Title: Black hole perturbations and gravitational wave source modelling

Speakers: Chris Kavanagh

Series: Strong Gravity

Date: January 07, 2021 - 1:00 PM

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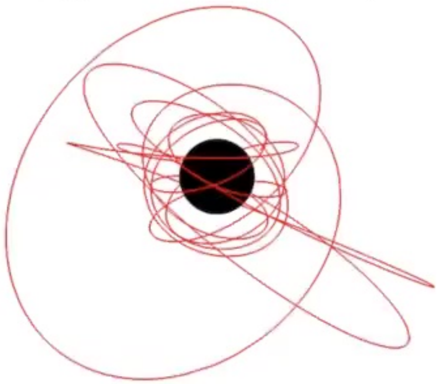
Abstract: Abstract: TBD

Black hole perturbation theory & GW source modelling

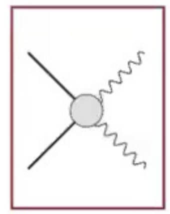
Chris Kavanagh
Max Planck Institute for Gravitational Physics (Albert Einstein Institute)
Potsdam

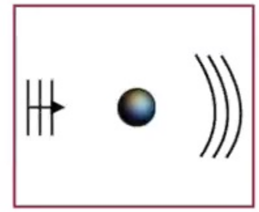
Perimeter Institute Strong Gravity seminar 07/01/2021

MOTIVATIONS



Kerr geodesic trajectory





Compton scattering ↔ Kerr wave scattering

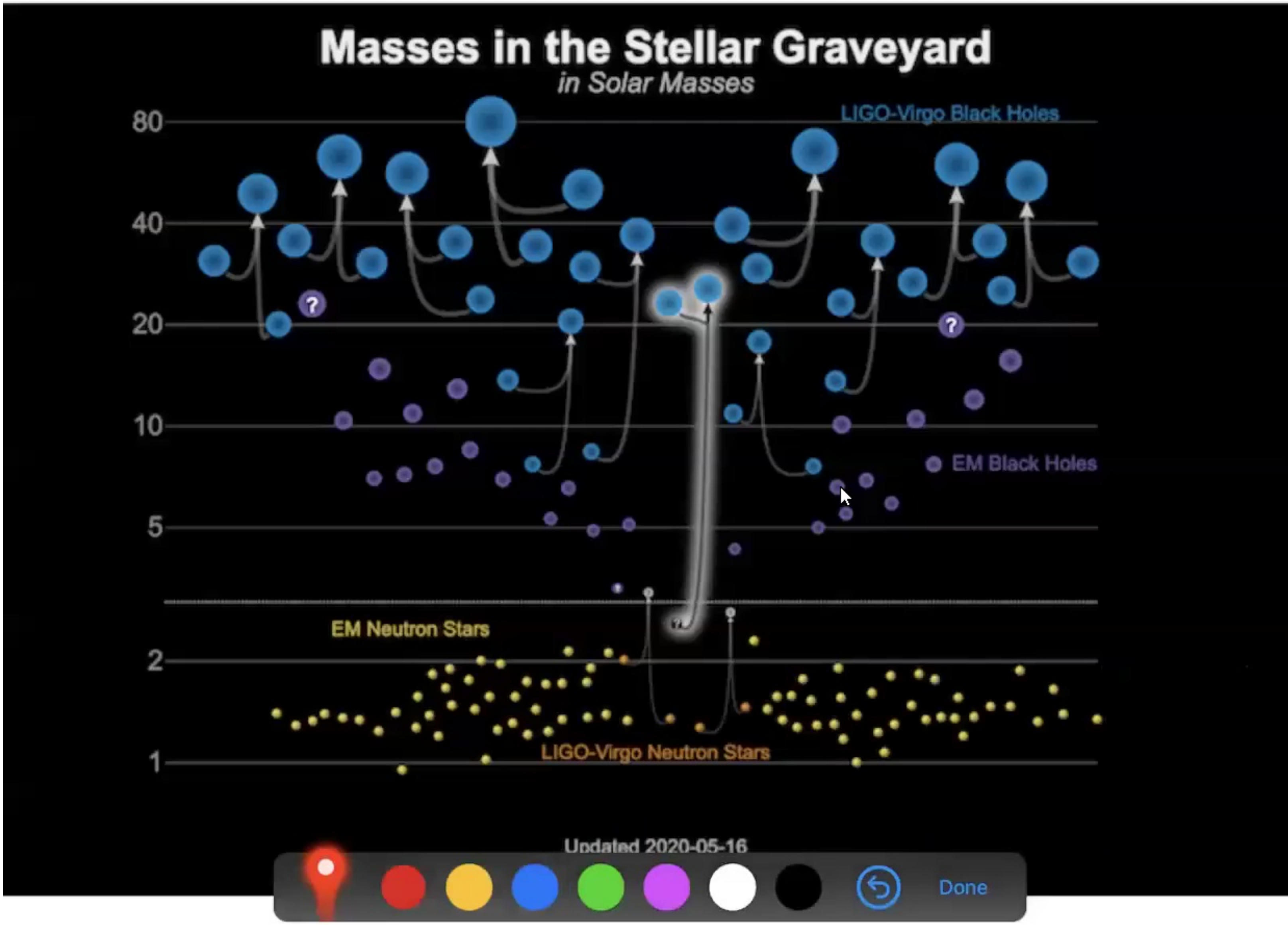
- Modelling extreme mass-ratio inspirals
- Focus on analytic methods/asymptotics
- interaction with weak field approximations
  - High post-Newtonian info
  - spin dependencies (e.g. Kerr)
  - quantum scattering methods

