Title: Decoherence vs space-time diffusion: testing the quantum nature of gravity

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Abstract: Consistent dynamics which couples classical and quantum systems exists, provided it is stochastic. This provides a way to study the back-reaction of quantum systems on classical ones and has recently been explored in the context of quantum fields back-reacting on space-time. Since the dynamics is completely positive and circumvents various no-go theorems this can either be thought of as a fundamental theory, or as an effective theory describing the limit of quantum gravity where the gravitational degrees of freedom are taken to be classical. In this talk we explore some of the consequences of complete positivity on the dynamics of classical-quantum systems. We show that complete positivity necessarily results in the decoherence of the quantum system, and a breakdown of predictability in the classical-phase space. We prove there is a trade-off between the rate of this decoherence and the degree of diffusion in the metric: long coherence times require strong diffusion relative to the strength of the coupling, which potentially provides a long-distance experimental test of the quantum nature of gravity We discuss the consequences of complete positivity on preparing superpositions of gravitationally different states. Each state produces different distributions of the gravitational field determined by the constraints of the theory. The overlap of these distributions imposes an upper bound on the degree of coherence of the superposition.

Decoherence vs space-time diffusion: testing the quantum nature of gravity

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Quantum foundations seminar, PI, 2020

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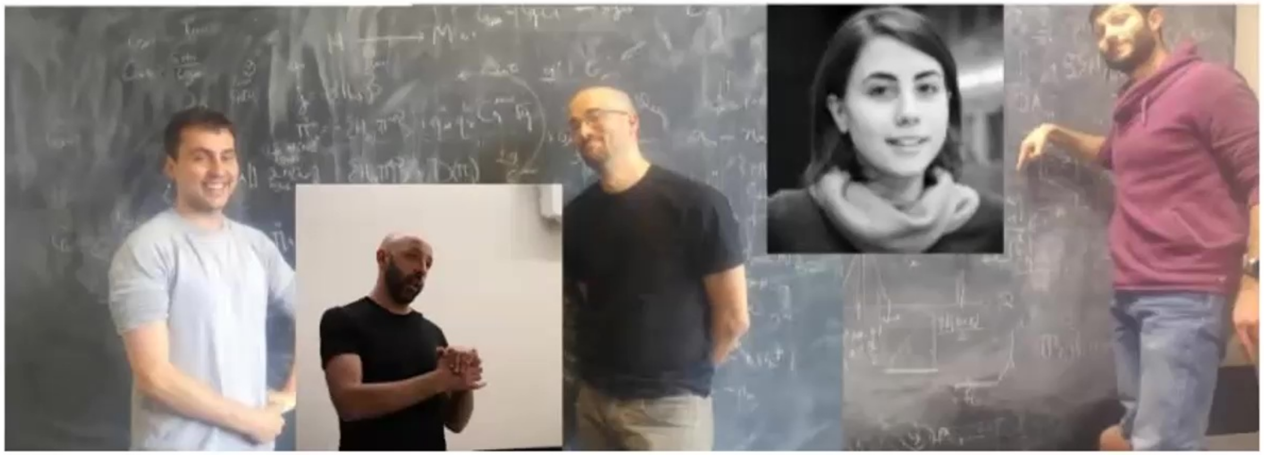
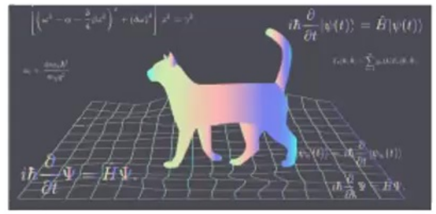


"Decoherence vs space-time diffusion: testing the quantum nature of gravity"

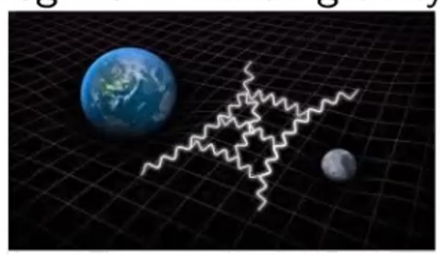
"Decoherence of quantum fields on classical space-time"

Motivation - why study classical quantum dynamics?

- Revisiting the question of whether or not gravity can be fundamentally classical



- Understanding the regime in which gravity behaves classically



(Feynman 1957 Chapman Hill, Eppley-Hannah 1977, Blanchard Jadczyk 1993, Diosi 1995)

- We can write down a classical-quantum master equation and use symmetry + complete positivity + physics input input to determine experimental and theoretical consequences

$$\frac{\partial \varrho}{\partial t} = \int dz' W^{\mu\nu}\left(z \mid z'\right) L_{\mu} \varrho\left(z'\right) L_{\nu}^{\dagger} - \frac{1}{2}\left\{ \int dz' W^{\mu\nu}\left(z' \mid z\right) L_{\nu}^{\dagger} L_{\mu}, \varrho(z) \right\}$$

- Not the same as semi-classical gravity
- In this talk I will discuss some of the consequences of **complete positivity** on the dynamics

Main Messages

Using positivity of the state and complete positivity of the dynamics

$$\int d\Delta d\Delta' \mathrm{Tr}_{A,B}\left[(f_A(\Delta)g_B(\Delta') - g_A(\Delta)f_B(\Delta'))^\dagger (f_A(\Delta)g_B(\Delta') - g_A(\Delta)f_B(\Delta'))T_A(\Delta)T_B(\Delta')\right] \quad (22)$$

A trade off between diffusion and decoherence

We will show there is a trade-off between the decoherence and diffusion: long coherence times require strong diffusion

Trade-off is experimentally bounding

The trade-off between decoherence and diffusion places strong experimental bounds on post-quantum theories of gravity, providing a potential indirect test for the quantum nature of gravity

Positivity of the state and gravitational decoherence

By demanding positivity of the CQ state, we can gain insight into some of the puzzling natures of the Diosi-Penrose decoherence rate

What is a classical-quantum state?

