

Title: Non-local quantum computation and holography

Speakers: Alex May

Series: Perimeter Institute Quantum Discussions

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Abstract: Relativistic quantum tasks are quantum computations which have inputs and outputs that occur at designated spacetime locations.

Understanding which tasks are possible to complete, and what resources are required to complete them, captures spacetime-specific aspects of quantum information. In this talk we explore the connections between such tasks and quantum gravity, specifically in the context of the AdS/CFT correspondence. We find that tasks reveal a novel connection between causal features of bulk geometry and boundary entanglement.

Further, we find that AdS/CFT suggests quantum non-local computations, a specific task with relevance to position-based cryptography, can be performed with linear entanglement. This would be an exponential improvement on existing protocols.



# Quantum tasks in holography

Alex May - The University of British Columbia

Based on:

arXiv 1902.06845

AM

1912.05649

AM, Jon Sonce,  
Geoff Penington



# "Quantum tasks in holography"

Alex May - The University of British Columbia

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arXiv 1902.06845\*

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## Q.I. and Q.G.

- Q.I. now plays a significant role in quantum gravity
  - ↳ most concretely in the AdS/CFT correspondence
- Often in Q.I., we don't think about how quantum information can be processed in a spacetime setting.

Understanding Q.I. in a spacetime setting leads to new insights in quantum gravity, and vice versa.



## Outline

- ① Recall a framework for thinking about quantum information processing in a spacetime context.
  - ↳ "Relativistic quantum tasks"
- ② Discuss how we can relate this to AdS/CFT, and results we obtain
- ③ Discuss what AdS/CFT tells us about quantum tasks
  - ↳ efficient quantum non-local computation?



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  - ↳ efficient quantum non-local computation ?



Relativistic quantum tasks



# Quantum Tasks

(see Kent 2012)

- Are quantum computations with inputs + outputs that occur at designated spacetime locations
- Involve two agencies,
  - ↳ Alice - who receives inputs and produces outputs
  - ↳ Bob - who gives inputs and verifies outputs



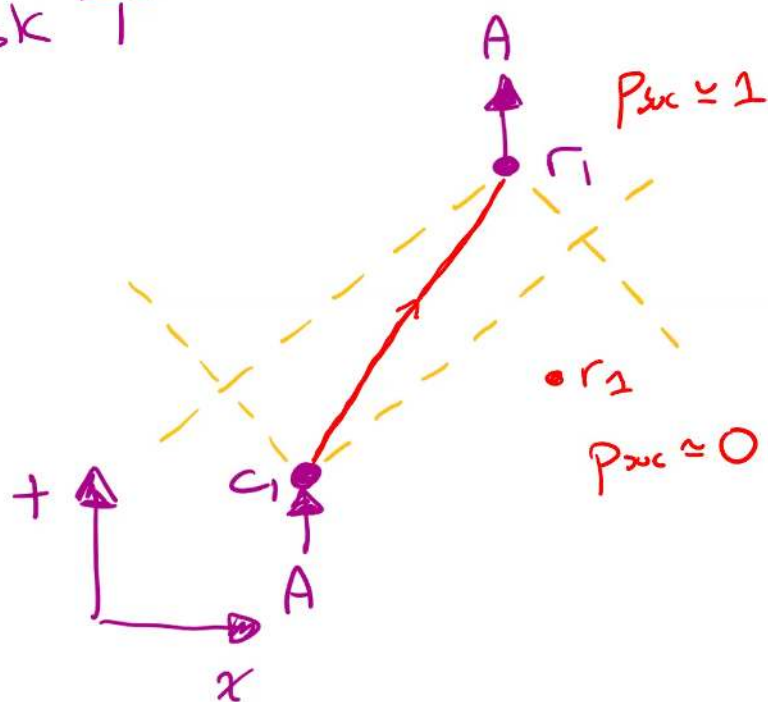
# Quantum Tasks (see Kent 2012).

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# Example: Receive + Return Task

Task T



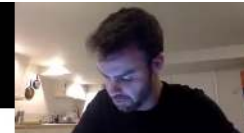
Question we might ask:

- With what probability is it possible?

$$P_{suc}(T) = ?$$

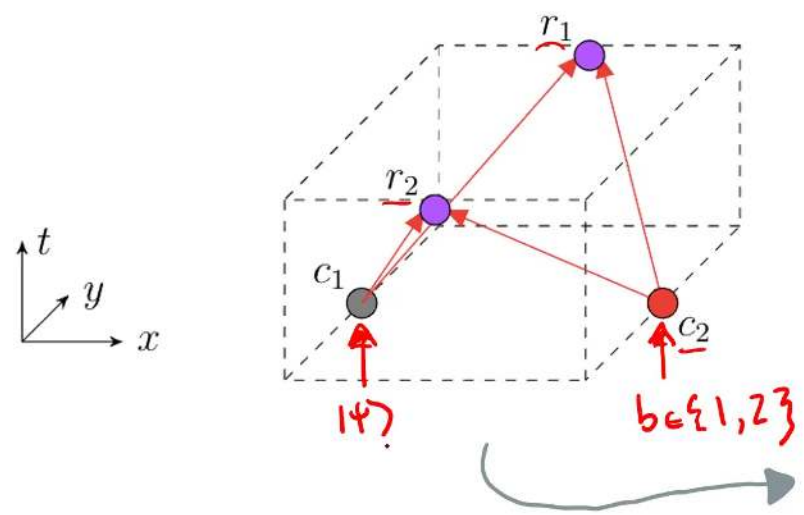
- What resources are necessary to achieve a given success probability?