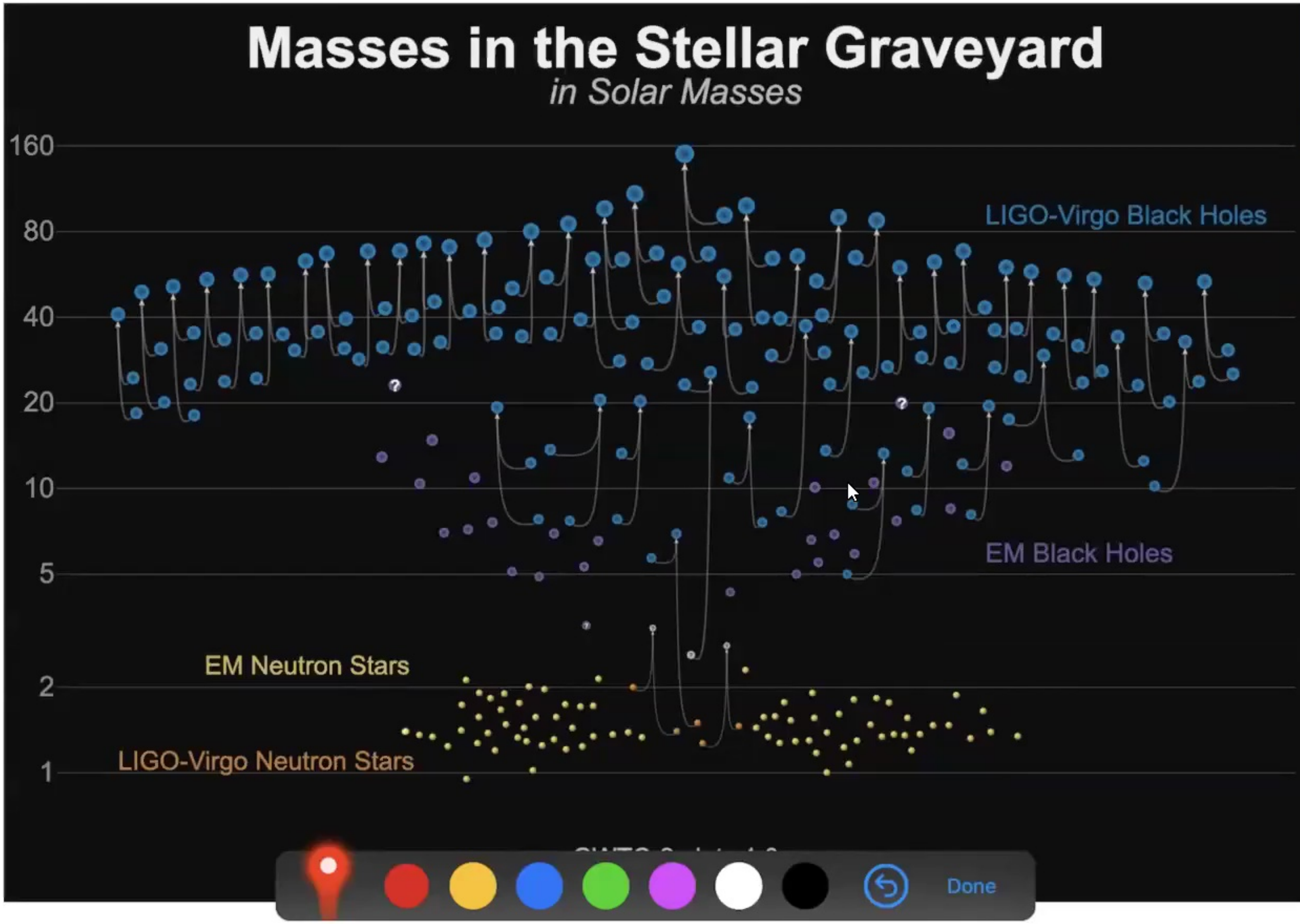
Done

GRAVITATIONAL WAVES AS A SCIENCE

LIGO—Virgo—KAGRA

- insight to energetic processes
- probe equation of state of nuclear matter
- inform models of stellar evolutions
- constraints on GR deviations
- beginning to measure H_0


Masses in the Stellar Graveyard
in Solar Masses
LIGO-Virgo Black Holes
EM Black Holes
EM Neutron Stars
LIGO-Virgo Neutron Stars
80
40
20
10
5
2
1
Updated 2020-05-16
Done


Masses in the Stellar Graveyard
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160
80
40
20
10
5
2
1
LIGO-Virgo Black Holes
EM Black Holes
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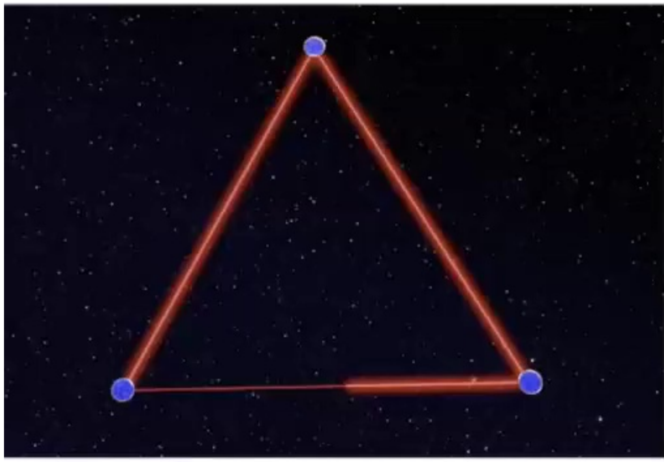
LIGO-Virgo Neutron Stars
Done

WE CAN LEARN MORE BY GOING TO SPACE:

Laser Interferometer Space Antenna (LISA)



- Selected for one of ESA's L3 missions (2017)
- Launch ~2034

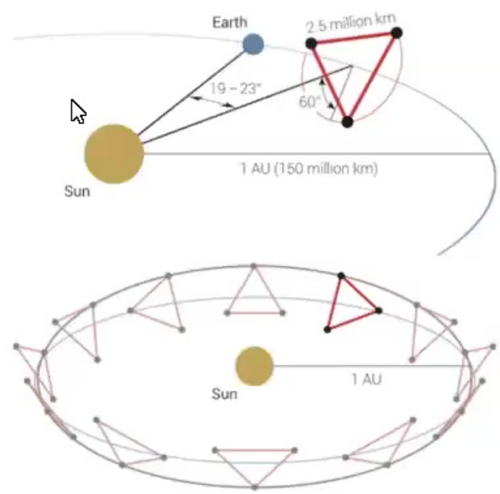


Figure 4: Depiction of the LISA Orbit.