Classical quantum states $\varrho(q,p)$

The state space consists of a Hilbert space at each point in phase space. Hybrid states are positive, $\rho(q,p) \geq 0$ and normalized $\int dqdp Tr[\varrho(q,p)] = 1$

Often take classical degrees of freedom to live in a phase space and will denote them by z

Example

A hybrid qubit with classical position and momentum q, p

$$\varrho(q,p,t) = \begin{pmatrix} u_0(q,p,t) & \alpha(q,p,t) \\ \alpha^*(q,p,t) & u_1(q,p,t) \end{pmatrix} \tag{1}$$

Conditions from positivity

The state must be **positive**, $\varrho(q,p) \geq 0$. For the hybrid qubit,

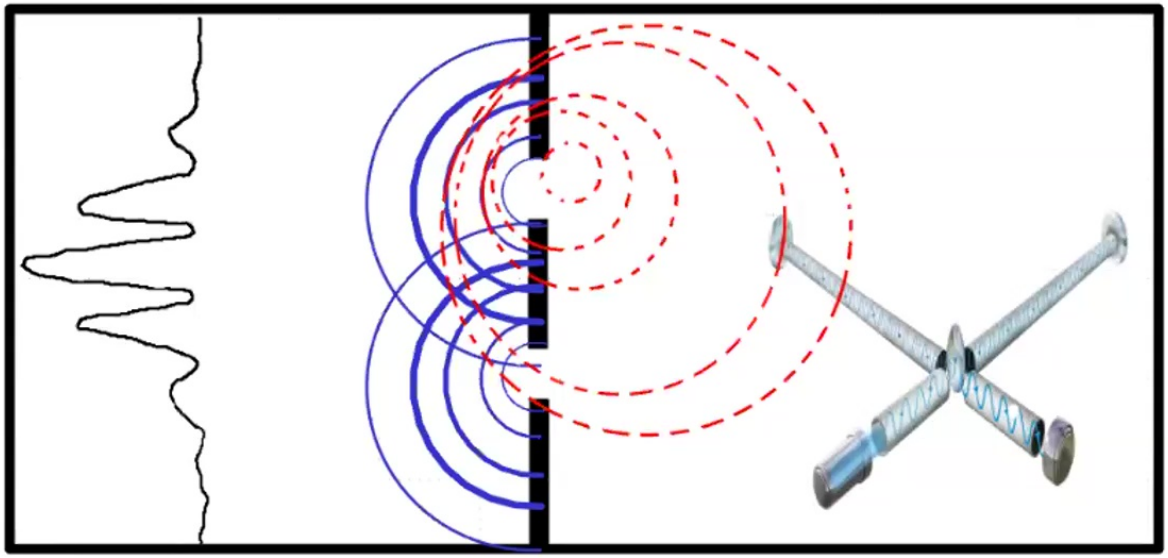
$$\varrho(q,p,t) = \begin{pmatrix} u_0(q,p,t) & \alpha(q,p,t) \\ \alpha^*(q,p,t) & u_1(q,p,t) \end{pmatrix} \quad (2)$$

this implies

$$|\alpha(q,p,q)|^2 \leq u_0(q,p,t)u_1(q,p,t) \quad (3)$$

So for **coherence**, the classical distributions $u_0(q,p,t), u_1(q,p,t)$ describing the populations **must** have **overlap**.

This has a somewhat intuitive explanation

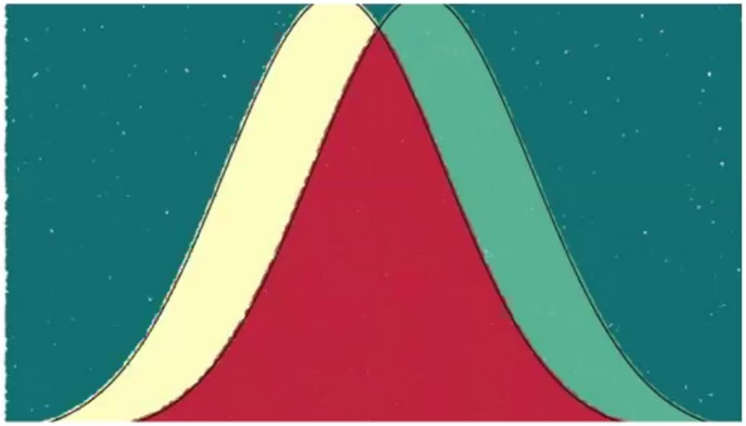


$$|\psi\rangle \rightarrow |L\rangle|E_L\rangle + |R\rangle|E_R\rangle$$

$$\rho = Tr_E(|\psi\rangle\langle\psi|) = \begin{pmatrix} \frac{1}{2} & \alpha \\ \alpha^* & \frac{1}{2} \end{pmatrix}$$

with $\alpha = \langle E_L|E_R\rangle$, we see interference patterns because the electromagnetic fields are **not** orthogonal

Classical states $|E_L\rangle, |E_R\rangle$ are perfectly distinguishable, **unless** they are **probability distributions**



What kind of dynamics is allowed?

