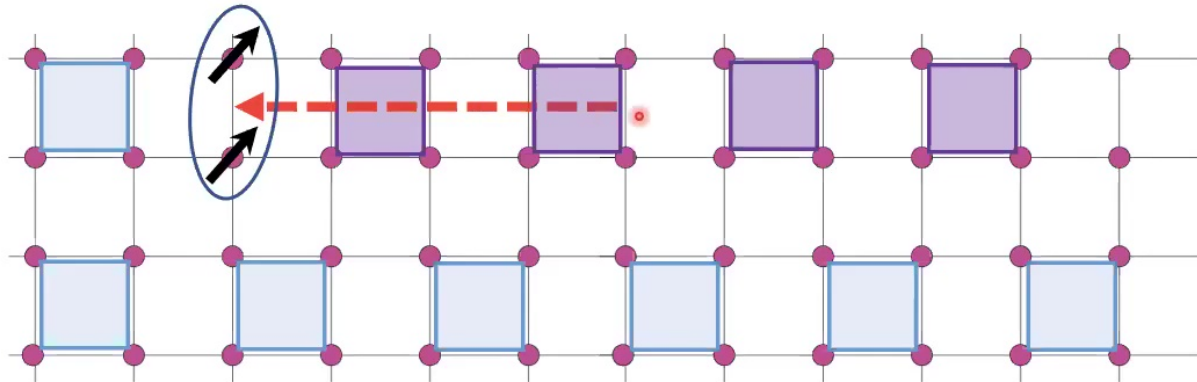
Michael Pretko, Xie Chen, Yizhi You

Fractons are a new type of quasiparticle which are immobile in isolation, but can often move by forming bound states.



How to move a spinon-pair (dipole)???



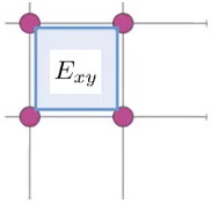


Moving dipole requires changing VPS pattern !

→ Emergent gauge field?

## Mapping (YY-2019)

VP  $\leftrightarrow$  Higher-rank gauge theory



•  
VP coverage  
=  $E_{xy}$  field

$$E_{xy}(\mathbf{r}) = (-1)^{i_r} P(\mathbf{r})$$

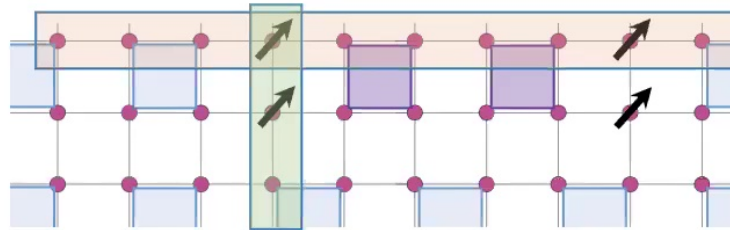
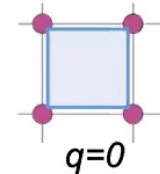
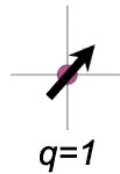
'Gauss Law'  $\rightarrow$  Charge Conservation

$$\partial_x \partial_y E_{xy}(\mathbf{r}) = (-1)^{i_r} (1 - q(\mathbf{r}))$$

Charge conservation on each row !

$$\int dx_i \rho = 0 \quad \text{Subsystem symmetry}$$

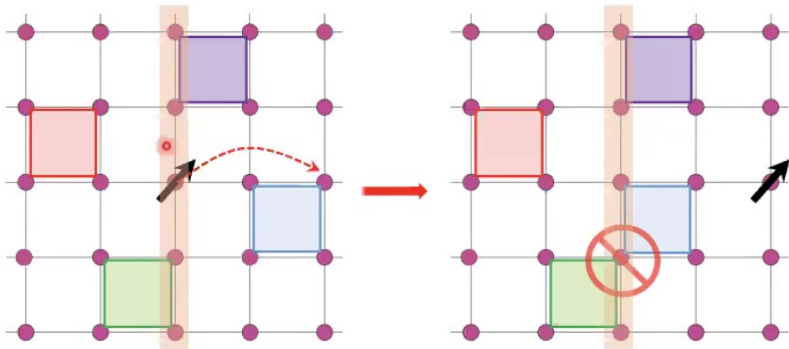
Emergent gauge field  $\leftrightarrow$  constraint !!!