- 2.5 million km arm length
  - lower frequency waves
- Earth trailing orbit

al, elisascience.org

A ZOO OF NEW SOURCES

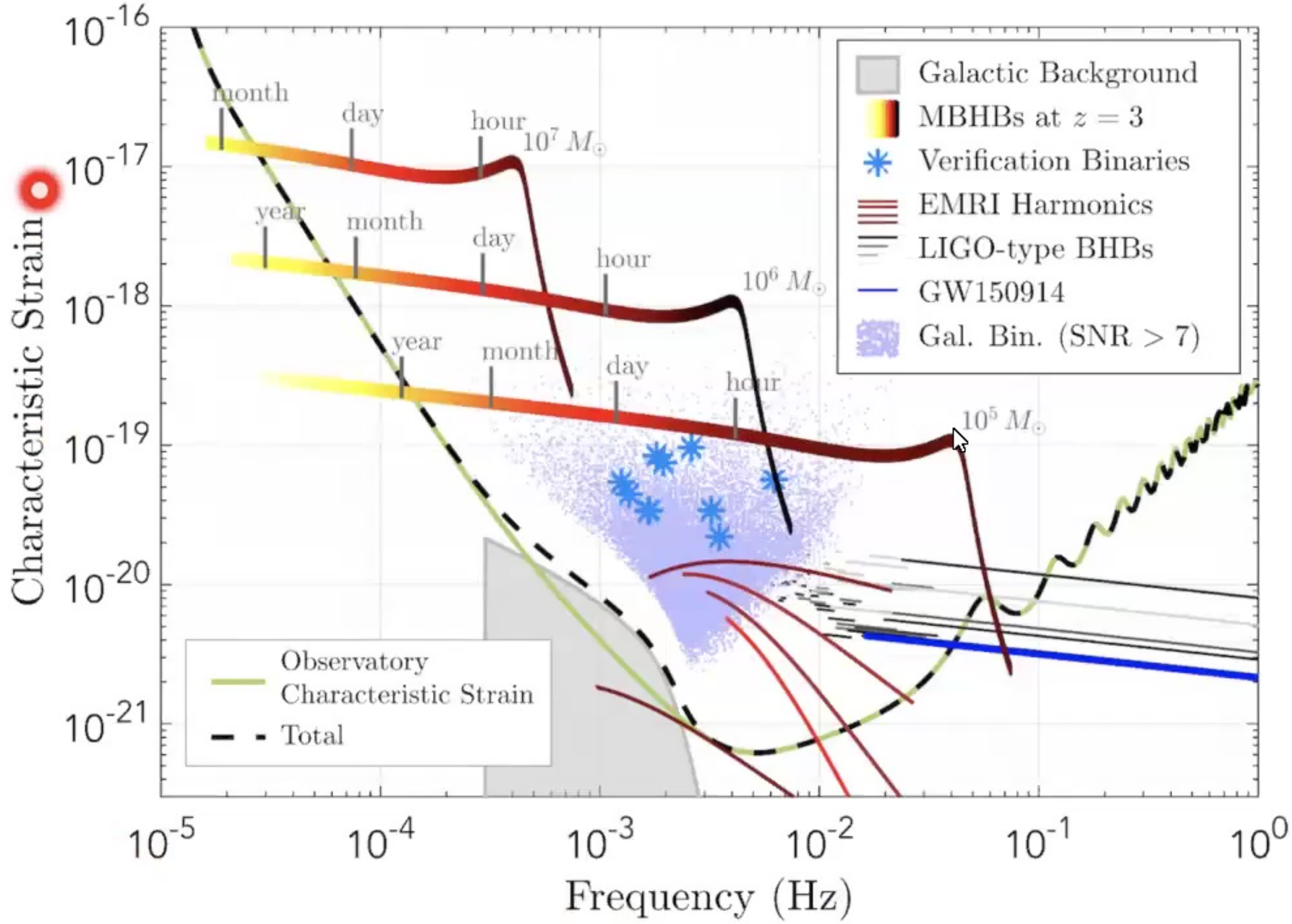


image: LISA L3 proposal, elisascience.org

1.Galactic Binaries:

Verification binaries

White Dw, NS, BHs & combo's

~25000

Low freq noise..

2. Massive Black Hole Binaries

10^3-10^8 $M_\odot$ binaries

Inspiral, Merger & Ringdown

3. Extreme Mass Ratio Inspirals

galactic nuclei

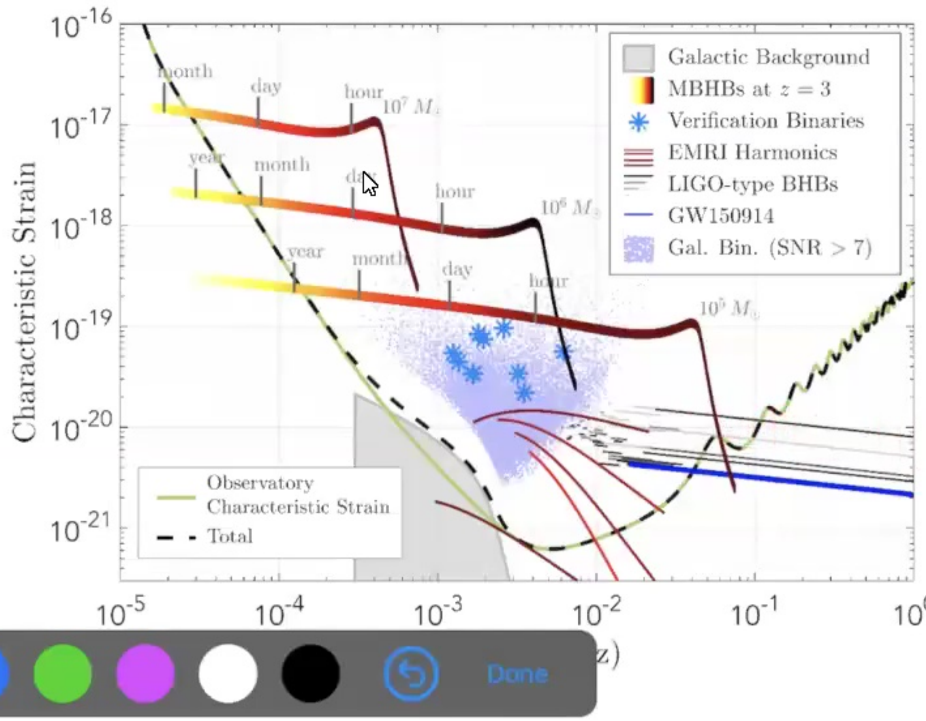
$10M_\odot$ orbiting $10^6 M_\odot$

4. "LIGO" Binaries

~100/yr[Sesana 16']

Multi-Messenger

Localisation for EM



1.Galactic Binaries:

Verification binaries
White Dw, NS, BHs & combo's
~25000
Low freq noise..

