Title: General constraints on metals

Speakers: Dominic Else

Series: Quantum Matter

Date: January 11, 2021 - 2:00 PM

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Abstract: Metals are ubiquitous in nature. One would like to determine the effective field theory that describe the low-energy physics of a metal. Many materials are successfully described by the so-called "Fermi liquid theory", but there is also much interest in "non-Fermi liquid metals" that evade such a description.

In this talk, I will present a very general perspective on metals that strongly constrains the possible effective field theories. The discussion is based on powerful theoretical concepts such as emergent symmetries and anomalies. From this perspective, combined with experimental observations, one can derive strong and unexpected conclusions about the nature of a particular kind of non-Fermi liquid metal, the "strange metal" observed in doped cuprates.

General constraints on metals

Dominic Else (MIT)

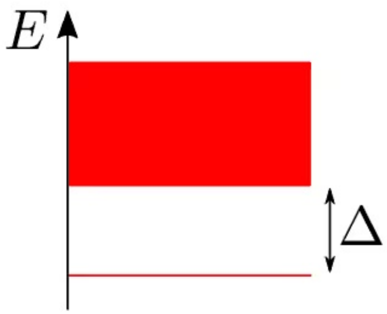
Colloquium, Perimeter Institute

January 11, 2021

An important dichotomy

Insulator

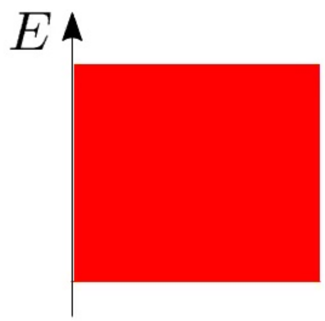
Conductivity $\sigma \sim e^{-T/\Delta}$



Gapped

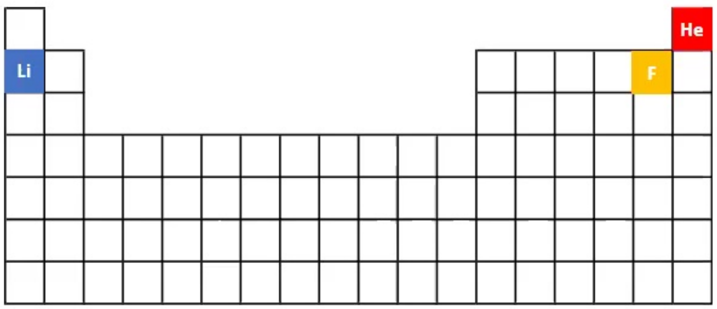
Metal

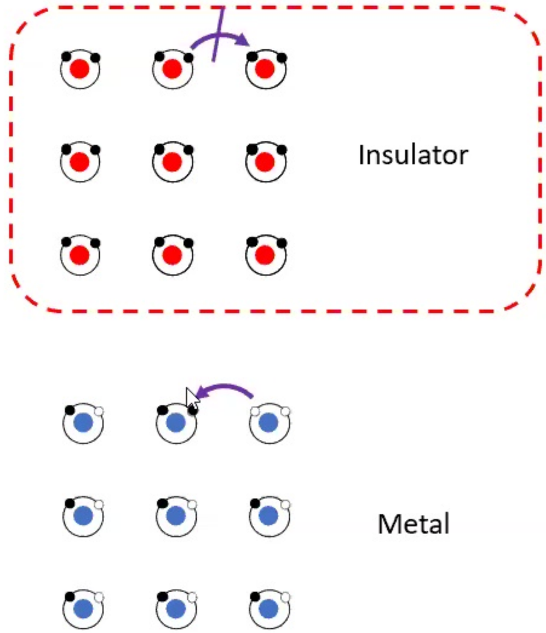
Conductivity $\sigma \sim T^{\alpha}$



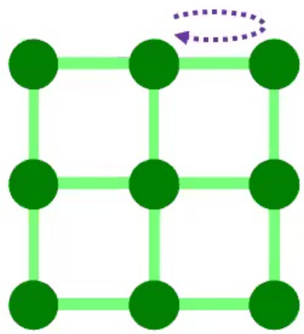
Gapless

What determines whether a material is a metal or an insulator?





