Title: Quantum spacetime and deformed relativistic kinematics

Speakers: Javier Relancio

Series: Quantum Gravity

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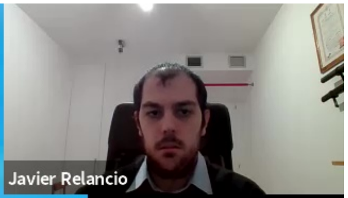
Abstract: In this seminar, I will consider a deformed kinematics that goes beyond special relativity as a way to account for possible low-energy effects of a quantum gravity theory that could lead to some experimental evidences. This can be done while keeping a relativity principle, an approach which is usually known as doubly (or deformed) special relativity. In this context, I will give a simple geometric interpretation of the deformed kinematics and explain how it can be related to a metric in maximally symmetric curved momentum space. Moreover, this metric can be extended to the whole phase space, leading to a notion of spacetime. Also, this geometrical formalism can be generalized in order to take into account a space-time curvature, leading to a momentum deformation of general relativity. I will explain theoretical aspects and possible phenomenological consequences of such deformation.



Quantum spacetime and deformed relativistic kinematics

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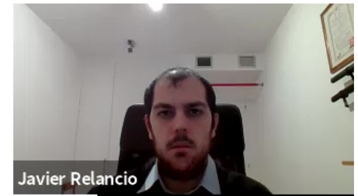
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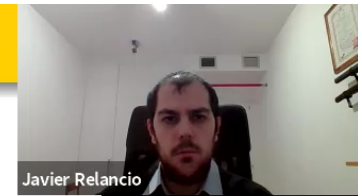
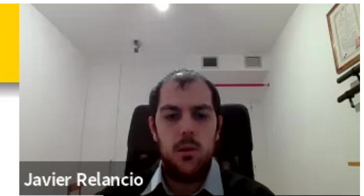
Motivation

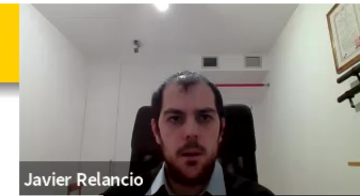


Space-time structure



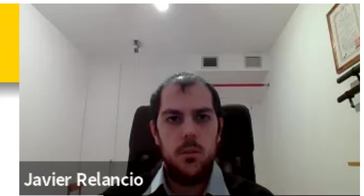
- Any answer has to include matter and also the space-time structure $\rightarrow$ Gravity
- If fundamental constituents of matter exist, does the same happen for spacetime?
- Do space "atoms" exist?

QFT and GR: incompatibilities



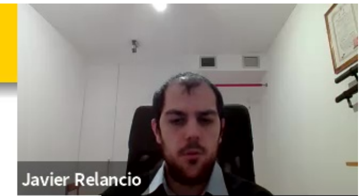
- One of the challenges for present physics is the unification of GR and QFT $\rightarrow$ QGT
- In QFT, one assumes a given spacetime and studies in detail the properties and motion of particles in it
- In GR, one assumes that the properties of matter and radiation are given and describes in detail the resultant spacetime (curvature)
- A QGT should be valid at any energy, but an interaction mediated by spin-2 particle (same equations of GR) is not renormalizable

Why do we need a QGT?



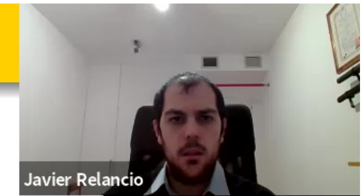
- Study of the first moments of the universe
- Black holes: information, singularity?

Quantum Gravity Theories



- Attempts of unification: string theory, quantum loop gravity, supergravity, causal set theory...
- In most of them a minimal length appears $\Longrightarrow$ Planck length (l_P)??
- This is closely related to an energy scale $\Longrightarrow$ Planck energy (Λ)??
- Problem: there are no experimental evidences of a fundamental QGT

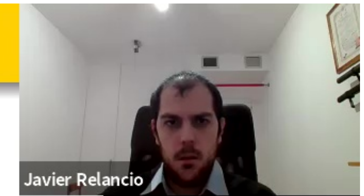
Spacetime: the last frontier



- Classical spacetime $\rightarrow$ "quantum" spacetime
- Symmetries? $\rightarrow$ LI should be broken/deformed at Planckian scales
- New effects $\rightarrow$ Micro black holes creation?
- Spacetime can be regarded as a "foam"

