Title: Photon Emission from Circular Equatorial Orbiters around Kerr Black Holes

Speakers: Delilah Gates

Series: Strong Gravity

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Abstract: We consider monochromatic and isotropic photon emission from circular equatorial Kerr orbiters. We calculate the critical curve delineating the region of photon escape from that of photon capture in each emitterâ€™s sky, allowing us to derive analytic expressions for the photon escape probability and the redshift-dependent total flux collected on the celestial sphere as a function of emission radius and black hole parameters. This critical curve generalizes to finite orbital radius the usual Kerr critical curve and displays interesting features in the limit of high spin. These results confirm that the near-horizon geometry of a high-spin black hole is in principle observable.

Photon Emission from Circular Equatorial Orbiters around Kerr Black Holes

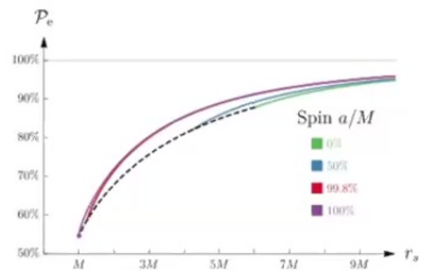
Delilah Gates
Harvard University

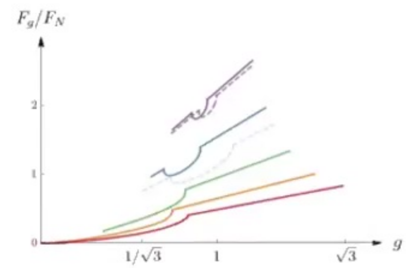
arXiv: 2010.07330
collaborators: S. Hadar & A. Lupsasca

Dec 17, 2020

Summary of Results

- Generalize the Kerr critical curve/BH shadow to circular equatorial orbiters
- Properties of isotropic, monochromatic emission from orbiters to celestial sphere:
  - Escape Probability
  - Flux as a function of redshift

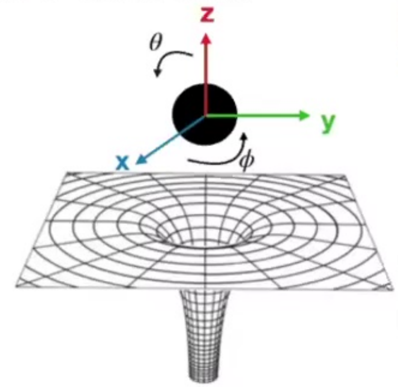




- Critical curve (and escape property and flux) in extremal regime
- Prospects for observability

1

Kerr Bound Photon Orbits & Critical Locus



- Kerr Metric (in Boyer-Linquist Coordinates)

$$ds^2 = -\frac{\Delta}{\Sigma}\left(\mathrm{d}t - a\sin^2\theta\,\mathrm{d}\phi\right)^2 + \frac{\Sigma}{\Delta}\,\mathrm{d}r^2 + \Sigma\,\mathrm{d}\theta^2 + \frac{\sin^2\theta}{\Sigma}\left[\left(r^2+a^2\right)\mathrm{d}\phi - a\,\mathrm{d}t\right]^2,$$

$$\Delta = r^2 - 2Mr + a^2, \quad \Sigma = r^2 + a^2\cos^2\theta.$$

$J = aM$

- Photon trajectory defined by conserved quantities E, L, Q (energy, angular momentum, Carter constant) $\lambda = \frac{L}{E}, \quad \eta = \frac{Q}{E^2}$
- Kerr bound photon orbits
  - photon shell $\tilde{r}_- \le \tilde{r} \le \tilde{r}_+, \quad \tilde{r}_\pm = 2M\left[1 + \cos\left(\frac{2}{3}\arccos\left(\pm\frac{a}{M}\right)\right)\right]$
  - tuned quantities $\tilde{\lambda}(\tilde{r}) = a + \frac{\tilde{r}}{a}\left[\tilde{r} - \frac{2\Delta(\tilde{r})}{\tilde{r}-M}\right], \quad \tilde{\eta}(\tilde{r}) = \frac{\tilde{r}^3}{a^2}\left[\frac{4M\Delta(\tilde{r})}{(\tilde{r}-M)^2} - \tilde{r}\right].$
- Critical locus $\mathcal{C} = \left\{\left(\tilde{\lambda}(\tilde{r}), \tilde{\eta}(\tilde{r})\right) \middle| \tilde{r}_- \le \tilde{r} \le \tilde{r}_+\right\}$

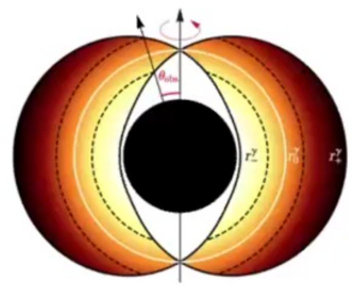


Image Ref: arXiv:1907.04329

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Critical Curve for Far Away Observer