Trivial vs. topological insulators

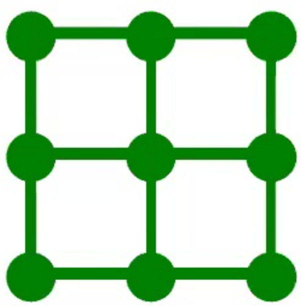


“Trivial” insulator

Ground state is entangled state

$$|\Psi\rangle \neq |\phi\rangle^{\otimes N}$$

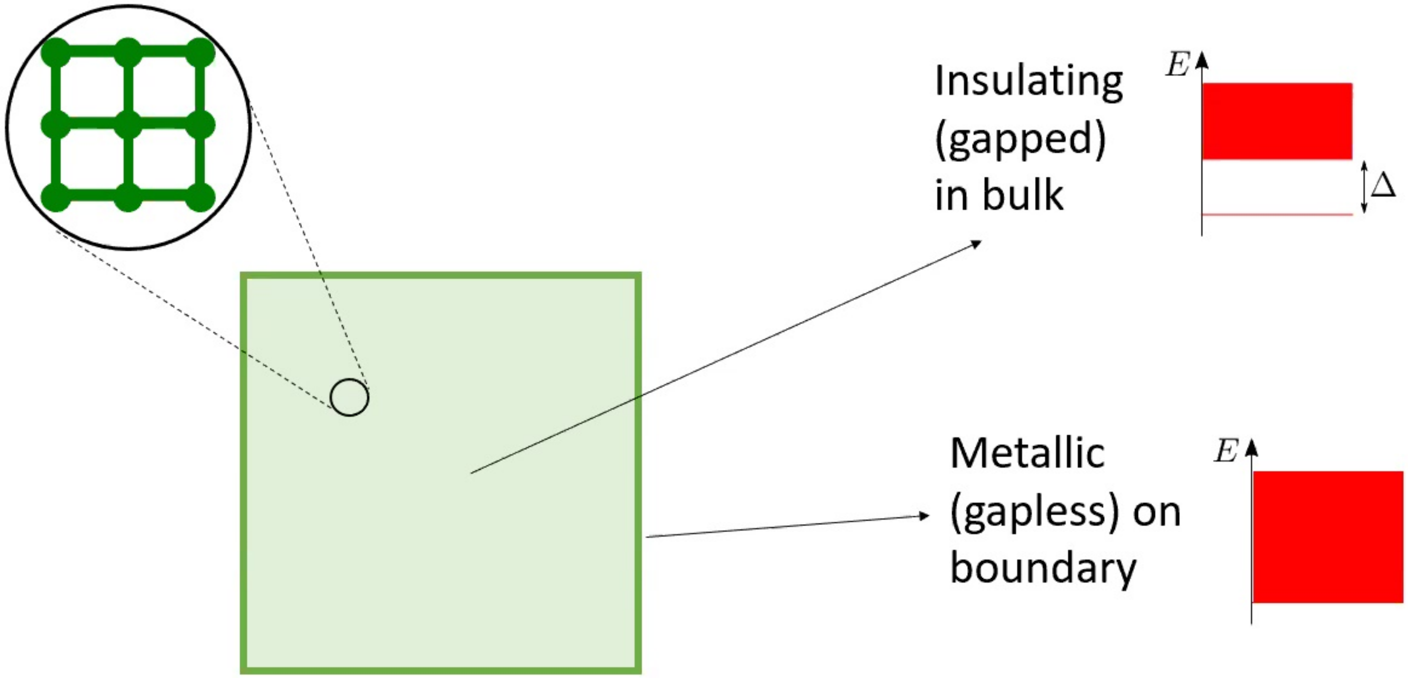
(but still a continuous deformation of a product state)



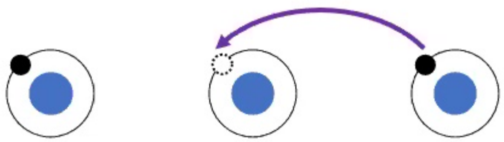
Topological Insulator

Ground state is in a *distinct phase of matter* from a trivial insulator

Topological insulator (“integer type”)



Metals



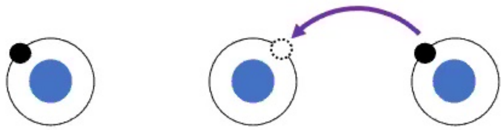
ν "Filling"

Average number of electrons per translation unit cell

For a metal, ν should not be an integer

What is the nature of the ground state?

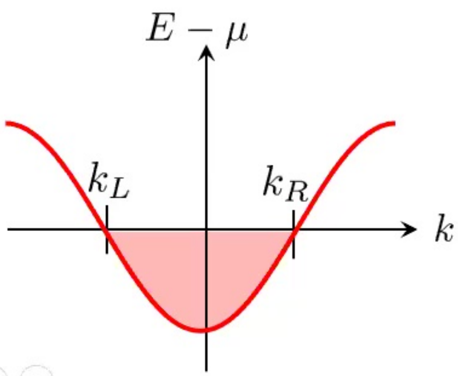
The non-interacting approximation



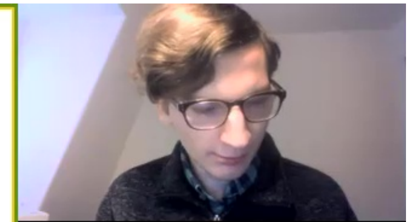
Ignore Coulomb repulsion between electrons

$$H = \sum_{i,j} t_{ij} c_i^\dagger c_j$$

Free electron problem

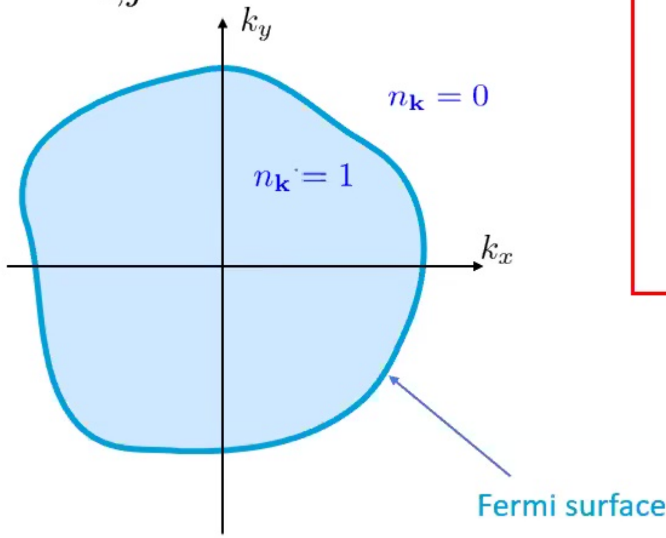


ν not being an integer ensures that the band is not fully filled



Fermi gas in 2D

$$H = \sum_{i,j} t_{ij} c_i^\dagger c_j$$



But what is the effect of including Coulomb repulsion?

