Title: Efficient Data Compression and Causal Order Discovery for Multipartite Quantum Systems

Speakers: Ge Bai

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Abstract: In this talk, I will discuss two problems: quantum data compression and quantum causal order discovery, both for multipartite quantum systems. For data compression, we model finitely correlated states as tensor networks, and design quantum compression algorithms. We first establish an upper bound on the amount of memory needed to store an arbitrary state from a given state family. The bound is determined by the minimum cut of a suitable flow network, and is related to the flow of information from the manifold of parameters that specify the states to the physical systems in which the states are embodied. We then provide a compression algorithm for general state families, and show that the algorithm runs in polynomial time for matrix product states.

For quantum causal order discovery, we develop the first efficient quantum causal order discovery algorithm with polynomial black-box queries with respect to the number of systems. We model the causal order with quantum combs, and our algorithm outputs the order of inputs and outputs that the given process is compatible with. Our method guarantees a polynomial running time for quantum combs with a low Kraus rank, namely processes with low noise and little information loss. For special cases where the causal order can be inferred from local observations, we also propose algorithms that have lower query complexity and only require local state preparation and local measurements. Our results will provide efficient ways to detect and optimize available transmission paths in quantum communication networks, as well as methods to verify quantum circuits and to discover the latent structure of multipartite quantum systems.

Efficient Data Compression & Causal Order Discovery for Multipartite Quantum Systems

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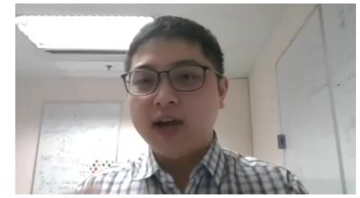
THE UNIVERSITY OF HONG KONG
DEPARTMENT OF COMPUTER SCIENCE

QICi
Quantum Information & Computation Initiative



Quantum Data Compression

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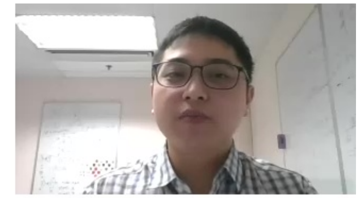


Quantum Memory is Essential

Classically hard problems can be solved efficiently on quantum computers

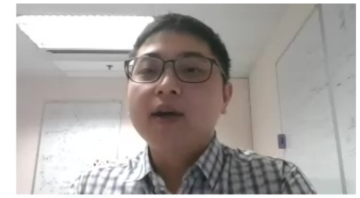
- Quantum supremacy [Arute et al. (Google) 2019, 54 qubits][Zhong et al. (USTC) 2020, 100-mode optical interferometer]

Harder problems require more memory

- Cracking 2048-bit RSA requires ~20 million qubits [Gidney, Ekerå 2019]

Quantum memories are useful but expensive

- Data are encoded in microscopic particles
- They are prone to errors
- They must be handled with extreme care

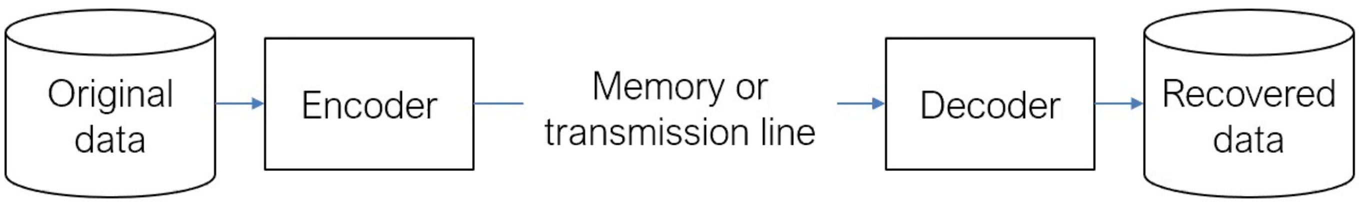


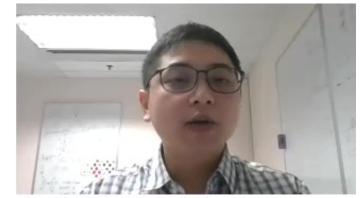
Quantum Data Compression

Compression: finds the minimal size of memory to carry the information

- Saves memory for quantum computers – more computing power
- Saves bandwidth for exchanging data with servers – more efficient networks

A sequence of pure states [Schumacher, 1993] and mixed states [Lo, 1995; Horodecki, 1998; Barnum et al. 2001]