- Critical curve: locus mapped onto observer sky
  - BH shadow/silhouette of backlight BH
  - delineates regions of photon escape and capture
- Bardeen observer screen coordinates & conserved quantities

$$\lambda = -\alpha \sin\theta_o$$

$$\eta = (\alpha^2 - a^2)\cos^2\theta_o + \beta^2$$

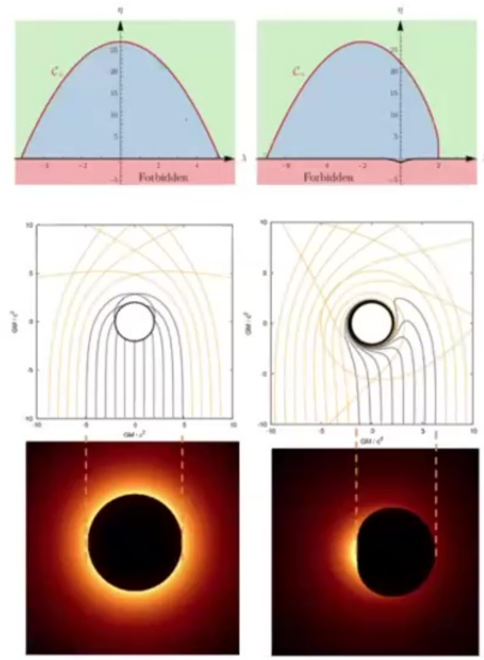
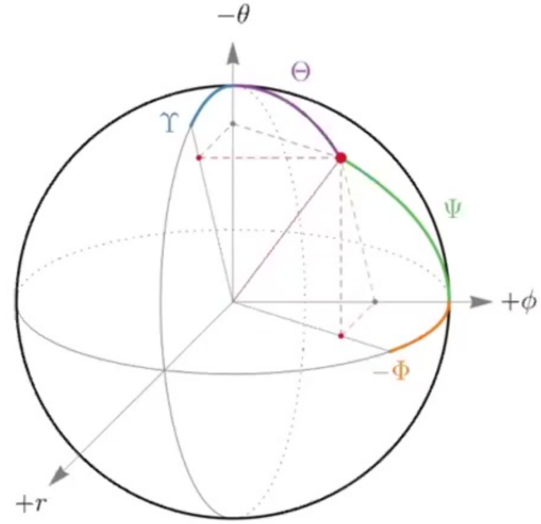


Image Ref: arXiv:1910.12881, http://spiro.fisica.unipd.it/~antonell/schwarzschild/scrots/bhscatter.png

3

Stable Circular Equatorial Observer & Orbiter Sky

- Stable circular equatorial orbiters for radii down to ISCO
- Define orbiter frame coordinate $(t), (r), (\theta), (\phi)$
- Orbiter sky: $\cos\Theta = -\frac{p^{(\theta)}}{p^{(t)}}$ $\quad \cos\Psi = \frac{p^{(\phi)}}{p^{(t)}}$



4

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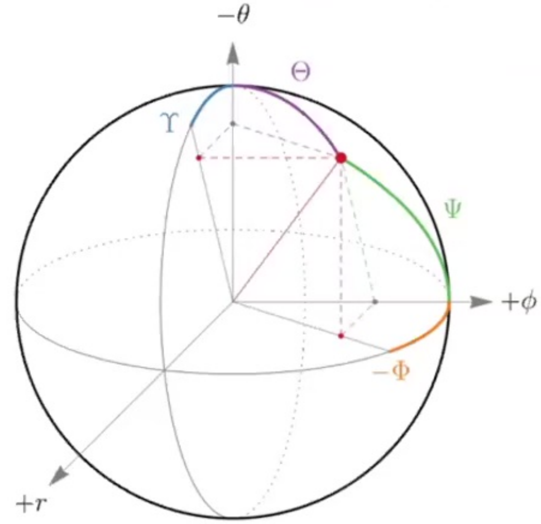
- Define orbiter frame coordinate $(t), (r), (\theta), (\phi)$
- Orbiter sky: $\cos\Theta = -\frac{p^{(\theta)}}{p^{(t)}}$ $\cos\Psi = \frac{p^{(\phi)}}{p^{(t)}}$
- Direction to BH: $\lambda = \eta = 0$ and $\pm_r = -1$ $\Theta_\bullet = -\Upsilon_\bullet = \frac{\pi}{2}$, $\Psi_\bullet = \Phi_\bullet$

Redshift: $g = \frac{E}{p^{(t)}}$

- if orbiter in ergosphere, we may have redshift nonpositive



$$\sin(\Phi - \Phi_\bullet) = -\frac{\lambda}{r_s} + \mathcal{O}\left(\frac{1}{r_s^2}\right),$$
$$\sin(\Theta - \Theta_\bullet) = \mp_\theta \frac{\sqrt{\eta}}{r_s} + \mathcal{O}\left(\frac{1}{r_s^2}\right).$$

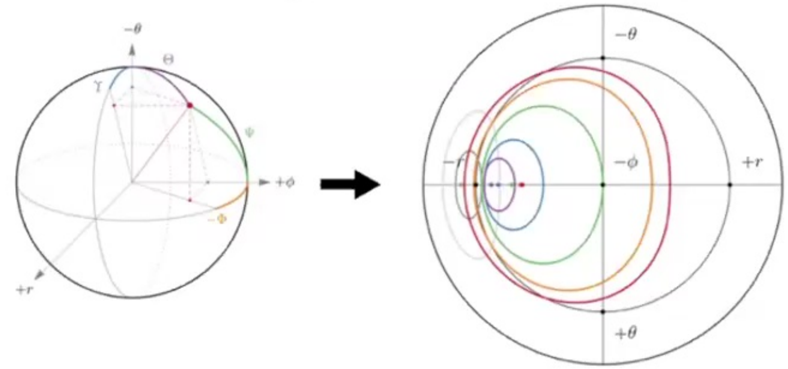
4

Area-Preserving Projection & Escape Probability

- Area-preserving, backside projection of orbiter sky

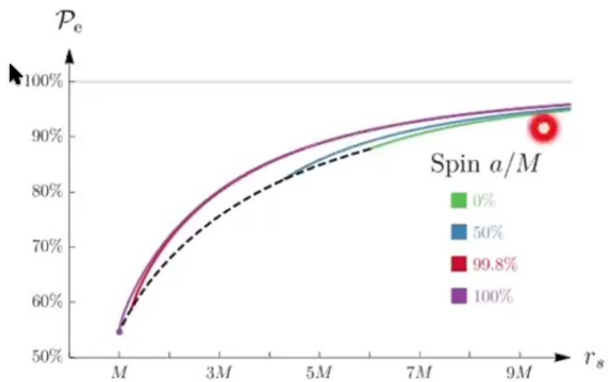
$$\mathrm{d}\mathcal{A} = \rho\,\mathrm{d}\rho \wedge \mathrm{d}\varphi = \sin\Psi\,\mathrm{d}\Psi \wedge \mathrm{d}\Upsilon = \mathrm{d}\Omega$$

$$\rho\,\mathrm{d}\rho = -\sin\Psi\,\mathrm{d}\Psi, \quad \mathrm{d}\varphi = -\,\mathrm{d}\Upsilon$$



