Title: Generalized entropy in topological string theory

Speakers: Gabriel Wong

Series: Quantum Fields and Strings

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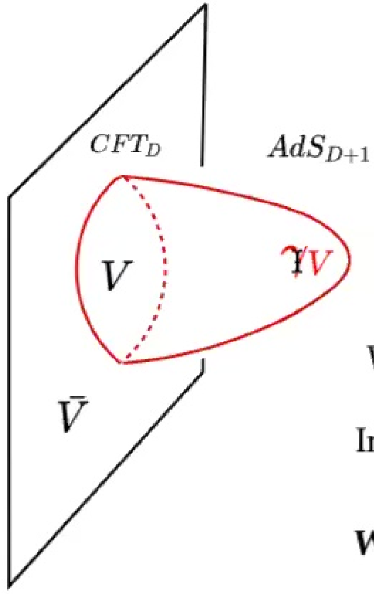
Abstract: The Ryu Takayanagi formula identifies the area of extremal surfaces in AdS with the entanglement entropy of the boundary CFT. However the bulk microstate interpretation of the extremal area remains mysterious. Progress along this direction requires understanding how to define entanglement entropy in the bulk closed string theory. As a toy model for AdS/CFT, we study the entanglement entropy of closed strings in the topological A model in the context of Gopakumar Vafa duality. We give a self consistent factorization of the closed string Hilbert space which leads to string edge modes transforming under a q-deformed surface symmetry group. Compatibility with this symmetry requires a q-deformed definition of entanglement entropy. Using the topological vertex formalism, we define the Hartle Hawking state for the resolved conifold and compute its q-deformed entropy directly from the closed string reduced density matrix. We show that this is the same as the generalized entropy, defined by prescribing a contractible replica manifold for the closed string theory on the resolved conifold. We then apply the Gopakumar Vafa duality to reproduce the closed string entropy from Chern Simons dual using the un-deformed definition of entanglement entropy. Finally we relate non local aspects of our factorization map to analogous phenomenon recently found in JT gravity.

Generalized entropy in topological string theory

Gabriel Wong

Fudan University

In collaboration with William Donnelly, Manki Kim, Yikun Jiang.
Based on hep-th https://arxiv.org/pdf/2010.15737.pdf
And a follow up hep-th 2012.XXXX



The HRRT / generalized entropy formula

is the basis for understanding space-time emergence from entanglement.

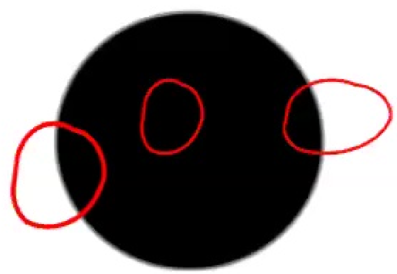
$$S_{CFT} = S_{\text{gen}} = \frac{\langle A(\gamma_V) \rangle}{4G} + S_{\text{bulk}} + \cdots$$ RT 2006, FLM 2013, HRT, Dong

What is the bulk microscopic interpretation of the area term?

Interesting because it measures the entanglement **of the spacetime** (Van Raamsdonk)

We want to understand this question from the bulk string theory

Bekenstein Hawking entropy as string entanglement entropy



$$S_{BH} = \frac{A}{4G}$$

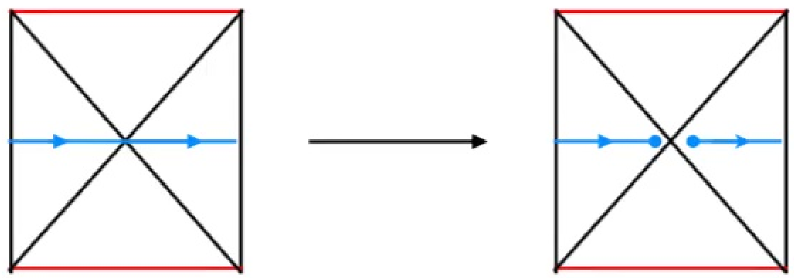
Susskind Uglum: BH entropy is the entanglement entropy of closed strings across the horizon

The factorization problem and edge modes

Understanding entanglement in the bulk requires factorization

Bulk factorization is hard because it is sensitive to UV d.o.f. (Harlow)

$$\mathcal{H}_{\text{bulk}} \longrightarrow \mathcal{H}_L \otimes \mathcal{H}_R$$



Boundary CFT factorizes. But the low energy theory in the bulk does not naively factorize

Need to introduce high energy bulk charges to factorize a Wilson line in thelow energy EFT.

Low energy theory must be extended to include **entanglement edge modes** that probe the bulk microstates.

A tempting conjecture:

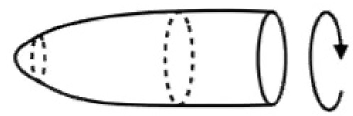
Perhaps the area term is the entanglement entropy of gravity edge modes

(J Lin, D. Harlow, Donnelly-Friedel, Donnelly-Wong)

Area term in generalized entropy

The generalized entropy is obtained via the **Euclidean gravity** path integral $Z(\beta)$ on a cigar geometry

$$S_{\text{gen}} = (1 - \beta\partial_\beta) \log Z(\beta)$$
$$= \frac{\langle A \rangle}{4G} + S_{\text{out}} + \cdots$$

$Z(\beta) =$  β

(Gibbons- Hawking)

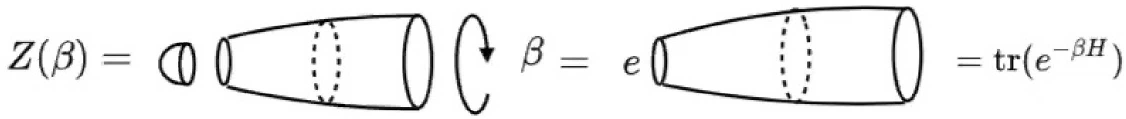
The area term comes from saddles where the circle shrinks in the interior to make a cigar geometry (See also recent discussion by Harlow- Shaghoulian, Jafferis-Kolchmeyer)

Can we give a canonical interpretation?