$$P_{suc}(T) \geq \alpha \Rightarrow \text{resources?}$$



# A "Summoning" task

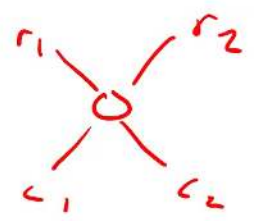
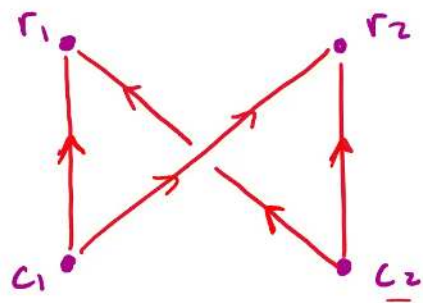
1101.4012 - Kent  
1210.0913 - AM, Patrick Hayden }  
}



- Alice gets (unknown) quantum state  $|\psi\rangle$  at  $c_1$
- Should bring  $|\psi\rangle$  to  $r_b$ ,  $b$  is announced at  $c_2$

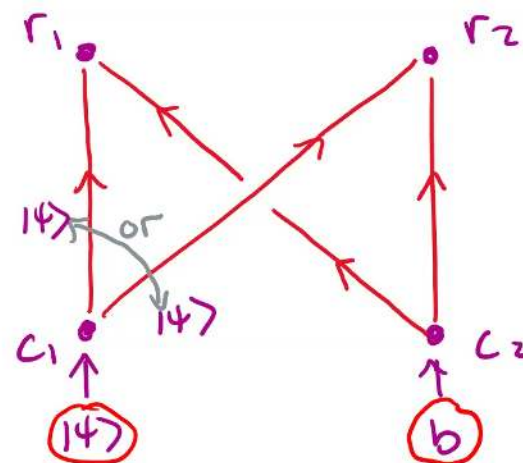
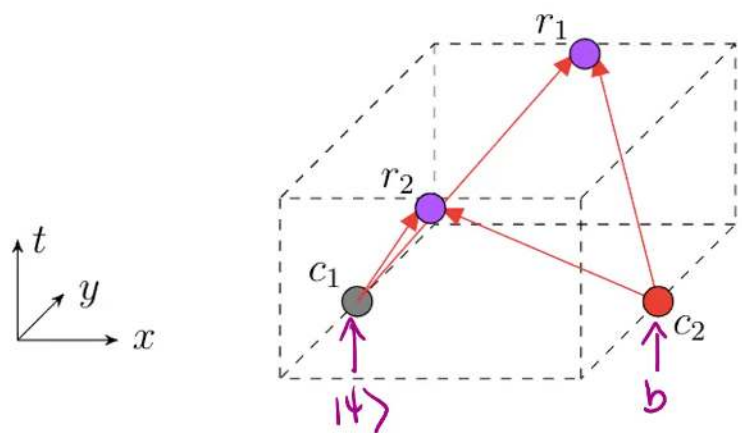
$\exists p$  s.t.

$$c_1, c_2 \prec p \prec r_1, r_2$$





# A "summoning" task

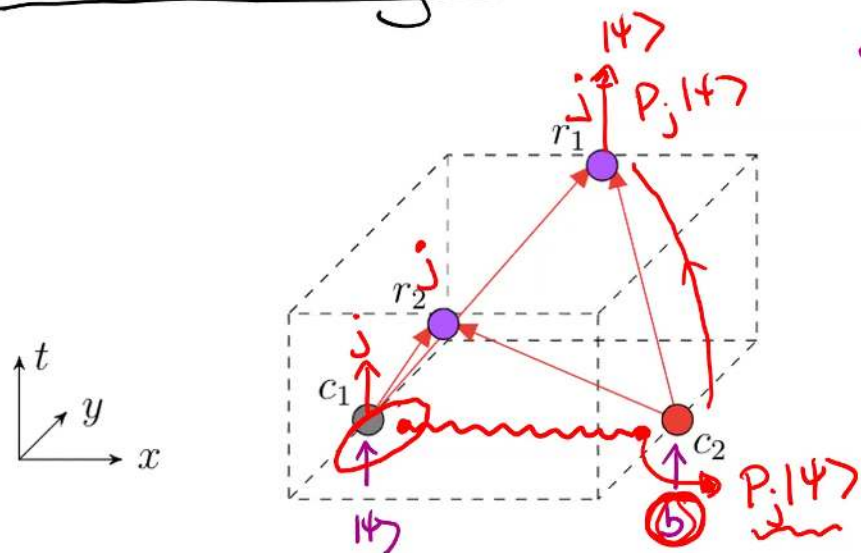


- Using simple (no entanglement) strategy

$$P_{\text{succ}} \sim 1/2$$



# A "summoning" task



• Using entanglement, can do:

- 1) Teleport  $|\psi\rangle$  from  $c_1$  to  $c_2$
- 2) Send classical measurement outcome  $j$  to both  $r_1$  and  $r_2$
- 3) Send quantum part, now at  $c_2$  along with  $b$ , to  $r_b$