Need to go beyond the non-interacting approximation

Effective field theory

UV Detailed microscopic physics:

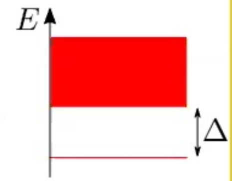
Electrons hopping between atoms + Coulomb repulsion

Renormalization-group (RG) flow

IR Low-energy, long-wavelength physics

can be described by an "effective field theory"

Trivial insulator:
"Null field theory"

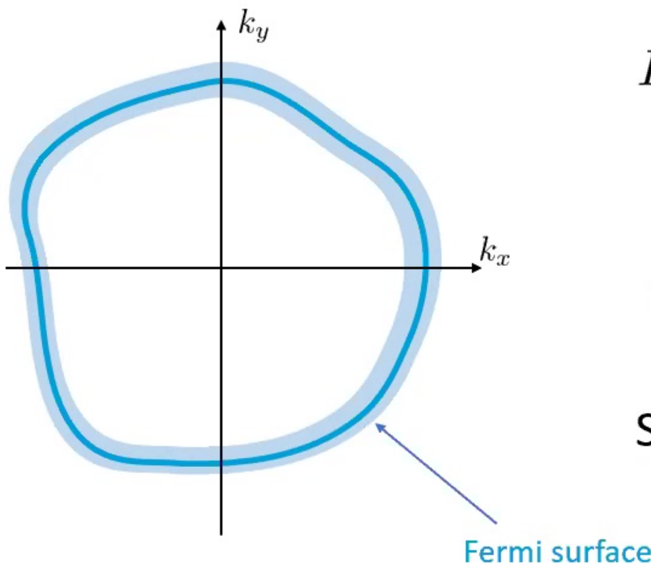


Topological insulator:
Topological quantum field theory

Metal
???

An effective field theory of a metal: Fermi liquid theory

(Landau, 1957)



$$H = \sum_k \epsilon_k \hat{n}_k + \sum_{k,k'} V_{k,k'} \hat{n}_k \hat{n}_{k'}$$

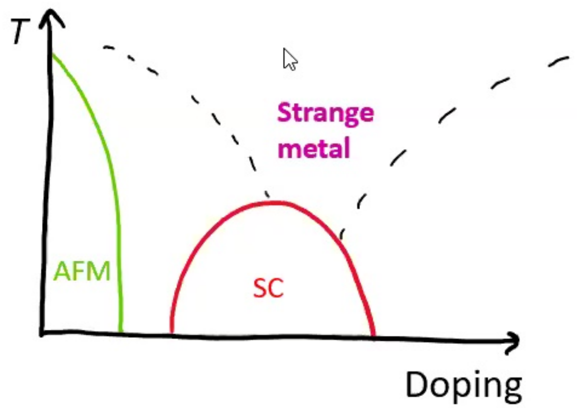
Long-lived quasiparticles on the Fermi surface

Successfully describes many metals

but not all

Strange metals

Doped cuprates (e.g. YBCO = Yttrium Barium Copper Oxide)
High temperature superconductors (YBCO has T_c ~ 93 K)



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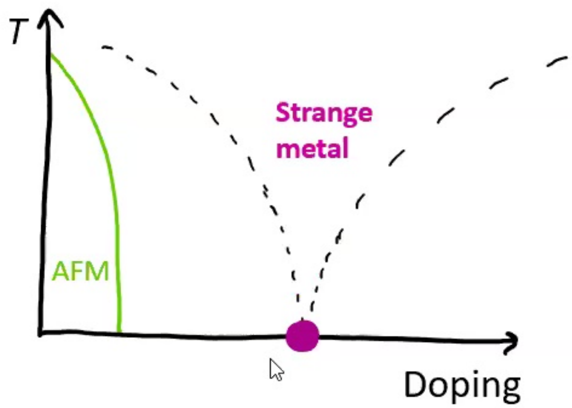

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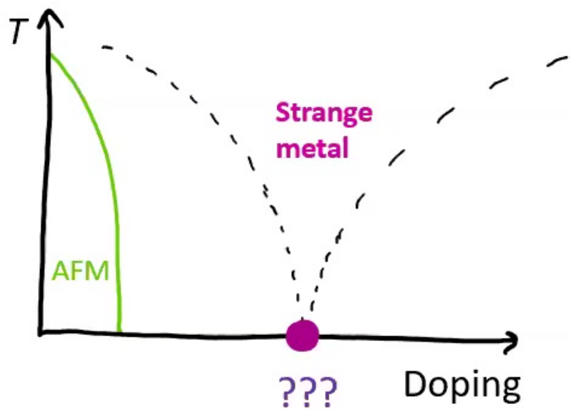


Fermi liquid theory	Strange metals
Resistivity $\rho \sim T^2$	Resistivity $\rho \sim T$

Filling constraints

[*DVE*, R. Thorngren, T. Senthil, arXiv:2007.07896]

Filling

UV Electrons, etc.

Symmetry $G = \mathbb{Z}^d \times \mathrm{U}(1)$

Lattice translation symmetry

Charge conservation symmetry

Filling ν

Average charge per unit cell

We consider only clean metals (no impurities)

???

IR Effective field theory

Prior result #1: LSMOH theorem

Symmetry $G = \mathbb{Z}^d \times \mathrm{U}(1)$

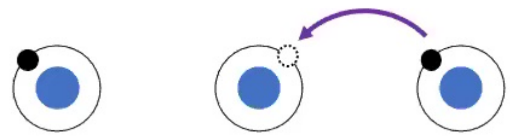
Lattice translation symmetry

Charge conservation symmetry

Assume not spontaneously broken

Filling ν

Average charge per unit cell

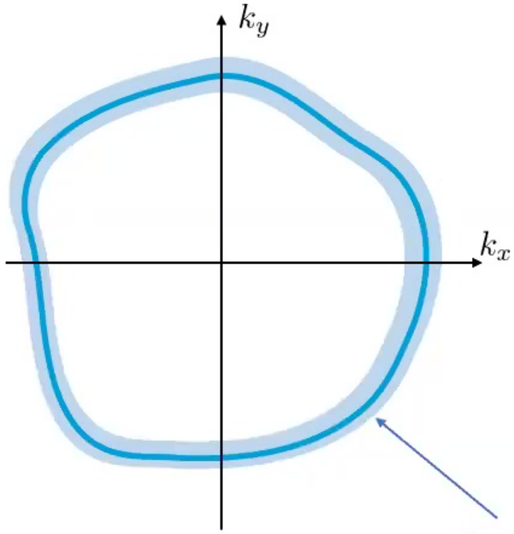


Theorem: Lieb-Schultz-Mattis-Oshikawa-Hastings (LSMOH)