- It must preserve the state space
$$\varrho(q,p) \geq 0, \ \int dqdp Tr[\varrho(q,p)] = 1$$
- We ask that it be completely positive on the quantum system

CQ dynamics (Oppenheim 2018)– CQ version of Kraus (87)

$$\varrho(z, t+\delta t) = \sum_{\mu\nu} \int dz' \Lambda^{\mu\nu}\left(z \mid z', \delta t\right) L_\mu \varrho\left(z', t\right) L_\nu^\dagger \qquad (4)$$

where positivity demands $\Lambda^{\mu\nu}(z|z')$ is a positive matrix for each z, z' and

$$\int dz \sum_{\mu\nu} \Lambda^{\mu\nu}\left(z \mid z', \delta t\right) L_\nu^\dagger L_\mu = \mathbb{I} \qquad (5)$$

due to normalization.

What kind of dynamics is allowed? continued

Quantum

Dynamics: $\sigma(t) = \sum_{\mu} \lambda^{\mu\nu} L_{\mu} \sigma(0) L_{\nu}^{\dagger}$

Normalization: $\sum_{\mu} \lambda^{\mu\nu} L_{\nu}^{\dagger} L_{\mu} = \mathbb{I}$

Positivity: $\lambda^{\mu\nu}$ positive matrix

Classical

Dynamics: $p(z,t) = \int dz' P(z \mid z', t) p(z', 0)$

Normalization: $\int dz P(z \mid z') = 1$

Positivity: $P(z|z')$ positive for each z, z'

CQ

Dynamics: $\varrho(z,t) = \int dz' \Lambda^{\mu\nu}(z \mid z', t) L_{\mu} \varrho(z', 0) L_{\nu}^{\dagger}$

Normalization: $\int dz \sum_{\mu\nu} \Lambda^{\mu\nu}(z \mid z', t) L_{\nu}^{\dagger} L_{\mu} = \mathbb{I}$

Positivity: $\Lambda^{\mu\nu}(z|z', t)$ positive matrix for each z, z'

Quantum

Master equation: $\frac{\partial \sigma}{\partial t} = -i[H,\sigma] + h^{\alpha\beta} L_\alpha \sigma L_\beta^\dagger - \frac{1}{2}\left\{ h^{\alpha\beta} L_\beta^\dagger L_\alpha, \sigma \right\}$

Positivity: $h^{\alpha\beta}$ positive matrix

Classical

Master equation:

$\frac{\partial p}{\partial t} = \int dz' W(z \mid z') p(z') - \int dz' W(z' \mid z) p(z)$

Positivity: $\delta(z,z') + \delta t W(z|z')$ positive

CQ

Master Equation: (In a basis of Lindblad operators $L_\mu = (I, L_\alpha)$)

$$\frac{\partial \varrho}{\partial t} = \int dz' W^{\mu\nu}(z \mid z') L_\mu \varrho(z') L_\nu^\dagger - \frac{1}{2}\left\{ \int dz' W^{\mu\nu}(z' \mid z) L_\nu^\dagger L_\mu, \varrho(z) \right\}$$

Positivity:

$$\Lambda^{\mu\nu}(z \mid z', \delta t) = \begin{bmatrix} \delta(z,z') + \delta t W^{00}(z \mid z') & \delta t W^{0\beta}(z \mid z') \\ \delta t W^{\alpha 0}(z \mid z') & \delta t W^{\alpha\beta}(z \mid z') \end{bmatrix}$$

a positive matrix

Kramers-Moyal expansion

- In classical dynamics we can perform a *Kramers-Moyal* expansion of the master equation

$$\frac{\partial p}{\partial t} = \int dz' W\left(z \mid z'\right) p\left(z'\right) - \int dz' W\left(z' \mid z\right) p(z)$$
$$\longrightarrow \sum_{n=1}^{\infty} (-1)^n \frac{\partial^n}{\partial z^n} \left[D_n(z) p(z,t)\right]$$

where

$$D_n(z) = \frac{1}{n!} \int dz' \left(z' - z\right)^n W\left(z' \mid z\right)$$

are the *moments* of the transition amplitude

Kramers-Moyal expansion continued

Example

Take $D_{1,q} = \frac{\partial H}{\partial p}$, $D_{1,p} = -\frac{\partial H}{\partial q}$, $D_{n\geq 2} = 0$

$$\frac{\partial p}{\partial t} = \{H, p\}$$

Example

Take $D_1 = \mu$, $D_2 = D$, $D_{n\geq 3} = 0$

$$\frac{\partial p}{\partial t} = -\frac{\partial}{\partial z}\left[\mu(z)p(z,t)\right] + \frac{\partial^2}{\partial z^2}\left[D(z)p(z,t)\right]$$

the *Fokker-Plank equation*

- D_1 characterizes the amount of Hamiltonian evolution in the system (more precisely the drift)
- D_2 characterizes the amount of diffusion in the system

