Title: Non-local quantum computation and holography

Speakers: Alex May

Series: Perimeter Institute Quantum Discussions

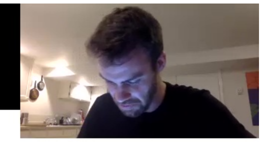
Date: December 16, 2020 - 4:00 PM

URL: http://pirsa.org/20120024

Abstract: Relativistic quantum tasks are quantum computations which have inputs and outputs that occur at designated spacetime locations.

Understanding which tasks are possible to complete, and what resources are required to complete them, captures spacetime-specific aspects of quantum information. In this talk we explore the connections between such tasks and quantum gravity, specifically in the context of the AdS/CFT correspondence. We find that tasks reveal a novel connection between causal features of bulk geometry and boundary entanglement.

Further, we find that AdS/CFT suggests quantum non-local computations, a specific task with relevance to position-based cryptography, can be performed with linear entanglement. This would be an exponential improvement on existing protocols.



Quantum tasks in holography

Alex May - The University of British Columbia

Based on:

arXiv 1902.06845 AM

1912.05649 AM, Jon Sorce, Geoff Penington

"Quantum tasks in holography"

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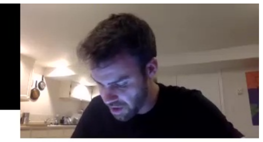


Q.I. and Q.G.

- Q.I. now plays a significant role in quantum gravity
  - ↳ most concretely in the AdS/CFT correspondence
- Often in Q.I., we don't think about how quantum information can be processed in a spacetime setting.

> Understanding Q.I. in a spacetime setting leads to new insights in quantum gravity, and vice versa.

Outline

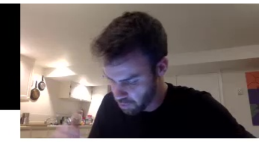
① Recall a framework for thinking about quantum information processing in a spacetime context.

↳ "Relativistic quantum tasks"

② Discuss how we can relate this to AdS/CFT, and results we obtain

③ Discuss what AdS/CFT tells us about quantum tasks

↳ efficient quantum non-local computation?



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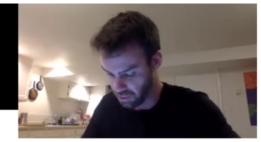
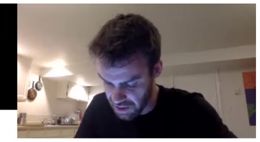


Relativistic quantum tasks



Quantum Tasks (see Kent 2012)

- Are quantum computations with inputs + outputs that occur at designated spacetime locations

- Involve two agencies,
  - ↳ Alice - who recieves inputs and produces outputs
  - ↳ Bob - who gives inputs and verifies outputs

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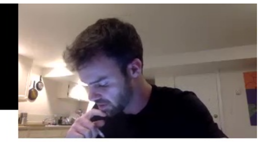
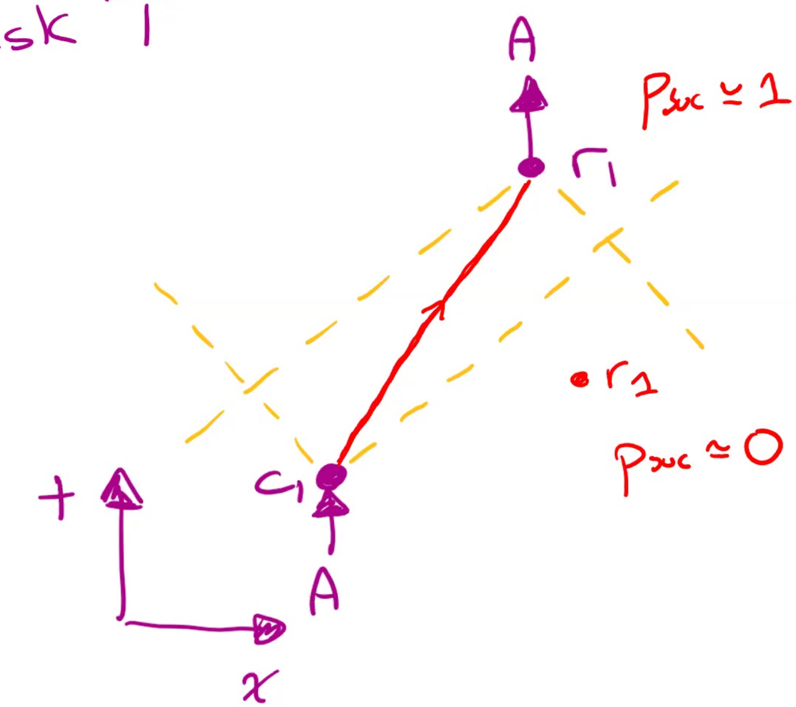


Example: Receive + Return Task

Task T



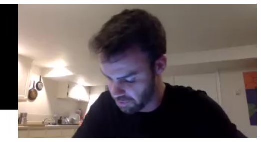
Question we might ask:

- With what probability is it possible?

$$\underbrace{p_{suc}(T)} = ?$$

- What resources are necessary to achieve a given success probability?

$p_{suc}(T) \geq \alpha \implies$ resources?