Spacetime: the last frontier

Towards a quantum gravity



- We can obtain l_P, t_P and M_P

$$l_P = \sqrt{\frac{\hbar G}{c^3}} = 1{,}6 \times 10^{-35}\,\mathrm{m}$$

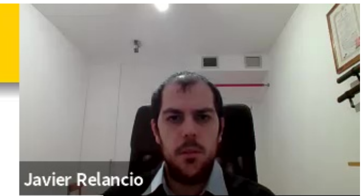
$$t_P = \sqrt{\frac{\hbar G}{c^5}} = 5{,}4 \times 10^{-44}\,\mathrm{s}$$

$$M_P = \sqrt{\frac{\hbar c}{G}} = 2{,}2 \times 10^{-8}\,\mathrm{kg} = 1{,}2 \times 10^{19}\,\mathrm{GeV}/c^2$$

- New approach $\rightarrow$ low energy limit of a QGT that could have experimental observations!
- No quantum or gravitational effects but

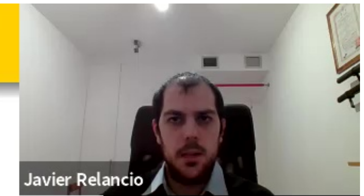
$$M_P = \lim_{\hbar, G \to 0} \sqrt{\frac{\hbar c}{G}} \neq 0$$

Lorentz invariance violation (LIV)



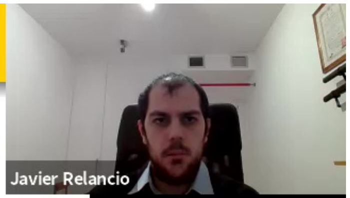
- This possibility was first considered in 60's
- There is a loss of the relativity principle
- There is a privileged observer $\rightarrow$ physical laws depending on the observer
- Formulated in the quantum field theory framework $\rightarrow$ standard model extension (SME)

Doubly Special Relativity (DSR)



- There is a relativity principle
- Two invariants in every inertial frame: speed of light c and Planck length l_P

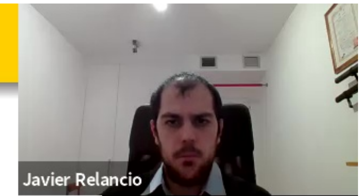
Kinematics



Ingoing particles ⇒ Interaction ⇒ Outgoing particles

past ∞   future ∞

Kinematics

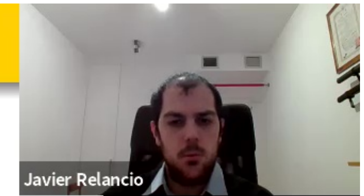


Ingoing particles ⇨ Interaction ⇨ Outgoing particles

past ∞ future ∞

- Ingoing and outgoing particles movement is described by the dispersion relation
- In the interaction, the conservation of total momentum holds

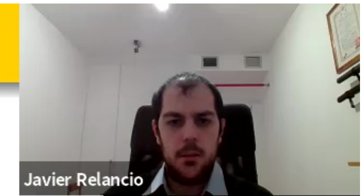
Kinematics in SR



- Dispersion relation

$$C(k) = k_0^2 - \vec{k}^2 = m^2$$

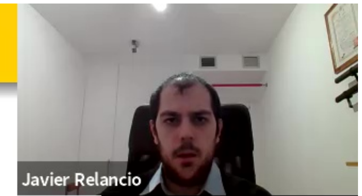
Kinematics in LIV



- Dispersion relation

$$C(k) = k_0^2 - \vec{k}^2 + \frac{k_0^3}{\Lambda} + \ldots = m^2$$

Kinematics in DSR



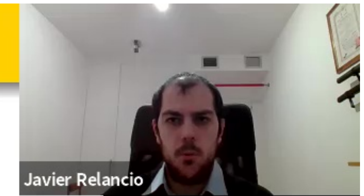
- Dispersion relation

$$C(k) = k_0^2 - \vec{k}^2 + \frac{k_0^3}{\Lambda} + \ldots = m^2$$

- Conservation law

$$\text{Total momentum} = (p \oplus q) = p + q + \frac{pq}{\Lambda} + \ldots$$

Kinematics in DSR



- Dispersion relation

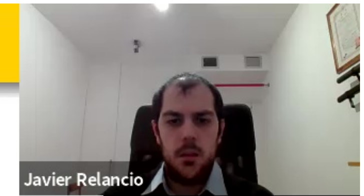
$$C(k) = k_0^2 - \vec{k}^2 + \frac{k_0^3}{\Lambda} + \ldots = m^2$$

- Conservation law

$$\text{Total momentum} = (p \oplus q) = p + q + \frac{pq}{\Lambda} + \ldots$$

- Dispersion relation and conservation law compatible with relativity principle $\rightarrow$ deformed Lorentz transformations

Kinematics in DSR



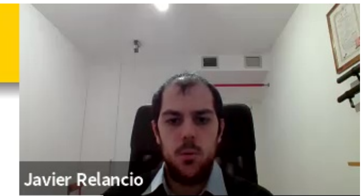
- κ-Poincaré: very much studied model appearing in the context of Hopf algebras [Majid and Ruegg (1994)]
- Deformed dispersion relation (DDR)

$$C(k) = \Lambda^2 \left(e^{k_0/\Lambda} + e^{-k_0/\Lambda} - 2 \right) - e^{k_0/\Lambda} \vec{k}^2$$