Quantum tasks capture spacetime-specific aspects of quantum information

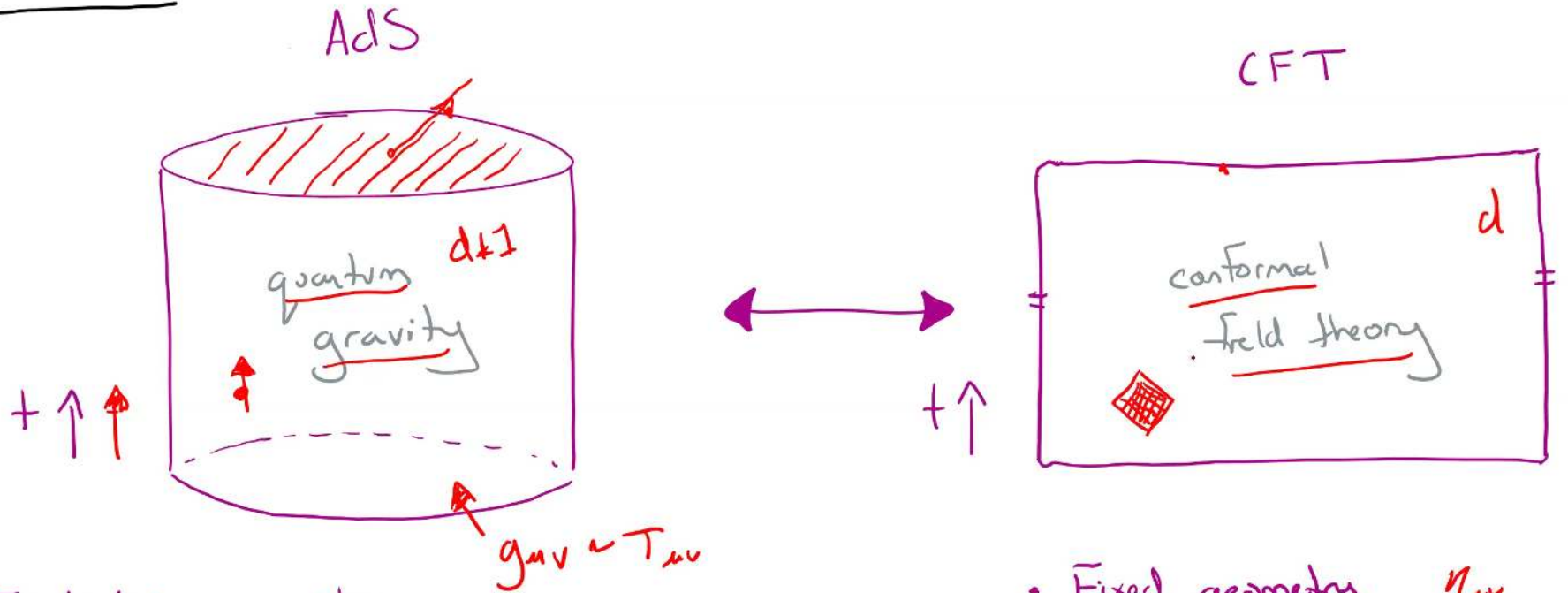
Same observation appears in:  
 "Teleporting an unknown quantum state ..."



Applying tasks to AdS/CFT



# AdS/CFT



- Includes gravity
- $d+1$  dimensional
- Includes quantum fields

- Fixed geometry  $\eta_{\mu\nu}$
- $d$  dimensional
- Includes quantum fields



## Applying tasks to holography

- Given a theory, associate a list of tasks we can define and their success probabilities

$$\text{Theory} \sim \{(T, p_{\text{succ}}(T))\} \sim \{(\quad)\}$$

- Given a duality, we should have:

some task  $\rightarrow T \leftrightarrow \hat{T} \leftarrow$  "same" task in dual theory

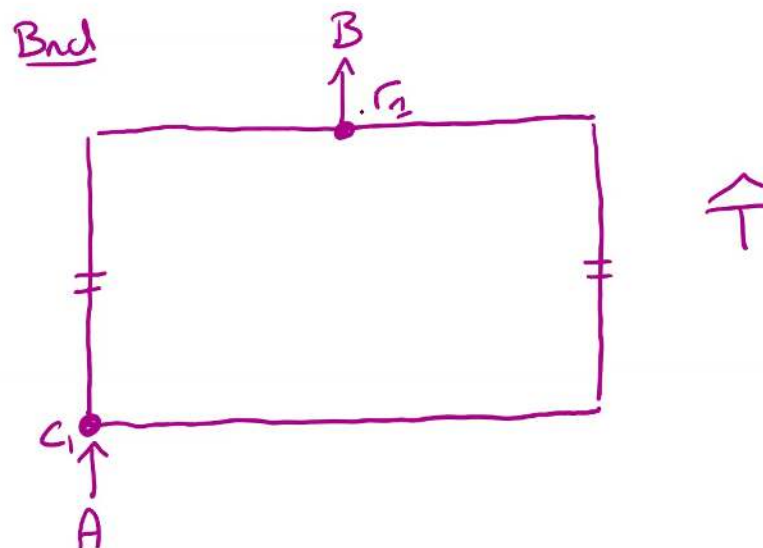
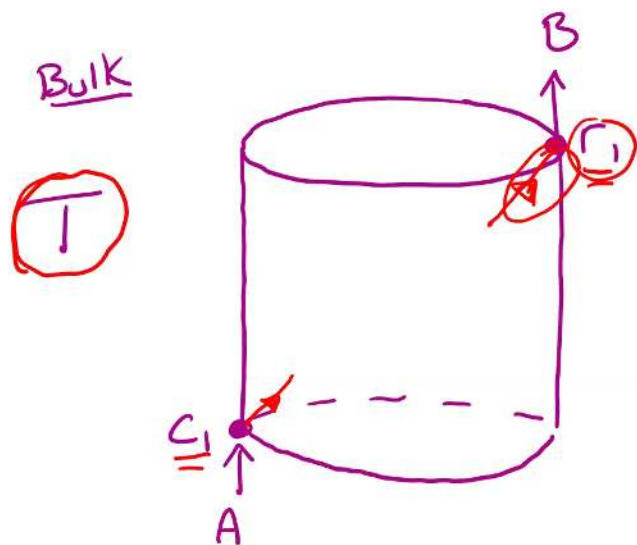
with:

$$p_{\text{succ}}(T) = p_{\text{succ}}(\hat{T})$$



# Dual tasks

$$T \leftrightarrow \hat{T}$$



• Restrict attention to  $T$  with points at the AdS boundary



Straight forward to identify input/output points in  $T, \hat{T}$

# The connected wedge theorem

arXiv: 1912.05649

AM, Jon Sorce,  
Geoff Penington

B84 Task +  
 $\mathbb{Z} \rightarrow \mathbb{Z}$  Connected wedge  
theorem



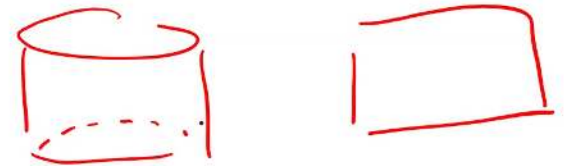
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# The connected wedge theorem

