

Title: Complexity phase diagrams

Speakers: Abinhav Deshpande

Series: Perimeter Institute Quantum Discussions

Date: January 06, 2021 - 4:00 PM

URL: <http://pirsa.org/21010002>

Abstract: In this talk, I argue that the question of whether a physical system can be simulated on a computer is important not just from a practical perspective but also a fundamental one. We consider the complexity of simulating Hamiltonians with respect to both dynamics and equilibrium properties. This gives us a classification and a phase diagram of the complexity. I will cover recent results in this topic, such as a dynamical complexity phase diagram for a long-range bosonic Hamiltonian and a complexity classification of the local Hamiltonian problem in the presence of a spectral gap. I will talk about the physical implications of these results and cover some of the basic proof ideas if time permits.



Complexity phase diagrams

•

January 6, 2021

Abhinav Deshpande



JOINT CENTER FOR
QUANTUM INFORMATION
AND COMPUTER SCIENCE



Perimeter Institute

1

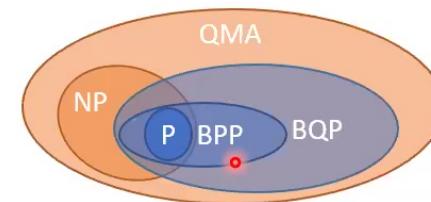


Looking for the quantum “signal”

- Quantum computing & quantum physics both interesting because of the possibility that there's something outside of classical physics/computing
- Quantifying non-classicality

When is a quantum system
efficiently simulable on a
classical computer?

- A question in the domain of complexity theory





Experimentally & theoretically interesting!

Experimental motivation

- Understanding regimes of simulability
→ Narrows down space where experiment can yield quantum advantage

Theoretical motivation

- Complexity theory: a classification tool
- A perhaps related, but different, way of characterizing non-classicality than entanglement
 - Entanglement has been successful at classifying phases of matter
 - Gapped/gapless¹, ergodic/not², topological/trivial³⁻⁶

¹ G. Vidal et al., PRL 90, 227902 (2003)

² J. H. Bardarson et al., PRL 109, 017202 (2012)

³ A. Kitaev & J. Preskill, PRL 96, 110404 (2006)

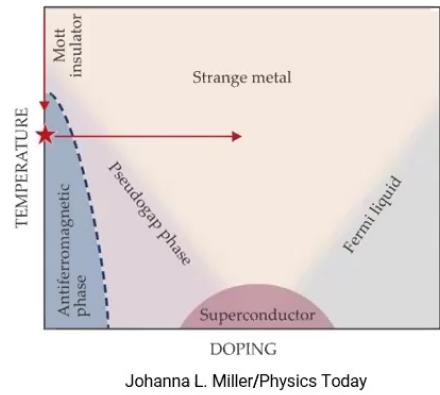
⁴ M. Levin & X.-G. Wen, PRL 96, 110405 (2006)

⁵ X. Chen et al., PRB 84, 235128 (2011)

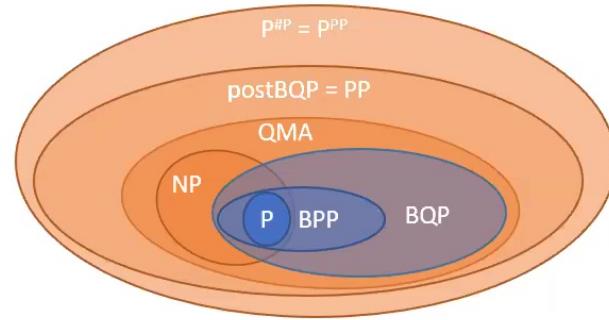
⁶ Schuch et al., PRB 84, 165139 (2011)