- Deformed conservation law (DCL)

$$(p \oplus q)_0 = p_0 + q_0 \,, \qquad (p \oplus q)_i = p_i + q_i e^{-p_0/\Lambda}$$

Kinematical interpretation



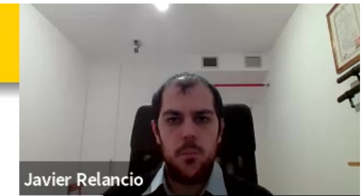
- In SR c is the same for every observer, so

$$u = \frac{v + u'}{1 + vu'}$$

- In DSR c and Λ are the same for every observer, so

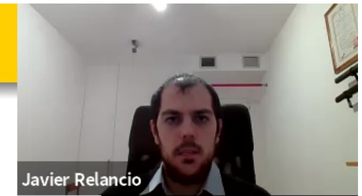
$$k_\mu = (p \oplus q)_\mu$$

Geometrical interpretation?



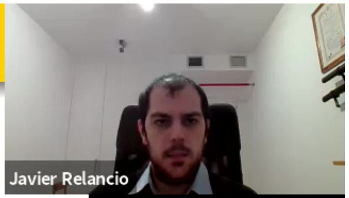
- Gravitational interaction can be described by a curved spacetime
- SR $\rightarrow$ GR $\implies$ flat spacetime $\rightarrow$ curved spacetime
- SR $\rightarrow$ DSR $\implies$ flat momentum space $\rightarrow$ curved momentum space?

Geometry on momentum space?



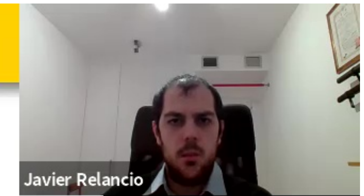
- First proposed by Born to avoid ultraviolet divergences in QFT
- Considered again in DSR framework [Amelino-Camelia et al. (2011)]
- Problem $\rightarrow$ not clear how to implement the relativity principle [Amelino-Camelia et al. (2011)]

Our perspective [Carmona et al. (2019)]



- Dispersion relation → Distance from the origin to k [Amelino-Camelia et al. (2011)]
- Translations, deformed "Lorentz" generators → 10 isometries of the metric!!

SR momentum space



- Flat Minkowski metric $\eta^{\mu\nu}$
- Dispersion relation $\rightarrow$ Square of the distance from the origin to k

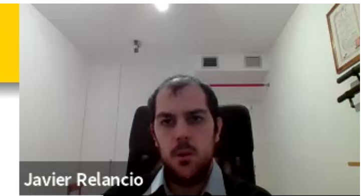
$$C(k) = k^2 = m^2$$

- Conservation law $\rightarrow$ 4 isometries of the metric corresponding to translations in momentum space forming a subgroup

$$q'_\mu = p_\mu + q_\mu$$

- Lorentz transformations $\rightarrow$ 6 isometries of the metric forming a subgroup

Construction of kinematics



- Start by a tetrad of a momentum metric

$$g_{\mu\nu}(k) = \varphi^a_\mu(k)\eta_{ab}\varphi^b_\nu(k)$$

- Compute the Lorentz transformation using

$$g_{\mu\nu}(k') = \frac{\partial k'_\mu}{\partial k_\rho} g_{\rho\sigma}(k) \frac{\partial k'_\nu}{\partial k_\sigma} \, ,$$

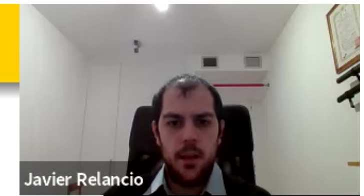
where

$$k'_\mu = k_\mu + \epsilon_{\alpha\beta}\mathcal{J}^{\alpha\beta}_\mu \, ,$$

- Compute the translations using

$$\varphi^a_\mu(p \oplus q) = \frac{\partial (p \oplus q)_\mu}{\partial q_\nu} \varphi^a_\nu(q)$$

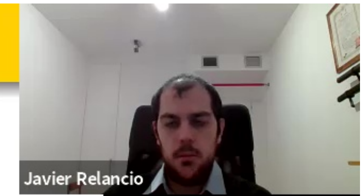
Construction of kinematics



- The previous equation is not satisfied for a generic tetrad $\rightarrow$ Which one should we take?
- A result of differential geometry shows that, in order to have a solution to

$$\varphi^a_\mu(p \oplus q) = \frac{\partial (p \oplus q)_\mu}{\partial q_\nu} \varphi^a_\nu(q)$$

the tetrad directional derivatives $\varphi^\mu_\nu \frac{\partial}{\partial k_\nu}$ must close an algebra $\rightarrow$ Right choice



de Sitter momentum space

- Particular choice of the tetrad

$$\varphi_0^0(k) = 1\,, \qquad \varphi_j^i(k) = \delta_j^i e^{-\frac{k_0}{\Lambda}}$$