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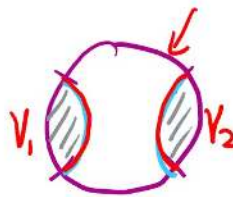
B84 Task +  
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theorem



## Mutual information and minimal surfaces

- $I(V_1; V_2) \equiv \underbrace{S(V_1)} + \underbrace{S(V_2)} - \underbrace{S(V_1 \cup V_2)}$

- Minimal surface enclosing  $\underbrace{V_1 \cup V_2}$  is   $\Leftrightarrow \underbrace{I(V_1; V_2) = O(1/b_N)}$

- Minimal surface enclosing  $\underbrace{V_1 \cup V_2}$  is   $\Leftrightarrow \underbrace{I(V_1; V_2) = O(1)}$



# $Z \rightarrow Z$ Connected Wedge Theorem

Thm 1 Pick  $\{c_1, c_2, r_1, r_2\}$ .

If the "scattering region" is non-empty, then  $I(V_1:V_2) \approx O(1/GN)$ .

$V_1 \cap V_2 \neq \emptyset$

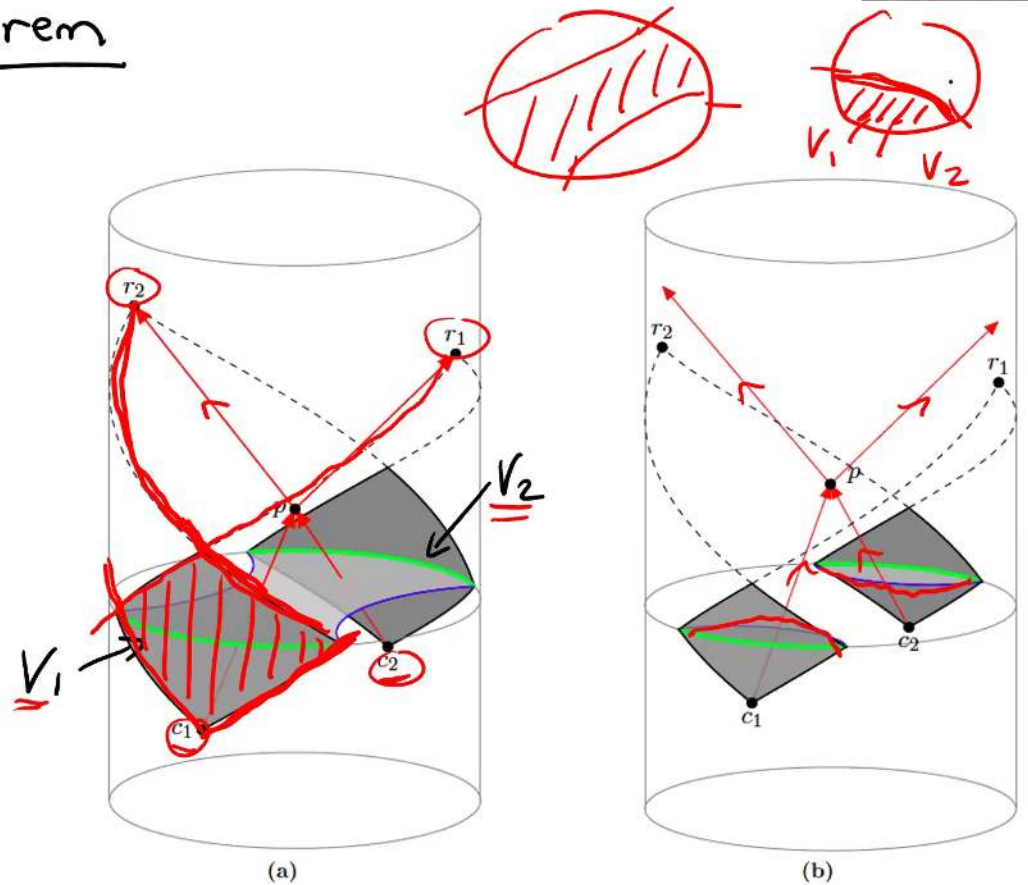
"input regions"

$$\left. \begin{aligned} V_1 &\equiv \hat{J}^+(c_1) \cap \hat{J}^-(r_1) \cap \hat{J}^-(r_2) \\ V_2 &\equiv \hat{J}^+(c_2) \cap \hat{J}^-(r_1) \cap \hat{J}^-(r_2) \end{aligned} \right\}$$

"scattering region"

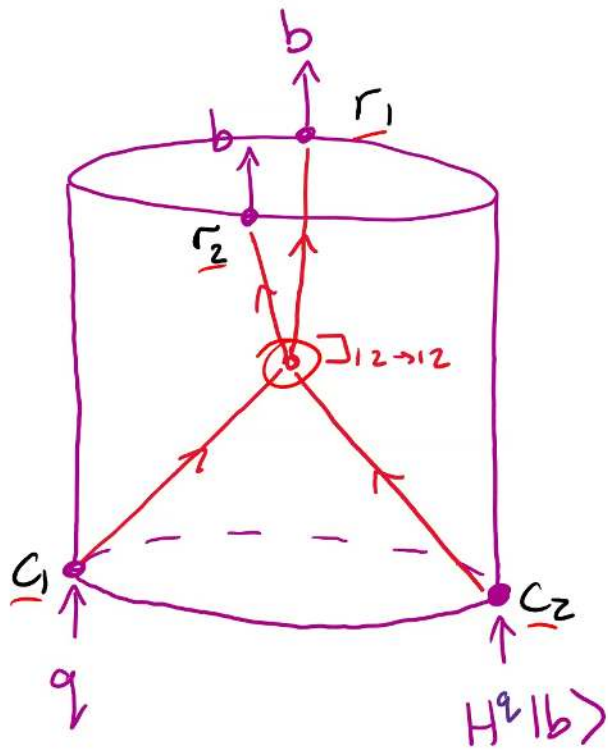
$$\hat{J}_{12 \rightarrow 12} \equiv \hat{J}^+(c_1) \cap \hat{J}^+(c_2) \cap \hat{J}^-(r_1) \cap \hat{J}^-(r_2) \quad \left. \right\}$$