Classification

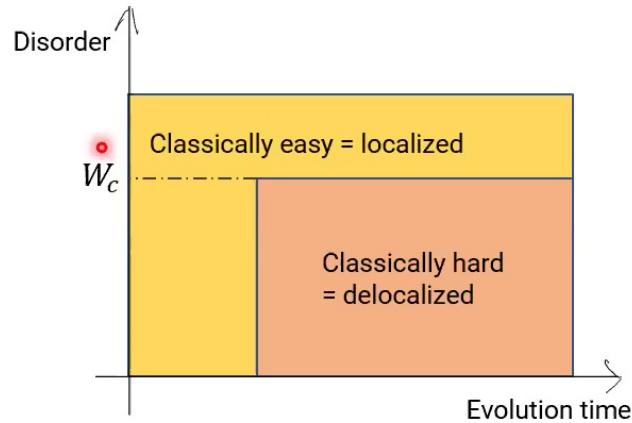


- Most questions involve classification
- Tool for classification





Classification



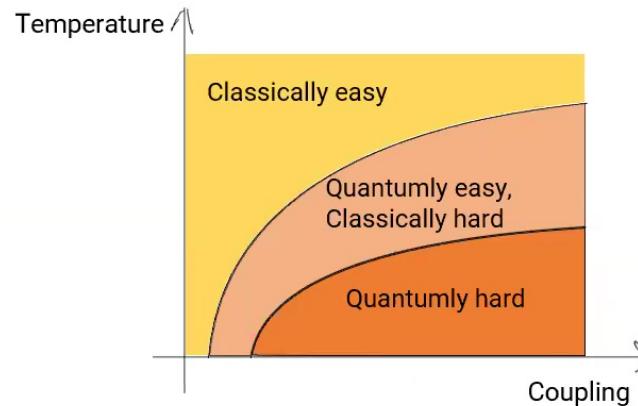
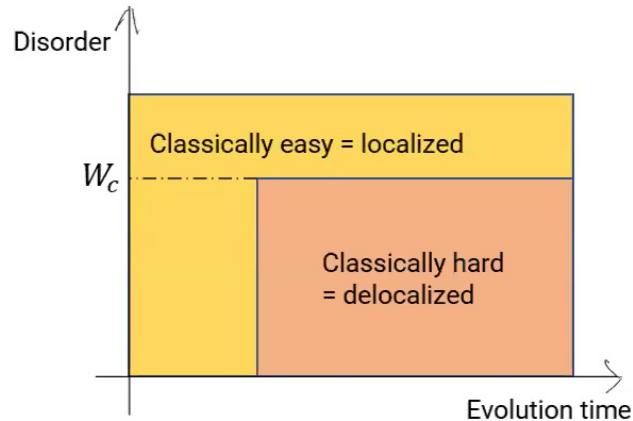
- Most questions involve classification
- Tool for classification

Can complexity theory help us classify phases?

4



Classification



- Most questions involve classification
- Tool for classification

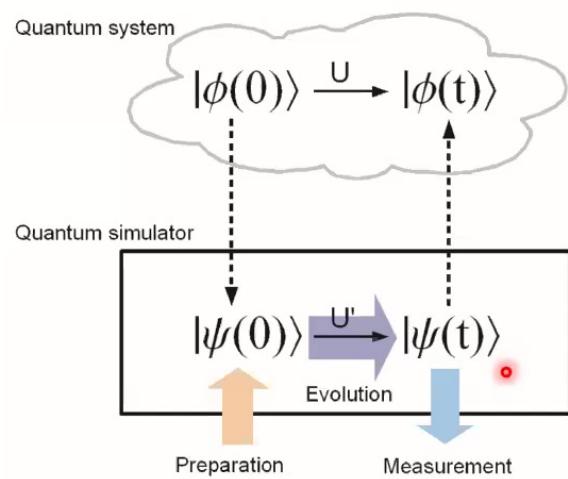
Can complexity theory help us classify phases?

4

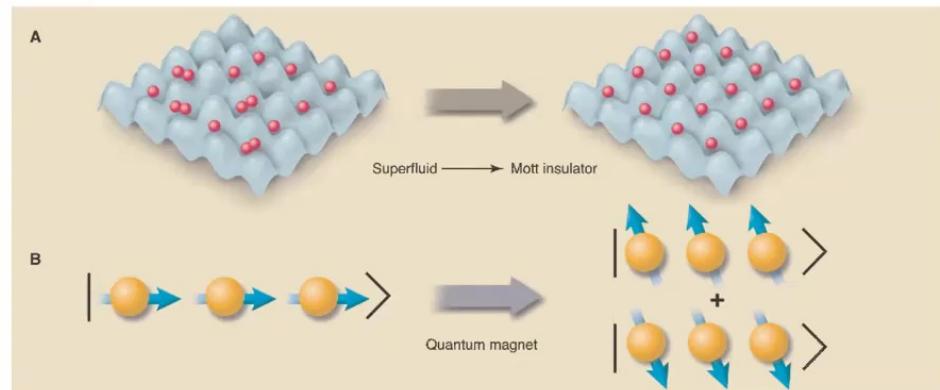


Simulating physics

Dynamics



Equilibrium



¹ Georgescu, Ashhab, Nori, RMP 86, 154 (2014)



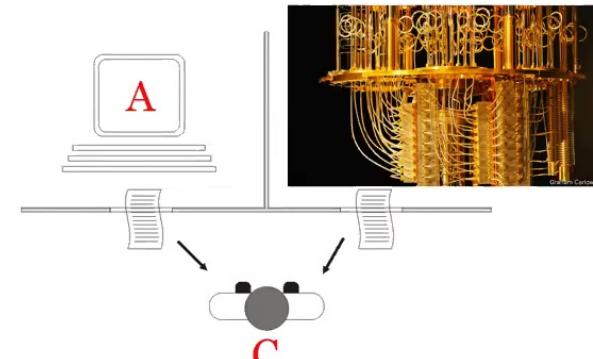
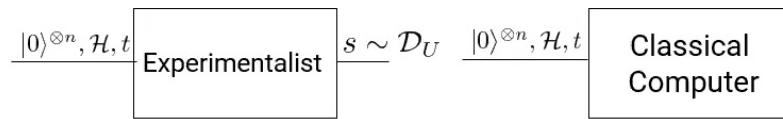
Outline

- Motivation & Background
- Dynamics
 - Interacting bosons
 - Localization
- Equilibrium
 - Ground states
- Outlook



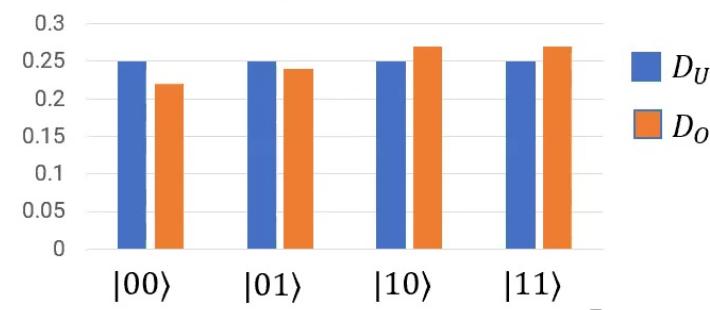
Defining simulation

- Mimic the experiment so that a referee cannot distinguish between the two.