CQ Kramers-Moyal expansion

CQ Kramers-Moyal expansion

$$\frac{\partial \varrho(z,t)}{\partial t} = \sum_{n=1}^{\infty}(-1)^n \left(\frac{\partial^n}{\partial z^n}\right)\left(D_n^{00}(z)\varrho(z,t)\right)$$
$$- i[H(z), \varrho(z)] + D_0^{\alpha\beta}(z) L_\alpha \varrho(z) L_\beta^\dagger - \frac{1}{2} D_0^{\alpha\beta}\left\{L_\beta^\dagger L_\alpha, \varrho(z)\right\}_+$$
$$+ \sum_{\mu\nu \neq 00} \sum_{n=1}^{\infty}(-1)^n \left(\frac{\partial^n}{\partial z^n}\right)\left(D_n^{\mu\nu}(z) L_\mu \varrho(z,t) L_\nu^\dagger\right)$$

Important moments for the talk

- $D_0^{\alpha\beta}$ characterizes the decoherence
- $D_1^{\mu\nu}$ characterizes the Hamiltonian part of the back-reaction on phase-space (drift)
- $D_2^{\mu\nu}$ characterizes the diffusion (spreading) in the phase-space

Main Results

We can use the conditions on positivity to prove the following

- A CQ Pawula theorem: unique (a.s) continuous master equation, or else one must have infinite moments (jumping)
- We **must** have a Decoherence term: For CQ back-reaction, we must have a decoherence term $D_0^{\alpha\beta}$. Interaction with a classical system **necessarily** causes **decoherence**
- We **must** have a Diffusion term $D_2^{\mu\nu}$: Interaction with a classical system **necessarily** results in a **loss of predictability** on the classical phase space

A trade off between diffusion and decoherence

There is a trade-off between the amount of diffusion and decoherence: long coherence times require strong diffusion

A CQ Pawula Theorem

CQ Pawula Theorem

For non-trivial CQ evolution, we must have infinitely many moments in the master equation (specifically, none of the even moments can vanish), or else the master equation takes the form

$$\frac{\partial \varrho(z,t)}{\partial t} = -i[H(z), \varrho(z,t)] + \sum_{n=1}^{n=2} (-1)^n \left(\frac{\partial^n}{\partial z^n}\right) \left(D_n^{00} \varrho(z,t)\right)$$
$$+ \frac{\partial}{\partial z}\left(D_1^{0\alpha} \varrho(z,t) L_\alpha^\dagger\right) + \frac{\partial}{\partial z}\left(D_1^{\alpha 0} L_\alpha \varrho(z,t)\right)$$
$$+ D_0^{\alpha\beta}(z) L_\alpha \varrho(z) L_\beta^\dagger - \frac{1}{2} D_0^{\alpha\beta} \left\{L_\beta^\dagger L_\alpha, \varrho(z)\right\}_+$$

Furthermore, $2D_{2,ii}^{00} \geq \left(D_0^{-1}\right)_{\alpha\beta} D_{1,i}^{0a} D_{1,i}^{0\beta *}$

Important fact for rest of the talk

We **must** have a decoherence term $D_0^{\alpha\beta}$ and a diffusion term $D_2^{\mu\nu}$

A trade-off between decoherence and diffusion

A trade-off between decoherence and diffusion

For **all** CQ master equations, we derive a trade-off between decoherence and diffusion depending on the *drift* in the system

- Two sources of drift with back-reaction, $D^{0\alpha}_{1,i}$, $D^{\alpha\beta}_{1,i}$
- The purely classical diffusion is bounded below by $2D^{00}_{2,ii} \geq \left(D_0^{-1}\right)_{\alpha\beta} D^{0\alpha}_{1,i} D^{0\beta *}_{1,i}$ (Diosi, 1995)
- The CQ diffusion term $D_2^{\alpha\beta}$ must satisfy the bound $\sum_\alpha 2D^{\alpha\alpha}_{2,ii} \sum_\beta D_0^{\beta\beta}(z) \geq \left|\sum_\alpha D^{\alpha\alpha}_{1,i}(z)\right|^2$

A trade-off between decoherence and diffusion

Physical input

If we want the dynamics to approximately reproduce Hamiltonian dynamics, we know what the first moment should be! (Oppenheim 2018)

$$Tr[\frac{\partial}{\partial z_i}(D^{\mu\nu}_{1,i} L_\mu \varrho(z) L^\dagger_\nu)] = Tr[\{H_m, \varrho\}]$$

- c.f two classical systems (z_1, z_2) interacting with a H_I

$$\frac{\partial p(z_1, z_2, t)}{\partial t} = \{H_1, \rho\} + \{H_2, \rho\} + \{H_I, \rho\}$$

- Integrating out the second system and defining $\bar{\rho}(z_1) = \int dz_2 \rho(z_1, z_2)$ we get an effective e.o.m

$$\frac{\partial \bar{\rho}(z_1)}{\partial t} = \{H_1, \bar{\rho}(z_1)\} + \int \mathrm{d}z_2 \{H_I, \rho(z_1, z_2)\}$$