$\hat{J}_{12 \rightarrow 12} \neq \emptyset$



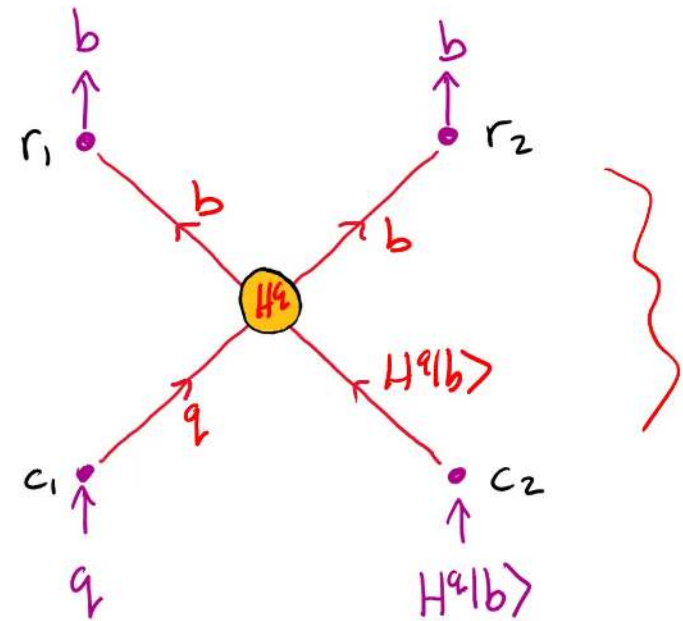


# Bulk perspective



$$\boxed{J_{12 \to 12} \neq \emptyset}$$

$\Rightarrow$

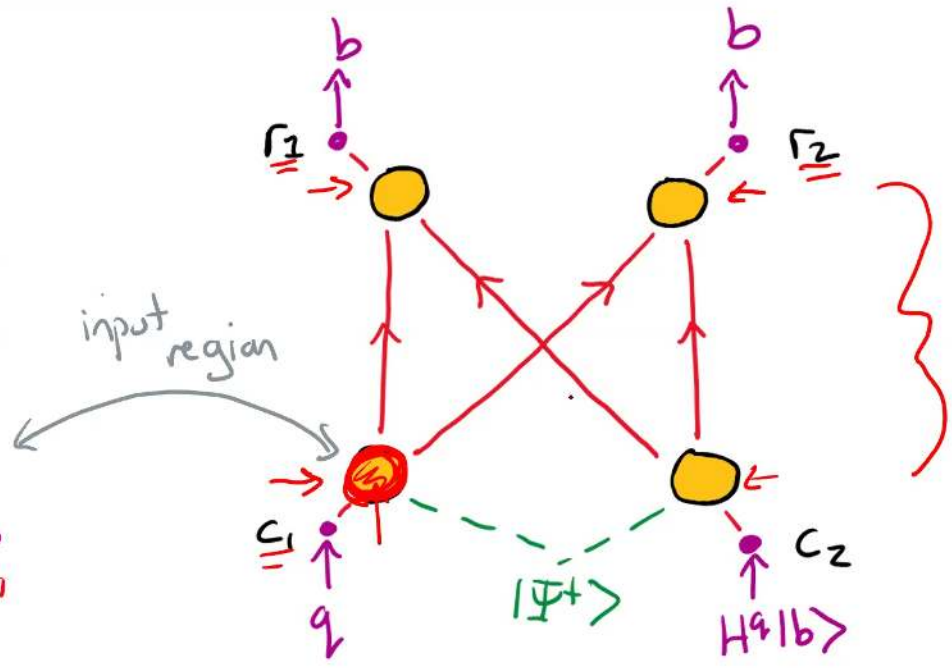
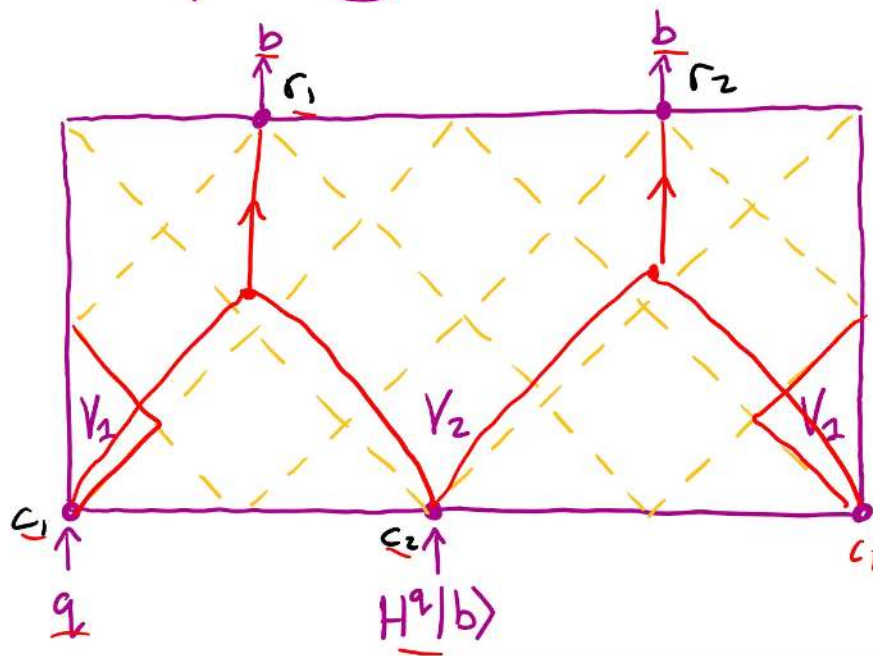


$$\boxed{P_{\text{suc}}(T) = 1}$$



# Boundary perspective

• Corresponding  $\hat{T}$ :