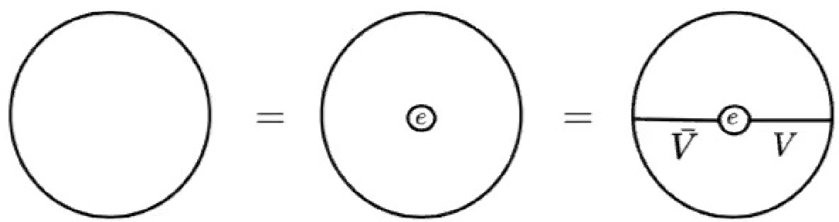
$$Z(\beta) = \text{tr}\, e^{-\beta H}?$$

Shrinkable boundary conditions and edge modes

Naively, we can give a trace interpretation if we have a **shrinkable boundary condition** e



This suggests generalized entropy is related to entanglement entropy:



$$\rho_V = e^{-\beta H}$$

In gravity, a topological constraint (Gauss Bonnet) implies the shrinkable boundary condition determined by the cap is non local in modular time (Kolchmeyer Jafferis). How do we interpret this?

Motivated by Susskind and Uglum, we look for an answer in string theory

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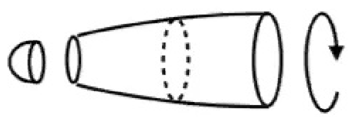
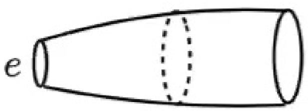
$Z(\beta) =$  $\beta =$ e  $= \mathrm{tr}(e^{-\beta H})$

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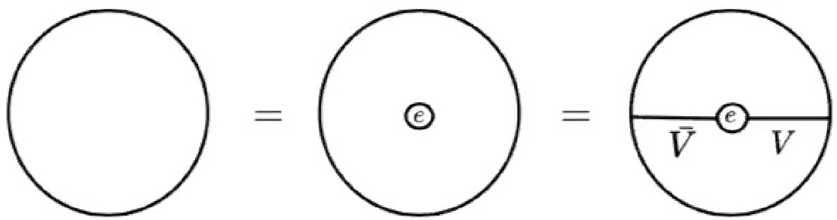

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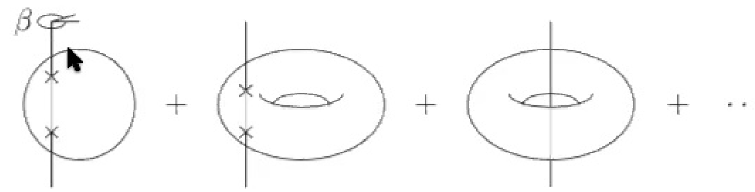
BH entropy from perturbative closed strings

Consider string theory on the flattened cigar geometry (Susskind Uglum)

$$S = (1 - \beta\partial_\beta)\,|_{\beta=2\pi}\log Z(\beta)$$

$$\log Z \; =$$



String theory path integral on manifold with conical angle

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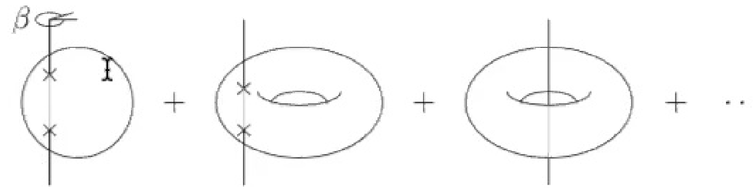
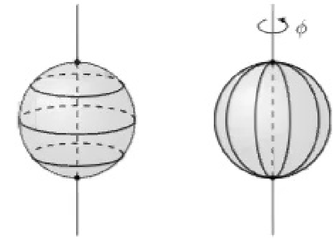
$\log Z \;=$ 

String theory path integral on manifold with conical angle

Open-closed string duality gives a canonical interpretation to the tree level entropy

$$S_{BH} = \frac{A}{4G} = O\left(\frac{1}{g_s^2}\right)$$

Contact term



$$Z_{\text{sphere}}(\beta) = \mathrm{tr}(e^{-H_{\text{open}}})$$

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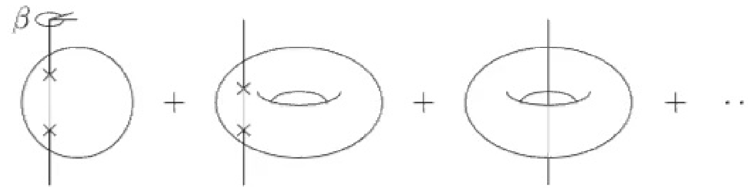

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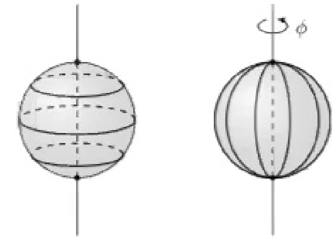

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String edge modes are the branes (non- perturbative objects!) needed to cut the string (Donnelly Wong 2016)

Susskind-Uglum : Generalized entropy =entanglement entropy of closed strings

Previous work

- Orbifold replica trick, String field theory, Topological string/Chern Simons gauge theory duality (Hubeny- Rangamani-Pius, Witten, Takayanagi et. al, Balasubramanian-Parrikar , Nassar ,Dabholkar, Strominger,...)

- String theory dual to 2DYM- Extended TQFT methods (Donnelly -Wong)

We apply extended TQFT methods to A model topological string

Important features:

- Perturbative amplitudes computable to all orders in string coupling

- String field theory is captured by a topological field theory !

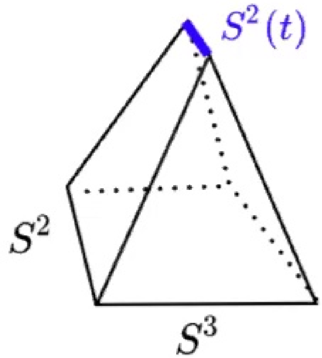
- UV completion from large N limit of q-deformed Yang Mills

  (Sums over topology changing processes corresponding to baby universes)

Overview

- We study entanglement entropy in top string theory using Gopakumar Vafa duality as a topological analog of AdS/CFT

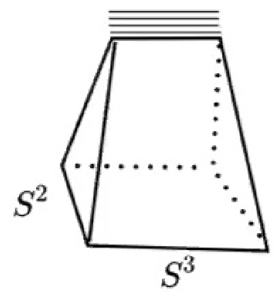
Closed strings on resolved Conifold $\longleftrightarrow$ **Open strings on deformed Conifold**



A Model TQFT

B field flux on $S^2(t)$

$$t = ig_s N$$



N branes on S^3

U(N) Chern Simons theory on S^3

- We defined a notion of generalized entropy for the topological string
- We apply extended TQFT methods to factorize the ``bulk" closed string Hilbert space
- Canonical calculation of generalized entropy on both sides of the duality. Edge modes are anyons transforming under a quantum group symmetry.
- Realization of Susskind Uglum: Generalized entropy of closed strings = Thermal entropy of open strings ending on entanglement branes