Trade-off provides an experimental bound

- There are experimental **upper bounds** on the decoherence rate λ

A trade-off between decoherence and diffusion

A trade-off between decoherence and diffusion

For **all** CQ master equations, we derive a trade-off between decoherence and diffusion depending on the *drift* in the system

- Two sources of drift with back-reaction, $D^{0\alpha}_{1,i}$, $D^{\alpha\beta}_{1,i}$
- The purely classical diffusion is bounded below by $2D^{00}_{2,ii} \geq \left(D_0^{-1}\right)_{\alpha\beta} D^{0a}_{1,i} D^{0\beta *}_{1,i}$ (Diosi, 1995)
- The CQ diffusion term $D_2^{\alpha\beta}$ must satisfy the bound $\sum_\alpha 2D^{\alpha\alpha}_{2,ii} \sum_\beta D_0^{\beta\beta}(z) \geq \left| \sum_\alpha D^{\alpha\alpha}_{1,i}(z) \right|^2$
- Trade-off has important consequences!

Trade-off provides an experimental bound

- There are experimental **upper bounds** on the decoherence rate λ
- Using the physical input that we want the dynamics to approximately reproduce Hamiltonian dynamics
- Give us an experimental **lower bounds** on the amount of diffusion in the classical system – at a relevant energy scale

Brief aside on post-quantum gravity

- In order to study the trade-off in a concrete setting we shall study a toy model of a non-relativistic quantum field interacting with a classical Newtonian potential
- I will therefore give a very brief tour of post-quantum gravity

Summary of post-quantum gravity

- Take classical degrees of freedom to be Riemmanian 3-metric g_{ab} and their conjugate momenta π^{ab}
- Couple to QM and consider dynamics of the state $\varrho(g_{ab}, \pi^{ab})$ in an ADM formalism
- Can study independence of the dynamics of the lapse and shift to find Momentum and Hamiltonian constraints (Oppenheim, ZWD 2011.15112)

$$
\begin{aligned}
\mathcal{L}_{constraint} = & \int d^3x M_a D_b N \big[\{g^{ab}(y), \mathcal{H}(y)\varrho\} + (-2iC_H^{ab}h^{\boldsymbol{\alpha\beta}} + \frac{i}{2}C_N^{ab}W_0^{\boldsymbol{\alpha\beta}} - \frac{i}{2}C_J^{ab}W^{\alpha\beta})[L_{\boldsymbol\beta}^\dagger L_{\boldsymbol\alpha}, \varrho] \\
& +2(C_J^{ab}h^{\boldsymbol{\alpha\beta}} + C_H^{ab}W^{\boldsymbol{\alpha\beta}})L_{\boldsymbol\alpha}\varrho L_{\boldsymbol\beta}^\dagger - (C_N^{ab}h^{\boldsymbol{\alpha\beta}} + C_H^{ab}W_0^{\boldsymbol{\alpha\beta}})\{L_{\boldsymbol\beta}^\dagger L_{\boldsymbol\alpha}, \varrho\}_+\big] \\
& \int d^3x N M_a \mathcal{H}_b(\{g^{ab}, \mathcal{H}_b \bar{\mathcal{L}}(\varrho)\} - \bar{\mathcal{L}}(\{g^{ab}, \mathcal{H}_b\varrho\}) \\
& + (\frac{i}{2}\mathcal{R}_{NN}^{\boldsymbol{\alpha\beta}} - \frac{i}{2}\mathcal{R}_{JJ}^{\boldsymbol{\alpha\beta}} - 2i\mathcal{R}_{HH}^{\boldsymbol{\alpha\beta}})[L_{\boldsymbol\beta}^\dagger L_{\boldsymbol\alpha}, \varrho] + (2\mathcal{R}_{HJ}^{\boldsymbol{\alpha\beta}} + 2\mathcal{R}_{JH}^{\boldsymbol{\alpha\beta}})L_{\boldsymbol\alpha}\varrho L_{\boldsymbol\beta}^\dagger - (\mathcal{R}_{NH}^{\boldsymbol{\alpha\beta}} + \mathcal{R}_{HN}^{\boldsymbol{\alpha\beta}})\{L_{\boldsymbol\beta}^\dagger L_{\boldsymbol\alpha}\}_+ \\
& + \int d^3x N M_a \big[(C_{JN}^{ab}(W_0^{\phi\phi} - 2ih^{\phi\phi}) - C_J^{ab}W_0^{\phi\phi} + C_N^{ab}W^{\phi\phi})D_b\phi\varrho\phi + \\
& + (C_{JN}^{ab}(W_0^{\phi\phi} + 2ih^{\phi\phi}) + C_J^{ab}W_0^{\phi\phi} - C_N^{ab}W^{\phi\phi})\phi\varrho D_b\phi \\
& + (C_{JN}^{ab}(-W_0^{\pi\pi} + 2ih^{\pi\pi}) - W_0^{\pi\pi}C_J^{ab} + C_N^{ab}W^{\pi\pi})D_b\pi\varrho\pi + (C_{JN}^{ab}(-W_0^{\pi\pi} - 2ih^{\pi\pi}) + W_0^{\pi\pi}C_J^{ab} - C_N^{ab}W^{\pi\pi})\pi\varrho D_b\pi \\
& + (C_{JN}^{ab}(-W_0^{\phi\phi} - 2ih^{\phi\phi}) + C_N^{ab}W^{ef} - W_0^{ef}h^{ef}C_J^{ab})(D_e D_f\phi)\varrho D_b\phi \\
& + (C_{JN}^{ab}(-W_0^{\phi\phi} + 2ih^{\phi\phi}) - C_N^{ab}W^{ef} + W_0^{ef}C_J^{ab})D_b\phi\varrho(D_e D_f\phi)\big] \\
& \int d^3x - N M_b C_{JN}^{ab}W^{\phi\phi}\{D_b\phi\phi, \varrho\}_+ + NW^{\pi\pi}\{\pi D_b(M_a\pi C_{JN}^{ab}, \varrho)\}_+ + M_a C_{JN}^{ab}\{D_b\phi D_d(NW^{cd}D_c\phi, \varrho)\}_+ \\
& - 2\int d^3x \big[NW_0^{\pi\pi}\pi D_b(C_{JN}^{ab}M_a\varrho)\pi + M_b D_d N W_0^{cd}C_{JN}^{ab}D_c\phi\varrho D_b\phi\big] \approx 0 \qquad (97)
\end{aligned}
$$