'Fundamental physics'

No hair, Kerr quadrupole moment
Lorentz violation, graviton mass
Hubble parameter
Stochastic GW background-early universe
Cosmic strings, unknowns

2. Massive Black Hole Binaries

10^3-10^8 $M_\odot$ binaries
Inspiral, Merger & Ringdown

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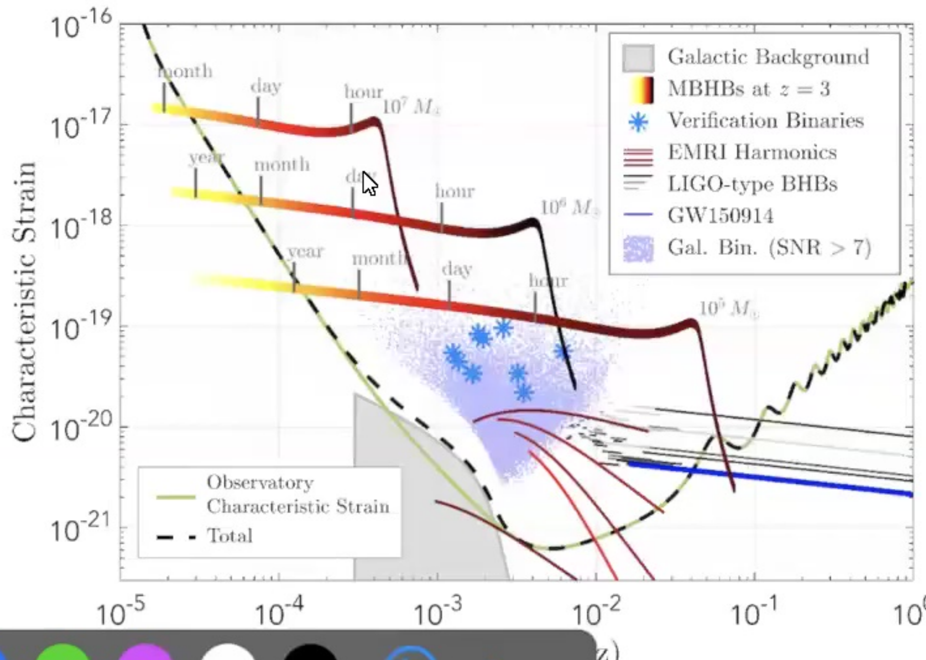


NEW SOURCES MEANS NEW CHALLENGES

Compared with waveform models for ground based detectors:

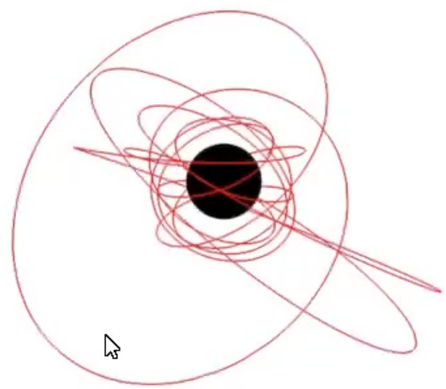
- More eccentricity
- longer signals
- higher SNR (errors stand out)
- need deeper spin information
- mass ratios: moderate to extreme

Data analysis: Massive number of overlapping signals— open problem
LISA data challenges (LDC)

EMRI SCIENCE

- In lisa band for years
- $\sim 10^4$-10^5 wave cycles detected
- tri-periodic $\Omega_\phi, \Omega_r, \Omega_\theta$ slowly evolving
- can exhibit resonances

need *high accuracy* waveforms
— 2nd order in perturbation theory



Credit: made using the Black Hole
Perturbation Toolkit
bhptoolkit.org

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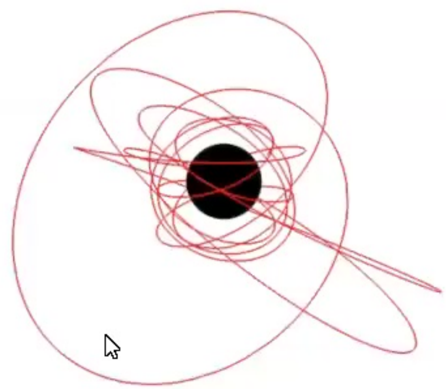

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bhptoolkit.org

- 4 digit precision on masses, eccentricity
- 3 digits on spin of bigger BH

Cleanly probe the spacetime geometry

EMRI SCIENCE: Outcomes

Massive black holes:

- Massive BH mass function— unconstrained by EM
- occupation fraction of massive BHs in low mass galaxies—clues to the origins of massive BHs
- Spin distributions of massive BHs- e.g. formation mechanisms

Galactic centres

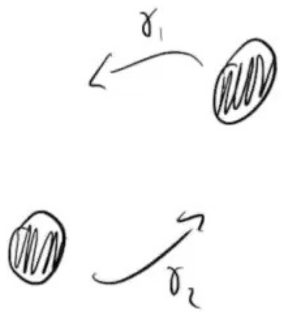
- Probe densities of stellar mass BHs in galactic nuclei via event rates
- eccentricity/inclination measurements can inform formation channels
- May see waveform deviations due to environment: accretion/3rd body interaction

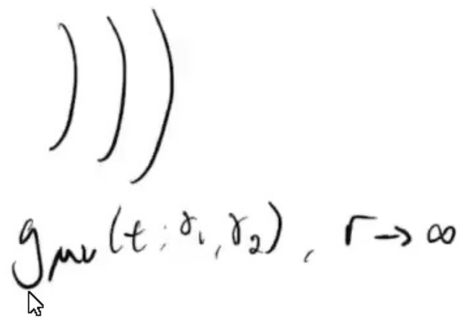
GR

- Precision tests of GR
- Test the Kerr nature of astrophysical BHs i.e. no hair theorem
- other fields, horizon, echoes?

Done

GW SOURCE MODELLING





Solve Einstein field equations

$$(\mathscr{E}g)_{\mu\nu} = 4\pi T_{\mu\nu}$$

Essential to have vast array of waveforms for data analysis!

GW SOURCE MODELLING: NUMERICAL RELATIVITY

$$(\mathscr{E}g)_{\mu\nu} = 4\pi T_{\mu\nu}$$

- GR on a grid- finite differencing/spectral methods

✔

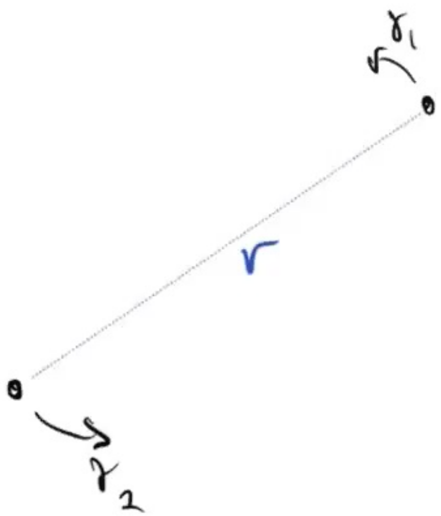
- Full non-linear GR
- inspiral, mergers, ringdowns
- control of errors