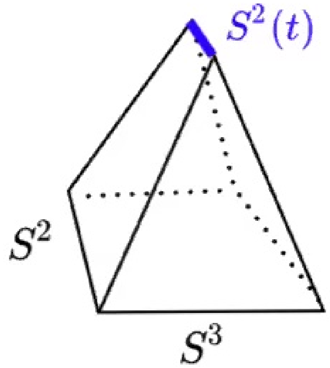
Outline

- Hartle Hawking state in string theory
- Formulating the factorization problem in extended TQFT
- A model closed TQFT and generalized entropy
- D brane edge modes and the canonical calculation of generalized entropy

A model topological strings

The resolved conifold

$t = A + iB$ Kahler modulus

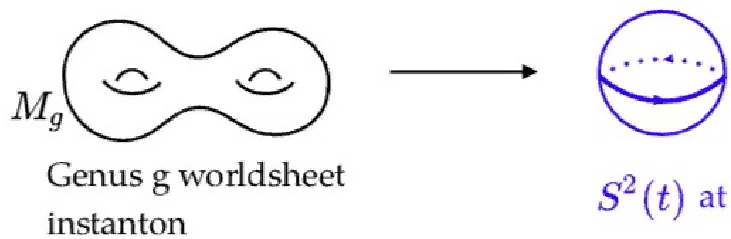


The free energy is an instanton sum

2- homology class

$$F = \sum_{g,n} g_s^{2g-2} N_{g,n} e^{-nt}.$$

Gromov Witten invariants



Target space = 6 dimensional Kahler manifold (Not necessarily Calabi Yau)

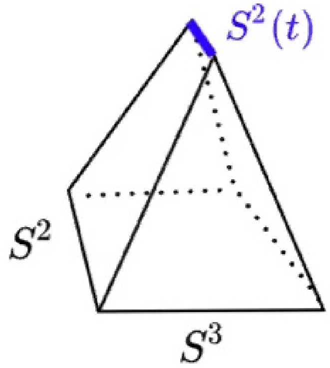
Closed string theory localizes to worldsheets instantons wrapping minimal volume 2 cycles

Amplitudes depend only on Kahler modulus

A model topological strings

The resolved conifold

$t = A + iB$ Kahler modulus



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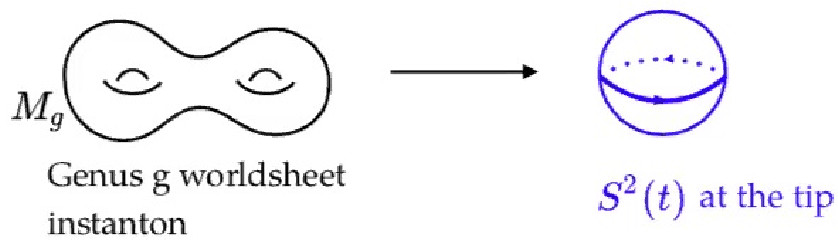
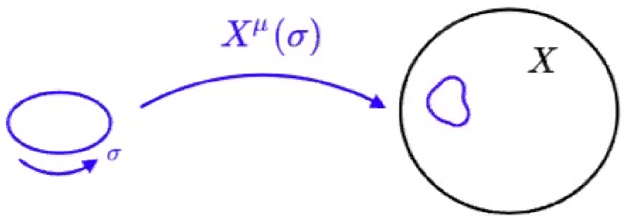


The resolved conifold partition function.

$$Z = e^{-F} = \exp\left(\sum_{n=1}^{\infty} \frac{1}{n(2\sin(\frac{ng_s}{2}))^2} e^{-nt}\right)$$

Time slices in string theory

In the first quantized (single string) theory, a state is a wavefunctional of closed loop



Single string wavefunctional

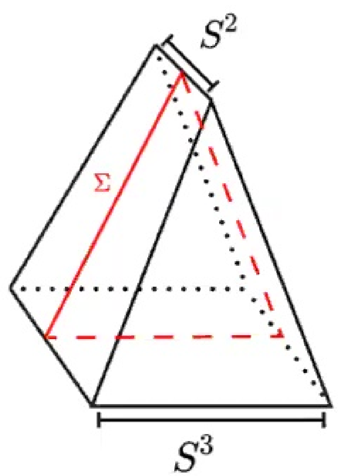
$$\Psi[X^\mu(\sigma)]$$

$X(\sigma)^\mu \in \mathcal{F}$ =loop space of X

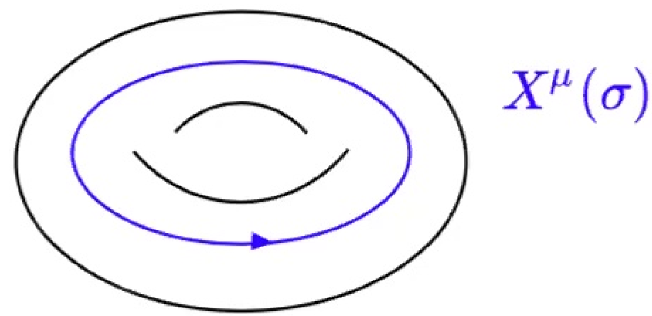
In the second quantized theory, the string field is an operator valued function on $\mathcal{F}$

$$\Psi[X^\mu(\sigma)] \longrightarrow \hat{\Psi}[X^\mu(\sigma)]$$

Degrees of freedom lives on $\mathcal{F} \longrightarrow$ a **time slice is a subset of** $\mathcal{F}_\Sigma \subset \mathcal{F}$



$$\mathcal{L} = \mathbb{C} \times S^1 \subset \Sigma$$



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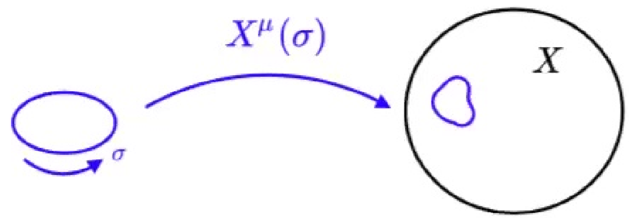

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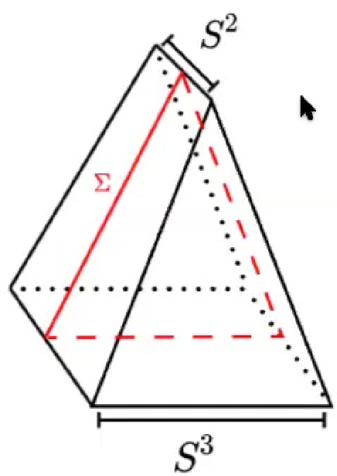

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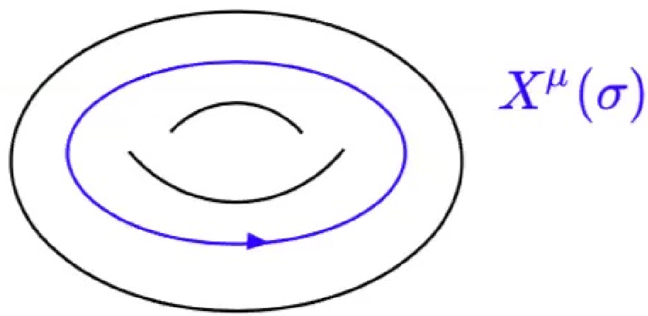


Closed TQFT and the category of 2D cobordisms

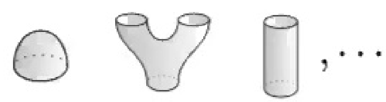
The cobordism description is a "generator-relations" approach to the path integral.