If ν is not an integer, then* the ground state must be metallic

*there are some loopholes that I won't go into

Prior result #2: Luttinger's theorem for Fermi liquids



Volume enclosed by Fermi surface

Microscopic filling

$$\frac{\mathcal{V}_F}{(2\pi)^2} = \nu \quad [\mathrm{mod}\, 1]$$

Prior result #1: LSMOH theorem

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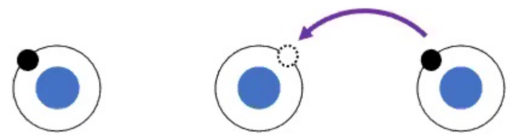

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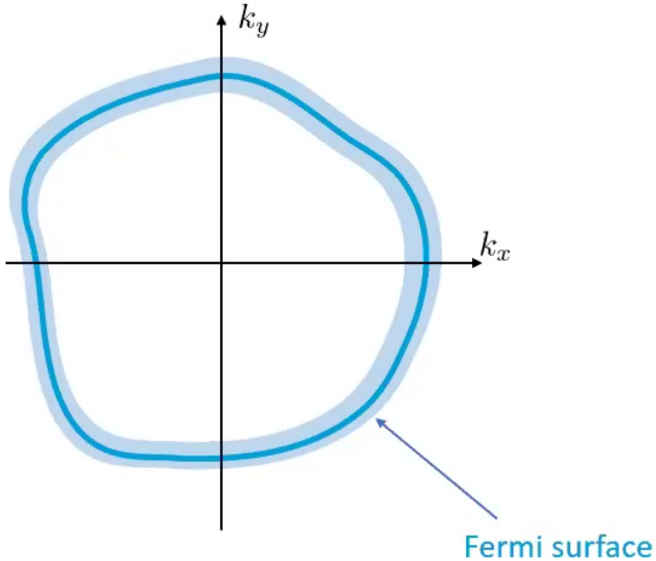

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Filling

UV Electrons, etc.

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Lattice translation symmetry

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Average charge per unit cell

???

Will vastly generalize Luttinger's theorem and Lieb-Schultz-Mattis-Oshikawa-Hastings theorem

IR Effective field theory

Filling

UV Electrons, etc. Symmetry $G = \mathbb{Z}^d \times \mathrm{U}(1)$

Lattice translation symmetry

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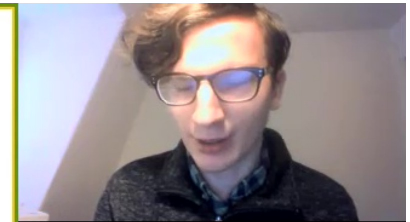
Average charge per unit cell

IR Effective field theory

Emergent symmetry G_{IR}

Anomaly



Axial anomaly

Massless Dirac fermion

$$\mathcal{L} = \overline{\psi}\gamma^\mu \partial_\mu \psi$$

Two conserved currents:

$$j^\mu = \overline{\psi}\gamma^\mu \psi$$

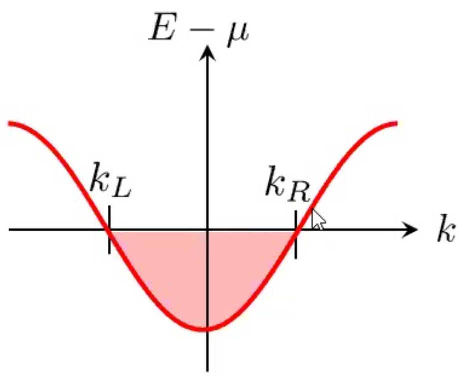
$$j_A^\mu = \overline{\psi}\gamma^5 \gamma^\mu \psi$$

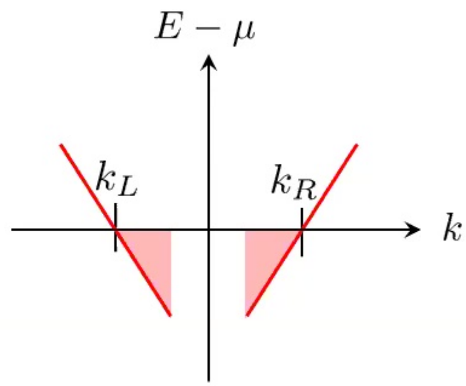
$\mathrm{U}(1) \times \mathrm{U}(1)$ symmetry

In the presence of background electromagnetic field, the axial current is not conserved:

$$\partial_\mu j_A^\mu \propto \mathbf{E} \cdot \mathbf{B}$$

Axial anomaly in a 1D Fermi gas





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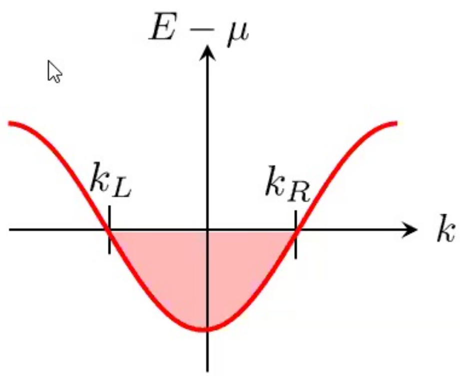