✘

- long run times
- difficulties with disparate scales

Done

GW SOURCE MODELLING: WEAK-FIELD+BOUND



$$r = |\mathbf{r}_1 - \mathbf{r}_2|$$

$$\frac{GM}{r} \ll c^2$$

$$v \ll c$$

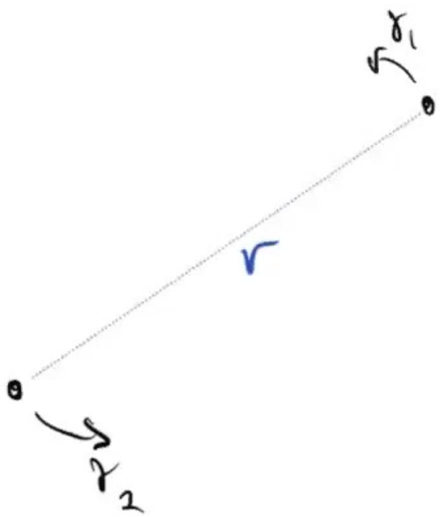
Use $\frac{1}{c}$ as a small parameter

-Perturbative corrections about flat spacetime

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{c^2} h_{\mu\nu}^{0PN} + \frac{1}{c^4} h_{\mu\nu}^{1PN} + \dots$$

GW SOURCE MODELLING: WEAK-FIELD+BOUND

Post-Newtonian



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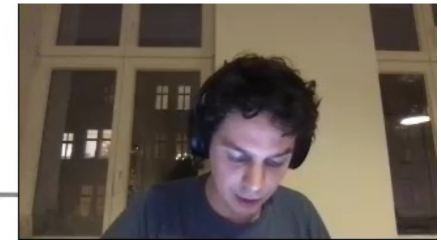
State of the art: 4PN Hamiltonian (time symmetric piece)

[Damour, Jaranowski + Schaefer 2016, Bernard, Blanchet, Bohe, Faye + Marsat 2016]

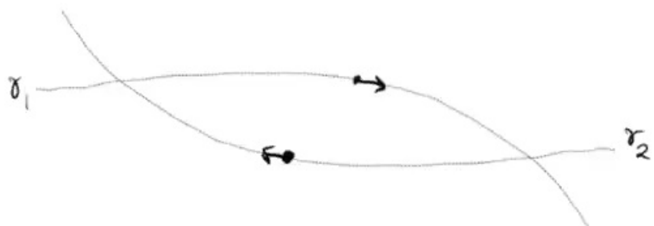
3.5PN radiation (time asymmetric) (+ other pieces?)

[Blanchet, Faye+ many authors]



GW SOURCE MODELLING: WEAK-FIELD+UNBOUND

Post-Minkowski



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Use G as a small parameter

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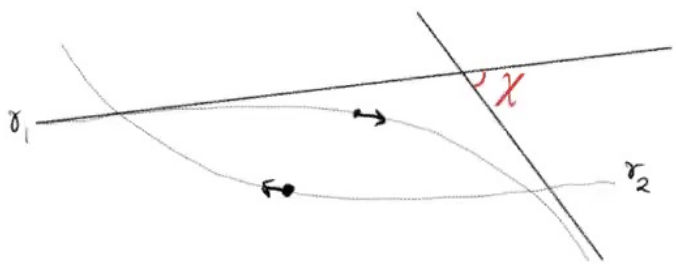
$$g_{\mu\nu} = \eta_{\mu\nu} + G h^{1PM}_{\mu\nu} + G^2 h^{2PM}_{\mu\nu} + G^3 h^{3PM}_{\mu\nu} + \dots$$

Done

GW SOURCE MODELLING: WEAK-FIELD+UNBOUND



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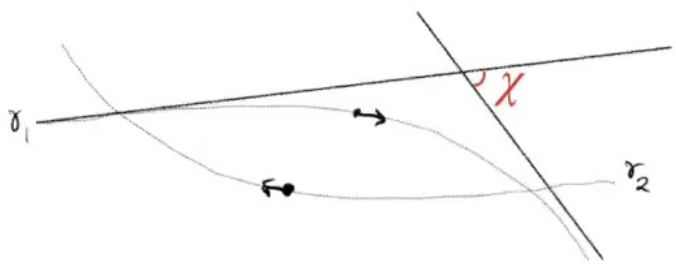
GW SOURCE MODELLING: WEAK-FIELD+UNBOUND

Emphasised by Damour (2016−): $\chi \leftrightarrow H$



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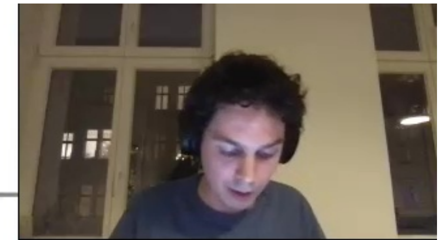
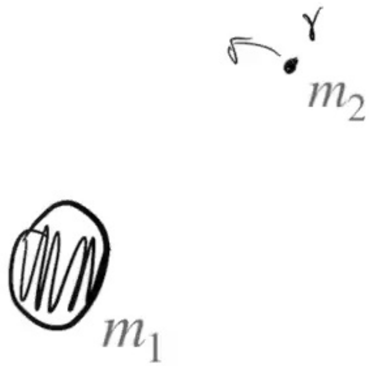
State of the art: 3PM conservative dynamics [Bern++ 2019, also by Cheung, Solon 2019, Kalin, Liu + Porto 2020] + spin [Bini, Damour 2018+, Bern++]

Done



GW SOURCE MODELLING: SMALL MASS-RATIO

Self-Force



$$\epsilon = m_2/m_1 \ll 1$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \epsilon h^{1SF}_{\mu\nu} + \epsilon^2 h^{2SF}_{\mu\nu} + \mathcal{O}(\epsilon^3)$$

0th order: *geodesics*

$$u^\beta \nabla_\beta u_\alpha = 0$$

1st order:

[Mino Sasaki & Tanaka, Quinn & Wald]

$$u^\beta \nabla_\beta u_\alpha = \epsilon F_\alpha[h_{\mu\nu}] + \mathcal{O}(\epsilon^2)$$

$$(\delta \mathcal{E} h)_{\mu\nu} = 4\pi T_{\mu\nu}$$

See reviews: [Bara

GW SOURCE MODELLING: SMALL MASS-RATIO

Self-Force computational strategy

$T_{\mu\nu}$ - source information, orbital trajectory i.e. u_α, multipolar structure

$$u^\beta \nabla_\beta u_\alpha = \epsilon F_\alpha[h_{\mu\nu}] + \mathcal{O}(\epsilon^2)$$

$$(\delta \mathcal{E} h)_{\mu\nu} = 4\pi T_{\mu\nu}$$

- Assume $\gamma = \bar{\gamma} + O(\epsilon)$ – calculate F_α along geodesics

This idea is formalised using a multi-timescale procedures
see Flanagan&Hinderer 08, Pound+ 2010—present

- Orbit average $\langle F_\alpha^{(1)} \rangle$- Dominant contribution, determined by asymptotic fluxes

Done

GW SOURCE MODELLING: SMALL MASS-RATIO

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- $(F_\alpha^{(1)} - \langle F_\alpha^{(1)} \rangle), \langle F_\alpha^{(2)} \rangle$- Subdominant, but needed for phase accuracy: e.g. Conservative SF, second order asymptotic fluxes..