Cobordisms form a category:

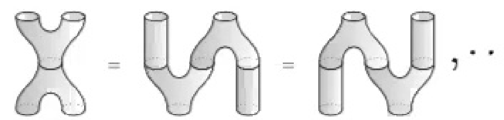
Objects= co-dim 1 manifolds

$\mathcal{H}_{\text{closed}}$

Morphisms =Cobordisms



Sewing relations



A 2D closed TQFT is a rule that assigns Hilbert space to circles and linear maps to cobordism. The composition of linear maps satisfies the sewing relations.

These cobordisms are in the **target space**

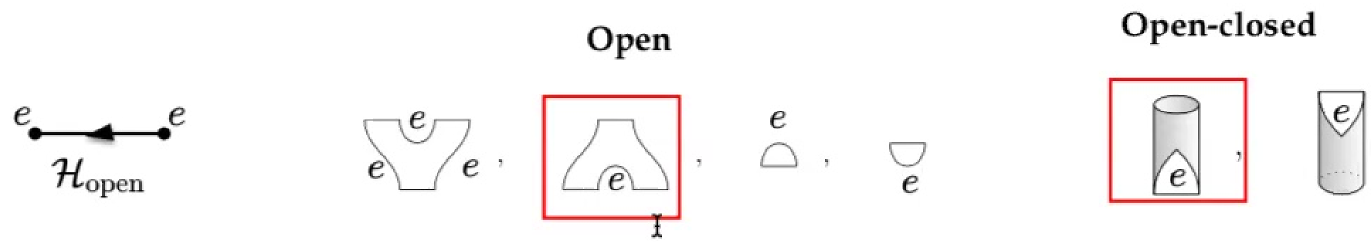
2D TQFT is a **Frobenius Algebra** with the multiplication of states defined by

$$\begin{array}{c} \mathcal{H}_{\text{closed}} \otimes \mathcal{H}_{\text{closed}} \\ \big\downarrow \\ \mathcal{H}_{\text{closed}} \end{array}$$

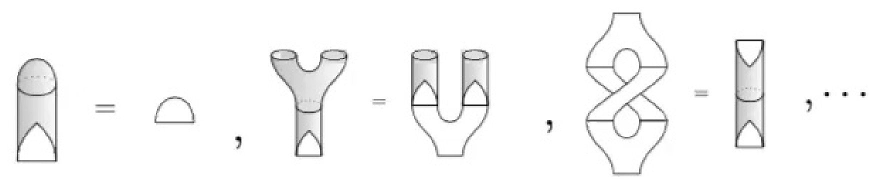
2D Extended TQFT

To describe entanglement and factorization we need to cut open the co-dim 1 time slices

This requires an **extension of the TQFT** that introduces codim 1 manifolds with boundaries that support edge modes



We treat the factorization maps as elements of the cobordism data subject to **local constraints** given by **open-closed sewing relations** (Moore Segal, Lazariou, Pfeiffer and Lauda)



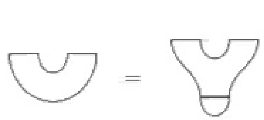
2D Extended TQFT

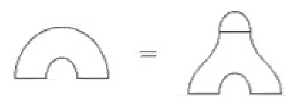
The extended TQFT assigns an open string Hilbert space $\mathcal{H}_{\text{open}}$ to the interval

Trace and adjoint operation on $\mathcal{H}_{\text{open}}$:

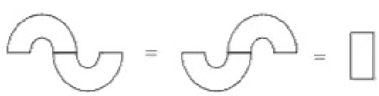
The open cobordisms include an invertible bilinear form:

$$\mathcal{H}_{\text{open}} \otimes \mathcal{H}_{\text{open}} \longrightarrow \mathbb{C}$$

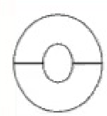




They determine a propagator (modular flow), and define a **canonical trace function**



$= e^{-\beta H}$

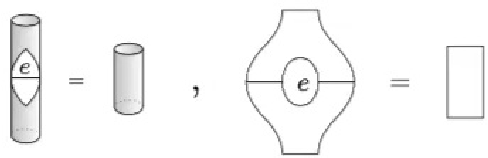


$$= \text{“}\,\mathrm{tr}\,\text{”}\, e^{-\beta H_{\text{open}}}$$

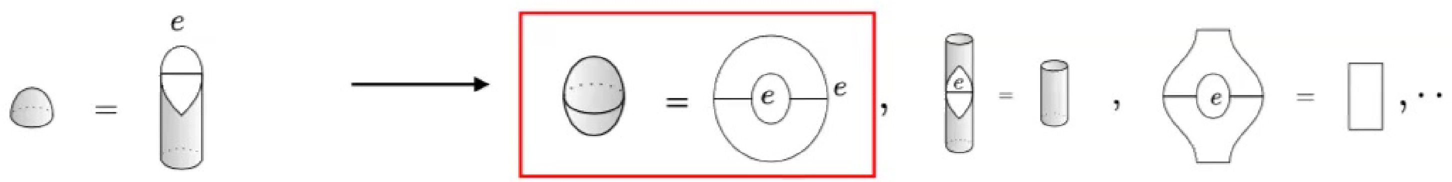
Area dependent QFT (Runkel et.al)

The E Brane axiom

Factorization should not change the state:



We introduced the **E brane axiom** to axiomatize the shrinkable boundary condition (Donnelly-Wong)



This is a reformulation of the Susskind Uglum's open-closed string duality in the **target space**

The E brane axiom and the sewing relations give a complete set of constraints which can be solved to obtain the factorization maps and edge modes by `bootstrap"

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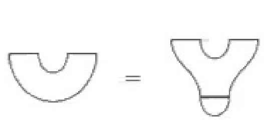