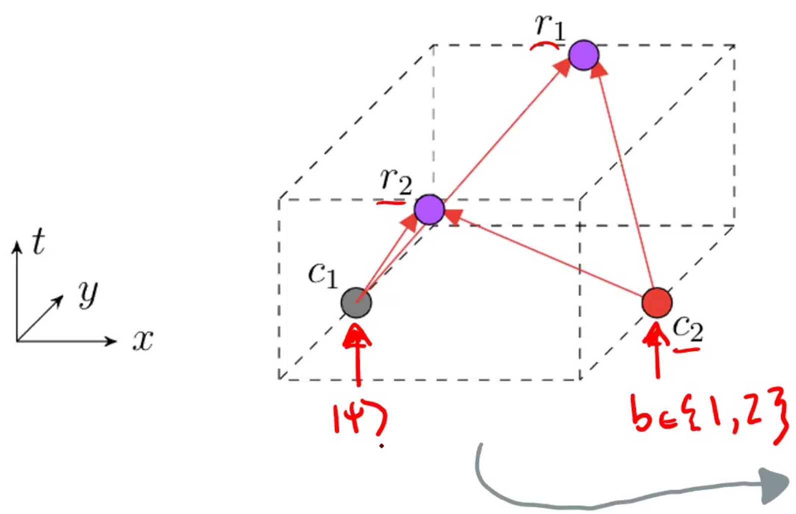
A "summoning" task

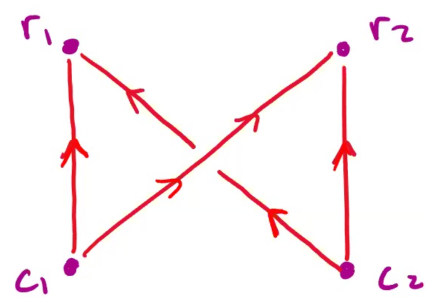
1101.4612 - Kent

1210.0913 - AM, Patrick Hayden

- Alice gets (unknown) quantum state $|\psi\rangle$ at c_1
- Should bring $|\psi\rangle$ to r_b,

  b is announced at c_2

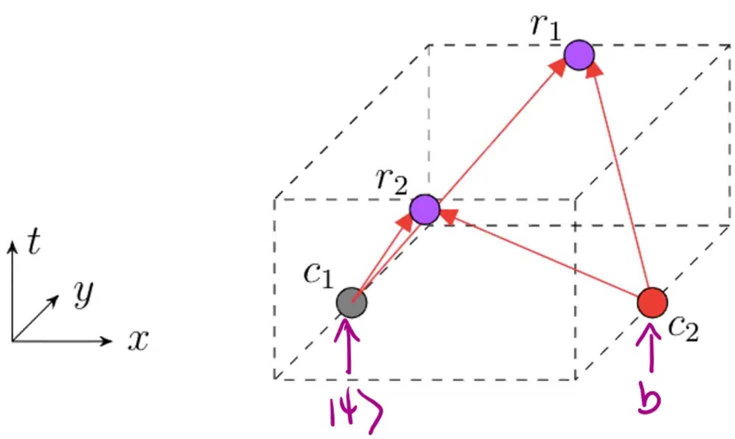


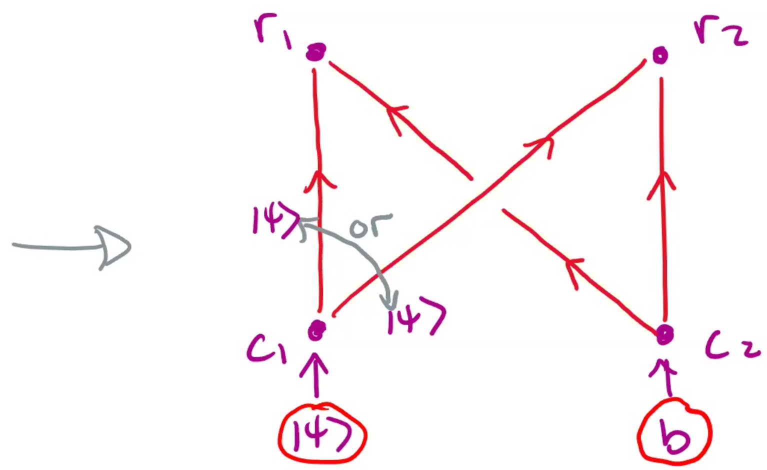


$\nexists\, p$ s.t.

$c_1, c_2 < p < r_1, r_2$

A "summoning" task

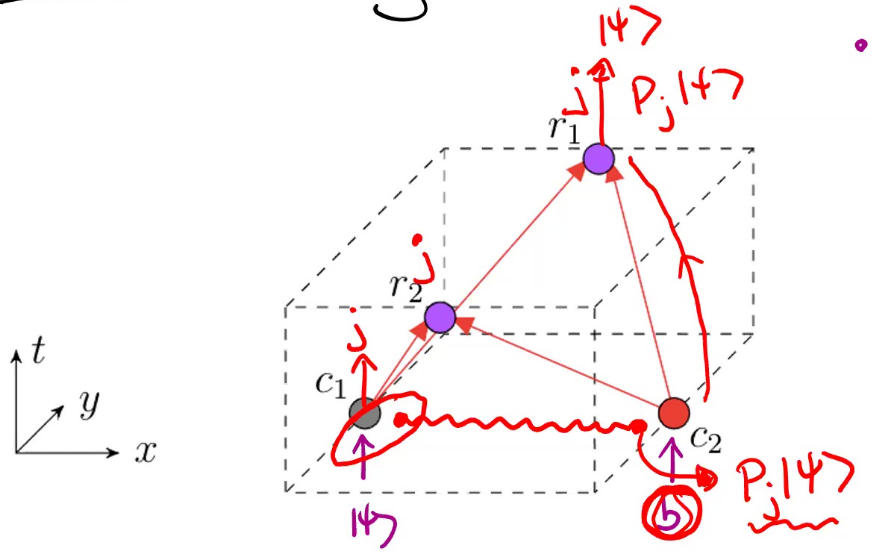




- Using simple (no entanglement) strategy

$$\boxed{P_{suc} \sim 1/2}$$

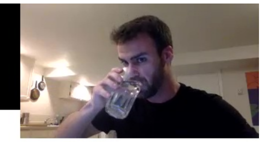
A "summoning" task



- Using entanglement, can do:
  1) Teleport $|\psi\rangle$ from c_1 to c_2
  2) Send classical measurement outcome j to both r_1 and r_2
  3) Send quantum part, now at c_2 along with b, to r_b

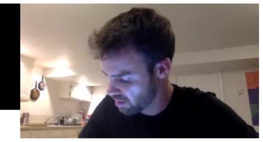
Same observation appears in: "Teleporting an unknown quantum state ..."