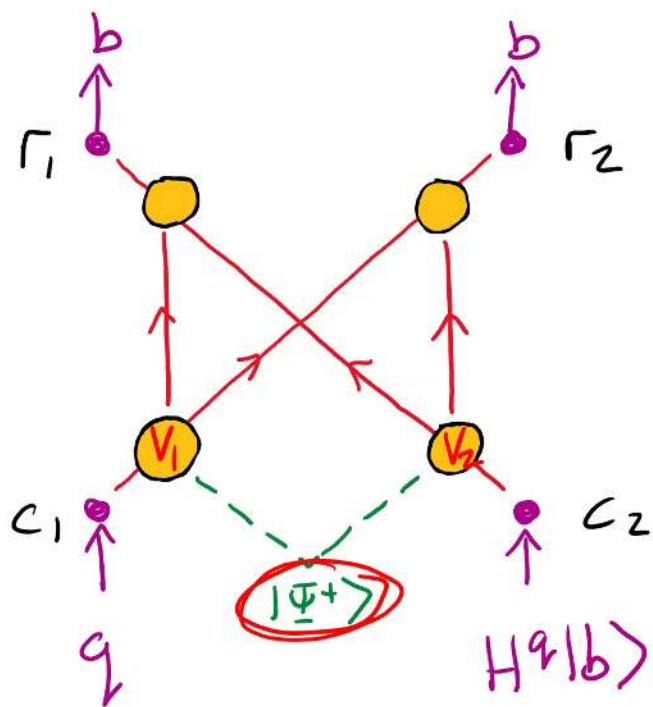


↳ Scattering region is empty!

⇒ forced to use "non-local" strategy.



# Comparing bulk + boundary



Recall: ↙ non-empty scattering regions

$$1 = P_{\text{suc}}(T) = \boxed{P_{\text{suc}}(\hat{T}) = 1}$$

For non-local strategy to have

$$P_{\text{suc}}(\hat{T}) = 1$$

requires  $V_1$  and  $V_2$  to share entanglement!

(can make this quantitative)



# $Z \rightarrow Z$ Connected Wedge Theorem

Thm: Pick  $c_1, c_2, r_1, r_2$ . Then  
if  $J_{12 \rightarrow 12} \neq \emptyset$ ,  $\underbrace{I(V_1; V_2)} = O(1/b_N)$ .

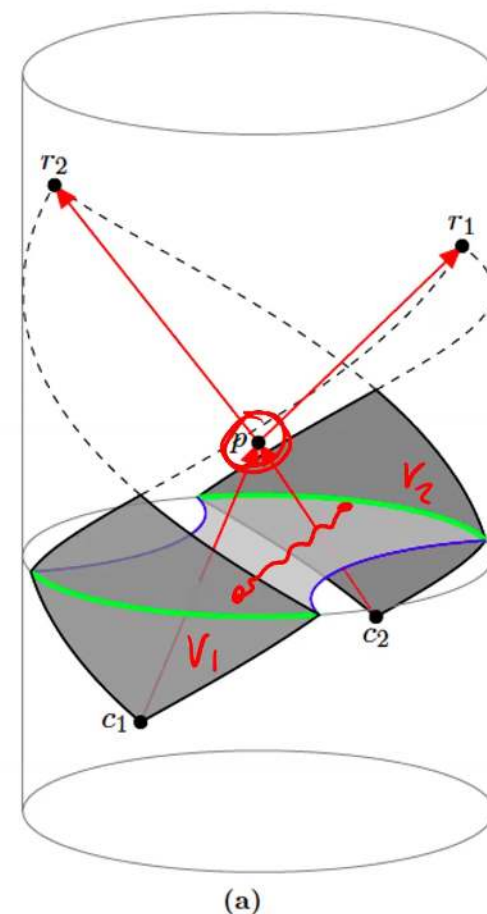
entanglement  $\sim$  causal features  
space-like surfaces

Recall:

$$V_1 = \hat{J}^+(c_1) \cap \hat{J}^-(r_1) \cap \hat{J}^-(r_2)$$

$$V_2 = \hat{J}^+(c_2) \cap \hat{J}^-(r_1) \cap \hat{J}^-(r_2)$$

$$J_{12 \rightarrow 12} = J^+(c_1) \cap J^+(c_2) \cap J^-(r_1) \cap J^-(r_2)$$



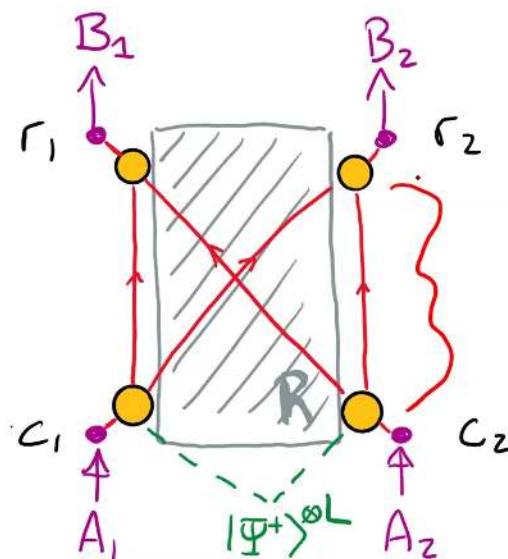
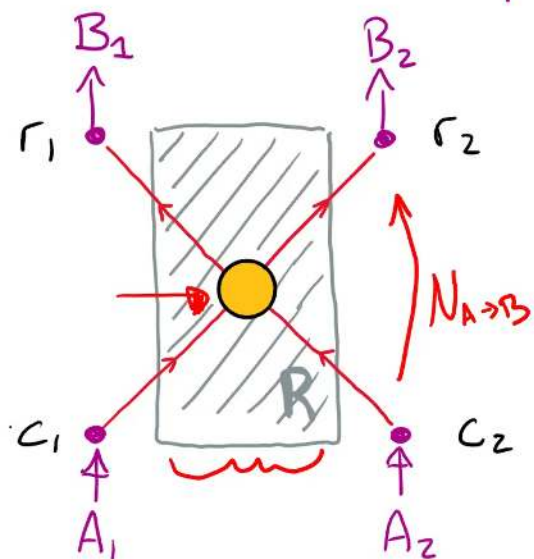


Towards efficient non-local  
quantum computation.