- One can see that

$$\left\{\varphi_\rho^0 \frac{\partial}{\partial k_\rho}, \varphi_\sigma^i \frac{\partial}{\partial k_\sigma}\right\} = \frac{1}{\Lambda}\varphi_\sigma^i \frac{\partial}{\partial k_\sigma}$$

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de Sitter momentum space

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- Particular choice of the tetrad

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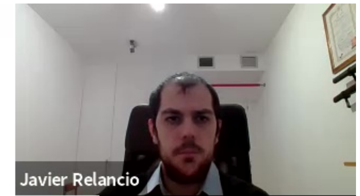
$$\left\{\varphi_\rho^0 \frac{\partial}{\partial k_\rho}, \varphi_\sigma^i \frac{\partial}{\partial k_\sigma}\right\} = \frac{1}{\Lambda}\varphi_\sigma^i \frac{\partial}{\partial k_\sigma}$$

- Using this tetrad one obtains the same kinematics of κ-Poincaré in the bicrossproduct basis!!
- We have understood how DSR kinematics is obtained from a curved momentum space!!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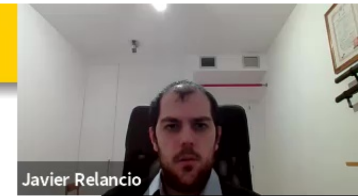
Geometry in phase space

- SR $\rightarrow$ GR $\Longrightarrow$ flat spacetime $\rightarrow$ curved spacetime
- SR $\rightarrow$ DSR $\Longrightarrow$ flat momentum space $\rightarrow$ curved momentum space!
- GR $\rightarrow$ DGR?$\rightarrow$ curved phase space?

Momentum in GR



- Trajectories in GR are described by geodesic equation
- Same result from Hamilton equations with a Casimir

$$C(x,k) = k_\mu g^{\mu\nu}(x) k_\nu$$

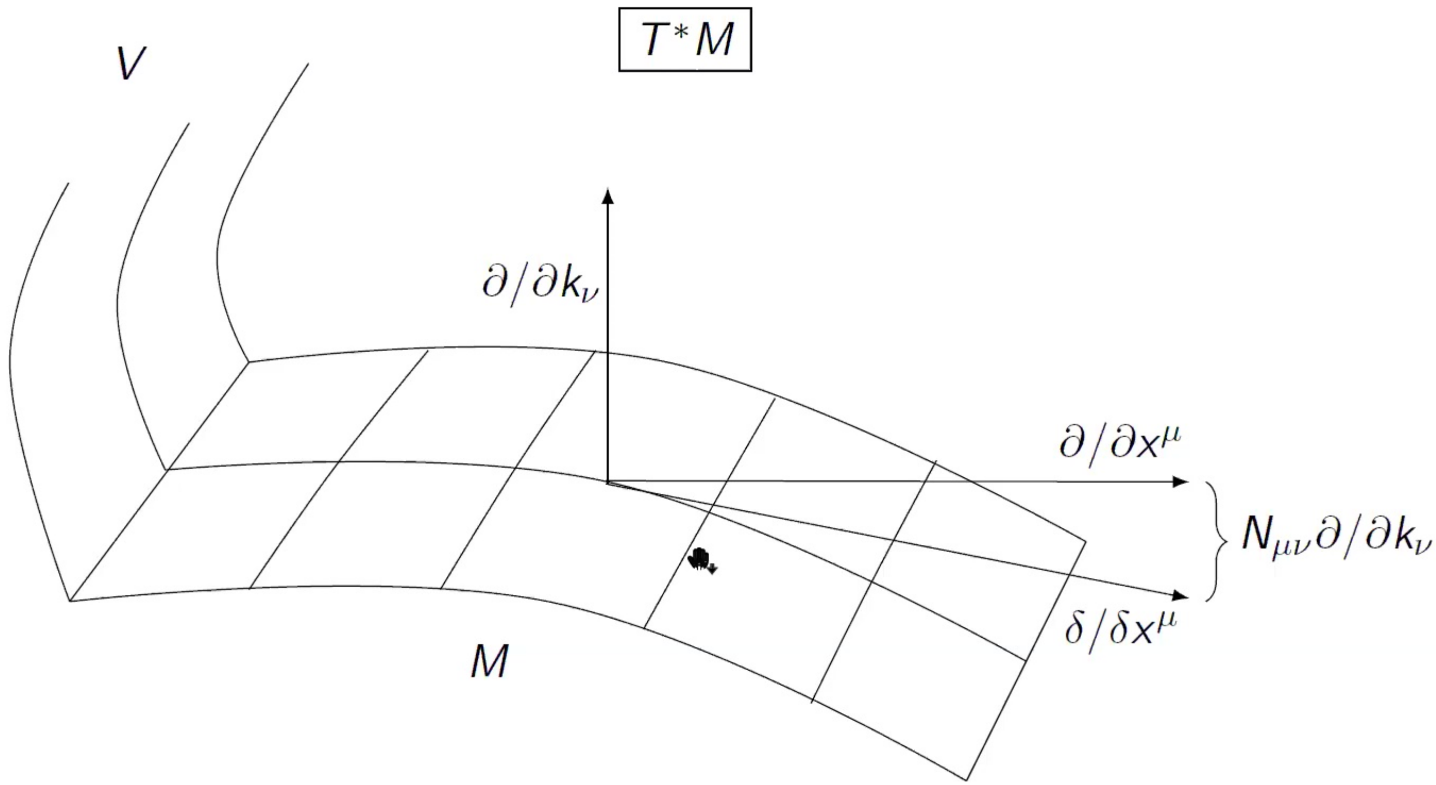
- Relation between momentum and velocity in GR