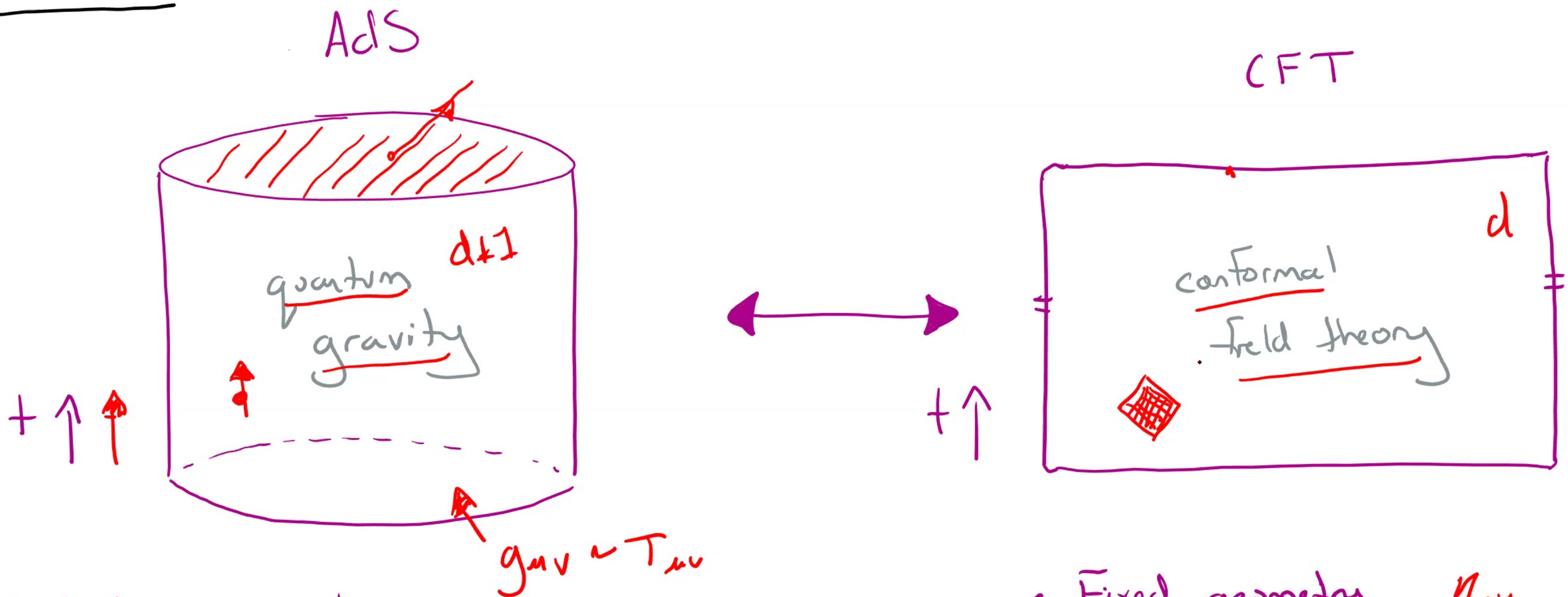
Quantum tasks capture spacetime-specific aspects of quantum information



Applyng tasks to AdS/CFT

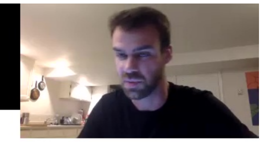
AdS/CFT





- Includes gravity
- d+1 dimensional
- Includes quantum fields

- Fixed geometry $\eta_{\mu\nu}$
- d dimensional
- Includes quantum fields



Applying tasks to holography

- Given a theory, associate a list of tasks we can define and their success probabilities

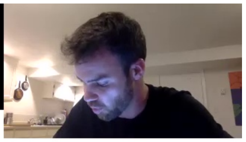
$$\text{Theory} \sim \{(T, p_{suc}(T))\} \sim \{(\qquad)\}.$$

- Given a duality, we should have:

$$\text{some task} \longrightarrow T \longleftrightarrow \hat{T} \longleftarrow \text{"same" task in dual theory}$$

with:

$$\boxed{p_{suc}(T) = p_{suc}(\hat{T})}$$

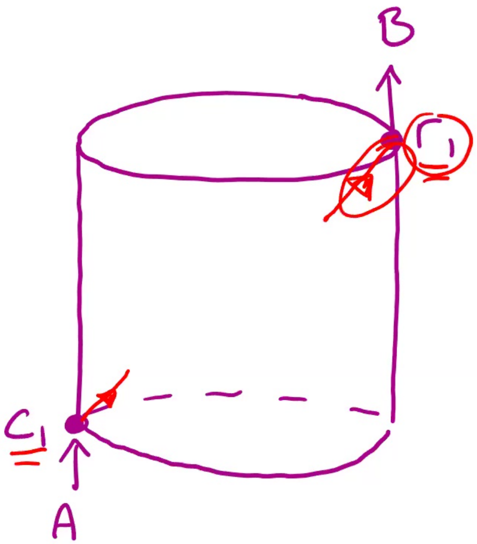


Dual tasks

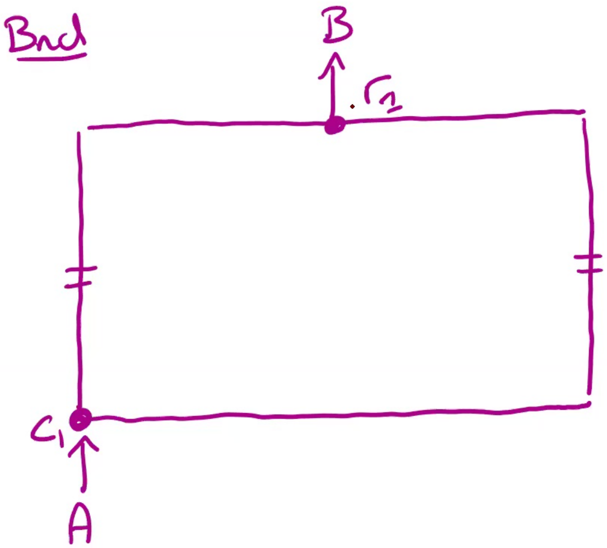
$T \leftrightarrow \hat{T}$

Bulk

T



Bnd



$\hat{T}$

- Restrict attention to T with points at the AdS boundary $\longrightarrow$ Straightforward to identify input/output points in T, $\hat{T}$