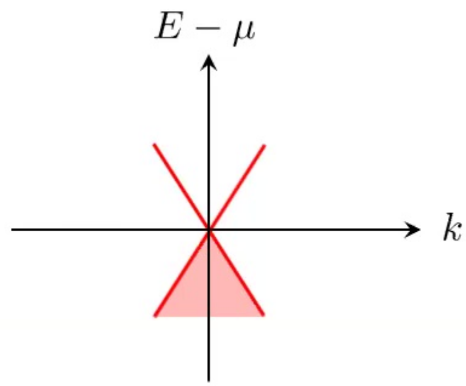

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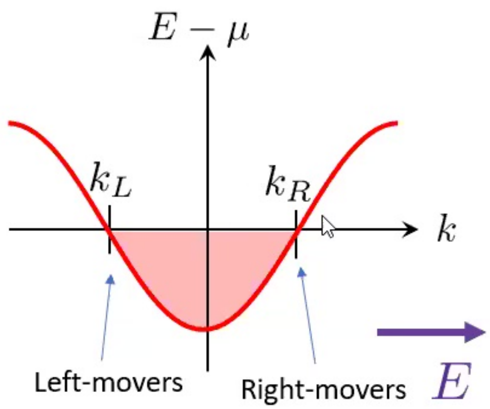
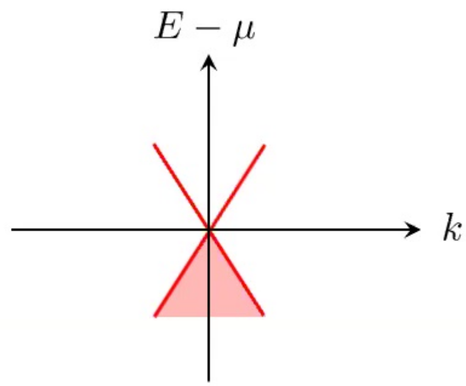
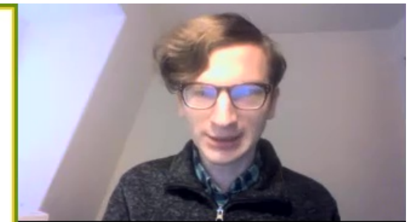




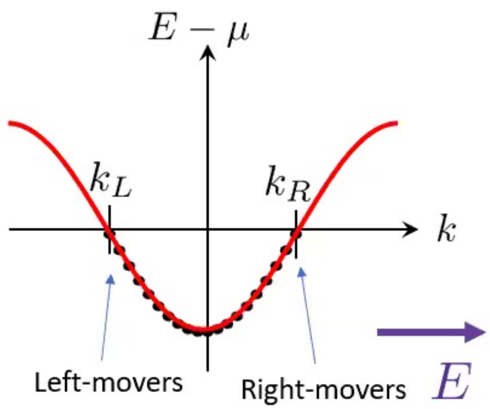


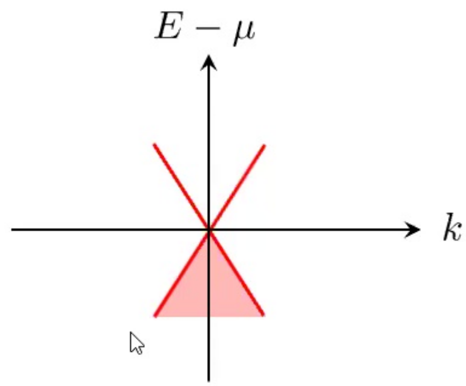
Emergent $\mathrm{U}(1) \times \mathrm{U}(1)$ symmetry

Left-moving and right-moving charges

$$\partial_\mu (j^L)^\mu = -E/(2\pi)$$
$$\partial_\mu (j^R)^\mu = E/(2\pi)$$

Axial anomaly in a 1D Fermi gas





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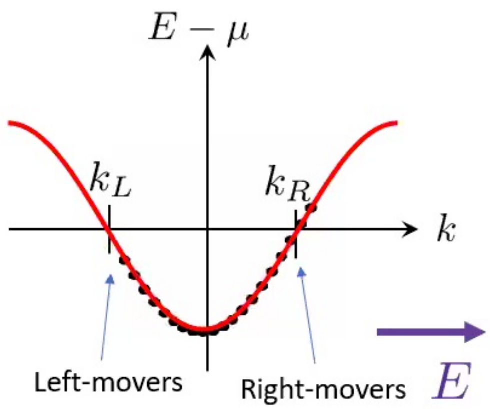

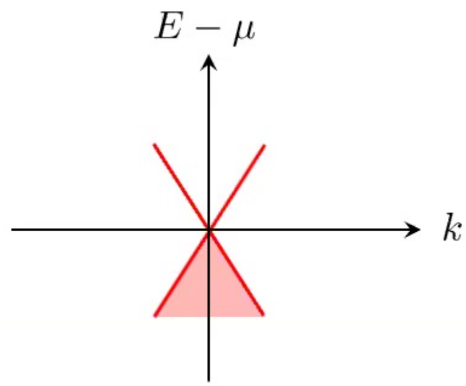

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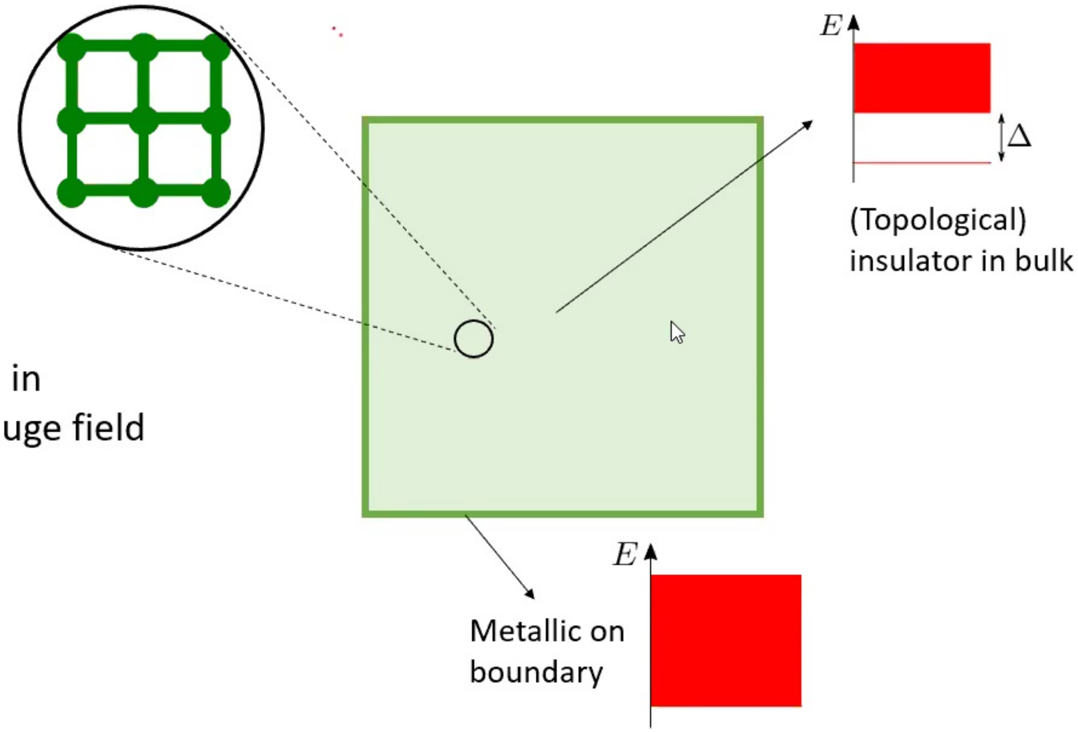
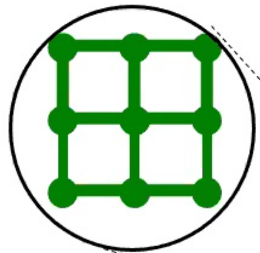
Example of a ‘t Hooft anomaly