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Accuracy throughout parameter space

- Orbit average $\langle F_\alpha^{(1)} \rangle$- Dominant contribution, determined by asymptotic fluxes — ~8 digits
- $(F_\alpha^{(1)} - \langle F_\alpha^{(1)} \rangle), \langle F_\alpha^{(2)} \rangle$- Subdominant, but needed for phase accuracy: e.g. Conservative SF, second order asymptotic fluxes.. — ~3 digits

Done

BLACK HOLE PERTURBATION THEORY

$$(\delta\mathcal{E}h)_{\mu\nu} = -\bar{g}_{\mu\nu}h^{\rho\sigma}R_{\rho\sigma} + h_{\mu\nu}R - h_{\nu}{}^{\sigma}{}_{;\mu\sigma} - h_{\mu}{}^{\sigma}{}_{;\nu\sigma} + h_{\mu\nu}{}^{;\sigma}{}_{;\sigma} + \bar{g}_{\mu\nu}h^{\rho\sigma}{}_{;\rho\sigma}$$

— spacetime symmetries and gauge invariance

Null tetrad approach: Newman & Penrose (1962)

'capture the light cone structure of spacetime'

$$e^{\mu}_{A} = \{l^{\mu}, n^{\mu}, m^{\mu}, \bar{m}^{\mu}\} : \qquad e^{\mu}_{A}e_{A,\mu} = 0$$

Work with scalars formed by projecting onto the tetrad, e.g. the Weyl (curvature) tensor:

$$\Psi_0 = C_{lmlm}, \quad \Psi_1 = C_{lnlm},$$
$$\Psi_2 = C_{lm\bar{m}n}, \quad \Psi_3 = C_{ln\bar{m}n},$$

Black hole spacetimes: l^μ, n^μ be principle null directions (radial in and outgoing)

$$\Psi_0 = \Psi_1 = \Psi_3 = \Psi_4 = 0$$

In Kerr spacetime:

$\delta\Psi_0 \equiv \psi_0$ and $\delta\Psi_4 \equiv \psi_4$ are to linear order gauge-invariant

and…….. satisfy *separable* PDEs

AND….. EACH gives ALL information about
non-trivial perturbations!!
[Wald, Cohen & Kegeles, Chrzanowski]

[Teukolsky 73]

$$\mathcal{O}\psi = T$$

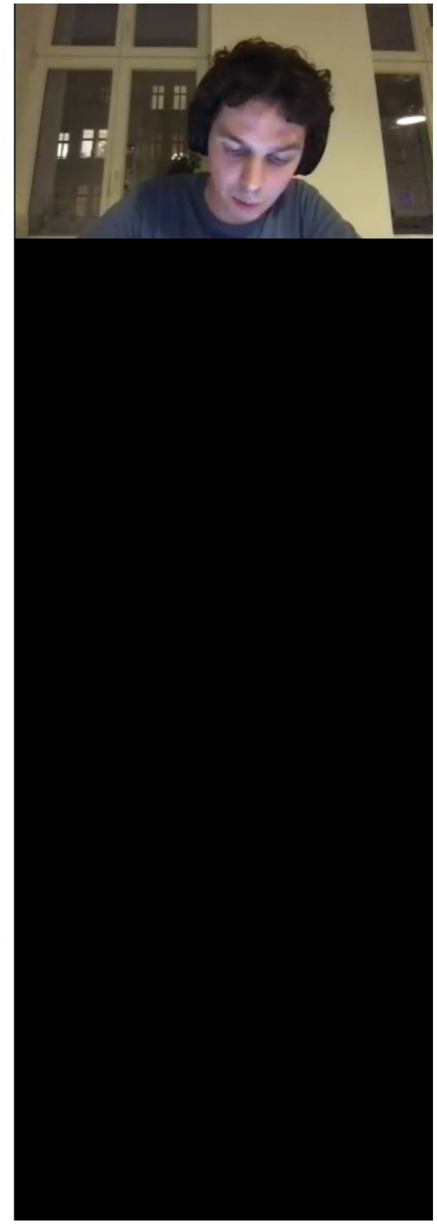
$\mathcal{O}$ – 2nd order differential operator

$T \sim \partial_\mu \partial_\nu T^{\mu\nu}$

$$\psi(t,r,\theta,\phi) = \sum_{lm} \int d\omega e^{-i\omega t} {}_sS_{lm}(\theta,\phi;a\omega) {}_sR_{lm}(r)$$

${}_sS_{lm}(\theta,\phi;a\omega)$: Spin-weighted spheroidal harmonics $\left(\rightarrow {}_sY_{lm}(\theta,\phi), a=0 \right)$

$$\left(\Delta^{-s} \frac{d}{dr} \left(\Delta^{s+1} \frac{d}{dr} \right) + {}_sV_{lm\omega}(r) \right) {}_sR_{lm\omega}(r) = {}_sT_{lm\omega}[T_{\mu\nu}]$$



Leaver and Mano, Suzuki, Tagoshi (MST) give required solutions analytically, satisfying physical boundary conditions

$$R^-_{lm} = A_1(\omega) \sum_{n=-\infty}^{\infty} a_n(\omega)\, {}_2F_1(\omega, r) \qquad R^-_{lm} \sim \Delta^{-s} e^{-i\omega r^*}, \quad r \to r_+$$

$$R^+_{lm} = A_2(\omega) \sum_{n=-\infty}^{\infty} a_n(\omega)\, U(\omega, r) \qquad R^+_{lm} \sim r^{-2s-1} e^{i\omega r^*}, \quad r \to \infty$$