The connected wedge theorem

arXiv: 1912.05649

AM, Jon Sorce,
Geoff Pennington

B84 Task +
2→2 connected wedge
theorem

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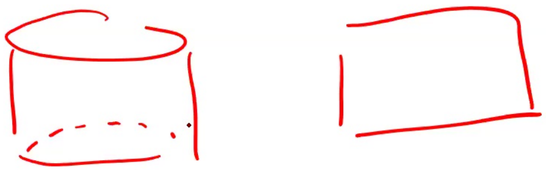

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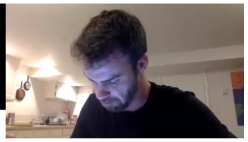

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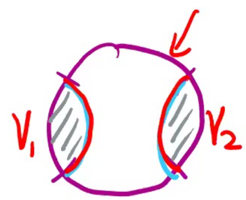
B84 Task +

2→2 connected wedge theorem

Mutual information and minimal surfaces

- $I(V_1 : V_2) \equiv \underbrace{S(V_1) + S(V_2)}\, - \underbrace{S(V_1 \cup V_2)}$

- Minimal surface enclosing $\underbrace{V_1 \cup V_2}$ is  $\iff \underbrace{I(V_1 : V_2) = O(1/G_N)}$

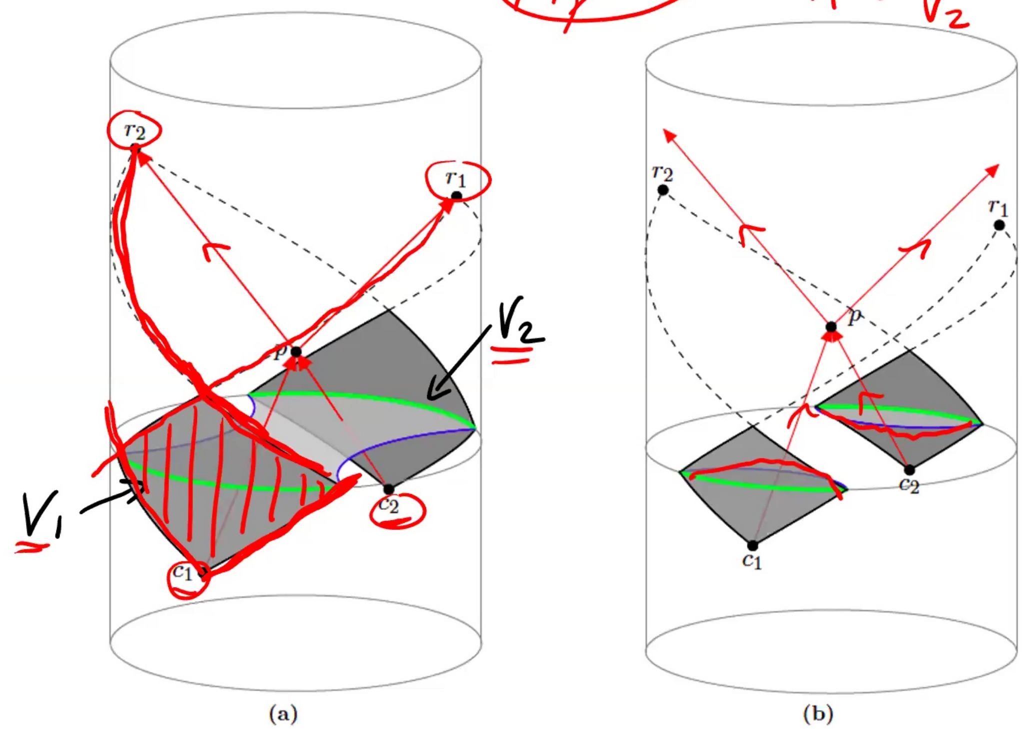
- Minimal surface enclosing $V_1 \cup V_2$ is  $\iff \underbrace{I(V_1 : V_2) = O(1)}$

2 → 2 Connected Wedge Theorem

Thm | Pick $\{c_1, c_2, r_1, r_2\}$.

If the "scattering region" is non-empty, then
$I(V_1 : V_2) \simeq O(1/G_N)$.

$V_1 \cap V_2 \neq \emptyset$



"input regions"

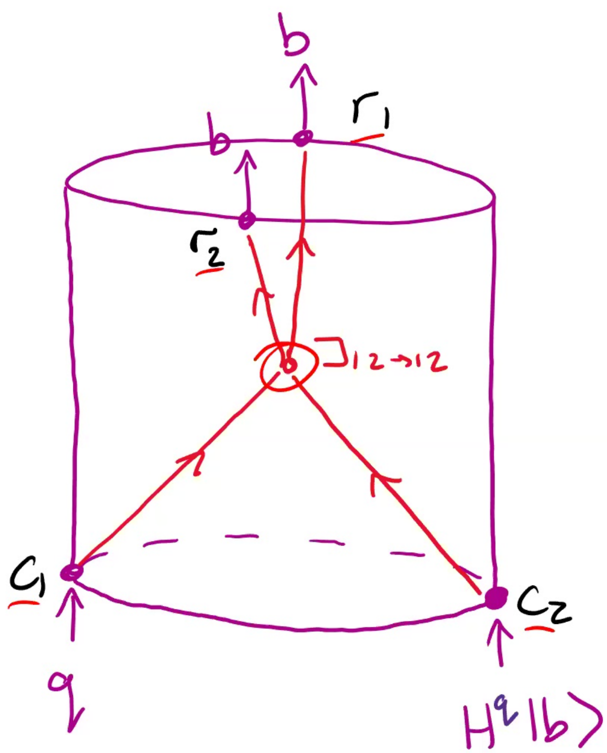
$$V_1 \equiv \hat{J}^+(c_1) \cap \hat{J}^-(r_1) \cap \hat{J}^-(r_2)$$
$$V_2 \equiv \hat{J}^+(c_2) \cap \hat{J}^-(r_1) \cap \hat{J}^-(r_2)$$