- Escape probability

$$\mathcal{P}_\mathrm{e} = 1 - \frac{\mathcal{A}}{4\pi}$$



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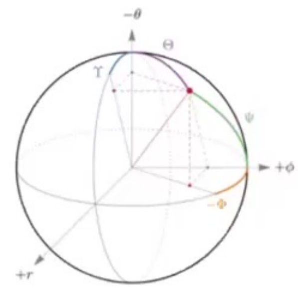
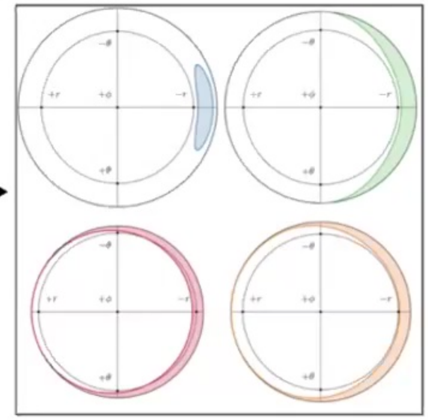
Redshift Weighted Projection & Flux

- Redshift weighted-preserving, frontside projection of orbiter sky

$$\mathrm{d}\mathcal{A}_g = \rho_g\,\mathrm{d}\rho_g \wedge \mathrm{d}\varphi_g = g(\Psi)\sin\Psi\,\mathrm{d}\Psi \wedge \mathrm{d}\Upsilon = g\,\mathrm{d}\Omega$$

$$\rho_g\,\mathrm{d}\rho_g = g(\Psi)\sin\Psi\,\mathrm{d}\Psi, \quad \mathrm{d}\varphi_g = \mathrm{d}\Upsilon$$





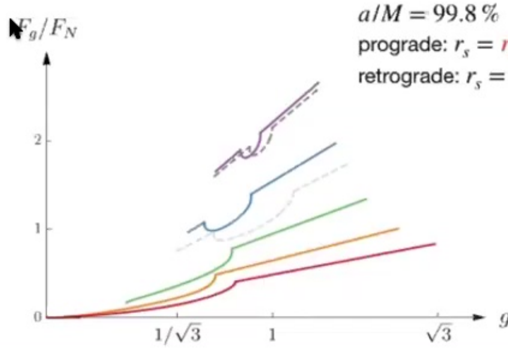
$a/M = 99.8\,\%$
prograde: $r_s = r^+_{\mathrm{ms}}, 2M, \tilde{r}_+, 10M, 20M$
retrograde: $r_s = r^-_{\mathrm{ms}}, 20M$

- Flux as a function of redshift

$$F = \frac{F_N}{4\pi}\int_{\mathcal{E}} \mathrm{d}\mathcal{A}_g = \left|\int_{\hat{g}}^{g_0} F_g\,\mathrm{d}g\right|$$

$$\Rightarrow \quad \frac{F_g}{F_N} = \frac{1}{4\pi}\frac{g\xi_s}{\sqrt{M\Delta(r_s)}}\int_{-\varphi_{\mathcal{E}}(g)}^{\varphi_{\mathcal{E}}(g)} \mathrm{d}\varphi$$

$$\xi_s = \sqrt{r_s^3 - 3Mr_s^2 \pm 2a\sqrt{M}r_s^{3/2}}$$



6

Kerr & Near Horizon Near Extreme Kerr

- Kerr Metric (in Boyer-Linquist Coordinates)

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$$\Delta = r^2 - 2Mr + a^2, \quad \Sigma = r^2 + a^2\cos^2\theta.$$

- Near horizon near extreme limit

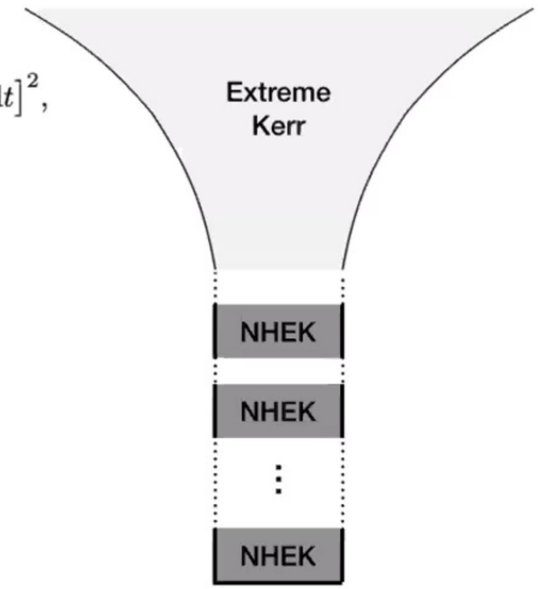
$$t = \frac{2MT}{\kappa^p}, \quad r = r_+(1+\kappa^p R), \quad \phi = \Phi + \frac{T}{\kappa^p},$$

$$0 < p \leq 1 \qquad a = M\sqrt{1-\kappa^2}, \quad 0 < \kappa \ll 1.$$

- Near Horizon Near Extreme Kerr (NHEK)

$$\frac{d\hat{s}^2}{2M^2\Gamma} = -R^2\,\mathrm{d}T^2 + \frac{\mathrm{d}R^2}{R^2} + \mathrm{d}\theta^2 + \Lambda^2(\mathrm{d}\Phi + R\,\mathrm{d}T)^2,$$

$$\Gamma = \frac{1+\cos^2\theta}{2}, \quad \Lambda = \frac{2\sin\theta}{1+\cos^2\theta}.$$