$$|\psi\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

Probability distributions

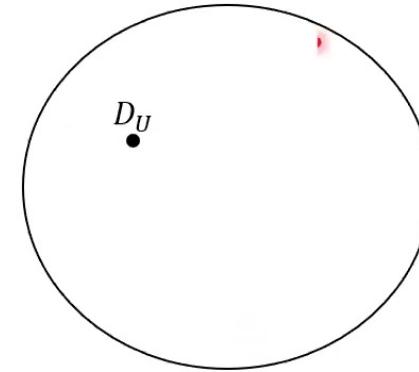


- Many modern quantum experiments sample from some probability distribution.



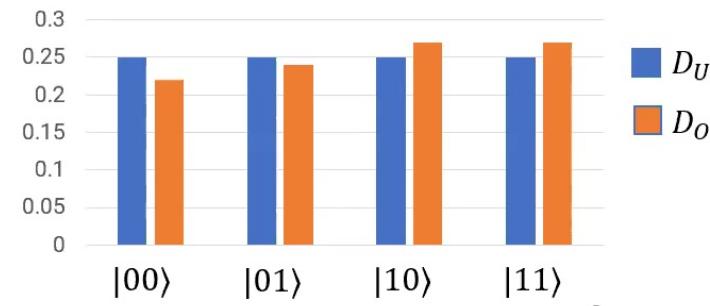
Sampling complexity

D_U : ideal distribution



$$|\psi\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

Probability distributions



8



Sampling complexity

D_U : ideal distribution

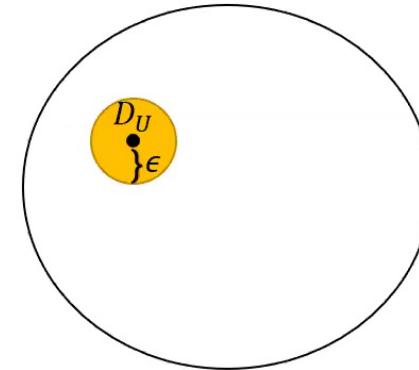
D_O : sampled distribution (by algorithm)

Error: variation distance between distributions

$$\epsilon = \left| |D_U - D_O| \right| = \frac{1}{2} \sum_x |p_U(x) - p_O(x)|.$$

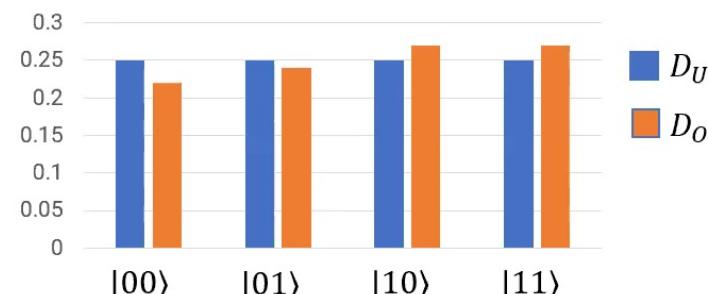
Easy: there exists a classical algorithm that can sample from some ϵ -close distribution D_O

- has runtime polynomial in n (number of qubits/particles)
- has poly-small ϵ .



$$|\psi\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

Probability distributions



8

Sampling complexity

D_U : ideal distribution

D_O : sampled distribution (by algorithm)

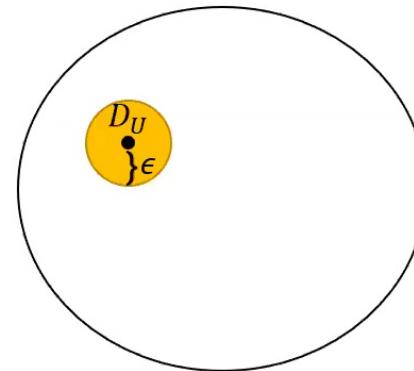
Error: variation distance between distributions

$$\epsilon = \left| |D_U - D_O| \right| = \frac{1}{2} \sum_x |p_U(x) - p_O(x)|.$$

Easy: there exists a classical algorithm that can sample from some ϵ -close distribution D_O

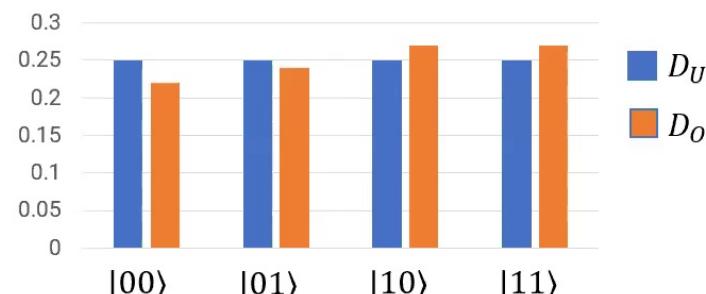
- has runtime polynomial in n (number of qubits/particles)
- has poly-small ϵ .