# Position based cryptography

- Bob chooses  $N_{A_1 A_2 \rightarrow B_1 B_2}$  in an attempt to force Alice to do operations inside  $R$



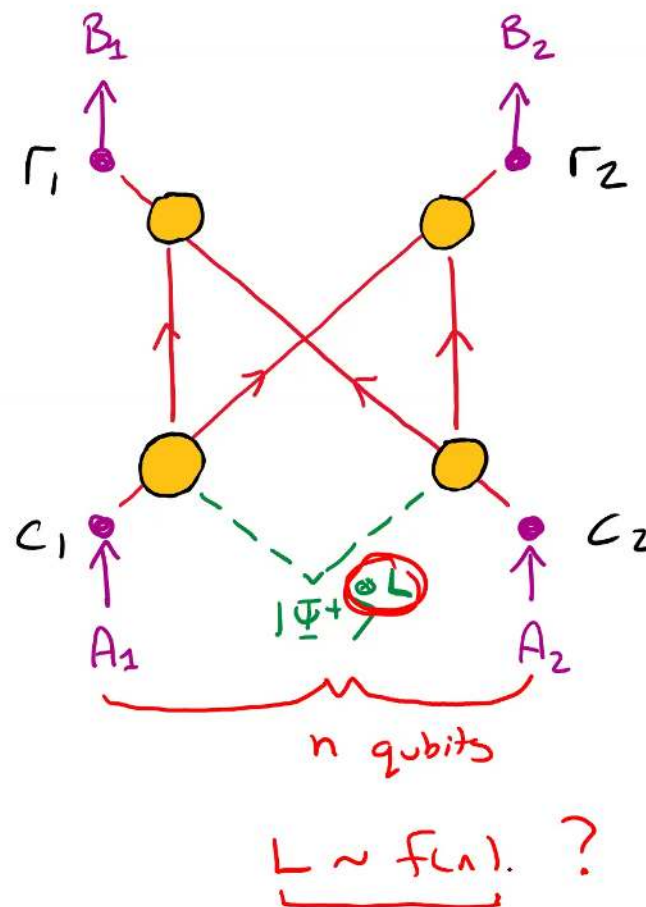
- Alice tries to cheat, by acting only outside  $R$ .



# Quantum non-local computation (QNLC)

- Can Alice always cheat?  
↳ Given enough entanglement, yes!

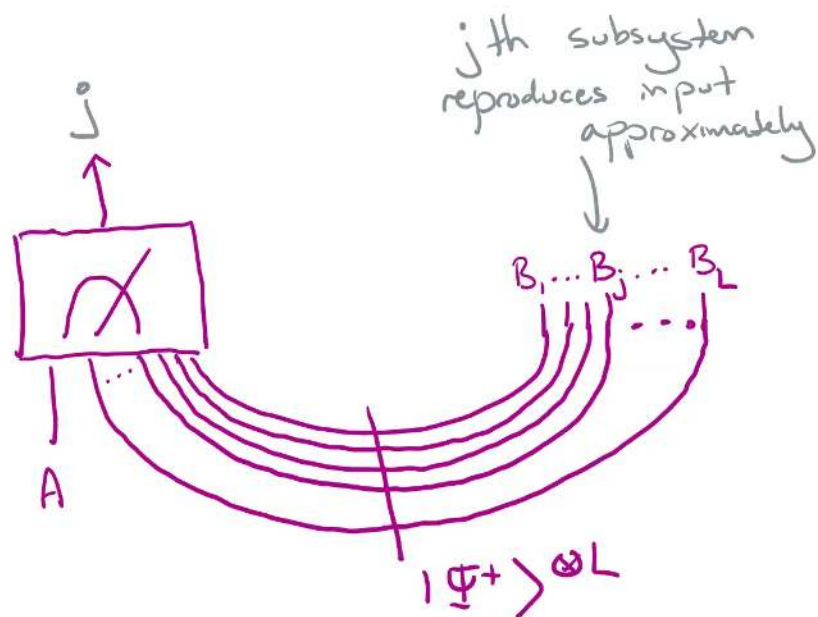
- Given  $n = \log \dim A$ ,  
how big do we need  $L$   
for  $p_{suc} \approx 1$ ?





# Port-teleportation

(Ishizaka + Hiroshima 2009)



- Define channel  $\mathcal{E}_{A \rightarrow B_j}$

- For appropriate measurement

$$\|\mathcal{E}_{A \rightarrow B_j} - \mathbb{I}_{A \rightarrow B_j}\|_{\diamond} \leq \frac{2^{2n+2}}{\sqrt{L}}$$