Newtonian limit - state space

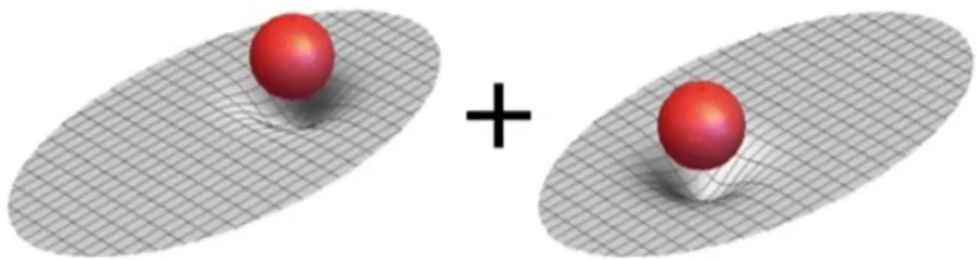
- We consider the case of a non-relativistic quantum field in a partially decohered super-position of approximately orthogonal $|L\rangle$, $|R\rangle$

$$|L/R\rangle = \int d^3x f_{L/R}(x)\psi^\dagger(x)|0\rangle$$

$f_L(x)f_R(x) \approx 0$, $\psi(x) = \int \frac{d^3p}{(2\pi)^3} a_{\vec{p}} e^{i\vec{p}\cdot\vec{x}}$

- Take classical d.o.f to be Newtonian potential Φ and its canonical conjugate π_g
- The state space is then

$$\varrho(\Phi, \pi_g, t) = \begin{pmatrix} u_L(\Phi, \pi_g, t) & \alpha(\Phi, \pi_g, t) \\ \alpha^*(\Phi, \pi_g, t) & u_R(\Phi, \pi_g, t) \end{pmatrix}$$



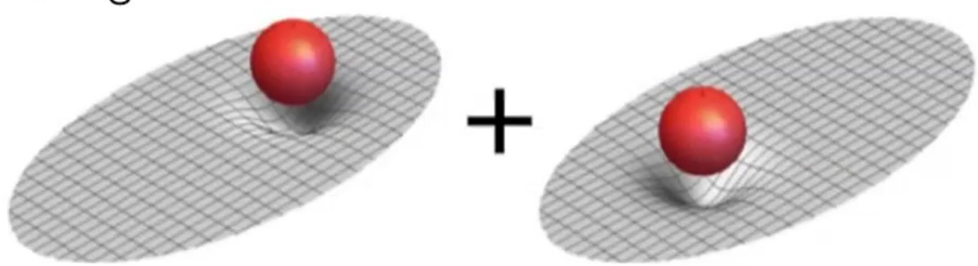
Positivity constraints on the state

Positivity of the state and gravitational decoherence

By demanding positivity of the CQ state, we can gain insight into some of the puzzling natures of the Diosi-Penrose decoherence rate $\lambda_D = \frac{\Delta E_D}{\hbar}$

$$\Delta E_D = \int d^3x d^3x' \frac{[m_L(x) - m_R(x)][m_L(x') - m_R(x')]}{|x - x'|}$$

- Non-local, mass in the left branch interacts with the mass in the right branch



Positivity bounds coherence

Classical-quantum decoherence rates

We see that we can arrive at the Diosi-Penrose decoherence using local dynamics. We find it is **not** a dynamical effect, but instead is a constraint imposed by demanding that the density matrix be **positive**.

$$\varrho(\Phi, \pi_g, t) = \begin{pmatrix} u_L(\Phi, \pi_g, t) & \alpha(\Phi, \pi_g, t) \\ \alpha^*(\Phi, \pi_g, t) & u_R(\Phi, \pi_g, t) \end{pmatrix} \tag{6}$$

At $t = 0$ we take the marginal distributions for the populations

$$u_{L/R}(\Phi) = \frac{\mathcal{N}}{2} \exp\left[- \int d^3x \frac{(\Phi(x) - \Phi_{L/R}(x))^2}{2\sigma^2} \right] \tag{7}$$

i.e, Gaussian's peaked around the value of the Newtonian potential Φ_L, Φ_R which satisfies Poisson's equation