Hard if not easy.



$$|\psi\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

Probability distributions

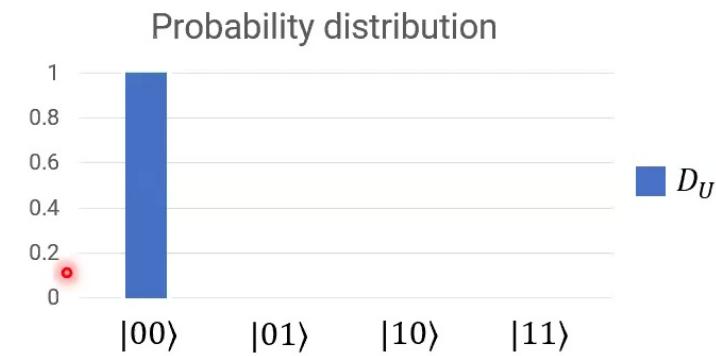




Dynamical phase transition

- Initial state:

$$|\psi(t=0)\rangle = |00\dots 0\rangle \text{ (easy)}$$





Dynamical phase transition

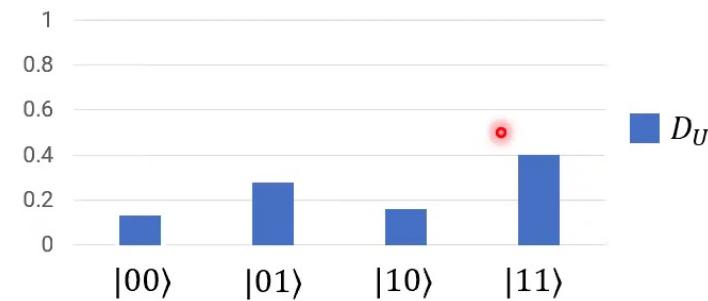
- Initial state:

$$|\psi(t=0)\rangle = |00\dots 0\rangle \text{ (easy)}$$

- After some time t :

$$|\psi(t)\rangle = e^{-iHt} |00\dots 0\rangle \text{ (hard)}$$

Probability distribution



9



Dynamical phase transition

- Initial state:

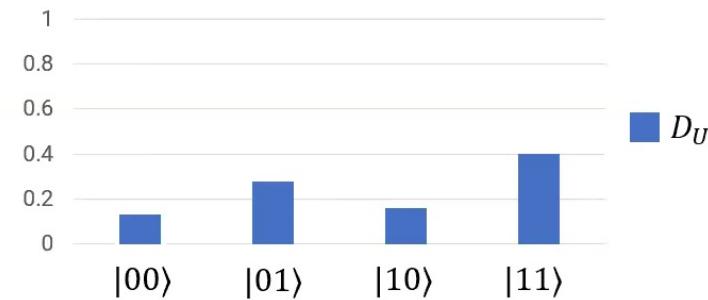
$$|\psi(t=0)\rangle = |00\dots 0\rangle \text{ (easy)}$$

- After some time t :

$$|\psi(t)\rangle = e^{-iHt} |00\dots 0\rangle \text{ (hard)}$$

- For every function $t(n)$, either easy or hard.

Probability distribution





Dynamical phase transition

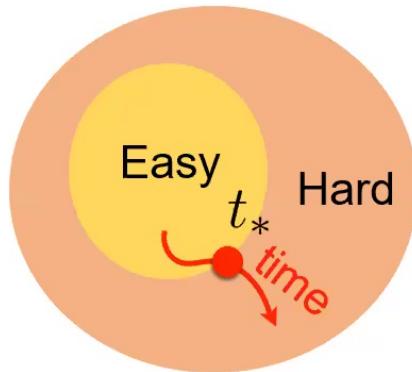
- Initial state:

$$|\psi(t=0)\rangle = |00\dots 0\rangle \text{ (easy)}$$

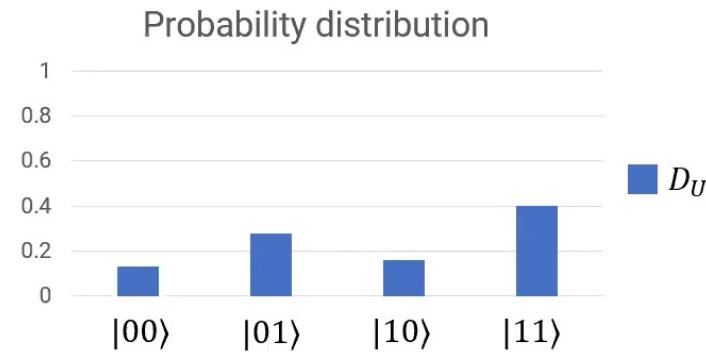
- After some time t :

$$|\psi(t)\rangle = e^{-iHt} |00\dots 0\rangle \text{ (hard)}$$

- For every function $t(n)$, either easy or hard.



What is the timescale $t_*(n)$?
Can it be used to classify Hamiltonians?
A “dynamical phase transition”



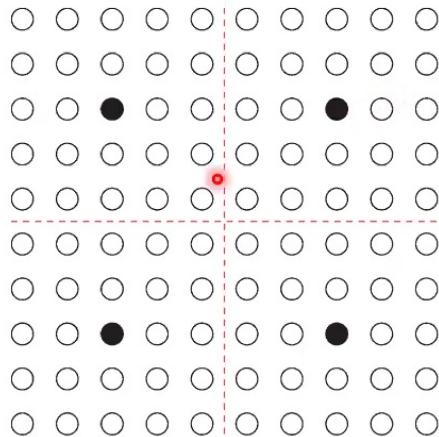


Bosons on a lattice

Consider an interacting bosonic Hamiltonian

$$H = \sum_{i,j} J_{ij}(t) a_i^\dagger a_j + V \sum_i \frac{n_i(n_i-1)}{2}$$

n bosons, $m = \Theta(n^\beta)$ sites, D dimensions.