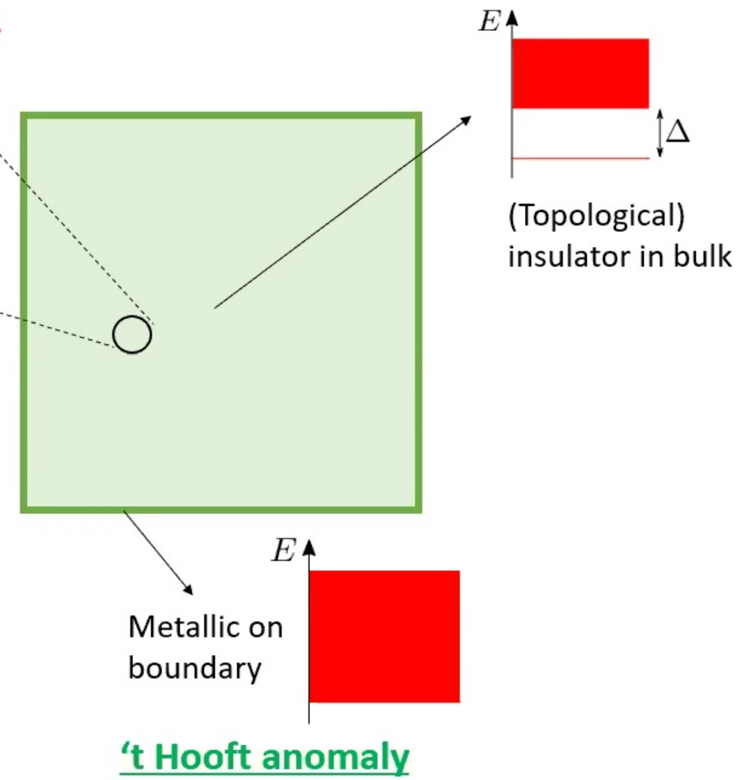
Remarks on ‘t Hooft anomalies



- Nonconservation of charge in response to background gauge field

Remarks on ‘t Hooft anomalies





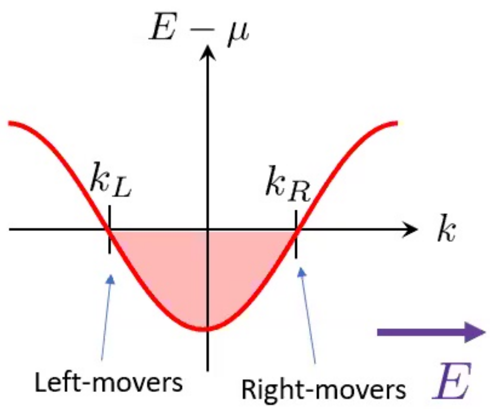
- Nonconservation of charge in response to background gauge field

- (‘t Hooft anomalies in *d* spatial dimensions)

$\leftrightarrow$

(Topological insulators in d+1 spatial dimensions)

Axial anomaly in a 1D Fermi gas



Emergent $\mathrm{U}(1)\times\mathrm{U}(1)$ symmetry

Left-moving and right-moving charges

$$\partial_\mu (j^L)^\mu = -E/(2\pi)$$
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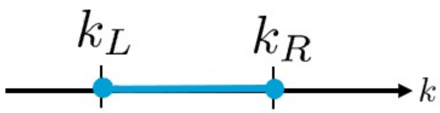
Example of a 't Hooft anomaly

Axial anomaly implies Luttinger's theorem

In the IR theory



(Assuming microscopic continuous translation symmetry for simplicity)

$$\hat{P} \sim k_L \hat{N}_L + k_R \hat{N}_R$$

Generators of $\mathrm{U}(1) \times \mathrm{U}(1)$

$$\partial_\mu (j^L)^\mu = -E/(2\pi)$$
$$\partial_\mu (j^R)^\mu = E/(2\pi)$$

$$\dot{n}_L = -E/(2\pi)$$
$$\dot{n}_R = E/(2\pi)$$

Rate of change of momentum density: $\dot{p} = E(k_R - k_L)/(2\pi)$

Microscopically

$\dot{p} = E\rho$

ρ is the microscopic charge density [continuous translation symmetry analog of filling]

$$(k_R - k_L)/(2\pi) = \rho$$

Luttinger's theorem

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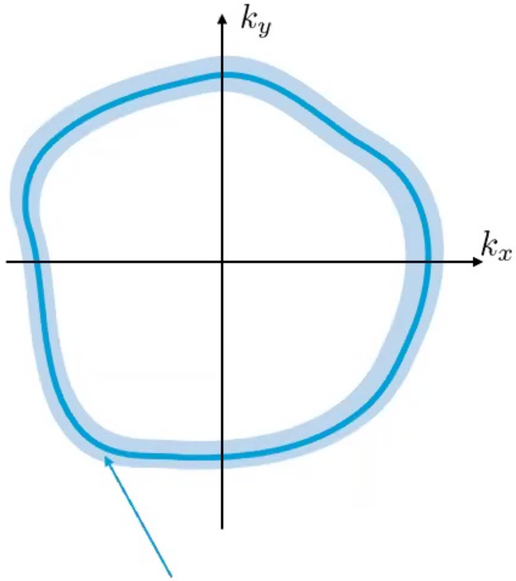
Emergent symmetry of a 2-D Fermi liquid

[*DVE*, R. Thorngren, T. Senthil, arXiv:2007.07896]



Charge at <u>every</u> point on the Fermi surface is conserved separately!