$$k_\mu = g_{\mu\nu}(x)\dot{x}^\nu$$

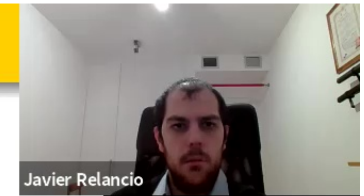
- Along geodesics the momentum changes

$$\dot{k}_\mu = k_\rho \Gamma^\rho_{\mu\nu}(x)\dot{x}^\nu = N_{\mu\nu}(x,k)\dot{x}^\nu$$

Cotangent bundle geometry in a nutshell



Cotangent bundle geometry in a nutshell [Miron (2012)]



- The horizontal distribution is constructed by

$$\frac{\delta}{\delta x^{\mu}} \doteq \frac{\partial}{\partial x^{\mu}} + N_{\nu\mu}(x,k)\frac{\partial}{\partial k_{\nu}}$$

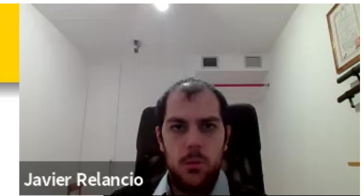
- $N_{\nu\mu}(x,k)$ determined by symmetry and compatibility of affine connection with metric
- Line element in the cotangent bundle

$$\mathcal{G} = g_{\mu\nu}(x,k)dx^{\mu}dx^{\nu} + g^{\mu\nu}(x,k)\delta k_{\mu}\delta k_{\nu}$$

where

$$\delta k_{\mu} = dk_{\mu} - N_{\nu\mu}(x,k)\,dx^{\nu}$$

Cotangent bundle geometry in a nutshell



- Horizontal curves

$$ds^2 = g_{\mu\nu}(x,k)dx^\mu dx^\nu$$

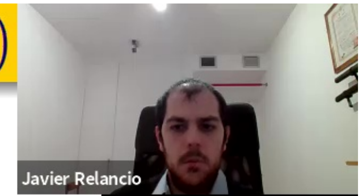
$$\frac{d^2x^\mu}{d\tau^2} + H^\mu_{\nu\sigma}(x,k)\frac{dx^\nu}{d\tau}\frac{dx^\sigma}{d\tau} = 0\,, \qquad \frac{\delta k_\lambda}{d\tau} = \frac{dk_\lambda}{d\tau} - N_{\sigma\lambda}(x,k)\frac{dx^\sigma}{d\tau} = 0$$

- Vertical curves

$$d\sigma^2 = g^{\mu\nu}(x,k)dk_\mu dk_\nu$$

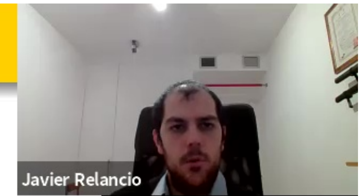
$$x^\mu(\tau) = x_0^\mu\,, \qquad \frac{d^2k_\mu}{d\tau^2} + C_\mu^{\nu\sigma}(x_0,k)\frac{dk_\nu}{d\tau}\frac{dk_\sigma}{d\tau} = 0$$

Relationship metric-action [Relancio and Liberati (2020b)]



- Hamilton equations $\Longleftrightarrow$ geodesic motion
- Distance is conserved along horizontal curves
- Which function represents the Casimir? $\rightarrow$ Square of the distance