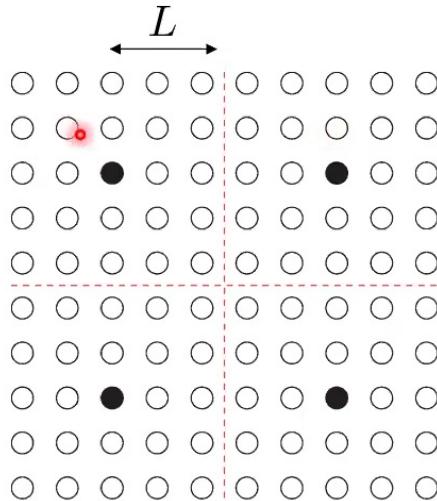
Bosons on a lattice

Consider an interacting bosonic Hamiltonian

$$H = \sum_{i,j} J_{ij}(t) a_i^\dagger a_j + V \sum_i \frac{n_i(n_i-1)}{2}$$

n bosons, $m = \Theta(n^\beta)$ sites, D dimensions.

Start with sparse initial state, so that lattice can be divided into clusters of radii $L = \Theta\left(\frac{m}{n}\right)^{1/D} = \Theta(n^{(\beta-1)/D})$.





Bosons on a lattice

Consider an interacting bosonic Hamiltonian

$$H = \sum_{i,j} J_{ij}(t) a_i^\dagger a_j + V \sum_i \frac{n_i(n_i-1)}{2}$$

n bosons, $m = \Theta(n^\beta)$ sites, D dimensions.

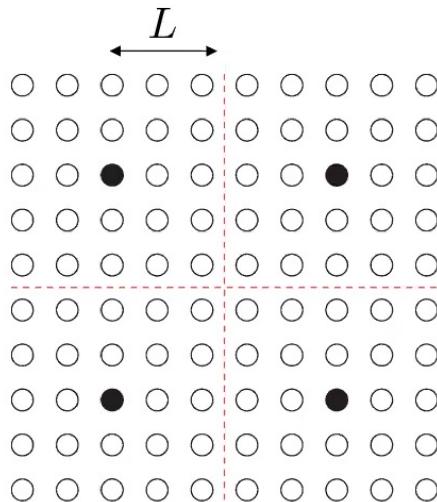
Start with sparse initial state, so that lattice can be divided into clusters of radii $L = \Theta\left(\frac{m}{n}\right)^{1/D} = \Theta(n^{(\beta-1)/D})$.

Long-range hops allowed: $|J_{ij}| \leq \frac{1}{d(i,j)^\alpha}$

$\alpha \rightarrow \infty$: nearest-neighbor

$\alpha = 0$: all-to-all hopping

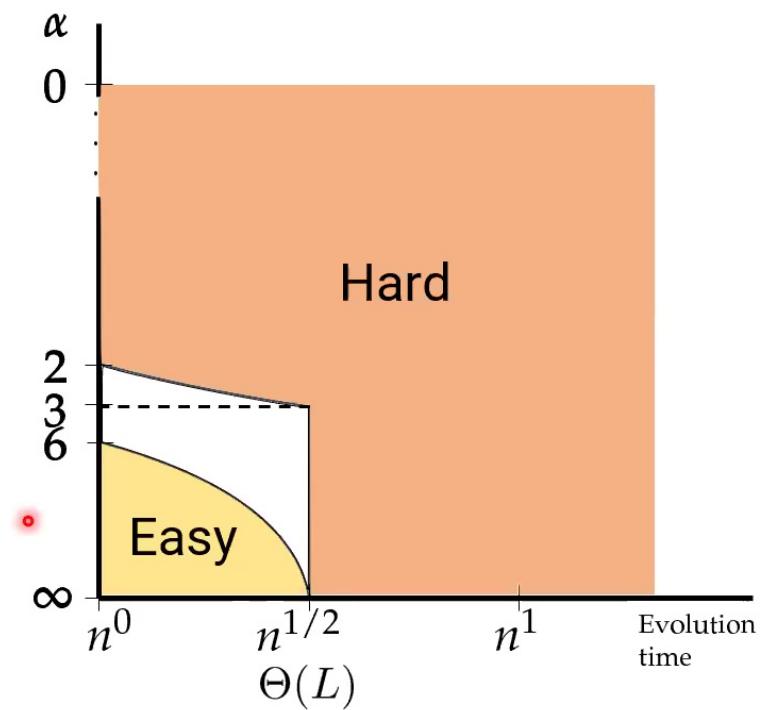
α finite, $V \rightarrow \infty$: long-range interacting spin systems



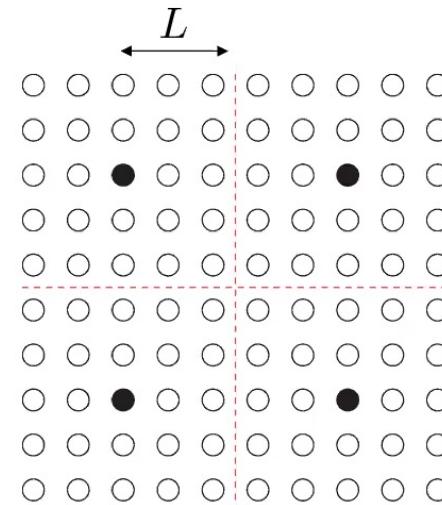
Models several experiments
Ultracold atoms, Rydberg
atoms with van-der Waals
interactions, dipole-dipole
interactions, ion traps