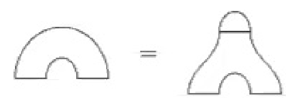

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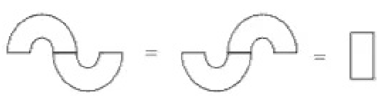

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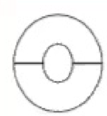

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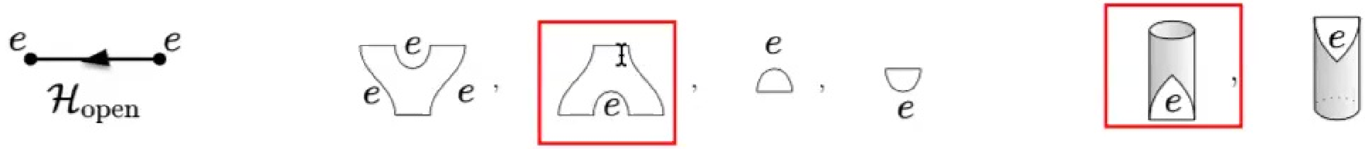
Open **Open-closed**



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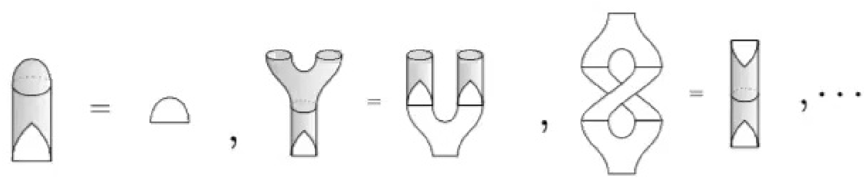

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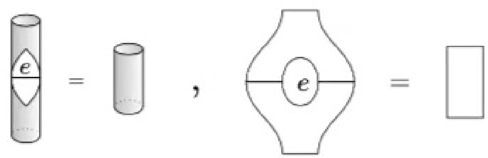

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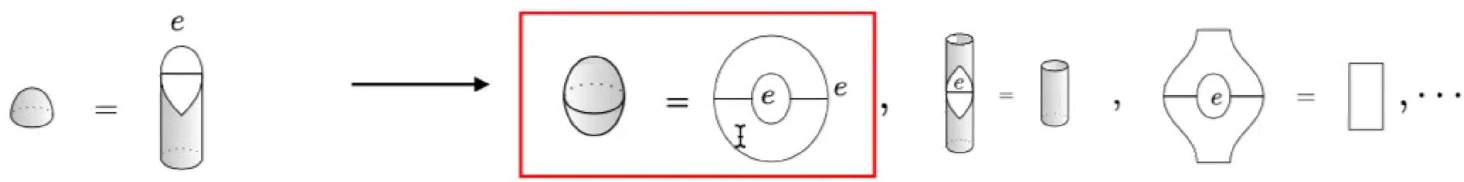

This is a reformulation of the Susskind Uglum's open-closed string duality in the **target space**

The E brane axiom and the sewing relations give a complete set of constraints which can be solved to obtain the factorization maps and edge modes by `bootstrap"

A model string field theory as an extended TQFT

Consider A model string theory on a sum of line bundles over a Riemann surface (with boundary)

$$X = L_1 \oplus L_2 \to \mathcal{S} \qquad \text{(Bryan, Pandharipande)}$$

The sewing rules of the multi-string amplitudes are the same as a closed TQFT. The cobordisms are now labelled by Chern classes (k_1, k_2) which captures the higher dimensional geometry

(k_1, k_2)

6D Target space=  $\mathcal{S}$

Resolved conifold

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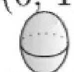
$Z = \langle HH^* | HH \rangle =$ 

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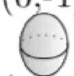
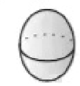
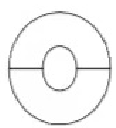
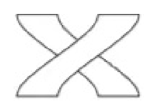
$Z = \langle HH^* | HH \rangle =$ (0,-1)  (-1,0) $=$ $(-1,-1)$ 

The new ingredients in the sewing relations are the **Chern class labelling** and a q-deformation of the edge modes

Quantum trace

 $= \mathrm{tr}_q e^{-\beta H_{\text{open}}}$

Braiding



The A model TQFT on Calabi-Yau manifolds

The Calabi Yau condition for $X = L_1 \oplus L_2 \to \mathcal{S}$ with Chern class (k_1, k_2) is

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(Aganagic- Ooguri Saulina Vafa)

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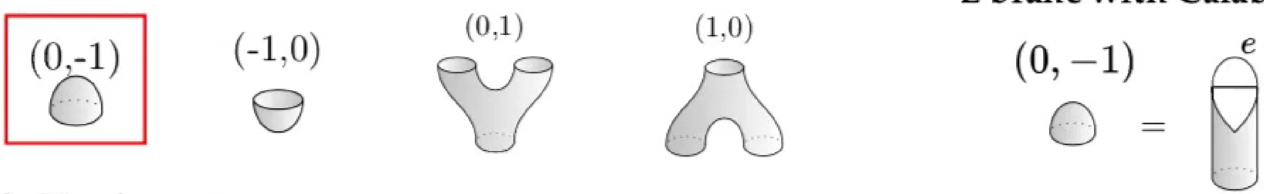

Hartle Hawking state

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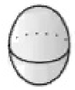
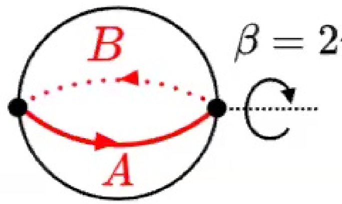
``Generalized entropy'' of the Hartle Hawking state

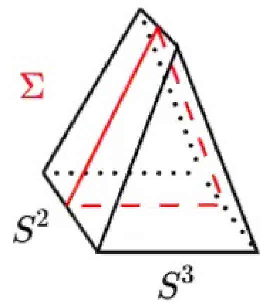
$(-1,-1)$

$Z(t) =$  $= \sum_R (d_q(R))^2 e^{-tl(R)}$ The resolved conifold

We replicate the angle around two anti podal points on the sphere

$Z(\beta) =$ 

$\beta = 2\pi n$



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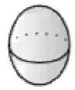
$Z(t) =$  $= \sum_R (d_q(R))^2 e^{-tl(R)}$