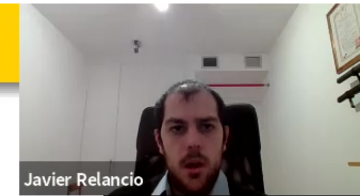
Relationship between metric and distance



- Simple way to relate metric and distance

$$D(0,k) = \frac{\partial D(0,k)}{\partial k_\mu} g_{\mu\nu}(k) \frac{\partial D(0,k)}{\partial k_\nu}$$

Relationship between metric and distance



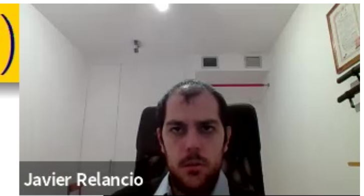
- Simple way to relate metric and distance

$$D(0,k) = \frac{\partial D(0,k)}{\partial k_\mu} g_{\mu\nu}(k) \frac{\partial D(0,k)}{\partial k_\nu}$$

- Then, the Casimir (square of the distance) satisfies

$$C(k) = \frac{1}{4} \frac{\partial C(k)}{\partial k_\mu} g_{\mu\nu}(k) \frac{\partial C(k)}{\partial k_\nu}$$

Construction of the metric [Relancio and Liberati (2020c)]



- Ansatz: including curvature on spacetime by $k_\mu \to \bar{k}_\mu = k_\alpha e^\alpha_\mu(x)$
- New Casimir $\to$ squared distance from $(x, 0)$ to (x, k)

$$C(\bar{k}) = \frac{1}{4}\frac{\partial C(\bar{k})}{\partial \bar{k}_\alpha} g_{\alpha\beta}(\bar{k}) \frac{\partial C(\bar{k})}{\partial \bar{k}_\beta} = \frac{1}{4}\frac{\partial C(\bar{k})}{\partial k_\mu} g_{\mu\nu}(x, k) \frac{\partial C(\bar{k})}{\partial k_\nu}$$

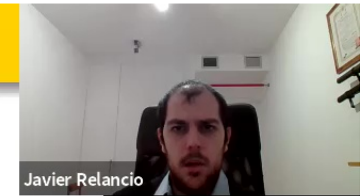
with metric

$$g_{\mu\nu}(x, k) = \Phi^\alpha_\mu(x, k)\eta_{\alpha\beta}\Phi^\beta_\nu(x, k)$$

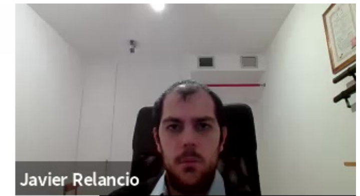
where

$$\Phi^\alpha_\mu(x, k) = e^\lambda_\mu(x)\varphi^\alpha_\lambda(\bar{k})$$

Properties of the metric



- Invariant under spacetime diffeomorphisms
- Also, if the starting momentum space is maximally symmetric we can define:
  - Modified translations
  - Modified Lorentz transformations
  - Modified dispersion relation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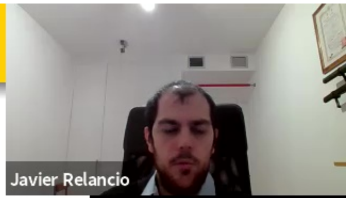
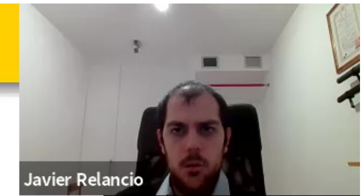
Quantum Gravity Phenomenology



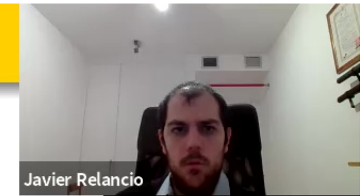
- Planck energy $\rightarrow 10^{19}$ GeV
- Particle accelerators $\rightarrow 1.3 \times 10^{4}$ GeV
- Cosmic rays $\rightarrow 10^{11}$ GeV
- Phenomenology? $\rightarrow$ Amplifications at low energies

LIV vs DSR phenomenology



Theory	Deviation to SR threshold	Energy
LIV	$\frac{m^2}{E^2}\,\frac{E}{\Lambda}$	$E^3 \sim \Lambda m^2$
DSR	$\frac{m^2}{E^2}\,\frac{m^2}{E\Lambda}$	$E \sim \Lambda$

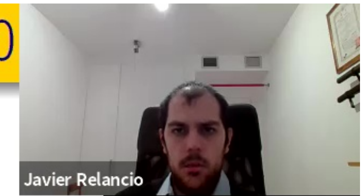
LIV vs DSR phenomenology



Theory	Deviation to SR threshold	Energy
LIV	$\frac{m^2}{E^2}\,\frac{E}{\Lambda}$	$E^3 \sim \Lambda m^2$
DSR	$\frac{m^2}{E^2}\,\frac{m^2}{E\Lambda}$	$E \sim \Lambda$

DSR only observable through time delays!