(Slice of) Phase diagram



$$\beta = 2, D = 2$$



n bosons, $m = \Theta(n^2)$ sites

$$|J_{ij}| \leq \frac{1}{d(i,j)^\alpha}, L = \Theta(\sqrt{n})$$

¹ A. D. et al., PRL 121, 030501 (2018) ² G. Muraleedharan et al., NJP 21, 055003 (2019) ³ N. Maskara*, A. D.* et al., arXiv:1906.04178



Localized systems

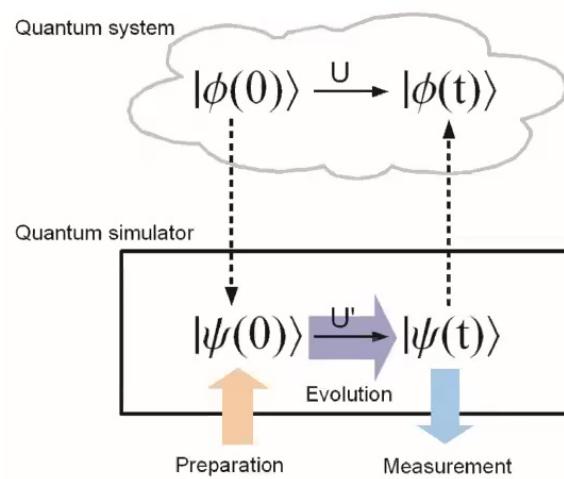
- Corollary: If Anderson-localized, $t_* = \infty$ (always easy)
- What about many-body localization (MBL)?
- Upcoming work for 1D MBL Hamiltonian on n spins
 - Sampling is in quasipolynomial time (quasi-easy) for $t = O(n^c)$ (any c).
 - Hard for exponential time $t = \exp(\Omega(n^\delta))$ (any $\delta > 0$)
- Difference between Anderson and many-body localization is detectable through sampling complexity as well¹

¹ M. C. Bañuls et al., PRB 174201 (2017)

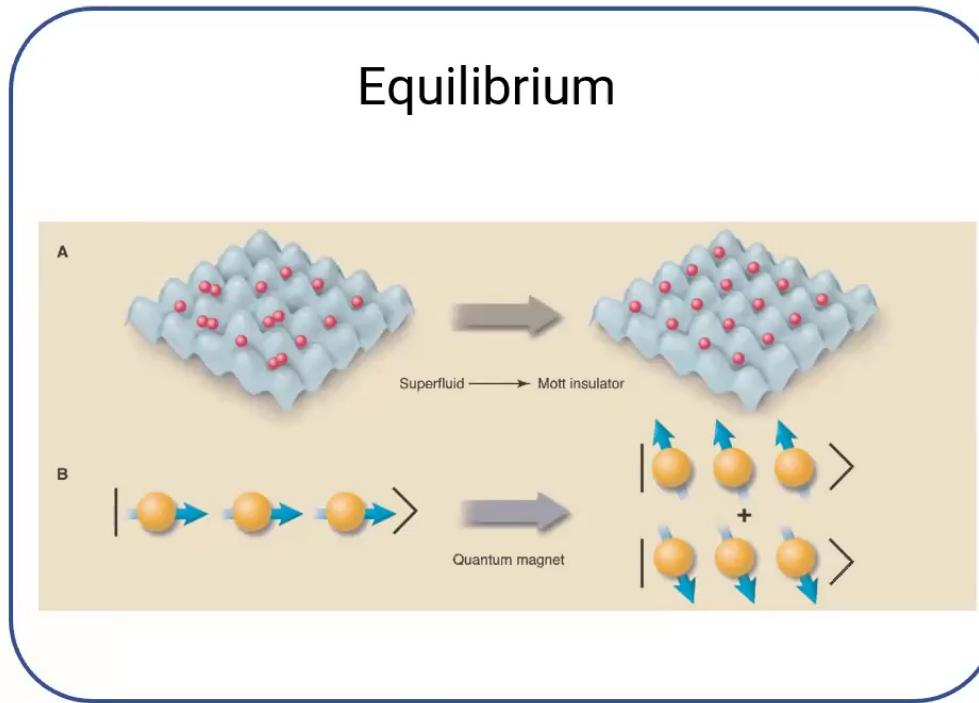


Simulating physics

Dynamics



Equilibrium



¹ Georgescu, Ashhab, Nori, RMP 86, 154 (2014)



Outline

- Motivation & Background
- Dynamics
 - Interacting bosons
 - Localization
- Equilibrium
 - Ground states
- Outlook

•

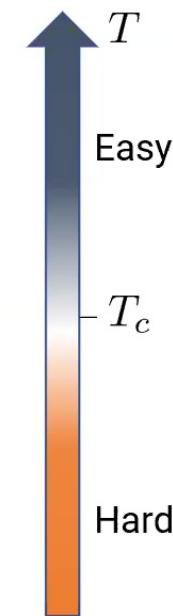
14



Simulating equilibrium physics

- Task of simulating physics at temperature T
- At high enough temperature: easy
- Ground states: hard?