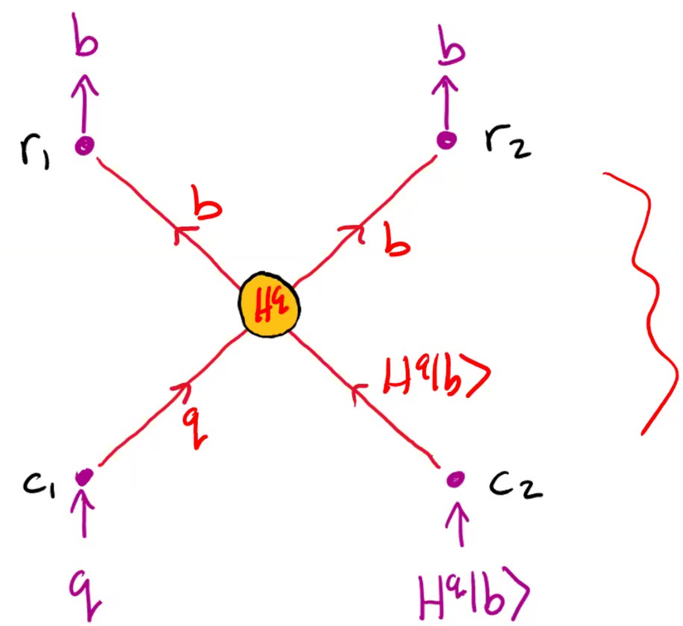
"scattering region"