Spinon inside VPS defect appear in quartet

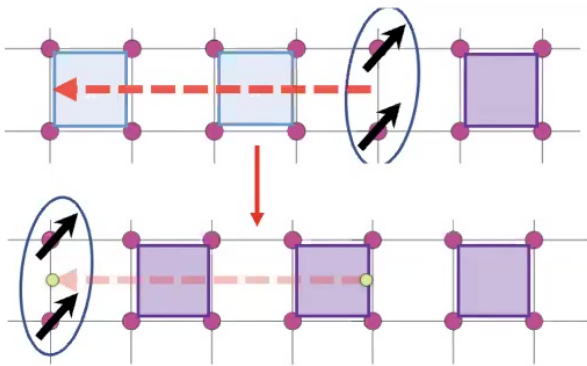
Conserved in each row/column!





Spinon **is a fracton**:  
Cannot move!

Charge conserved on  
row/columns



✓ A pair of spinon (dipole) **only**  
fluctuate along the transverse  
stripe!

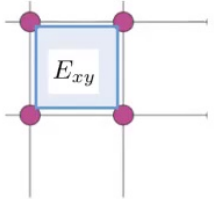
✓ Such dynamics is accompanied  
by the plaquette flipping along the  
path





## Mapping

VP  $\leftrightarrow$  Higher-rank U(1) gauge theory



VP coverage  
=  $E_{xy}$  field

$$E_{xy}(\mathbf{r}) = (-1)^{i_r} P(\mathbf{r})$$

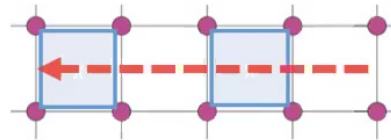
'Gauss Law'  $\rightarrow$  Charge Conservation

$$\partial_x \partial_y E_{xy}(\mathbf{r}) = (-1)^{i_r} (1 - q(\mathbf{r}))$$

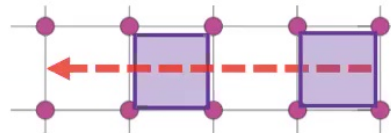
Charge conservation on each row !

$$\int dx_i \rho = 0 \quad \text{Subsystem symmetry}$$

$$A_{xy} \rightarrow A_{xy} + \partial_x \partial_y \alpha, \quad \text{Create/annihilate VP}$$



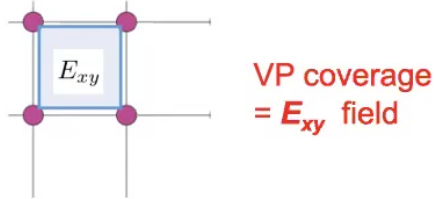
$$e^{i \int dy A_{xy}} \quad \text{VPS Pattern flip}$$





## Mapping

VP  $\leftrightarrow$  Higher-rank U(1) gauge theory



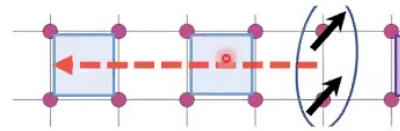
$$E_{xy}(\mathbf{r}) = (-1)^{i_r} P(\mathbf{r})$$

'Gauss Law'  $\rightarrow$  Charge Conservation

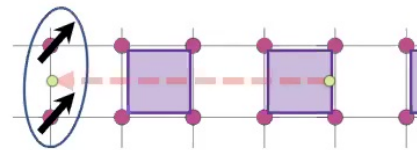
$$\partial_x \partial_y E_{xy}(\mathbf{r}) = (-1)^{i_r} (1 - q(\mathbf{r}))$$

Charge conservation on each row !

$$\int dx_i \rho = 0$$



$$z_r^\dagger z_{r+e_x} e^{i \int dy A_{xy}} z_{r+y} z_{r+e_x+y}^\dagger$$

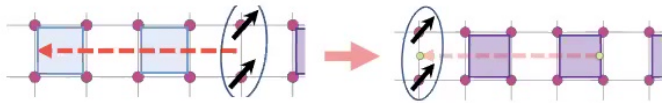


✓ Moving dipole requires changing VPS pattern along the path!