•





Ground-state complexity

- Task: find the ground-state energy of a local Hamiltonian H to $1/\text{poly}(n)$ additive error.
- QMA-hard in general^{1,2}
- QMA is the quantum generalization of NP: likely intractable
- Hardness results persist even for “natural” Hamiltonians³⁻⁶.

If the ground states of interacting QFTs are so complicated, how did Nature find them?

Asked 7 years, 10 months ago Active 7 years, 10 months ago Viewed 1k times

Scott Aaronson/physics.stackexchange

What breaks?

Hardness results might be missing structure.

¹ Kitaev, Shen, Vyalyi (2002)

² Kempe, Kitaev, Regev, arXiv:quant-ph/0406180.

³ Aharonov et al. arXiv:0705.4077

⁴ Oliveira and Terhal, arXiv:quant-ph/0504050

⁵ Gottesman and Irani, arXiv:0905.2419

⁶ Cubitt and Montanaro, arXiv:1311.3161



Spectral gap

- Physics

- Guarantees area laws¹, tensor networks¹, rigorous algorithms² (proven in 1D)
- Performance of adiabatic algorithm

- Hamiltonian complexity

- Existing techniques can't give hard instances with $\Theta(1)$ spectral gap^{3,4}
- Even $1/\text{poly}(n)$ spectral gaps aren't understood⁵
- Our work⁶: evidence that spectral gap makes problem easier

¹ M. Hastings, J. Stat. Mech., 08024 (2007)

³ González-Guillén and Cubitt, arXiv:1810.06528

⁵ Aharonov et al., arXiv:0810.4840

17

² Landau, Vazirani, Vidick, Nat. Phys. (2015)

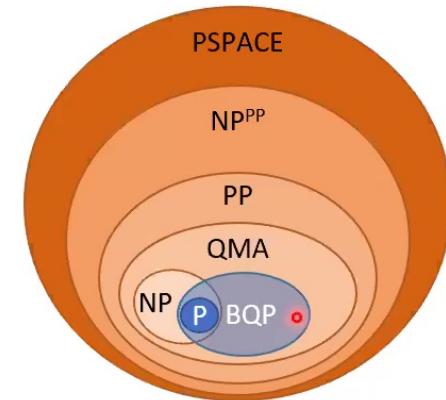
⁴ Crosson and Bowen, arXiv:1703.10133

⁶ A. D. Gorshkov, and Fefferman, arXiv:2007.11582



Main result 1

- Provable setting where the spectral gap affects the complexity
- Informally: compute ground-state energy of Δ -gapped Hamiltonian to inverse-exponential precision
- $\Delta \geq 0$ case (no promise): PSPACE-complete¹



18

¹ Fefferman and Lin, arXiv:1601.01975, arXiv:1604.01384

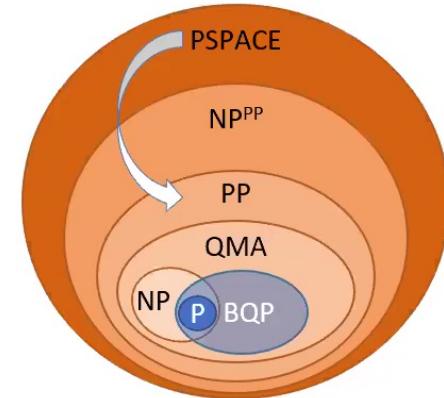


Main result 1

- Provable setting where the spectral gap affects the complexity
- Informally: compute ground-state energy of Δ -gapped Hamiltonian to inverse-exponential precision
- $\Delta \geq 0$ case (no promise): PSPACE-complete¹
- We show: $\Delta \geq 1/\text{poly}(n)$ case is PP-complete.

Result 1

The spectral gap provably makes the problem easier* in this setting



18

¹ Fefferman and Lin, arXiv:1601.01975, arXiv:1604.01384

Main result 1

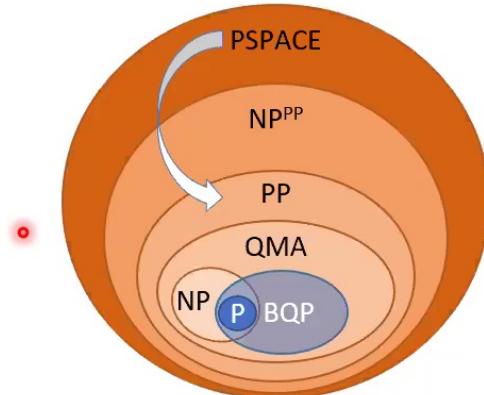
- Provable setting where the spectral gap affects the complexity
- Informally: compute ground-state energy of Δ -gapped Hamiltonian to inverse-exponential precision
- $\Delta \geq 0$ case (no promise): PSPACE-complete¹
- We show: $\Delta \geq 1/\text{poly}(n)$ case is PP-complete.