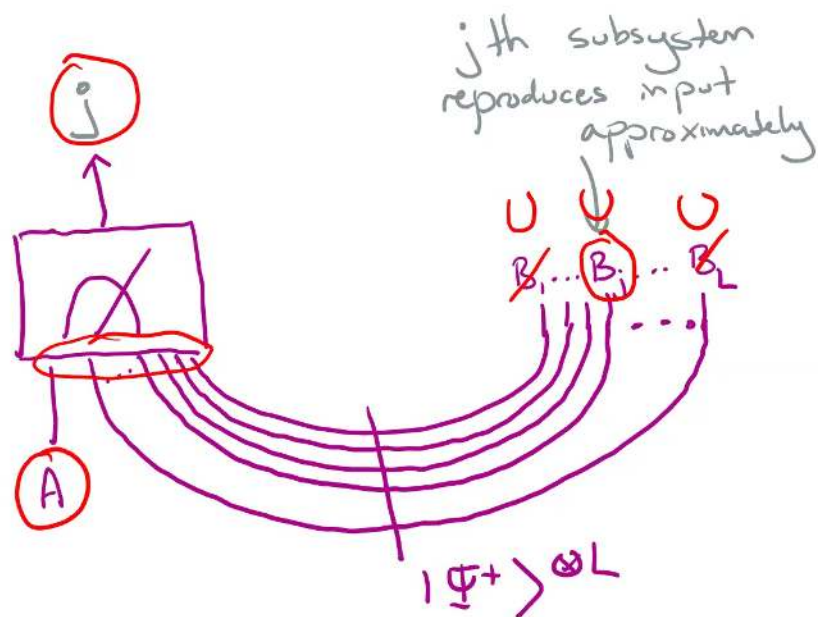
- Defines a teleportation that requires only trivial correction (trace out  $B_{i \neq j}$ )

- Connections to universal processors, transverse codes, ...



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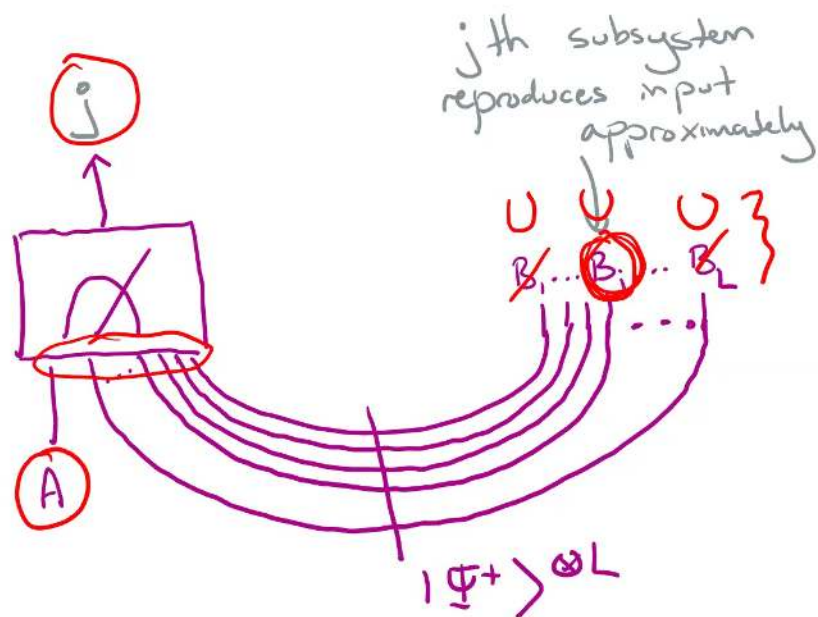
$$L \sim 2^{2n}$$

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# Port-teleportation

(Ishizaka + Hiroshima 2009)



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## Port-teleportation approach to QNLC

- Port-teleportation is involved in the most efficient known QNLC scheme (Bergi + König 2011)

• For

$$\| \Sigma_U - \cup \|_{\diamond} \leq \underline{\epsilon}$$

↑ applied channel
↑ intended channel

Their construction requires

$$L \sim \frac{nZ^n}{\epsilon^2}$$

\* Note: For Clifford operations port-teleportation is unnecessary, and

$$L \sim n$$



# Holographic approach to QNLC

- Set-up same task in AdS/CFT

- Can send in

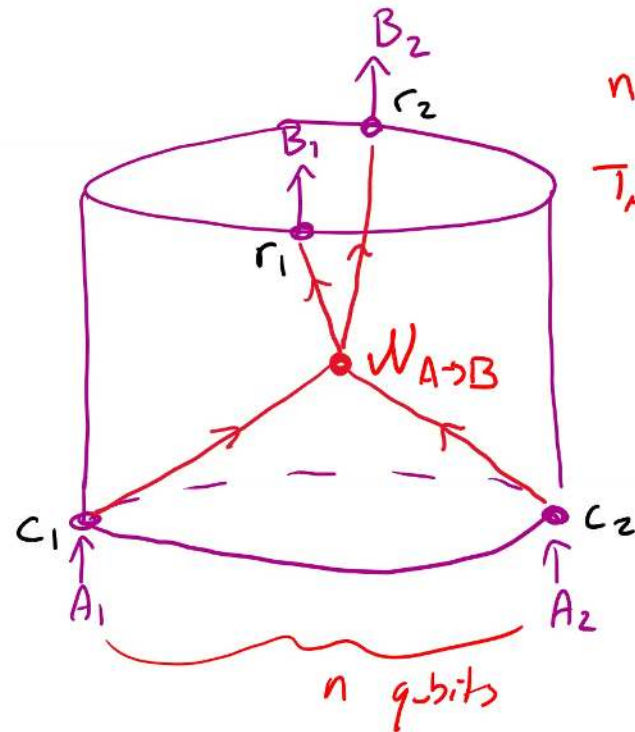
$$n \sim O(1/G_N)$$

qubits

- Have  $I(V_1:V_2) \sim O(1/G_N)$

$$L \sim n$$

$\Rightarrow$  AdS/CFT points to an efficient method for quantum non-local computations!



$$n \sim 1/G_N$$

$$T_{UV} G_N \sim G_{UV}$$

$$L$$



## Some caveats

- A more precise statement is that:

Any channel  $\mathcal{N}_{A \rightarrow B}$  that can be implemented inside of the spacetime region  $\mathcal{J}_{I_2 \rightarrow I_2}$  can be performed non-locally with linear entanglement.

- It's not clear to me what exactly this set of channels is... but it's almost certainly more than the Clifford group.



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Summary



## Summary

- The quantum tasks framework highlights spacetime specific aspects of quantum information.
- This perspective on quantum information is relevant to quantum gravity
  - ↳ Further motivates understanding quantum tasks
- AdS/CFT gives some new insights into quantum tasks, and perhaps quantum information more broadly.



Thanks !