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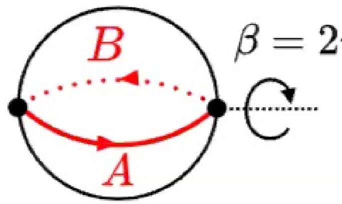
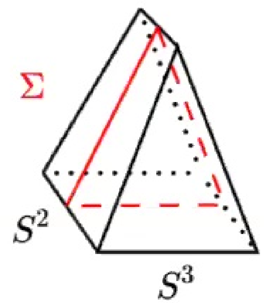
$$Z(\beta) =$$ 



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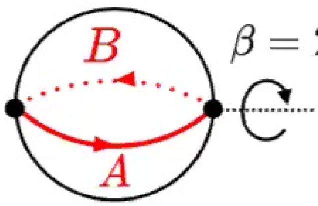
$Z(\beta) =$ 

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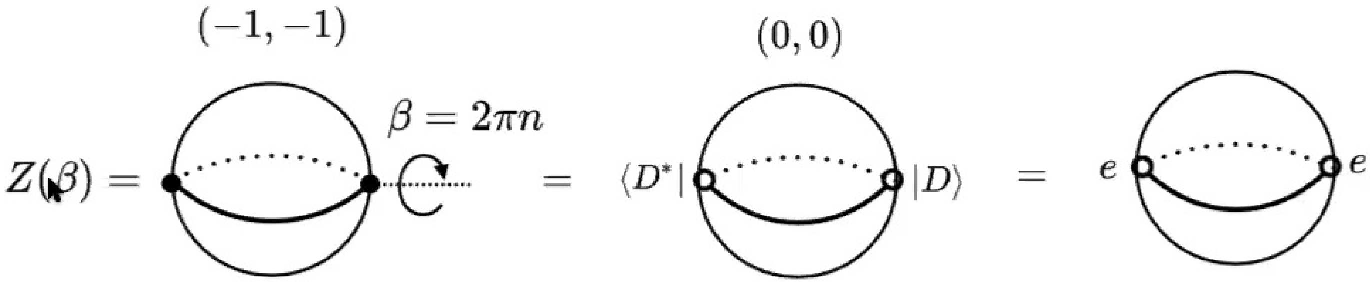
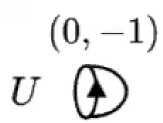
Shrinkable boundary condition from the Calabi-Yau cap



As in gravity, Calabi Yau caps imply a nonlocal shrinkable boundary condition



$$\langle U|D\rangle = \sum_R \boxed{(-i)^{l(R)} d_q(R) q^{\kappa_R/4}} \mathrm{tr}_R(U)$$
$$= \delta(U,D)$$

The worldvolume holonomy is a nontrivial diagonal matrix of phases

$$D_{ij} = \delta_{ij} q^{-i+\frac{1}{2}} \in U(\infty)$$

We will identify D with **Drinfeld** element of a quantum group

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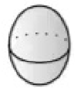
$$Z(t) = $$  $$= \sum_R (d_q(R))^2 e^{-tl(R)}$$

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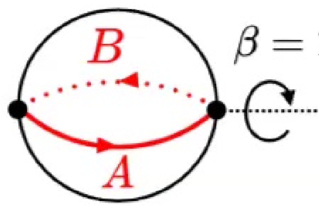
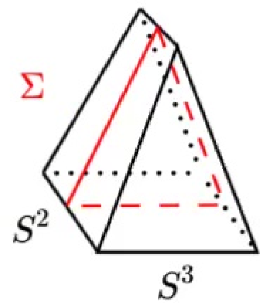
$$Z(\beta) = $$ 



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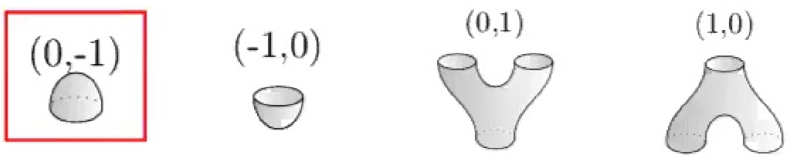

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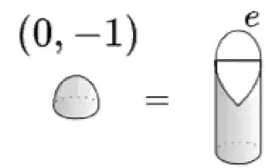

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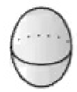
$Z(t) =$  $= \sum_R (d_q(R))^2 e^{-tl(R)}$ The resolved conifold

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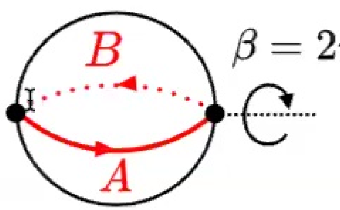
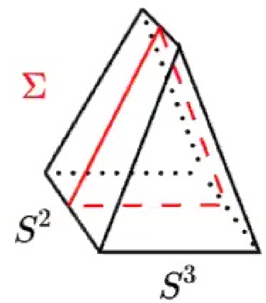
$Z(\beta) =$ 



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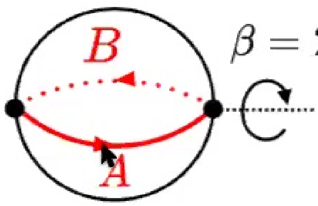

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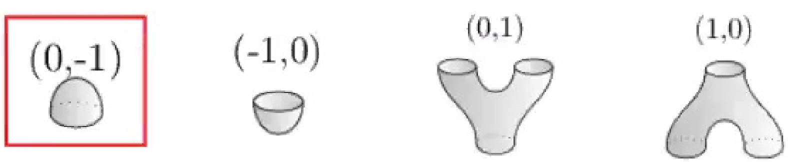

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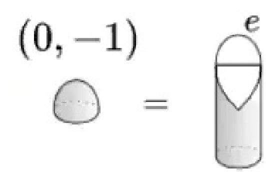

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Factorization, E brane axiom and edge mode symmetry

$$\langle U|HH\rangle = \overset{(0,\text{-}1)}{\text{}}$$  $$= \sum_R (-i)^{l(R)} d_q(R) q^{\kappa_R/4} e^{-tl(R)} \mathrm{tr}_R(U)$$

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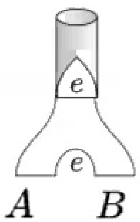
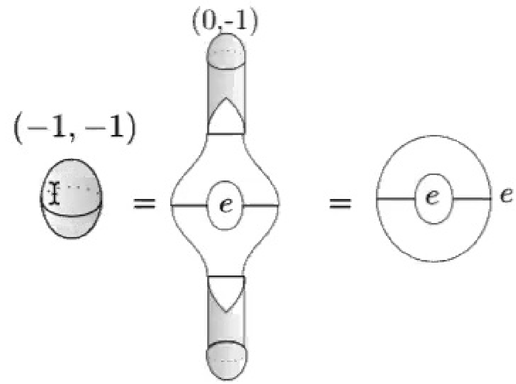
Factorization map



$$\mathcal{H}_{\text{closed}} \to \mathcal{H}_{\text{open}} \otimes \mathcal{H}_{\text{open}}$$