Result 1

The spectral gap provably makes the problem easier* in this setting

- Weak evidence it's true in general

¹ Fefferman and Lin, arXiv:1601.01975, arXiv:1604.01384

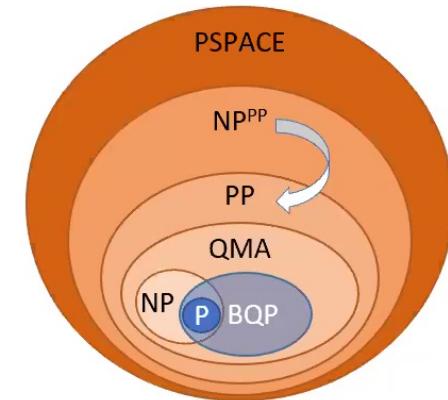




Main result 2

- Suppose there exists a polynomial-size circuit to prepare the ground state. “Low circuit-complexity promise”
- Again compute ground-state energy of Δ -gapped Hamiltonian to $1/\exp$ precision.
- $\Delta \geq 1/\text{poly}(n)$ case + low circuit-complexity promise again PP-complete.

Result 2
Promise of low circuit-complexity makes no difference if spectral-gap promise is already present



19



An interesting conjecture

- A candidate explanation for Result 2

Conjecture
Spectral gap implies polynomial-size circuit to
prepare low-energy state

- Conjecture implies most natural ground states have short circuits to prepare them!
- Explains why variational algorithms¹ seem to perform well

¹A. Peruzzo et al., Nat. Comm. 5, 4213 (2014)



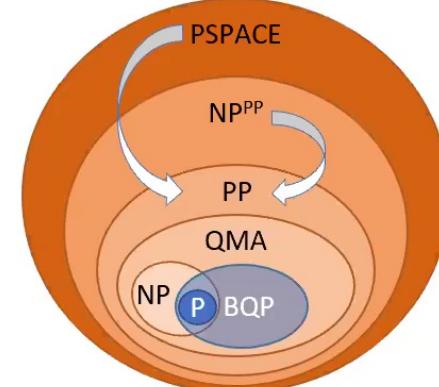
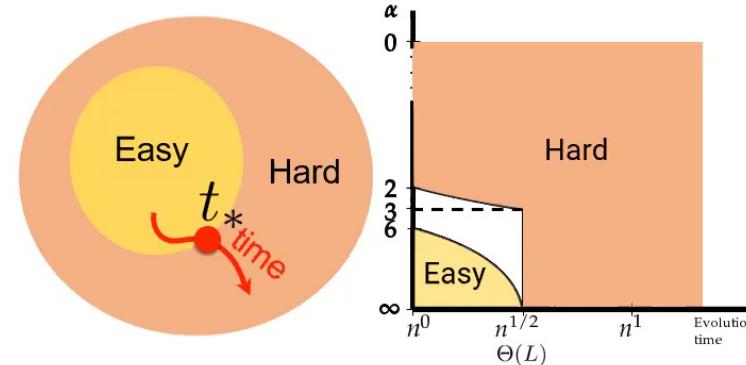
Outline

- Motivation & Background
- Dynamics
 - Interacting bosons
- Equilibrium
 - Ground states
- Outlook



Summary

- Complexity as a way of classifying dynamics
 - Long-range Hamiltonians
 - Anderson-localized systems
 - Many-body-localized systems
- Complexity of equilibrium states
 - Spectral gap affects ground-state complexity
 - Conjecture, if true, would explain why VQE works well

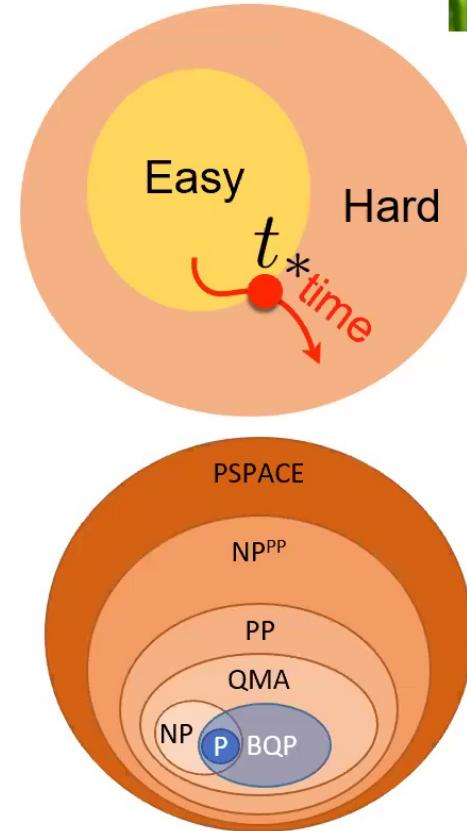


22



Outlook

- Classical & quantum algorithms for simulation
- More applications of complexity theory
 - Entanglement phase transitions¹⁻⁴?
 - Can noise make a system hard to simulate⁵?
- Other features of tractable Hamiltonians
 - $\Theta(1)$ -gapped ground states: connections to area laws, tensor-network descriptions
 - States at higher temperatures?



¹ Chan et al., PRB 99, 224307 (2019) ³ Li et al., PRB 98, 205136 (2020)

² Skinner et al., PRX 9, 031009 (2019) ⁴ Bao, Choi, Altman, PRB 101, 104301 (2020)

⁵ O. Shtanko, A. D., et al., arXiv:2005.10840



Thank You!

Collaborators on these works

Alexey Gorshkov



Chris Baldwin
Chi-Fang Chen
Adam Ehrenberg
Michael Foss-Feig
Zhexuan Gong
Michael Gullans
Andrew Guo

Bill Fefferman

Yifan Hong
Paul Julienne
Andrew Lucas
Nishad Maskara
Pradeep Niroula
Oles Shtanko
Minh Tran