$\rightarrow$  Dipole current minimal couple with emergent **higher-rank gauge field!**

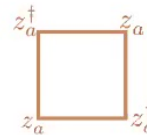


# What happens after plaquette melting? (YY-2020)



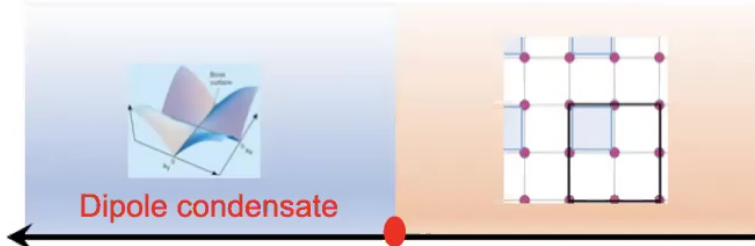
Gaussian theory

$$\mathcal{L} = \frac{K}{2} \sum_{a=1,2} (\partial_t \theta_a)^2 - \frac{K}{2} \sum_{a=1,2} (\partial_x \partial_y \theta_a + A_{xy})^2$$



Dipole fluctuation = ring-exchange

✓ Dipole: fluctuate in **1d**, condense?  
 → No long-range order → Mermin-Wagner theorem  
 → Quasi-long-range order between dipoles!



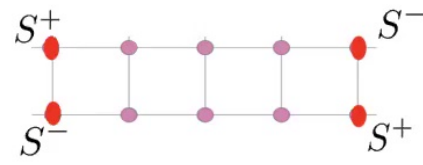
Algebraic liquid

VPS

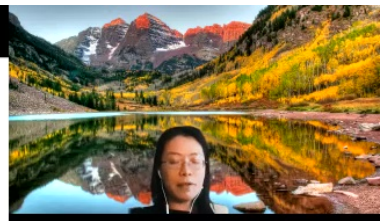
Fractonic KT transition

$$\langle S^+(r) S^-(r + e_y) S^+(r + x) S^-(r + e_y + x) \rangle$$

$$= \frac{1}{(x)^{1/(K\pi^2)}}$$



**Algebraic Liquid phase = Elusive 2D Bose metal**



## Elusive Algebraic Liquid phase: A Bose metal

**'Bose Fermi surface'!** (Fisher-2005, Balents-2004, YY-2019)

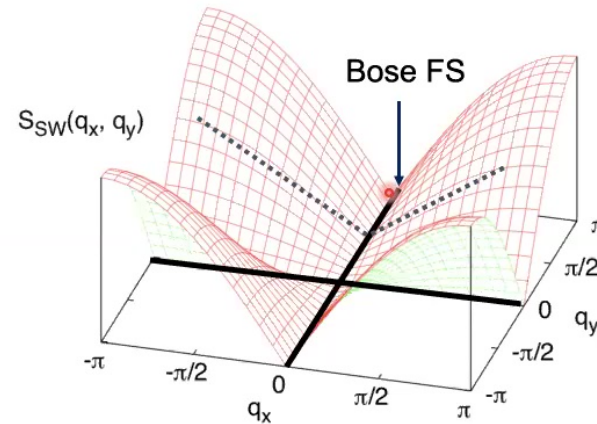
✓ **Zero-energy branch line on  $k_x, k_y$  axis**

Compare to 2D Fermi surface (zero energy points forms a close line)

**Transport: Specific heat**

$$C_v \sim T \ln(1/T)$$

✓ **Akin to marginal non-Fermi liquid**

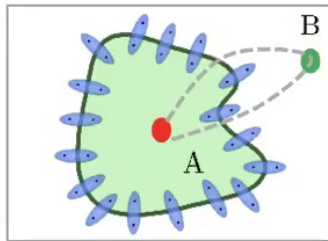


$$E \sim k_x k_y$$

**Get a 'Fermi surface' from boson!!!**



## Quantum information viewpoint – Entanglement & Mutual information



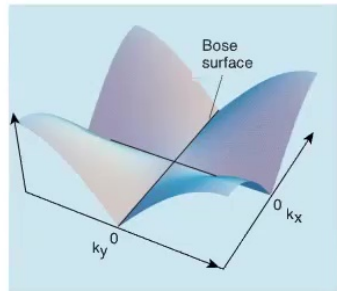
$$\rho_A = \text{Tr}_{\bar{A}} |\Psi\rangle\langle\Psi|$$