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$a = M$

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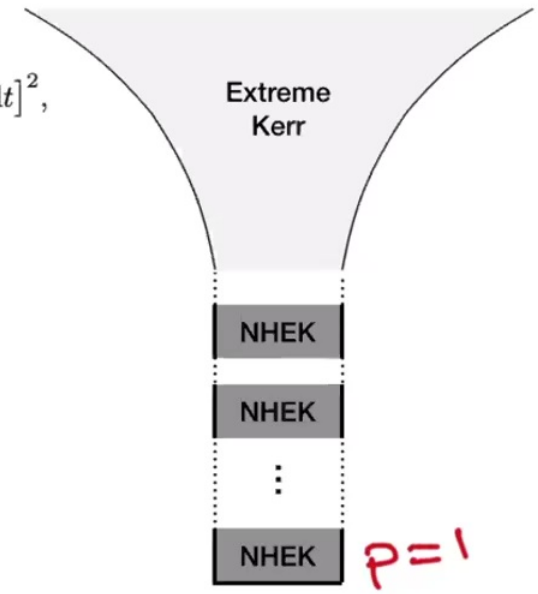
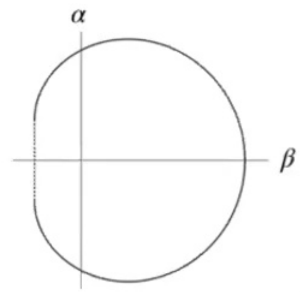
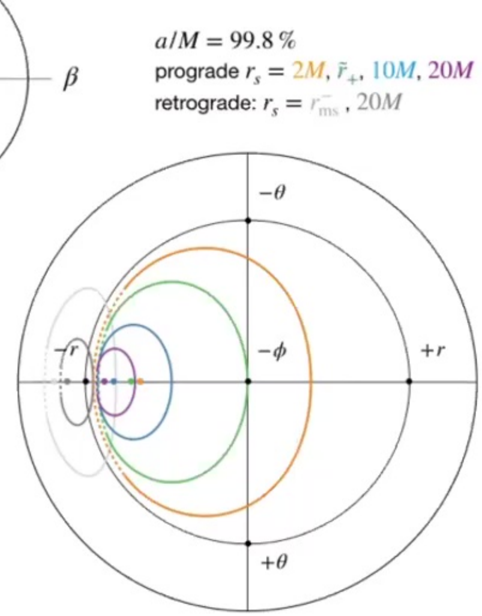


6

Critical Curve for Circular Equatorial Orbiter

- Critical Curve $\mathcal{C} = \left\{ \left(\tilde{\Psi}(\tilde{r}), \tilde{\Upsilon}(\tilde{r}) \right) \middle| \tilde{r}_- \leq \tilde{r} \leq \tilde{r}_+ \right\}.$
- Near Extreme limit (extreme Kerr orbiter)
  - $\mathcal{C} = \mathcal{C}_f \cup \mathcal{C}_n$
    - $a \to M$
    - $a = M\sqrt{1-\kappa^2}, \quad \tilde{r} = M\left(1 + \kappa \tilde{R}\right), \quad 0 < \kappa \ll 1$





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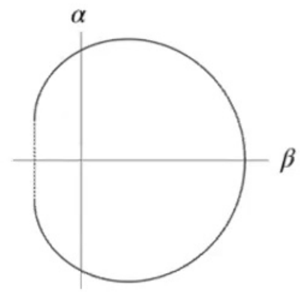
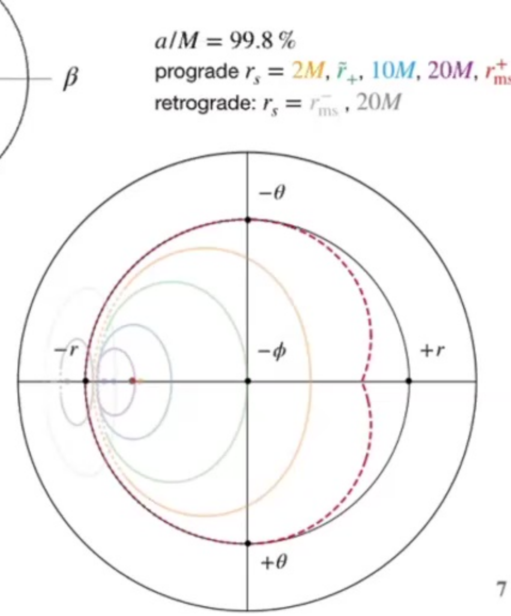
    - $a \to M$
    - $a = M\sqrt{1-\kappa^2}, \quad \tilde{r} = M\left(1 + \kappa \tilde{R}\right), \quad 0 < \kappa \ll 1$
- Near Extreme limit (NHEK orbiter)
  - $\mathcal{C} = \mathcal{C}^+ \cup \mathcal{C}^-$,
    - $a = M\sqrt{1-\kappa^2}, \qquad 0 < \kappa \ll 1$