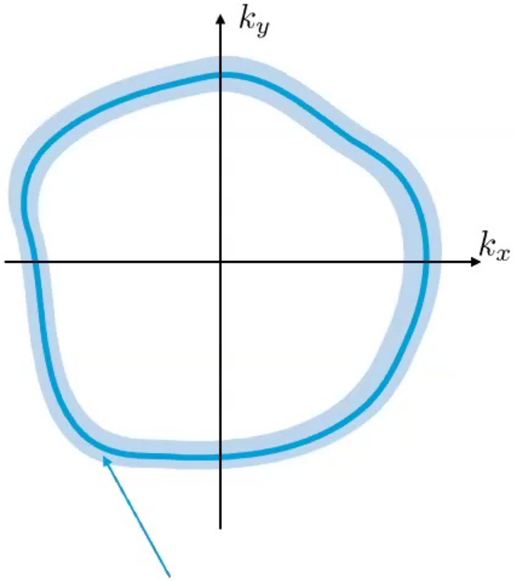
$$H = \sum_k \epsilon_k \hat{n}_k + \sum_{k,k'} V_{k,k'} \hat{n}_k \hat{n}_{k'}$$

G_{IR}
$= \{$ Smooth functions from $S^1 \to \mathrm{U}(1)$ $\}$
$= \mathrm{LU}(1)$

"Loop group"

Emergent symmetry of a 2-D Fermi liquid

[*DVE*, R. Thorngren, T. Senthil, arXiv:2007.07896]



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"Loop group"

't Hooft anomalies of loop group
→ Luttinger's theorem

Filling

[*DVE*, R. Thorngren, T. Senthil, arXiv:2007.07896]

UV Electrons, etc. Symmetry $G = \mathbb{Z}^d \times \mathrm{U}(1)$

Lattice translation symmetry

Charge conservation symmetry

Filling ν

Average charge per unit cell

IR Effective field theory *Emergent* symmetry G_{IR}

't Hooft anomaly

The general filling formula

[***DVE***, R. Thorngren, T. Senthil, arXiv:2007.07896]

1. The emergent symmetry G_{IR}
2. The group homomorphism $\varphi : \mathbb{Z}^d \times \mathrm{U}(1) \to G_{\mathrm{IR}}$.
3. The 't Hooft anomaly $\alpha \in \mathcal{C}^d(G_{\mathrm{IR}})$

General formula $\longrightarrow$ Filling ν [mod 1]

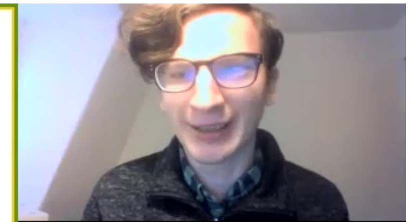
't Hooft anomaly of G_{IR} in *d* spatial dimensions
↳ G_{IR} SPT in *d* + 1 spatial dimensions
↳ Topological action for a G_{IR} gauge field in *d*+2 space-time dimensions

e.g. $S_M[A] = \frac{1}{24\pi^2} \int_{M \times S^1} A \wedge dA \wedge dA$

Set $M = T^d \times S^2$. Choose a G_{IR} gauge field on M as follows:

- Each of the non-trivial cycles on T^d carries a gauge holonomy of $\varphi(\mathbb{T}_k)$, $k = 1, \cdots, d$, where $\mathbb{T}_1, \cdots, \mathbb{T}_d$ are the generators of $\mathbb{Z}^d$.
- The S^2 carries a unit Chern number of the U(1) symmetry generated by $\varphi(\hat{Q})$, where $\hat{Q}$ is the generator of the microscopic U(1).

Then $\boxed{\nu = \frac{1}{2\pi} S_M[A] \quad [\mathrm{mod}\, 1].}$



Consequences

[***DVE***, R. Thorngren, T. Senthil, arXiv:2007.07896]

- Rederive past results
- The *compressibility theorem*

Theorem

If $d \geq 2$ and G_{IR} is a compact finite-dimensional Lie group, then ν is a rational number.

For irrational ν, G_{IR} must be an infinite-dimensional group – infinitely many conserved quantities!

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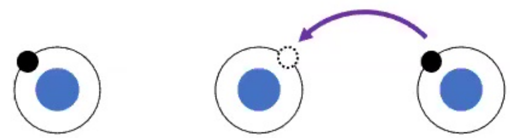

Theorem

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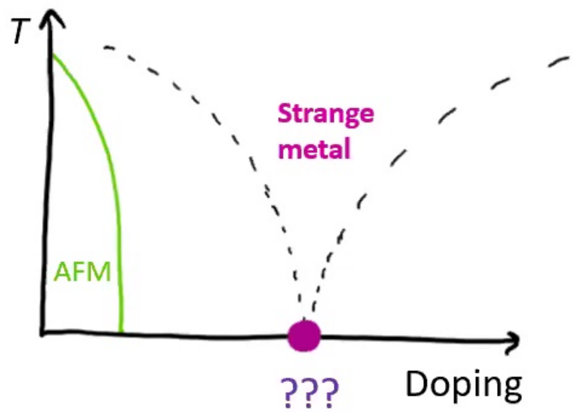
We expect this to hold in general for (clean) metals.