$$S(A) = -\text{Tr}_A (\rho_A \ln \rho_A)$$

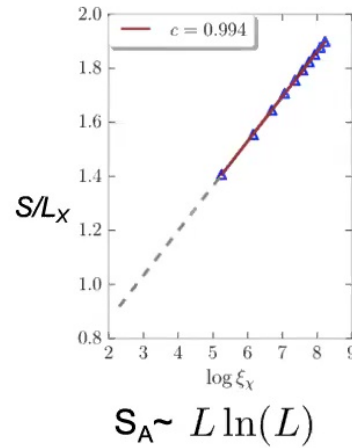
Entanglement Entropy

2D GS  $\rightarrow$  Entanglement area law

$EE \sim L$  (Length of the boundary)



Algebraic Liquid phase



### Algebraic Liquid phase

✓ Violation of the area law !  $L \ln(L)$

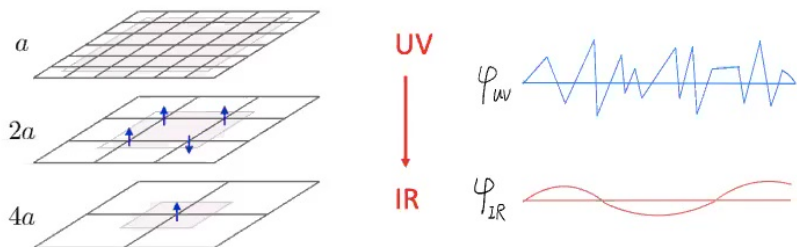
✓ long-range mutual information!

✓ Similar to 2d FS

(YY-Pollmann, YY-2019-PRR)

# Critical phase beyond Renormalization Perspective?

## Universality of critical phenomenon



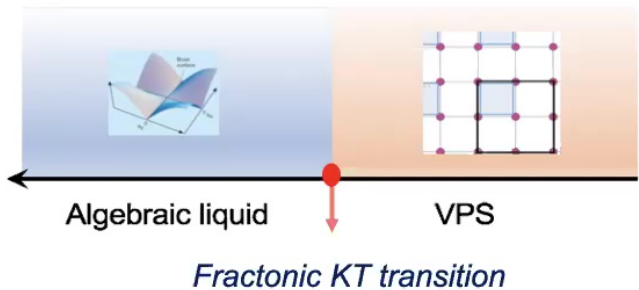
$$\langle C(0,0) C(\vec{r},0) \rangle = \frac{1}{r^{D-2+\eta}}$$

Universal scaling at QCP

Coarse grained = keep long wave-length

→ low energy = long wave-length

→ Scaling & critical exponent is universal



Charge correlator at the algebraic liquid phase

$$\langle \hat{n}(x,y,0) \hat{n}(0,0,0) \rangle = \frac{1}{(xy)^2}$$

$$\langle S^+(0) S^-(t) \rangle \sim e^{-a(\ln(t))^2}$$

$$\langle \partial_x S^+(0) \partial_x S^-(t) \rangle \sim \frac{1}{t^\alpha}$$

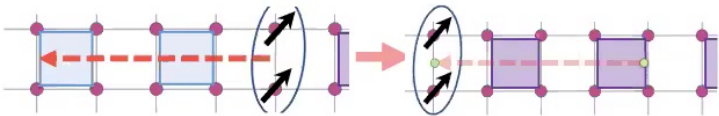
?