$$J_{12\to12} = J^+(c_1) \cap J^+(c_2) \cap J^-(\bar{r}_1) \cap J^-(\bar{r}_2)$$

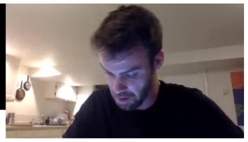
$J_{12\to12} \neq \emptyset$

Bulk perspective



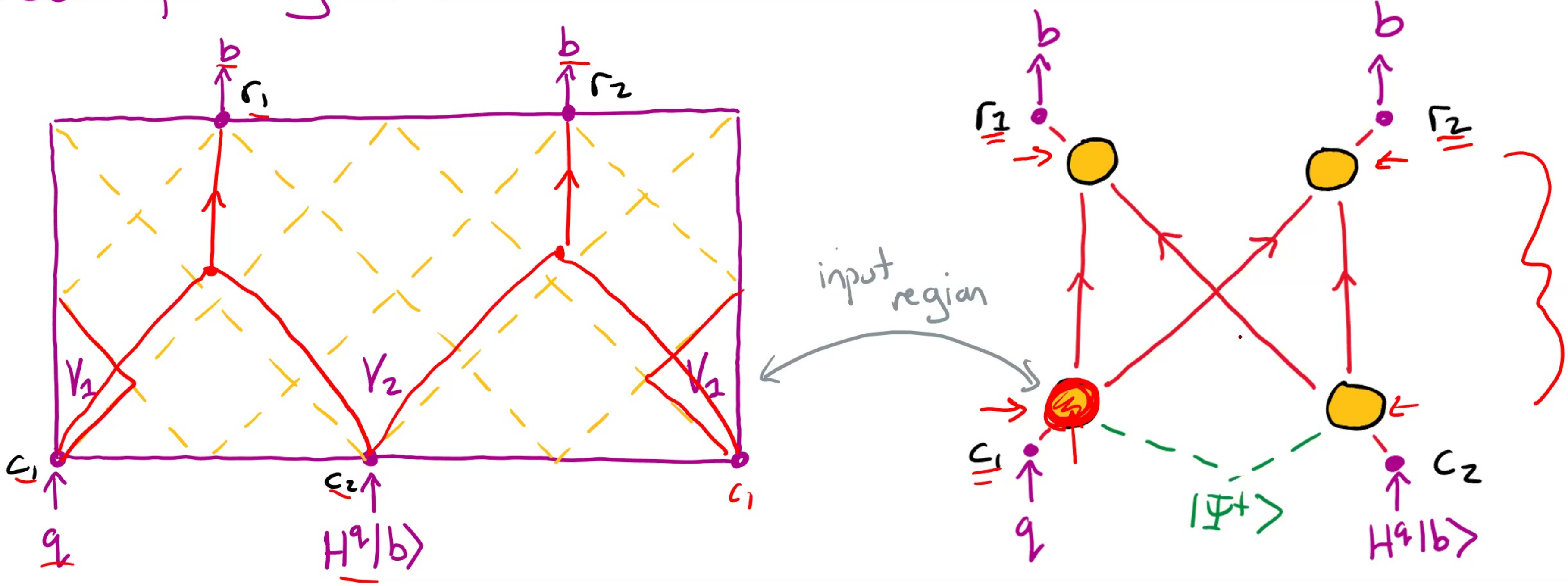


$$\boxed{\mathcal{J}_{12\to12} \neq \emptyset} \implies \boxed{p_{suc}(T) = 1}$$



Boundary perspective

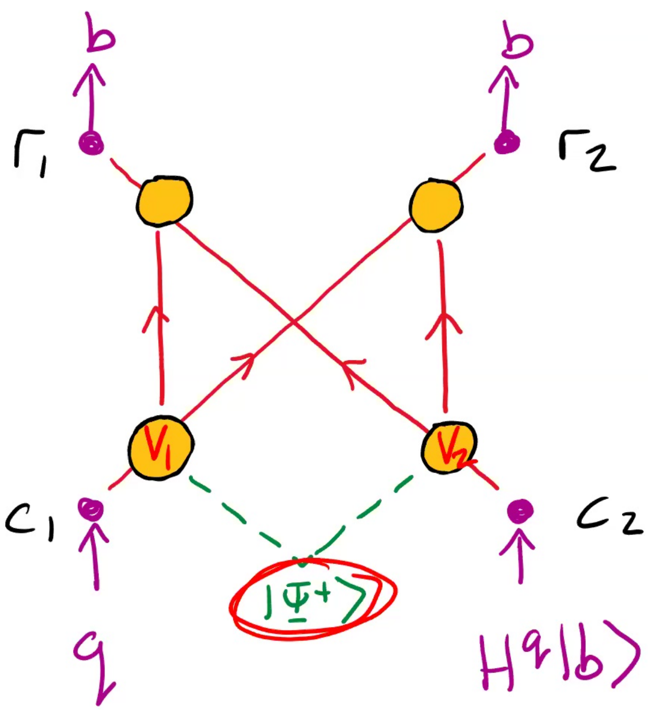
• Corresponding $\hat{T}$:



↳ Scattering region is empty!

$\Longrightarrow$ forced to use "non-local" strategy.

Comparing bulk + boundary



- Recall:

non-empty scattering regions

$$\underline{1 = p_{suc}(T)} = \boxed{p_{suc}(\hat{T}) = 1}$$

- For non-local strategy to have

$$p_{suc}(\hat{T}) = 1$$

requires $\underline{V_1 \text{ and } V_2}$ to share entanglement!

(can make this quantitative)

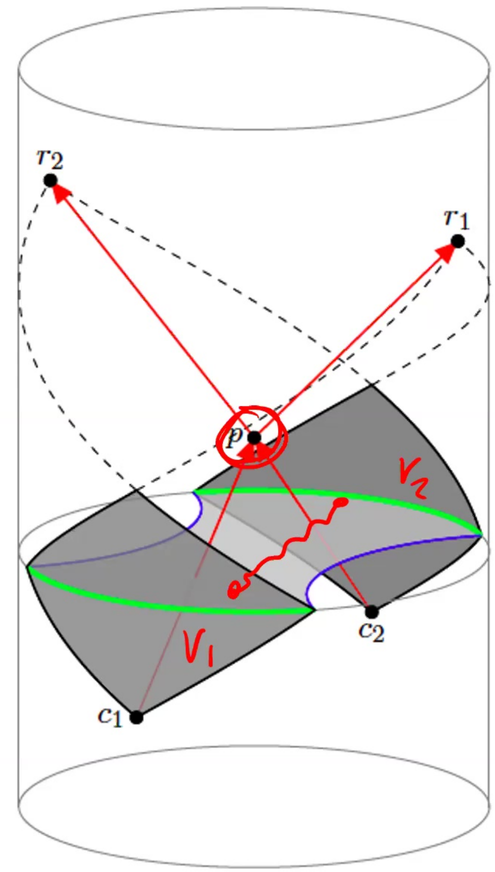
$2 \to 2$ Connected Wedge Theorem

Thm: Pick c_1, c_2, r_1, r_2. Then

if $J_{12\to 12} \neq \emptyset$, $\underbrace{I(V_1 : V_2)} = O(1/G_N)$.

entanglement ~ causal features

spacelike surfaces



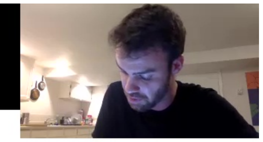
(a)

Recall:

$$V_1 = \hat{J}^+(c_1) \cap \hat{J}^-(r_1) \cap \hat{J}^-(r_2)$$

$$V_2 = \hat{J}^+(c_2) \cap \hat{J}^-(r_1) \cap \hat{J}^-(r_2)$$

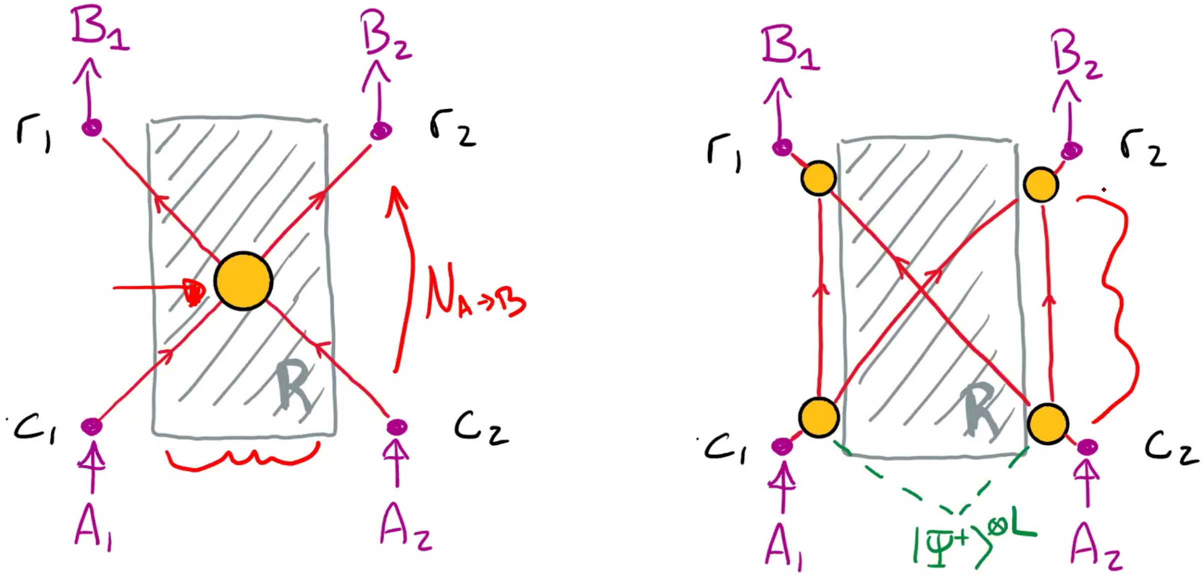
$$J_{12\to 12} = J^+(c_1) \cap J^+(c_2) \cap J^-(r_1) \cap J^-(r_2)$$