Applications to strange metals

[***DVE***, T. Senthil, arXiv:2010.10523]

Applications to strange metals

[***DVE***, T. Senthil, arXiv:2010.10523]



Assumptions about strange metals:

- Clean
- Exist at irrational filling

Imply infinitely many conserved quantities

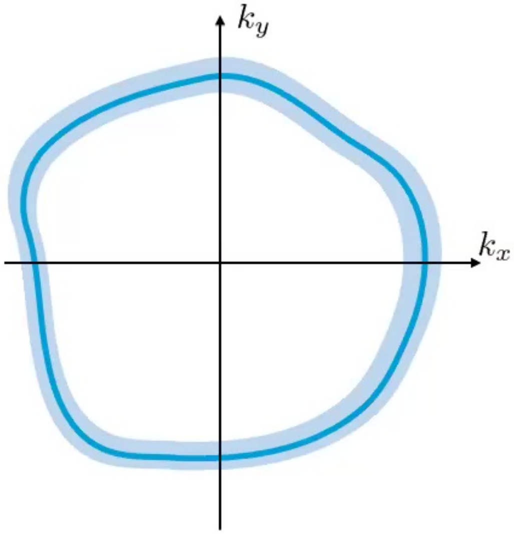
- Conductivity scaling

$$\sigma(\omega, T) = T\Sigma(\omega/T)$$

$\Sigma(0)$ Is finite

Conductivity of Fermi liquid theory

(Landau, 1957)



$$H = \sum_k \epsilon_k \hat{n}_k + \sum_{k,k'} V_{k,k'} \hat{n}_k \hat{n}_{k'}$$

$$\sigma(\omega, T) = D\delta(\omega)$$

- DC conductivity is infinite
- Scaling form

$$\sigma(\omega, T) = T\Sigma(\omega/T)$$

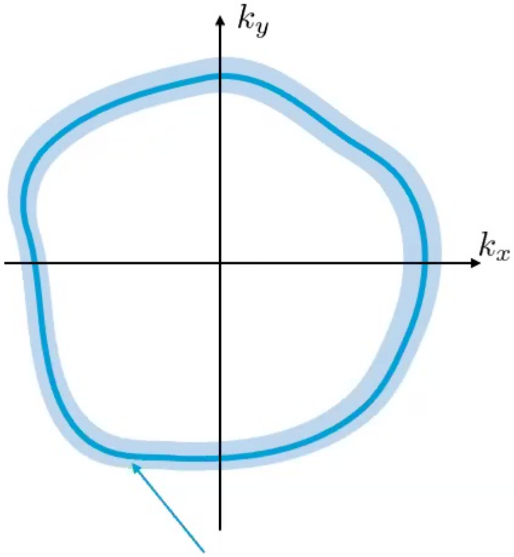
is satisfied with $\Sigma(0) = \infty$

Conductivity of Fermi liquid theory

(Landau, 1957)

$$H = \sum_k \epsilon_k \hat{n}_k + \sum_{k,k'} V_{k,k'} \hat{n}_k \hat{n}_{k'}$$

+ (dangerously irrelevant terms)



Charge at every point on the Fermi surface is conserved separately!

$$\sigma(\omega, T) = D\delta(\omega)$$

- DC conductivity is infinite

Comes from conserved quantities

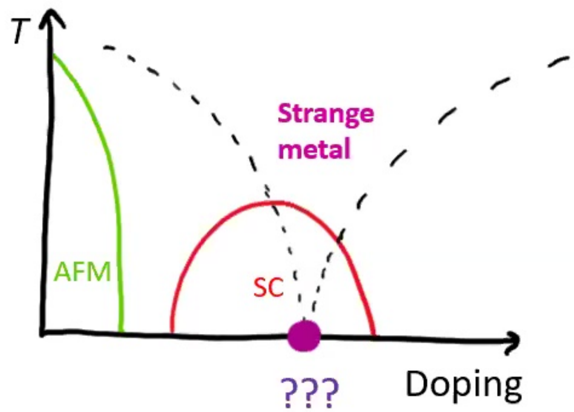
- Scaling form

$$\sigma(\omega, T) = T\Sigma(\omega/T)$$

is satisfied with $\Sigma(0) = \infty$

Applications to strange metals

[***DVE***, T. Senthil, arXiv:2010.10523]



Assumptions about strange metals:

- Clean
- Exist at irrational filling

Imply infinitely many conserved quantities

- Conductivity scaling

$$\sigma(\omega, T) = T\Sigma(\omega/T)$$

$\Sigma(0)$ Is finite

Conclusion:

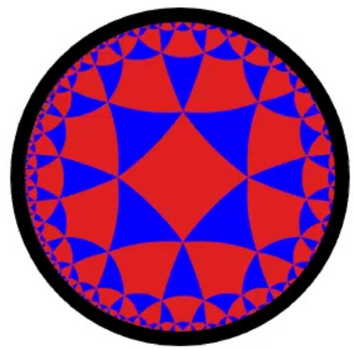
$$\mathcal{M} = \chi_{P_x P_x} = \infty$$

in the strange metal

Future directions: implement these properties in holography?

Field theory in d space-time dimensions ⟷ (Duality) ⟷ Quantum gravity in d+1 space-time dimensions in asymptotic AdS space

Strongly coupled field theory in d space-time dimensions ⟷ (Duality) ⟷ ***Classical*** gravity in d+1 space-time dimensions in asymptotic AdS space



[Image from Wikipedia]

Conclusion/outlook

UV Detailed microscopic physics:

Electrons hopping between atoms + Coulomb repulsion

Renormalization-group (RG) flow

IR Low-energy, long-wavelength physics

can be described by an “effective field theory”

Conclusion/outlook

UV Detailed microscopic physics:

Electrons hopping between atoms + Coulomb repulsion

Renormalization-group (RG) flow

Structural constraints

IR Low-energy, long-wavelength physics

can be described by an "effective field theory"

Thank you!

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