Variation of parameters gives a general inhomogeneous solution

$$R(r) = C_-(r) R^-(r) + C_+(r) R^+(r)$$

where

$$C_\pm = \frac{1}{W} \int R^\mp(r') T(r') dr'$$

In self-force $T \sim \delta(r - r_p)$, $C_\pm(r) = C_\pm(r_p)$

POST-NEWTONIAN SELF-FORCE

$$\frac{GM}{r_p} \ll c^2, \quad GM\omega \sim GM\Omega \ll c^3$$

$$R^-_{l=2} = 1 - \frac{2}{3} ir\omega \frac{1}{c} - \frac{11}{42} r^2\omega^2 \frac{1}{c^2} + O(c^{-3})$$

$$R^+_{l=2} = 1 + ir\omega \frac{1}{c} + \left(\frac{5}{r} - \frac{r^2\omega^2}{2}\right)\frac{1}{c^2} + O(c^{-3})$$

e.g. metric for a particle in a circular orbit

$$h^{l=2}_{tt} = 2y - \frac{39}{7} y^2 + \frac{23}{36} y^3 + O(y^4) \qquad y = (M\Omega)^{-2/3}$$

Mechanical to go to arbitrary post-Newtonian order

Parameter space

Fluxes, $\langle F_\alpha^{(1)} \rangle$: large-$r$, low e [Munna, Evans++, Fujita et al ++]

10-20PN, e^{10-20} Schwarzschild

4PN, e^6 generic orbits in Kerr,

$$\frac{dE}{dt} = \frac{32}{5} y^5 \left[1 - \left(\frac{1247}{336} + \frac{35}{12}\nu \right) y + 4\pi y^{3/2} + \left(-\frac{44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2 \right) y^2 + \ldots \right]$$

Utility limited by high numerical accuracy requirements on fluxes.

Conservative SF-PN [Bini, Damour, Geralico, Hopper, CK, Ottewill, Wardell, Le Tiec…]

(incomplete overview!)	**Circular**	**eccentric**	**inclined**
Schwarzschild	~20PN	~7-9PN, e^6	—
Kerr	~10PN	~8.5PN, e^2	

Lower accuracy requirements, equivalent to 2SF.

- These contributions are numerically more expensive -> mode sum convergence..
- More room f ce coverage

Parameter space

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Lower accuracy requirements, equivalent to 2SF.

- These contributions are numerically more expensive -> mode sum convergence..
- More room for PN expansions to drastically influence parameter space coverage

PN/PM Conservative dynamics

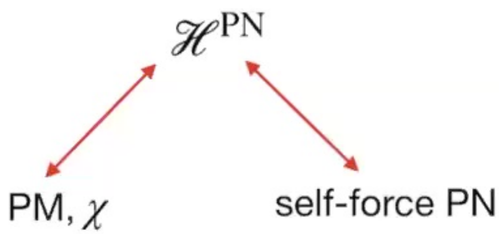
$$H^{PN} = H^{N} + \frac{1}{c^2} H^{1PN} + \ldots$$

(radiation appears at 2.5PN)

self-force PN: determines this to linear order in mass-ratio

Observation:[Bini, Damour Geralico 2019]

Post-Minkowski scattering dramatically constrains mass-ratio dependencies!!



3PM + 1sf: Full PN to 3PN

+ 2sf: to 5PN [Bini, Damour Geralico 2019]

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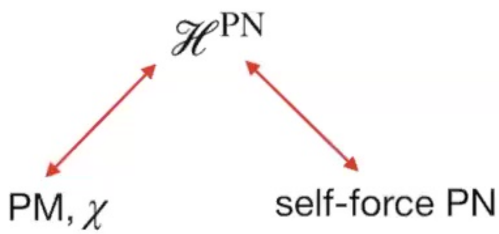


3PM + 1sf: Full PN to 3PN

+ 2sf: to 5PN [Bini, Damour Geralico 2019]

Adding spin to this picture
[Antonelli, CK, Khalil, Steinhoff, Vines 2020+]:

$$H + H_{SO} S_i + H_{SS} S_1 S_2 + O(S^2)$$

Completely fix new 3rd sub-leading terms in H_{SO} and H_{SS} .

PLANE WAVE SCATTERING: PROBING KERR SPACETIME

w/ Y F Bautista, A Guevara, J Vines

Kerr BH
(M, a)

Note:
$0 \leq \frac{a}{GM} \leq 1$ for
BHs with horizons

$\frac{d\sigma}{d\Omega}$ -differential cross-section

In GR

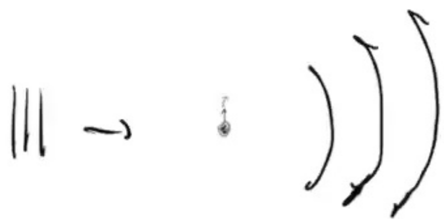
$$\frac{d\sigma}{d\Omega} \equiv \frac{dE^{\mathrm{S}}}{dtd\Omega} \Bigg/ \frac{dE^{\mathrm{PW}}}{dtd\Omega}.$$

Textbook BHPT:

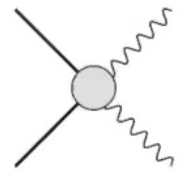
$$\frac{dE}{dtd\Omega} = \frac{1}{4\pi\omega} |\psi_4|^2$$

Done

$GM\omega = 2M\pi/\lambda \ll 1$



Gravitational Compton Amplitude



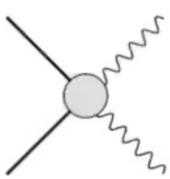
Conjecture:

classical limit `amplitude' calculations of scattering of spinning particles $s \to \infty$

$\longleftrightarrow$ scattering of Kerr BHs with $a/GM \to \infty$

[Guevara, Vines, Steinhoff, Buonanno, Ochirov, Chung, Huang, Kim, Lee, Maybee, O'Connell, Arkani-Hamed, Siemonsen , Bini, Damour]

[Arkani-Hamed,Huang, Huang 17'] [Guevara, Chivata 19']:

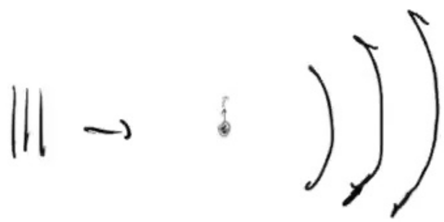


"Compton" amplitude for $s \leq 2$ i.e. will give Kerr to a^4, leading order in G

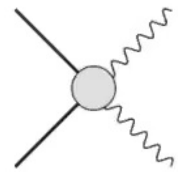
Spurious poles appear at a^5. Need to be fixed by 'counter terms'

- Question: Will this agree with Kerr to a^4 ?

$$GM\omega = 2M\pi/\lambda \ll 1$$



Gravitational Compton Amplitude



Conjecture:

classical limit `amplitude' calculations of scattering of spinning particles $s \to \infty$

⟷ scattering of Kerr BHs with $a/GM \to \infty$

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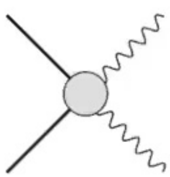

"Compton" amplitude for $s \leq 2$ i.e. will give Kerr to a^4, leading order in G

Spurious poles appear at a^5. Need to be fixed by `counter terms'

- Question: Will this agree with Kerr to a^4 ?
- Can wave scattering fix these counter terms...?