      $r_s = M(1 + \kappa^q R_s), \qquad 0 < q \leq \frac{2}{3}$

      $\tilde{r} = M\left(1 + \kappa^p \tilde{R}\right), \qquad 0 < p \leq 1 \qquad p(\mathcal{C}^+) = \frac{q}{2}, \quad p(\mathcal{C}^-) = 1$





7

High-Spin Perturbation Theory (ISCO)

Critical curve in near extreme limit

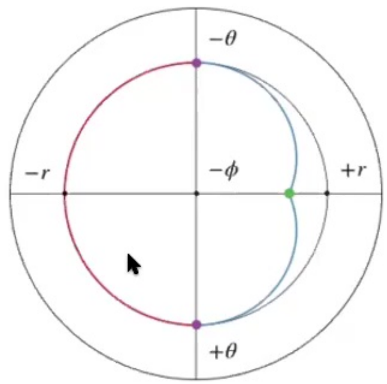
$$a = M\sqrt{1-\kappa^2}, \qquad 0 < \kappa \ll 1$$

$$r_s = M(1+\kappa^q R_s), \qquad 0 < q \le \frac{2}{3}$$

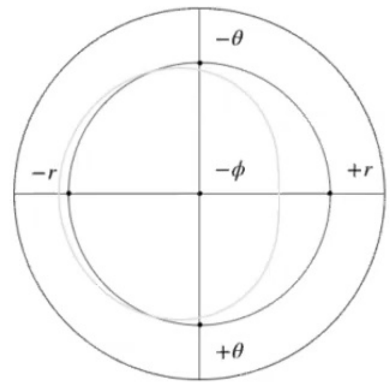
$$\tilde{r} = M\left(1+\kappa^p \tilde{R}\right), \qquad 0 < p \le 1$$

Extremal value & leading correction as $\kappa \to 0$						
NHEK p-band	$(0, \frac{q}{2})$	$\frac{q}{2}$	$(\frac{q}{2}, q)$	q	$(q, 1)$	1
Extreme $\cos\tilde{\Theta}$	0	κ^0	κ^0			κ^0
Correction	κ^{-2p+q}	$\kappa^{q/2}$	κ^{2p-q}	κ^q	$\kappa^q, \kappa^{2(1-p)}$	κ^q, κ^{1-q}
Extreme $\cos\tilde{\Psi}$	κ^0	κ^0	0			0
Correction	κ^{-2p+q}	κ^q	κ^{2p-q}	κ^q	κ^q, κ^{2-p-q}	κ^q, κ^{1-q}

Extremal ISCO



Near Extremal ISCO



$$p(\mathcal{C}^+) = \frac{q}{2}, \quad p(\mathcal{C}^-) = 1$$

8

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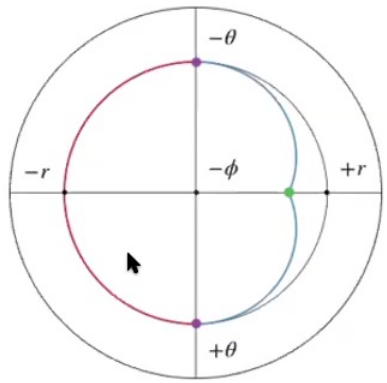
Extremal ISCO



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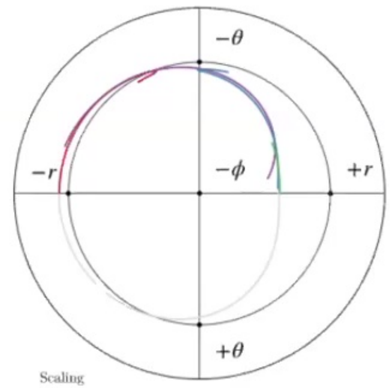
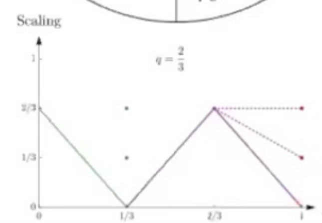
Near Extremal ISCO

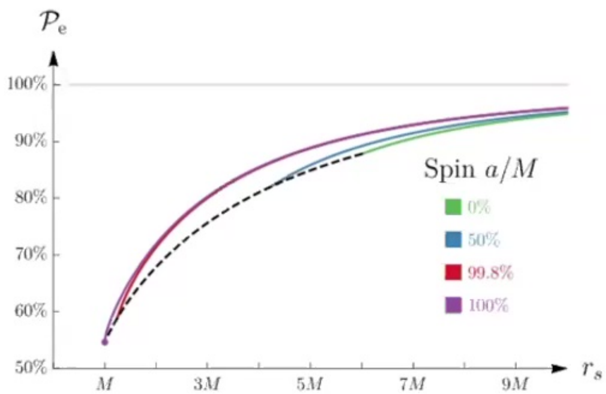




8

Escape Probability & Flux at Extremality

- Escape probability (purple curve)

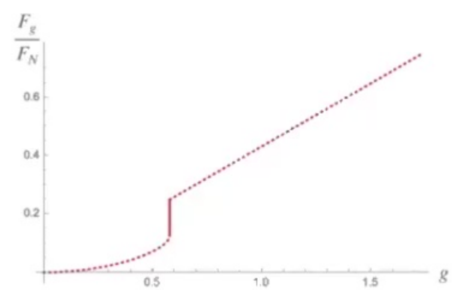


NHEK emitter Escape Probability: $\mathcal{P}_e = \frac{5}{12} + \frac{\arctan\sqrt{5/3}}{\sqrt{5}\pi} \approx 54.6455\%$