Towards efficient non-local quantum computation

Position based cryptography

- Bob chooses $\mathcal{N}_{A_1A_2 \to B_1B_2}$ in an attempt to force Alice to do operations inside R

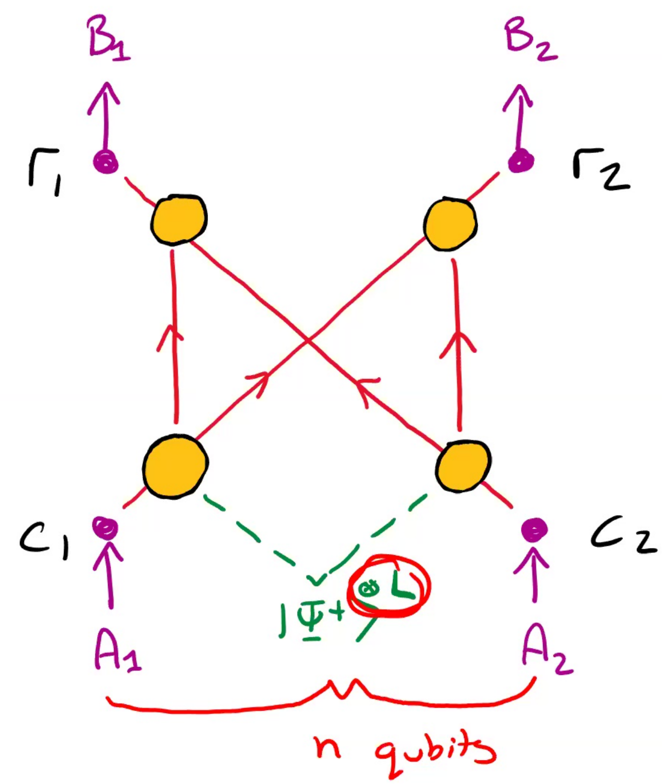


- Alice tries to cheat, by acting only outside R.

Quantum non-local computation (QNLC)

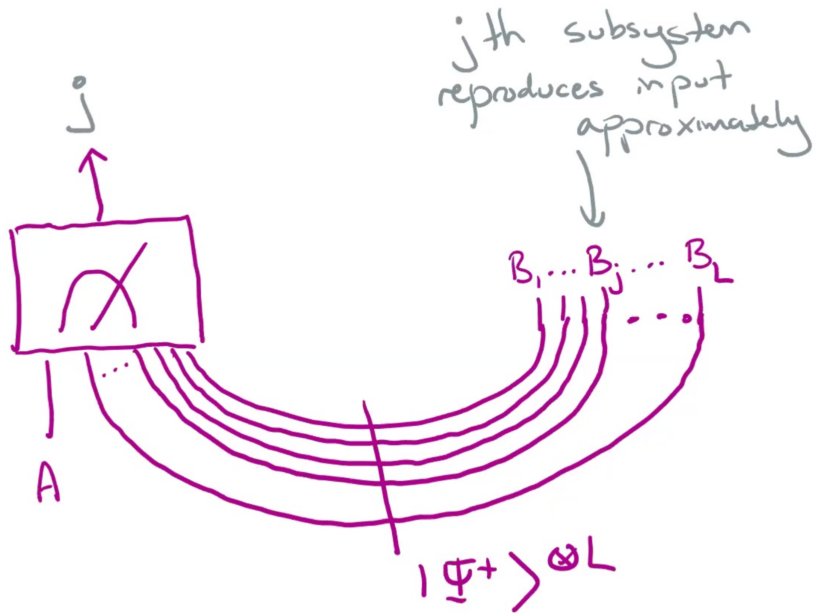
- Can Alice always cheat?
  ↳ Given enough entanglement, yes!

- Given $n = \log \dim A$,
  how big do we need L
  for $p_{suc} \simeq 1$?



$\underbrace{L \sim f(n)}$. ?

Port-teleportation (Ishizaka + Hiroshima 2009)



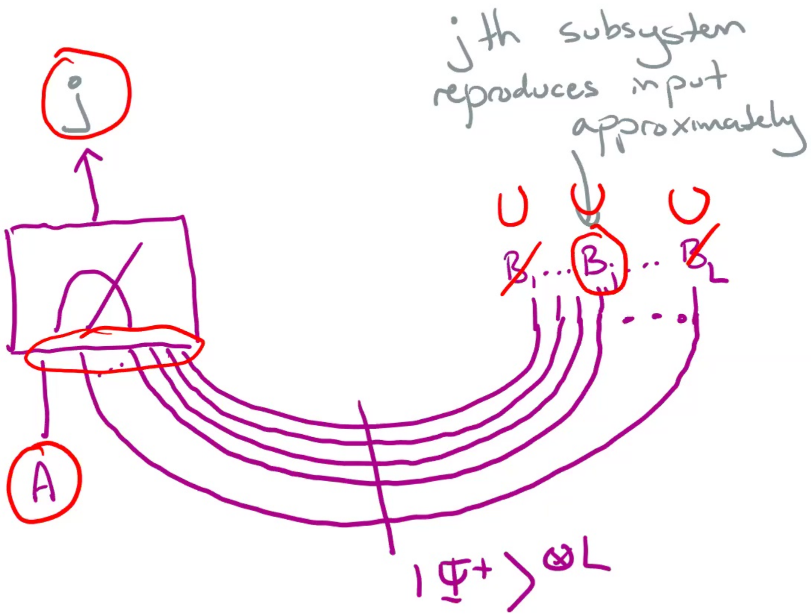
- Define channel $\mathcal{E}_{A\to B_j}$
- For appropriate measurement

$$\| \mathcal{E}_{A\to B_j} - \mathbb{I}_{A\to B_j} \|_{\diamond} \leq \frac{2^{2n+2}}{\sqrt{L}}$$

- Defines a teleportation that requires only trivial correction (trace out $B_{i\neq j}$)
- Connections to universal processors, transverse codes, ...