Following
[Matzner++ 70's, Dolan 07,08]

$$R^{\mathrm{PW}}_{lm}(r \to \infty) \sim A^{PW}(c_1(\gamma)e^{i\omega r_*} + c_2(\gamma)e^{-i\omega r_*})$$

flat spacetime plane wave
[Chrzanowski et al 76']

$$R^{S}_{lm}(r \to \infty) \sim A^{S}e^{i\omega r_*}$$

$$R^{-}_{lm} = R^{PW} + R^{S}$$

$$R^{-}_{lm} = A(\omega)\sum a_n(\omega)_2F_1(r,\omega) \sim A(B^{\mathrm{inc}}_{lm\omega}r^{-1}e^{-i\omega r_*} + B^{\mathrm{ref}}_{lm\omega}r^{3}e^{i\omega r_*})$$

using MST

Following [Matzner++ 70's, Dolan 07,08]

$$\frac{d\sigma}{dt} = |F|^2 + |G|^2$$

$$F(\theta,\phi) = \frac{1}{i\omega}\sum_{\ell} {}_{-2}S_{lm}(\gamma,\phi_0,a\omega)\,{}_{-2}S_{lm}(\theta,\phi,a\omega)(\mathscr{F}_{lm\omega}-1) \qquad \mathscr{F}_{\ell m\omega} = \frac{(-1)^{l+1}\mathscr{C}_{lm}}{16\omega^4}\frac{B^{\rm ref}_{lm\omega}}{B^{\rm inc}_{lm\omega}},$$

$$G(\theta,\phi) = \frac{1}{i\omega}\sum_{\ell} {}_{-2}S_{lm}(\gamma,\phi_0,a\omega)\,{}_{-2}S_{lm}(\pi-\theta,\pi-\phi,a\omega)\mathscr{G}_{\ell m\omega} \qquad \mathscr{G}_{\ell m\omega} = -\frac{3i}{4\omega^3}\frac{B^{\rm ref}_{lm\omega}}{B^{\rm inc}_{lm\omega}}.$$

$B^{\rm ref}_{lm\omega}, B^{\rm inc}_{lm\omega}$ - analytic functions of a,ω known from MST expansions

Strategy: expand in $GM\omega \ll 1$, fixing $q = a/GM$.

$$\mathscr{F} \sim a_0(q) + a_2(q)(GM\omega)^2 + a_3(q)(GM\omega)^3 + a_4(q)(GM\omega)^4 + a_5(q)(GM\omega)^5 + a_6(q)(GM\omega)^6 + O(GM\omega^7)$$

We find:

$a_i(q)$–polynomial in q, degree $i-1$, $i \leq 5$. So pick out highest power in q

Done

PRELIMINARY RESULTS

e.g. on axis scattering

$$\frac{d\sigma}{d\Omega} = \frac{G^2M^2}{\sin^4(\theta/2)}\Big(\cos^8(\theta/2)[1 - 4a\omega\sin^2(\theta/2) + 8a^2\omega^2\sin^4(\theta/2) - \frac{32}{3}a^3\omega^3\sin^6(\theta/2) + \frac{32}{3}a^4\omega^4\sin^8(\theta/2)$$

$$+\sin^8(\theta/2)[\omega \to -\omega]\Big)$$

agrees *exactly* with

$$\frac{d\sigma}{d\Omega} \sim \frac{A_4^{\mathrm{GR}}A_4^{*\mathrm{GR}}}{128\pi^2 M} + \mathcal{O}(a^5)$$

$$= \frac{G^2M^2}{\sin^4(\theta/2)}\Big(\cos^8(\theta/2)e^{-4a\omega\sin(\theta/2)} + \sin^8(\theta/2)[\omega \to -\omega]\Big)$$

where A_4^{GR} is the classical limit of the Compton amp

Done

PRELIMINARY RESULTS: a^5?

$$\mathcal{F} \sim a_0(q) + a_2(q)(GM\omega)^2 + a_3(q)(GM\omega)^3 + a_4(q)(GM\omega)^4 + a_5(q)(GM\omega)^5 + a_6(q)(GM\omega)^6 + O(GM\omega^7)$$

$a_i(q)$ − polynomial in q, degree $i-1$, $i \leq 5$. So pick out highest power in q

$$a_6^{l=2}(q) = \sum_{i=0}^{5} c^i q^i + \left(\frac{1}{225} + \frac{q^2}{75}\right)\left(4iq\log(2\epsilon\kappa) + 4iq\psi^{(0)}\left(\frac{2iq}{\kappa}\right) + 8iq\gamma_E + \kappa\right) \qquad \kappa = \sqrt{1-q^2}$$

analytically continue to $q \to \infty$

$$\frac{d\sigma}{d\Omega} = \frac{G^2M^2}{\sin^4(\theta/2)}\Bigg(\cos^8(\theta/2)[1 - 4a\omega\sin^2(\theta/2) + 8a^2\omega^2\sin^4(\theta/2) - \frac{32}{3}a^3\omega^3\sin^6(\theta/2) + \frac{32}{3}a^4\omega^4\sin^8(\theta/2) - \frac{128}{15}a^5\omega^5\sin^{10}(\theta/2)] + \sin^8(\theta/2)[\omega \to -\omega]\Bigg)$$

This series is given by..

$$\frac{d\sigma}{d\Omega} = \frac{G^2M^2}{\sin^4(\theta/2)}\left(\cos^8(\theta/2)e^{-4a\omega\sin(\theta/2)} + \sin^8(\theta/2)[\omega \to -\omega]\right)$$

PRELIMINARY RESULTS: a^5?

$$\mathscr{F} \sim a_0(q) + a_2(q)(GM\omega)^2 + a_3(q)(GM\omega)^3 + a_4(q)(GM\omega)^4 + a_5(q)(GM\omega)^5 + a_6(q)(GM\omega)^6 + O(GM\omega^7)$$

$a_i(q)-$polynomial in q, degree $i-1$, $i \leq 5$. So pick out highest power in q

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CONCLUSIONS/TAKE AWAYS

- PN-SF increasingly useful, a lot of potential still unexplored
- Results from 2sf are beginning to arrive, exciting to see how they will fit in with the picture- currently working on a SF-PN calculation at 2nd order.
- Scattering Amplitude calculations proving useful- future higher spin PN?
- direct crossover between BHPT and Amplitudes is looking fruitful (see upcoming paper)

LISA is <15 years away, there is a lot to do!