- Flux from NHEK emitter

$$\frac{F_g}{F_N} = \begin{cases} \frac{\sqrt{3}}{4\pi} g \arcsin\left(\frac{2g}{\sqrt{1+2g/\sqrt{3}-g^2}}\right) & 0 \le g < \frac{1}{\sqrt{3}}, \\ \frac{\sqrt{3}}{4} g & \frac{1}{\sqrt{3}} < g \le \sqrt{3} \end{cases}$$

$$\lim_{g\to(1/\sqrt{3})^+} F_g = 2 \lim_{g\to(1/\sqrt{3})^-} F_g$$



9

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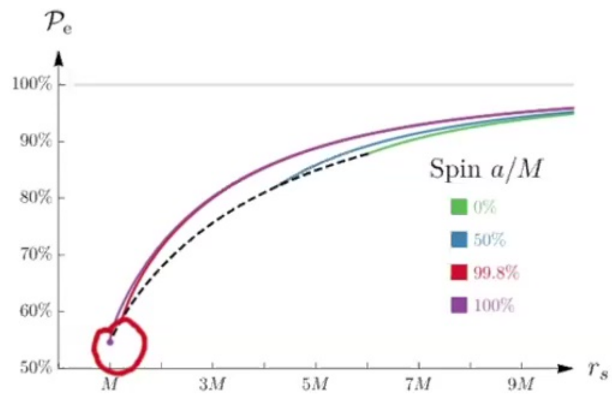

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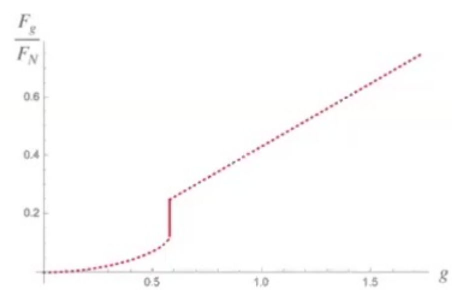
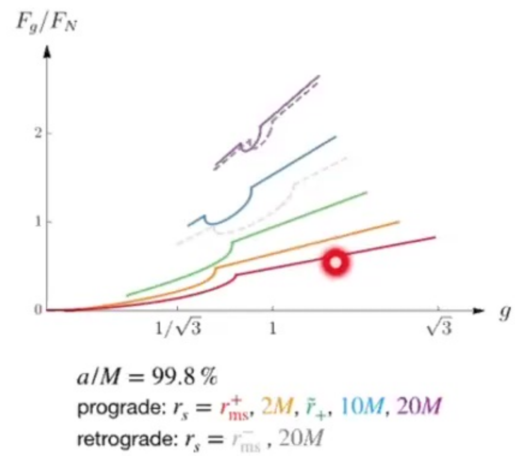


$g = \sqrt{3}$

9

Observability

- This is flux to the entire celestial sphere
- For total flux we need emissivity profile
- But: most blueshifted photon
  - comes from ISCO
  - emitted in direction of orbiter travel
  - reaches celestial sphere at equator
- BH + disk of circular equatorial orbiter, viewed from equator —> constrain spin with maximum observable blueshift



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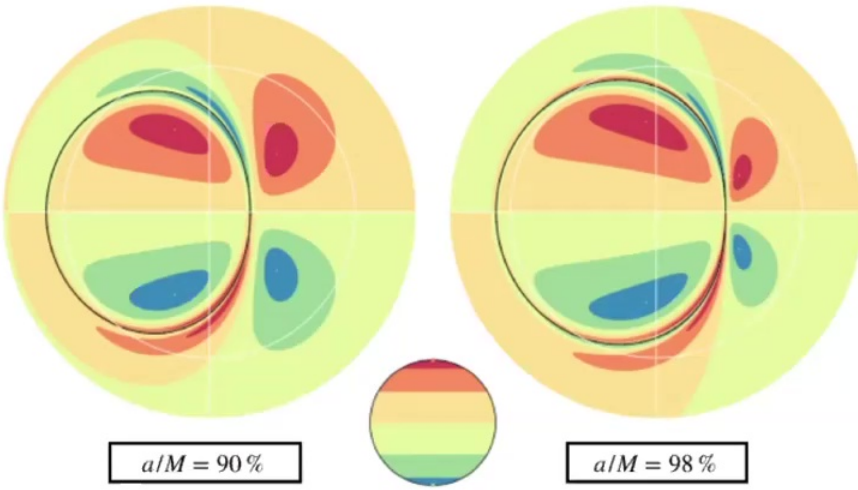
Flux to Fixed Observer: “Unfolding” Celestial Sphere



- Use Kerr geodesics to find curves of fixed observer angle θ_o
- Flux to fixed observer on celestial sphere (graphically)

$$F_{\hat{g}^{(i)}}(\hat{\theta}_o^{(j)}) = \sum_{m=0}^{\infty} F_{\hat{g}^{(i)}}^{(m)}(\hat{\theta}_o^{(j)}),$$