# Why UV blend with IR ?

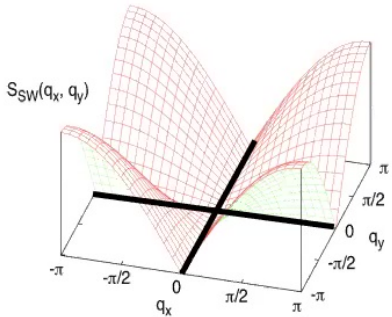
low energy modes from short wave-length physics



$$\mathcal{L} = \frac{K}{2} \sum_{a=1,2} (\partial_t \theta_a)^2 - \frac{K}{2} \sum_{a=1,2} (\partial_x \partial_y \theta_a + A_{xy})^2$$

Subsystem symmetry → local fluctuation with zero energy

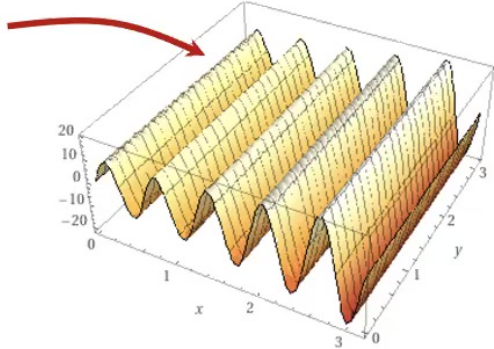
→ low energy modes from short wave-length physics → Bose surface (E=0 at large k)



Rough field fluctuation at low energy

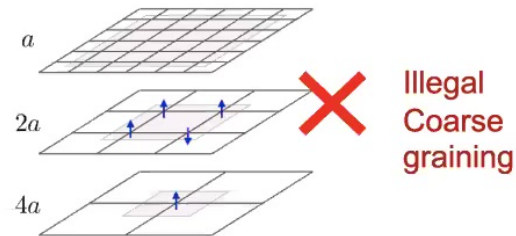
→ Short wave-length physics enter IR

UV-IR mixing! (Seiberg-2020)



## Short wave-length physics enter IR → UV-IR mixing!

Subsystem symmetry → local fluctuation zero energy  
→ low energy modes from short wave-length physics



UV-IR mixing → short wave-length physics survives at low energy  
→ IR theory affected by UV

✓ *Critical exponent independent of spacetime, emergent fractal dimension* Zhou, Pollmann, You, to appear

✓ *EFT Depends on UV cut-off*

✓ *Higher order operators could be more relevant!*

**Originate from Fracton !**

**New quantum field theory: IR blend with UV!**

(You-2019,  
Seiberg-2020,  
Karch-2020)



*Quasiparticle  
with restrict motions*



# Fracton



*Subsystem symmetry*

## New QFT



- + UV-IR Mixing
- + 'IR' theory: dominate by short wave-length
- + New fixed point and criticality beyond RG

## Novel phases



- + Fracton topological order
- + Fracton spin liquid, Fracton spin ice
- + Fracton topological insulator

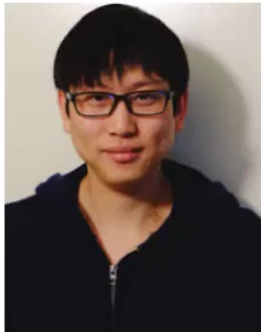
## Application



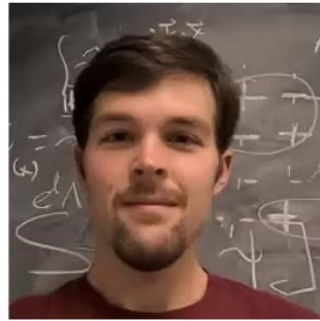
- + Non-ergodic behavior (MBL, Scar....)
- + Glassy dynamics
- + Quantum qubits for information storage



# Thank You!

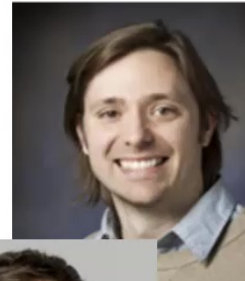


Zhen Bi (MIT)



Mike Pretko (Boulder)

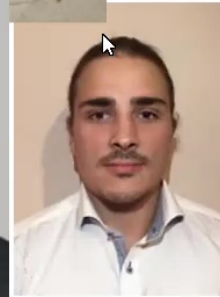
VPS transitions Phys. Rev. Research **2**,013162



Hughes  
(UIUC)



Pollmann  
(TUM)



Bibo  
(TUM)

*Emergent fracton at QCP,*  
[arXiv:2008.01746](https://arxiv.org/abs/2008.01746)