Port-teleportation

(Ishizaka + Hiroshima 2009)



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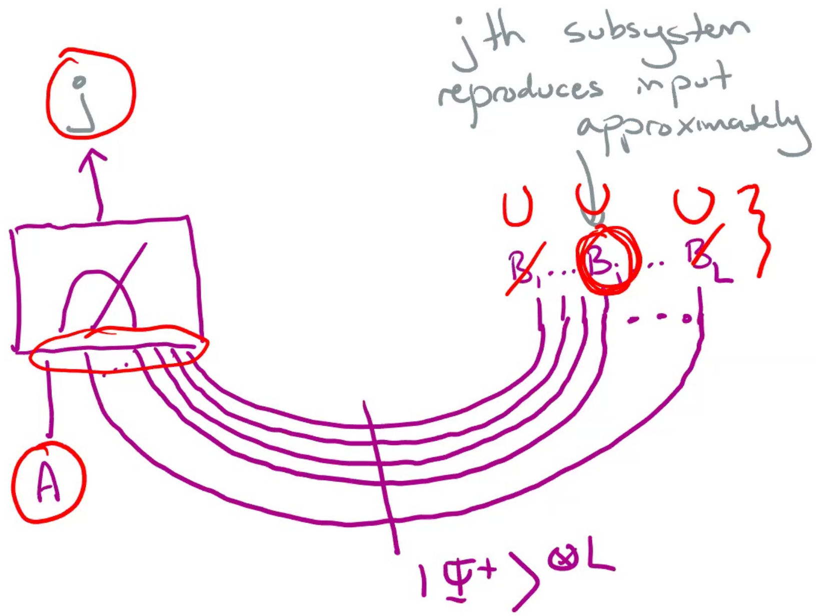
- Defines a teleportation that requires only trivial correction (trace out $B_{i\neq j}$) $\quad L \sim 2^{4n}$
- Connections to universal processors, transverse codes, ...

Port-teleportation

(Ishizaka + Hiroshima 2009)



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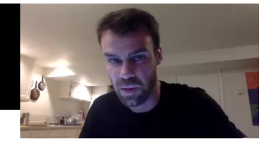


Port-teleportation approach to QNLC

- Port-teleportation is involved in the most efficient known QNLC scheme (Beigi + Konig 2011)

- For

$$\| \mathcal{E}_U - U \|_\diamond \leq \epsilon$$

applied channel   intended channel

Their construction requires

$$\boxed{L \sim \frac{nZ^n}{\epsilon^2}}$$

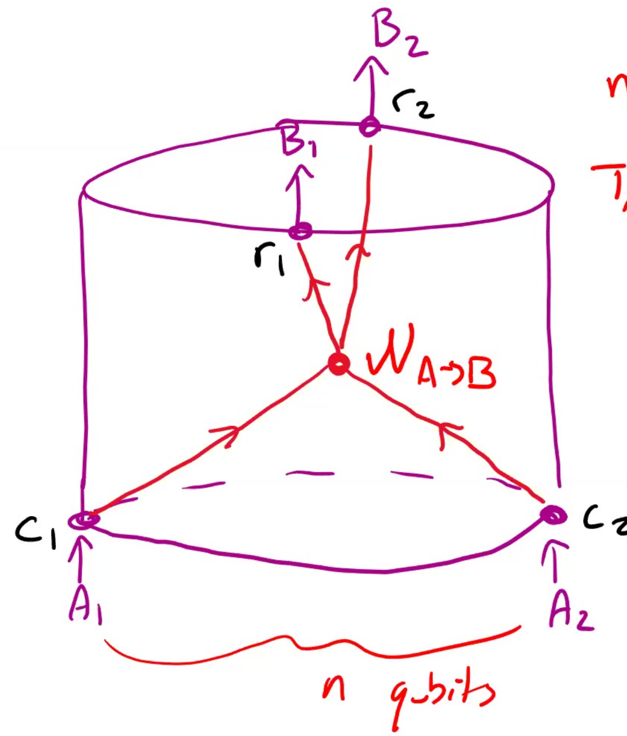
* Note: for Clifford operations port-teleportation is unnecessary, and

$$\boxed{L \sim n}$$

Holographic approach to QNLC

- Set-up same task in AdS/CFT
- Can send in $\boxed{n \sim O(1/G_N)}$ qubits
- Have $\underline{I(V_1 : V_2) \sim O(1/G_N)}$

$$\boxed{L \sim n}$$



$n \sim 1/G_N$

$T_{\mu\nu} G_N \sim G_{\mu\nu}$

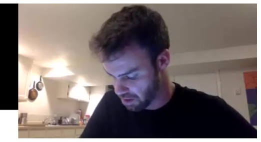
$\Rightarrow$ AdS/CFT points to an efficient method for quantum non-local computations!

Some caveats

- A more precise statement is that:

$\boxed{\text{Any channel } \mathcal{N}_{A\to B} \text{ that can be implemented inside of the spacetime region } J_{12\to 12} \text{ can be performed non-locally with linear entanglement.}}$

- Its not clear to me what exactly this set of channels is... but its almost certainly more than the Clifford group.



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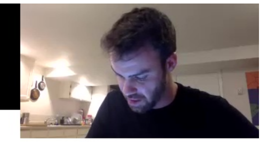
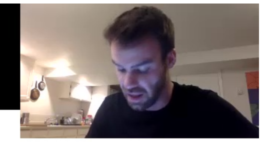


Summary



Summary

- The quantum tasks framework highlights spacetime specific aspects of quantum information.
- This perspective on quantum information is relevant to quantum gravity
  - ↳ Further motivates understanding quantum tasks
- AdS/CFT gives some new insights into quantum tasks, and perhaps quantum information more broadly.

Thanks!