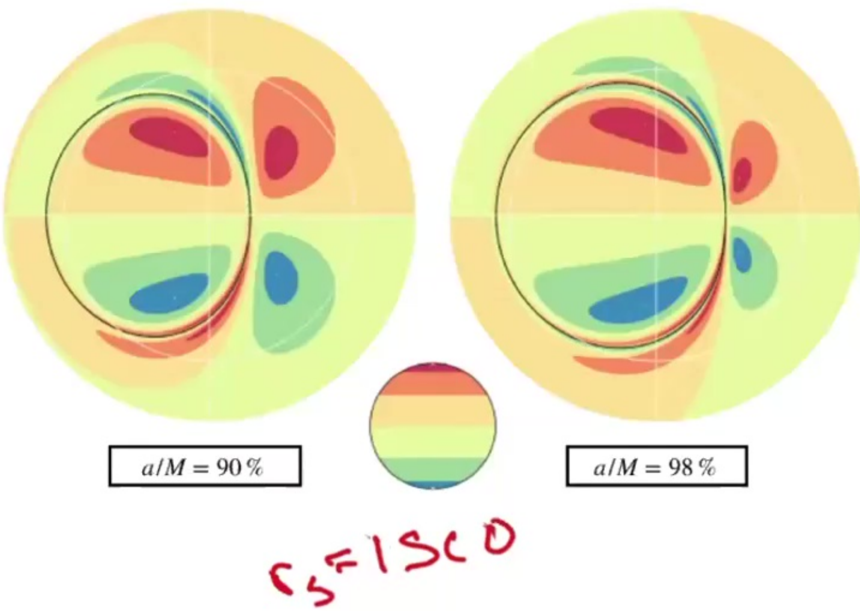
$$F_{\hat{g}^{(i)}}^{(m)}(\hat{\theta}_o^{(j)}) \approx \hat{g}^{(i)} \mathcal{A}^{(m)}(\hat{g}^{(i)}, \hat{\theta}_o^{(j)}) \approx \mathcal{A}_g^{(m)}(\hat{g}^{(i)}, \hat{\theta}_o^{(j)})$$

$$\hat{x}^{(i)} \equiv (x^{(i+1)} - x^{(i)})/2$$

- Maximum observable blueshift corresponds to largest radius on projection that θ_o curve reaches
- For observability:
  - difficult
  - needs to be done for orbiters at all source radii

10

Flux to Fixed Observer: "Unfolding" Celestial Sphere



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- For observability:
    - difficult
    - needs to be done for orbiters at all source radii

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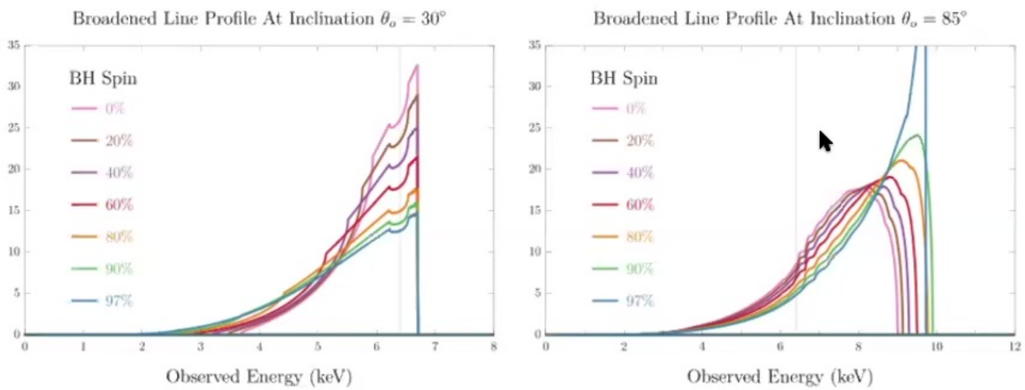
Broadened Line Emission & Maximum Observable Blueshift?

- Model: Kerr BH + equatorial disk, emitters on stable circular orbit
- Broadened Emission Line Profile
- X-ray reflection method of constraining BH spin



Plots courtesy of Laura Brenneman using RELLINE code by Dauser

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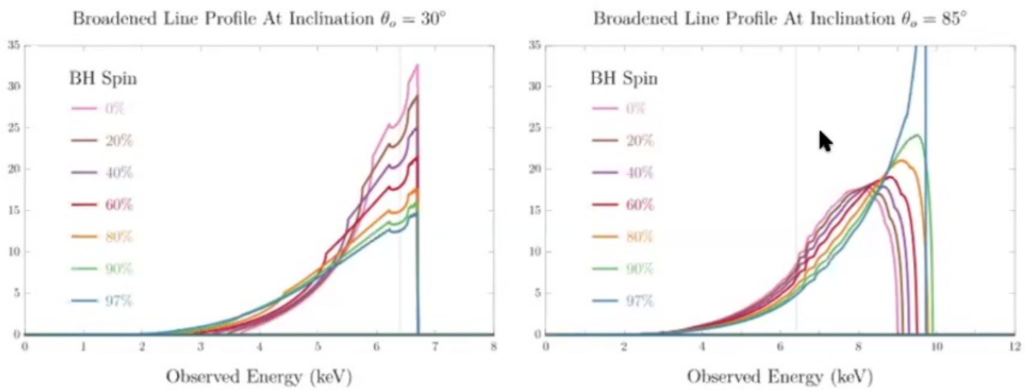

- Observations:
  - MOB has non-vanishing flux
  - low inclination: common, low valued MOB
  - high inclination: larger valued, spin dependent MOB (which is mostly increasing but not entirely monotonic in spin)