Positivity bounds coherence continued

- **Positivity** of the state tells us that at $t = 0$,

$$|\alpha(\Phi, \pi_g, 0)|^2 \leq u_L(\Phi, \pi_g, 0)\, u_R(\Phi, \pi_g, 0)$$

- For the Gaussian marginal distributions

$$|\alpha(\Phi, \pi_g, 0)|^2 \leq \frac{\mathcal{N}^2}{4} \exp\left[-\int d^3x \frac{(\Phi_L(x) - \Phi_R(x))^2}{4\sigma^2}\right] \times \exp\left[-\int d^3x \frac{\left(\Phi(x) - \frac{1}{2}(\Phi_L(x) + \Phi_R(x))\right)^2}{\sigma^2}\right]$$

- If $\Phi_{L/R}$ satisfy Poisson's equation

$\Phi_{L/R} = -4\pi G \int d^3x' \frac{\mu_{L/R}(x')}{|x-x'|}$

$$\int d^3x\, (\Phi_L(x) - \Phi_R(x))^2 \sim \int d^3x d^3x' \frac{[\mu_L(x) - \mu_R(x)][\mu_L(x') - \mu_R(x')]}{|x - x'|} \quad (8)$$

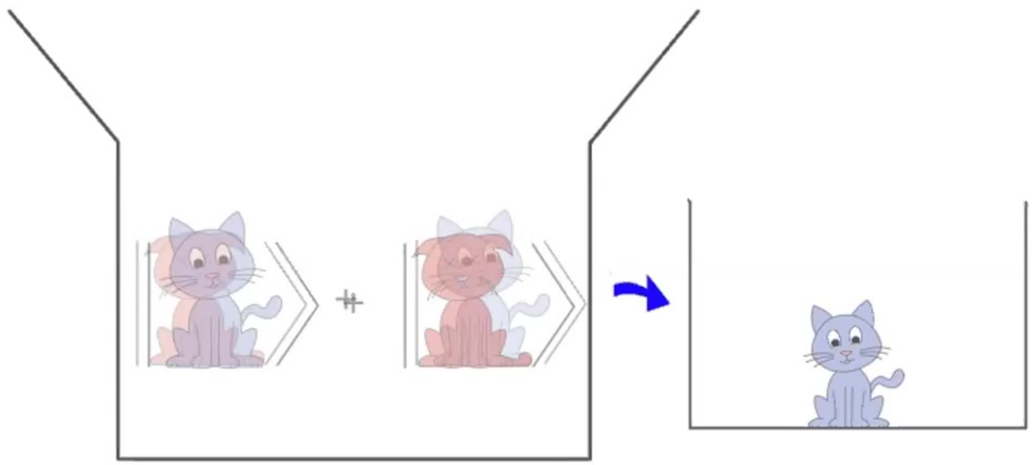
Positivity bounds coherence continued

Quantum decoherence on classical space-time

- We see that here DP is **not** a dynamical decoherence rate. Instead it is a **non-dynamical** effect which arises due to **positivity** of the density matrix.
- Gives a bound on the allowed coherence one can **prepare**: two masses with very different gravitational fields cannot be prepared in a coherent superposition.
- Since it is a condition on the allowed states one is allowed to prepare the non-locality is less of a problem than if this is a dynamical effect.

- When we include dynamics, we also have a **dynamical** contribution to the decoherence (model dependent)

$$\int D\Phi D\pi_g |\alpha(\Phi, \pi_g)| = \bar{\alpha}(t) = \frac{1}{4} \exp\left[-\int d^3x \frac{(\Phi_L(x) - \Phi_R(x))^2}{8\sigma^2}\right] \times \exp\left[-\int d^3x \left[\frac{\lambda(\Phi) t\, (\Phi_L(x) + \Phi_R(x))}{2}\right]\right]$$



Positivity bounds coherence continued

- **Positivity** of the state tells us that at $t = 0$,

$$|\alpha(\Phi, \pi_g, 0)|^2 \leq u_L(\Phi, \pi_g, 0)\, u_R(\Phi, \pi_g, 0)$$

- For the Gaussian marginal distributions

$$|\alpha(\Phi, \pi_g, 0)|^2 \leq \frac{\mathcal{N}^2}{4} \exp\left[-\int d^3x \frac{(\Phi_L(x) - \Phi_R(x))^2}{4\sigma^2}\right] \times \exp\left[-\int d^3x \frac{\left(\Phi(x) - \frac{1}{2}(\Phi_L(x) + \Phi_R(x))\right)^2}{\sigma^2}\right]$$

- If $\Phi_{L/R}$ satisfy Poisson's equation

$\Phi_{L/R} = -4\pi G \int d^3x' \frac{\mu_{L/R}(x')}{|x-x'|}$

$$\int d^3x\, (\Phi_L(x) - \Phi_R(x))^2 \sim \int d^3x d^3x' \frac{[\mu_L(x) - \mu_R(x)][\mu_L(x') - \mu_R(x')]}{|x - x'|} \quad (8)$$

- We take the pure gravity Hamiltonian to be

$$H_c(\Phi) = \int_x \left(-\frac{\pi G}{3} \pi_g^2 + \frac{(\nabla \Phi)^2}{4\pi G} \right)$$