Time delay in flat spacetime [Relancio and Liberati (2020



- We use this de Sitter metric

$$g_{00}(k) = 1\,, \qquad g_{0i}(k) = 0\,, \qquad g_{ij}(k) = \eta_{ij}\, e^{-2k_0/\Lambda}$$

- The line element is

$$ds^2 = dt^2 - dx^2 e^{-2k_0/\Lambda}$$

- The line element for photons with different energies is

$$ds^2 = dt^2 - dr^2 = dt^2 - dx_l^2 = dt^2 - dx_h^2\, e^{-2k_0/\Lambda} = 0$$

being

$$r = x_l = x_h\, e^{-k_0/\Lambda}$$

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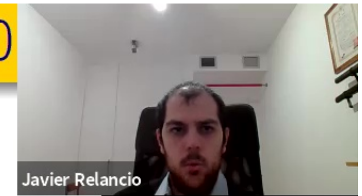

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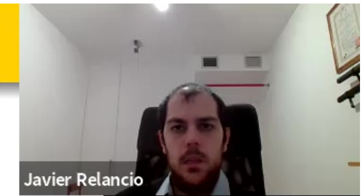
- Absence of time delays!!

Synchrotron radiation [Relancio and Liberati (2020a)]



- We use this de Sitter metric

$$g_{00}(k) = 1\,, \qquad g_{0i}(k) = 0\,, \qquad g_{ij}(k) = \eta_{ij}\, e^{2k_0/\Lambda}$$

- Critical frequency of synchrotron radiation

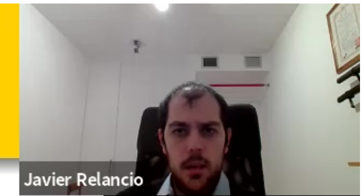
$$\omega_c = \frac{3Bek_0^2}{2m^3}\left(1 - \frac{2k_0}{\Lambda}\right)$$

- We find a maximum frequency

$$\omega_c^{\max} = \frac{Be\Lambda^2}{18m^3}$$

- Constraint on the scale: $\Lambda > 7{,}8 \times 10^3$ TeV

Phenomenology in an expanding universe [Relancio and Liberati (2020a)]



- We use the metric

$$g_{00}(x,k) = 1\,, \quad g_{0i}(x,k) = 0\,, \quad g_{ij}(x,k) = \eta_{ij}\, R^2(x^0)\, e^{\mp 2k_0/\Lambda}$$

- Absence of time delays!!
- Redshift $\rightarrow$ From AGN $\Lambda > 49$ keV
- Luminosity distance $\rightarrow$ From GW170817 $\Lambda > 8$ keV
- No stronger constraints

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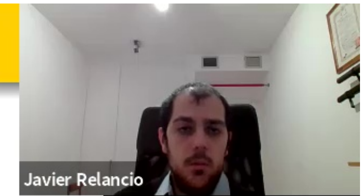
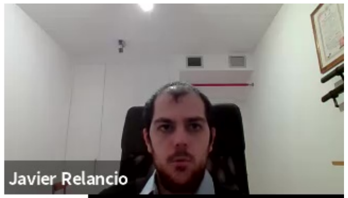
Schwarzschild black hole [Relancio and Liberati (2020c)]

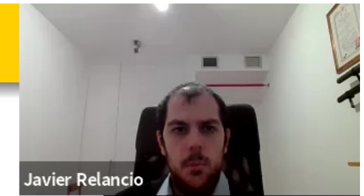


- Horizon independent of energy
- Surface gravity

$$\kappa_{\text{peeling}} = \frac{e^{k_0/\Lambda}}{4M}$$


Javier Relancio

Einstein equations [Relancio and Liberati (2020b)]



- Geodesic deviation $\longleftrightarrow$ commutator of two covariant derivatives
- Riemann tensor

$$R^{\mu}_{\nu\rho\sigma}(x,k) = \frac{\delta H^{\mu}_{\nu\rho}(x,k)}{\delta x^{\sigma}} - \frac{\delta H^{\mu}_{\nu\sigma}(x,k)}{\delta x^{\rho}} + H^{\lambda}_{\nu\rho}(x,k)H^{\mu}_{\lambda\sigma}(x,k) - H^{\lambda}_{\nu\sigma}(x,k)H^{\mu}_{\lambda\rho}(x,k)$$

- Einstein tensor [Miron (2012)] and Einstein equations

$$G_{\mu\nu}(x,k) = R_{\mu\nu}(x,k) - \frac{1}{2}g_{\mu\nu}(x,k)R(x,k) = 8\pi T_{\mu\nu}(x,k)$$