Plots courtesy of Laura Brenneman using RELLINE code by Dauser

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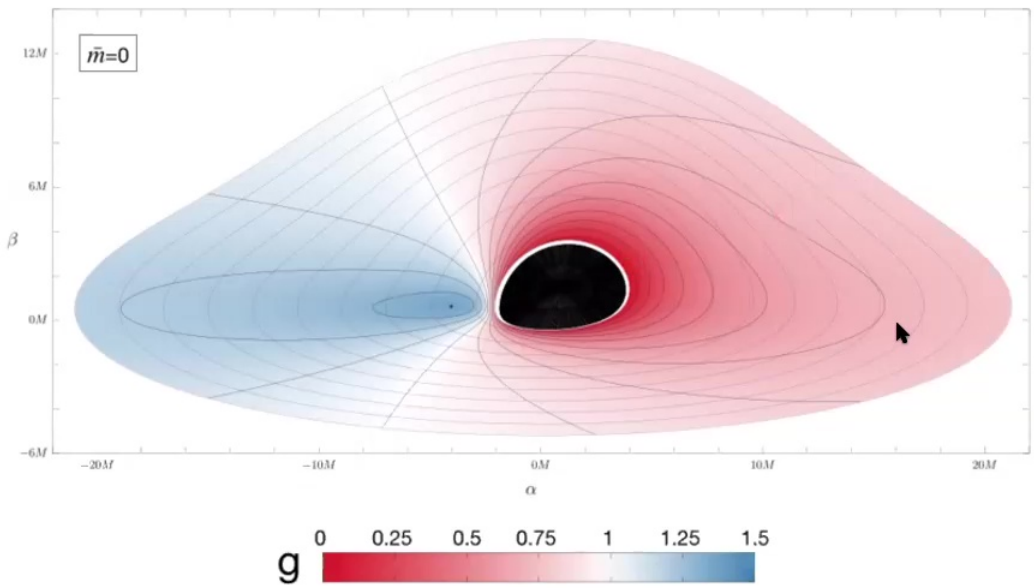
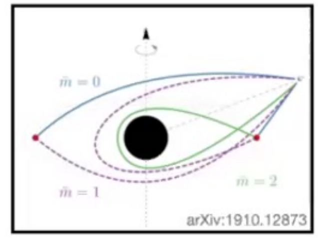
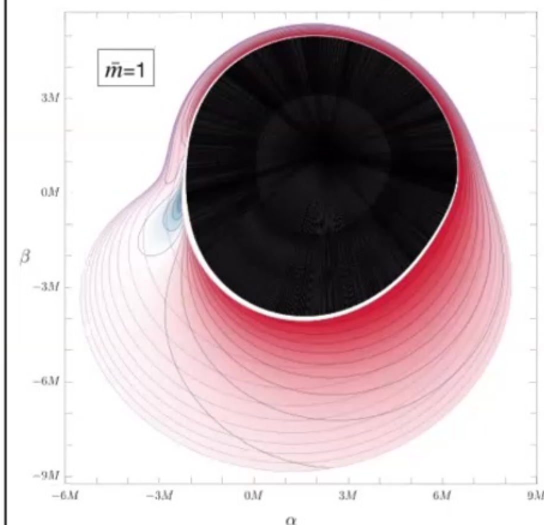
Disk Parameterization by Redshift and Emission Radius

$a/M = .998,\ \theta_o = 75^\circ$





2009.03316



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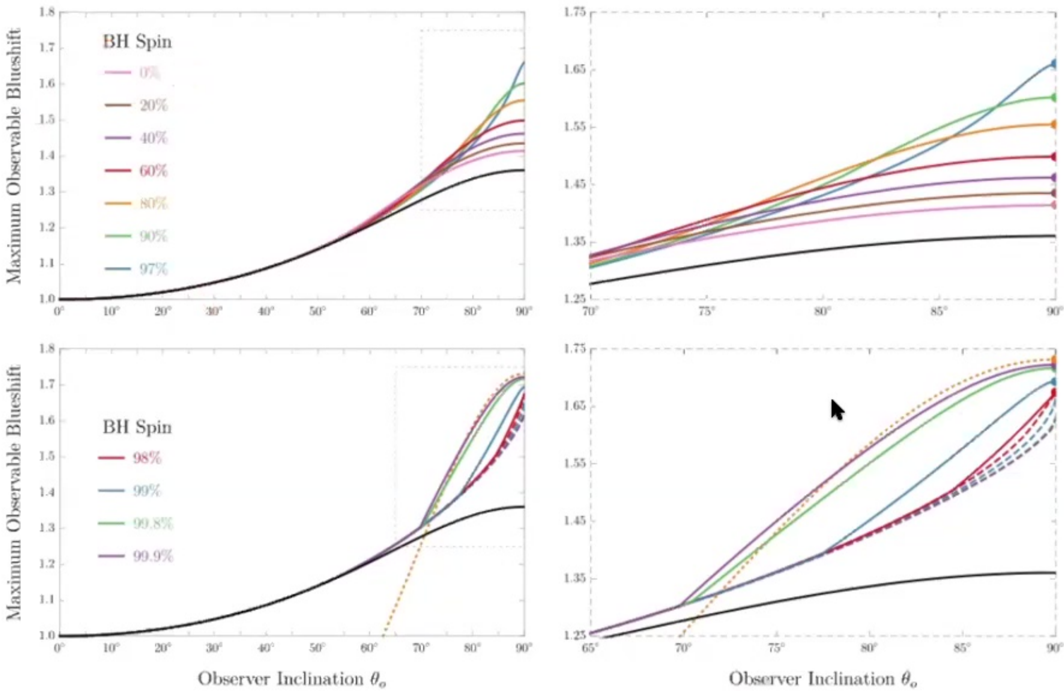
MOB (numerics)



- regimes:
  - any spin, low inclination
  - any spin, equatorial (max) inclination
  - low to moderate spin, high inclination
  - high spin, high inclination

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Summary of Results

- Generalize the Kerr critical curve/BH shadow to circular equatorial orbiters
- Properties of isotropic, monochromatic emission from orbiters to celestial sphere:
  - Escape probability
  - Flux as a function redshift
- Extremal regime
  - Hints at hight-spin perturbation theory
  - More than half of photons emitted from NHEK orbiters escape BH —> NHEK may be visible
  - Flux in the NHEK is universal
  - The most blueshifted photons
    - come from NHEK
    - can (potentially) be used to constrain BH spin for equatorial observer
- Observability
  - Maximum observable blueshift
    - finite flux, can constrain BH spin and observing angle

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Thank you