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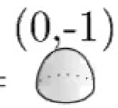
$\langle U|HH\rangle =$ 

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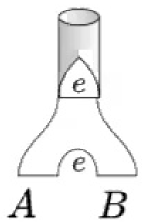

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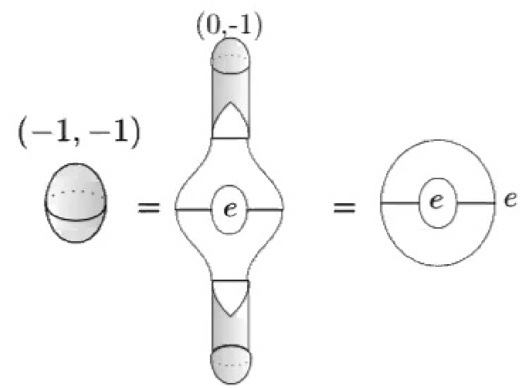


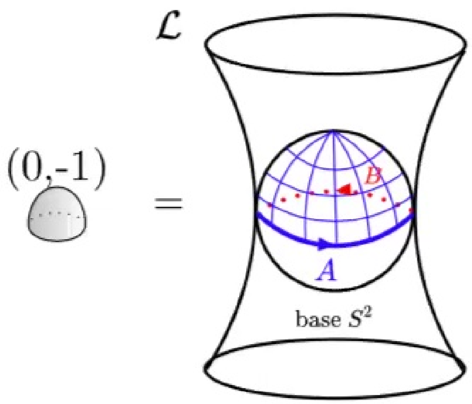
The edge mode symmetry group compatible with the E brane axiom is $U(\infty)_q$, a large N limit of the quantum group $U(N)_q$ with $q = e^{ig_s}$

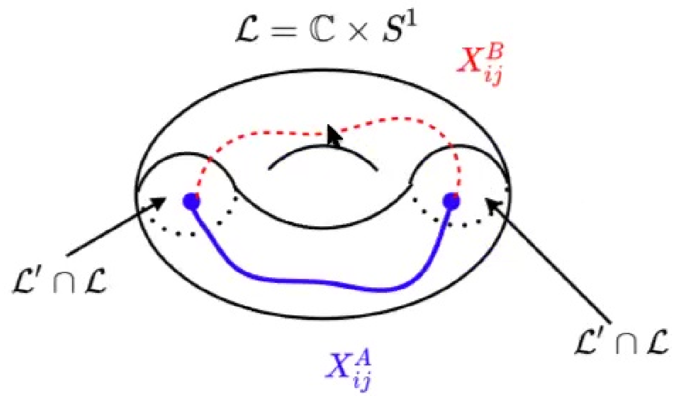
The open string edge modes are anyons !

Edge modes as D branes

Introduce N>>1 entanglement. branes on $\mathcal{L}'$, which intersects $\mathcal{L}$

This give a new sector of open strings ending on $\mathcal{L}' \cap \mathcal{L}$





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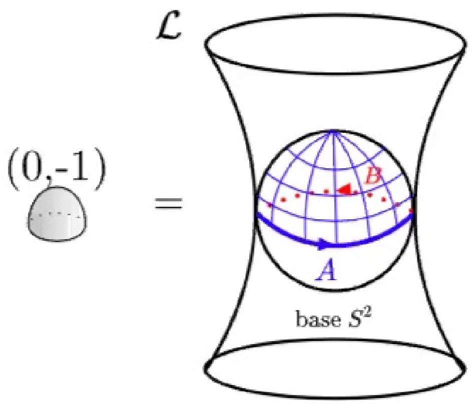
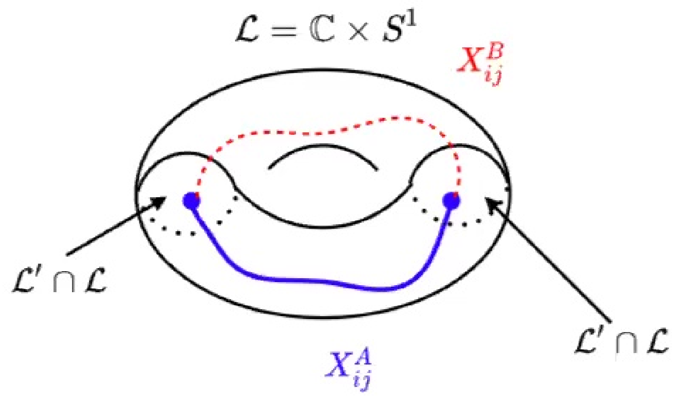




Factorization map

$$\mathcal{H}_{\text{closed}} \to \mathcal{H}_{\text{open}} \otimes \mathcal{H}_{\text{open}}$$

$$\mathrm{tr}^q_R(U) \to \mathrm{tr}^q_R(U^A U^B) = \sum_{ikj} (D_R^{-1})_{ij} R(U^A)_{ik} R(U^B)_{kj}$$

Drinfeld element

Representation Matrix

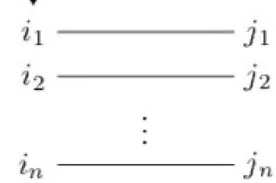
Open string wavefunctions

Consider the undeformed case q=1 corresponding to zero string coupling

$\mathcal{A}(U(N)) =$ Commutative algebra of functions on a U(N)

Generators $\boxed{\langle U|I,J\rangle = U_{i_1 j_1} U_{i_2 j_2} \cdots U_{i_n j_n}}$

Chan paton factors label E branes



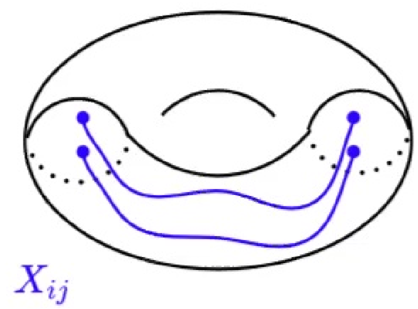
Bosonic statistics $U_{i_1 j_1} U_{i_2 j_2} = U_{i_2 j_2} U_{i_1 j_1}$

q-deformation gives the open string anyon statistics

$\mathcal{A}(U_q(N))$ non commutative algebra of functions

Anyonics statistics defined by R matrix $U_{i_1 j_1} U_{i_2 j_2} \neq U_{i_2 j_2} U_{i_1 j_1}$

$$\boxed{\mathcal{H}_{\mathrm{open}} = \mathcal{A}(U(\infty)_q)}$$



$$U_{ij} = P \exp \int X^*_{ij} A$$

The Drinfeld element of $U(N)_q$

A quantum group element g acts on the open string by a q-deformed version of $U \to gUg^{-1}$. This is the **edge mode symmetry**. The **invariant trace function** is given by

$$\mathrm{tr}_R^q(U) = \mathrm{tr}_R(\mathbf{u}^{-1}U) \qquad \mathbf{u_{ij}} = \delta_{ij} q^{N/2} q^{-i+1/2}$$ is the Drinfeld element

The quantum dimension is given by

$$\dim_q(R) = \mathrm{tr}_R^q(1)$$

It is the dimension of the collective Hilbert space of many anyons corresponding to the open string endpoints .