- The Quantum Hamiltonian is

$$H_m(\Phi, \phi) = m \int d^3x (1 + 2\Phi(x)) \psi^\dagger \psi := H_m^0 + H_I(\Phi)$$

- Using the theory of Oppenheim 2018 we consider a master equation

$$\frac{\partial \varrho}{\partial t} \approx \{H_c(\Phi), \varrho\} - i\left[H_m^0, g\right] + m \int d^3x D_0 \left[\psi \varrho \psi^\dagger - \frac{1}{2}\left\{\psi^\dagger \psi, \varrho\right\}\right]$$

$$+ 2m \int d^3x \psi \frac{\delta \varrho}{\delta \pi_g} \psi^\dagger + m \int \psi \frac{\delta^2}{\delta \pi_g^2} (D_2 \varrho) \psi^\dagger + \cdots$$

- We take the pure gravity Hamiltonian to be

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- This approximately reproduces the Newtonian interaction

$$\mathrm{Tr}\left[\{H_I, \varrho\}\right] = -2m \int d^3x\, \mathrm{Tr}\left[\psi^\dagger\psi\frac{\delta\varrho}{\delta\pi_g}\right],\ m\psi^\dagger(x)\psi(x) = m(x)$$

Trade-off between decoherence and diffusion

- The drift results in a lower bound for the diffusion in the momenta conjugate to Φ, π_g
- This results in gravitational kinetic energy production

$$\Delta E = \int d^3x \frac{Gc^2\pi}{3} \langle \pi_g^2 \rangle \geq \int d^3x \frac{2tc^2 G\pi |\langle m(x) \rangle|^2}{3\lambda}$$

(where λ is the decoherence rate and $\langle m(x) \rangle$ is the expectation value of the mass density.)

- We can get an order of magnitude estimate for the kinetic energy production from *Gerlich et al. 2007*
- The decoherence rate is $\lambda < 10^{-5}s^{-1}$ for clusters of nucleons of mass 10^{-24} and typical radius $r \sim 10^{-9}$. Taking the experiment to be conducted on the order of seconds we find gravitational kinetic energy production

$$\Delta E \sim 10^{-11} J, \; mc^2 \sim 10^{-8} J$$

- Complete positivity gives us a **trade-off** between decoherence, drift and diffusion
- In toy models, this seems to be a very relevant **prediction** of treating the gravitational field classically

General lesson

Treating gravity classically leads to diffusion in the gravitational field which should be experimentally testable – but need to understand this more generally

Summary

Quantum decoherence in classical space-time

- We have shown that positivity of the CQ state gives rise to upper bounds on the coherence.
- A dynamical term, due to the decoherence term D_0
- A non-dynamical term, due to positivity of the state, which gives rise to a Diosi-Penrose like bound

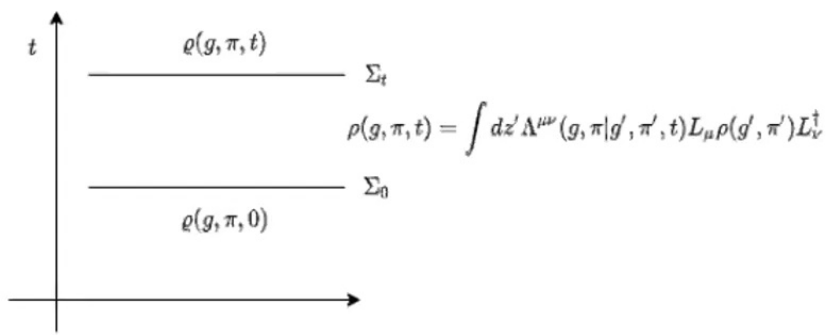
A trade-off between decoherence and diffusion

- We have shown there is a trade-off between decoherence and diffusion: long coherence times require strong diffusion

- In simple toy model of quantum fields interaction with classical gravity we have seen this is a non-negligible effect and potentially puts fundamental CQ theories in danger of running afoul of experimental observations.

Discussion and future outlook

- Better understand the validity of the toy model we use, so that we can strengthen statements and begin thinking about possible experiments
- Understand what general lessons can be learned: a lot of the results rely on complete positivity, are largely independent of the specifics of the dynamics, and we might expect them to generalize to the non-Markovian case.



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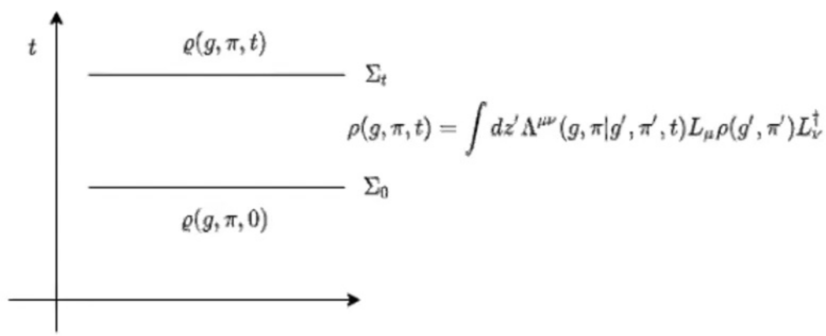

- Better understand when can expect the theory to hold as an *effective theory*, for example by understanding how we arrive at the CQ limit.


Carlo Rovelli


Thank
You