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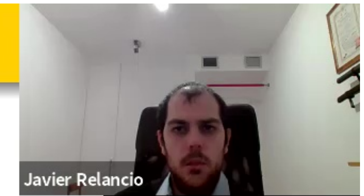
Einstein equations [Relancio and Liberati (2020b)]



- Geodesic deviation $\longleftrightarrow$ commutator of two covariant derivatives
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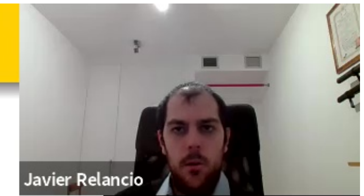
$$G_{\mu\nu}(x,k) = R_{\mu\nu}(x,k) - \frac{1}{2}g_{\mu\nu}(x,k)R(x,k) = 8\pi T_{\mu\nu}(x,k)$$

- From its conservation

$$G^{\mu\nu}(x,k)_{;\mu} = 0$$

selects one basis!!

Physical basis [Relancio and Liberati (2020b)]



- Physical momentum metric

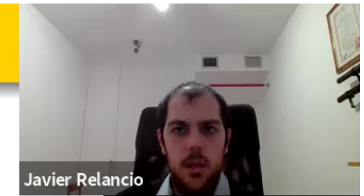
$$g_{\mu\nu}(k) = \eta_{\mu\nu} \left(1 - \frac{k_0^2 - \vec{k}^2}{4\Lambda^2}\right)^2$$

- The Casimir is a function of k^2

$$\mathcal{C}(k) = 4\Lambda^2 \operatorname{arccoth}^2 \left(\frac{2\Lambda}{\sqrt{k_0^2 - \vec{k}^2}}\right)$$

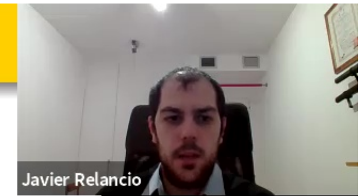
- Einstein tensor independent on momentum!

Friedmann equations [Relancio and Liberati (2020b)]



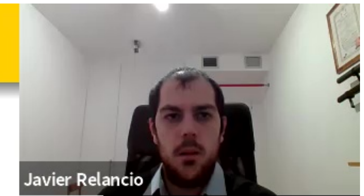
- Expanding universe $\rightarrow$ energy momentum tensor of a perfect fluid
- Usual Friedmann equations
- Consistent with conservation of energy-momentum tensor
- Consistent Raychaudhuri equation

Conclusions: curved momentum space



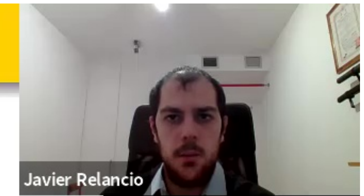
- We have developed a geometrical interpretation of a relativistic deformed kinematics
- We obtain the κ-Poincaré kinematics from a de Sitter momentum space
- Other possible kinematics can be obtained though this framework and also for anti de Sitter

Conclusions: phase space geometry



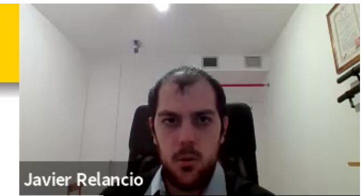
- We have proposed a simple way to include both space-time and momentum curvature $\rightarrow$ geometry in phase space
- Hamilton equations $\Longleftrightarrow$ geodesic motion
- Distance is conserved along horizontal curves

Conclusions: phenomenological results



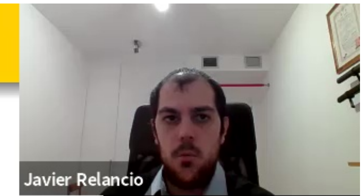
- Synchrotron radiation $\Lambda \geq 1$ PeV
- Universe expanding
  - Absence of time delays
  - Luminosity distance and redshift $\Lambda \geq 100$ keV
- Schwarzschild black hole
  - Same horizon $\rightarrow$ different from LIV scenario
  - Surface gravity $\rightarrow$ energy dependent Hawking temperature?
- New phenomenology in high-energy particle and astroparticle physics

Conclusions: choice of momentum variables

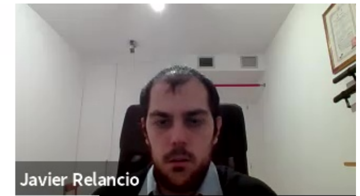


- Conservation of Einstein tensor $\rightarrow$ privileged momentum basis
- Properties
  - Einstein tensor is momentum independent
  - Also compatible with Raychaudhuri's equation

Outlook and future prospects



- New phenomenological implications from our model $\rightarrow$ Stronger constraints?
- Formal developments $\rightarrow$ Consistency checks
- Multiparticle system $\rightarrow$ Time delays?



Thanks for your attention!!!