For n>>1 anyons of type R their collective Hilbert space has dimension

$$\lim_{n\to\infty} \dim H(n) \to (\dim_q(R))^n$$

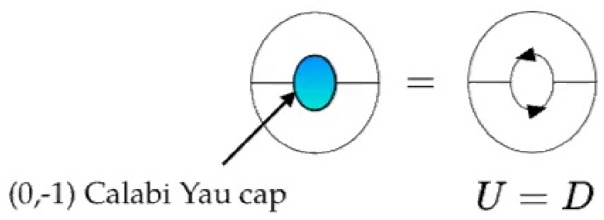
The large N limit of the Drinfeld element

The large N limit of the Drinfeld element is subtle: it requires an analytic continuation of q and converts U(N) quantum dimensions into symmetry group quantum dimensions with phases:

$$D = q^{-N/2}\mathbf{u} \qquad \lim_{N\to\infty} \mathrm{tr}_R(D) = (-i)^{l(R)} d_q(R) q^{\kappa_R/4}$$

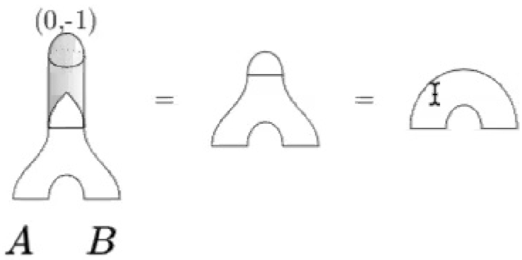
The limit plays an important role in relating A model string theory to q-deformed 2D Yang Mills, as well as the Gopakumar Vafa duality related the closed string theory to open string theory with a large N number of D branes

D is also the holonomy that defines the shrinkable boundary condition in the presence of the Calabi Yau constraint



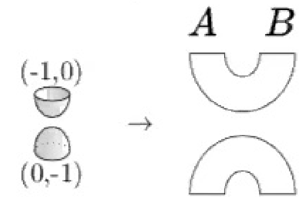
The canonical calculation of entanglement entropy

$|HH\rangle \rightarrow$

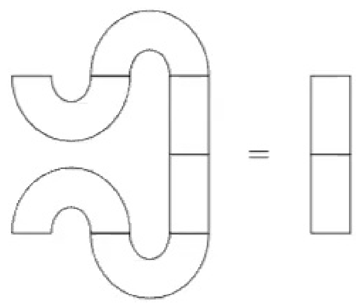


Un normalized density matrix

$\tilde{\rho} = |HH\rangle\,\langle HH^*| =$



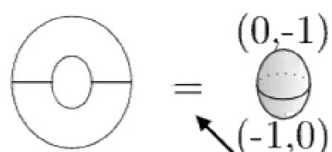
Reduced density matrix $\tilde{\rho}_A = \mathrm{tr}_B\,\tilde{\rho} =$



$= \sum_{R,i,j} e^{-tl(R)} |Rij\rangle\,\langle Rij|$

Degenerate for fixed R

Edge modes $\mathrm{tr}_q \tilde{\rho}_A =$



$= \sum_R (d_q(R))^2 e^{-tl(R)}$

Quantum trace : invariant under q-deformed edge mode symmetry

Summing over branes gives a closed string theory! (large N open-closed string duality)

q-deformed entanglement entropy as generalized entropy

The quantum trace defines a q-deformed entanglement entropy (used non-unitary systems).

$$S_q = -\mathrm{tr}_q(\rho_A \log \rho_A) = -\mathrm{tr}(D\rho_A \log \rho_A) = \sum_R p(R)(-\ln p(R) + 2\ln d_q(R))$$

Matches the generalized entropy !

$$p(R) = \frac{(d_q(R))^2 e^{-tl(R)}}{Z}$$

Analogous formula for the generalized entropy in JT (Jafferis-Kolchmeyer , Kitaev-Suh)

We identified the ``defect operator" D with the Drinfeld element of $U(\infty)_q$ which defines the quantum trace.

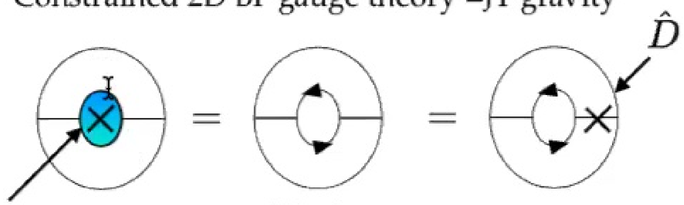
Questions and future directions

- What does this teach us about entanglement in general string theories? Are we probing a universal sector of the edge modes ?
- We used the target space TQFT to define E branes. What is the worldsheet description? Maybe this would give a hint of how holes emerges on the worldsheet
- Is it possible to relate the large N open-closed string duality involve E branes with the "physical " large N Gopakumar Vafa duality?
- The relation to JT suggests suggests we should look for a q-deformed description of JT gravity and seek a symmetry description of the defect operator
- We used a categorical notion of trace to give the micro state interpretation. Is category theory useful in formulating quantum gravity? Perhaps we gain some flexibility in address puzzles like factorization, e.g. the use of quantum traces.
- What about the UV completion via q2DYM and the sum over topologies?

Non-locality and factorization in topological strings

Shrinkable boundary condition can be local in modular time

Constrained 2D BF gauge theory =JT gravity



$$Z(\beta) = \mathrm{tr}(\hat{D} e^{-\beta H})$$

$$S_q = -\mathrm{tr}(\hat{D} \rho_A \log \rho_A)$$

This is the same result as in Jafferis Kolchmeyer, but we give a symmetry perspective on the ``defect operator" D as the Drinfeld element.

Question: Is this non local boundary condition an artifact of a low energy effective theory? Or can the microscopic Hilbert space of quantum gravity be fundamentally non-local?

Recent discussion: (Harlow- Shaghoulian)