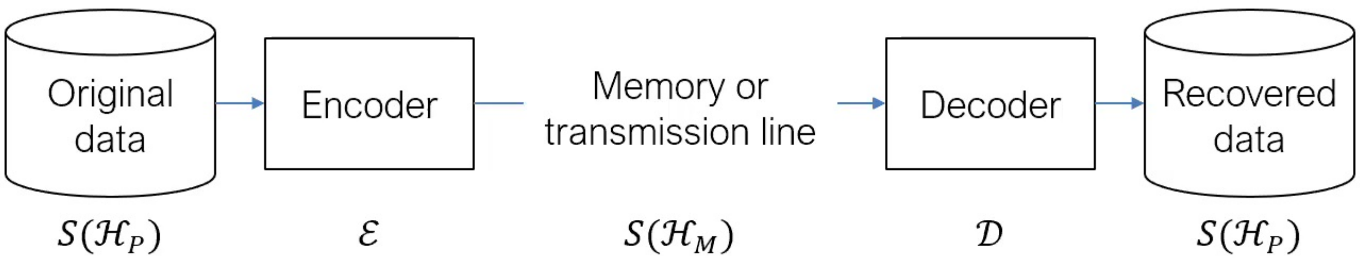
Quantum Data Compression

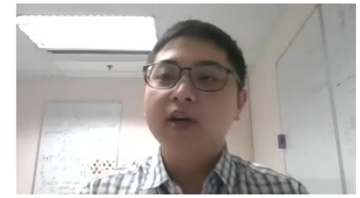
A state family is a set of parameterized states $\{\rho_x\}_{x \in \mathbf{X}} \subset S(\mathcal{H}_P)$ ← States on Hilbert space $\mathcal{H}_P$

A compression protocol $(\mathcal{E}, \mathcal{D})$ consists of two quantum channels: encoder $\mathcal{E}$: $S(\mathcal{H}_P) \to S(\mathcal{H}_M)$ and decoder $\mathcal{D}$: $S(\mathcal{H}_M) \to S(\mathcal{H}_P)$ s.t

$$(\mathcal{D} \circ \mathcal{E})(\rho_x) = \rho_x, \qquad \forall x \in \mathbf{X}$$

The memory size of the protocol is $\lceil \log \dim \mathcal{H}_M \rceil$ qubits – to be minimized

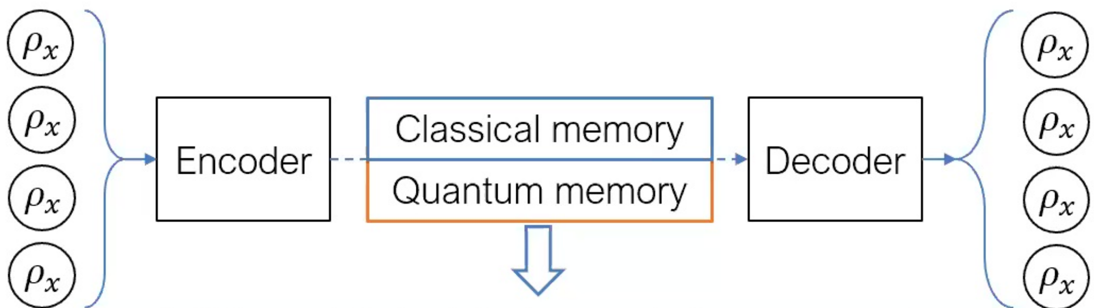




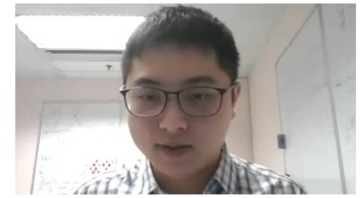
Review of Results

Previous results: only special cases of independent identically prepared (i.i.p.) states $\rho_x^{\otimes n}$ [Plesch & Buzek 2010] [Yang et al. 2016 & 2018]

Result 1: optimal compression of general case i.i.p. states
[Yang, **Bai**, Chiribella, Hayashi, IEEE Trans. on Information Theory, 2018]



$\sim 1/2 \log n$ bits/qubits for each free parameter
Arbitrarily small fraction, but non-zero quantum memory



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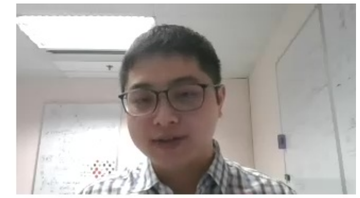
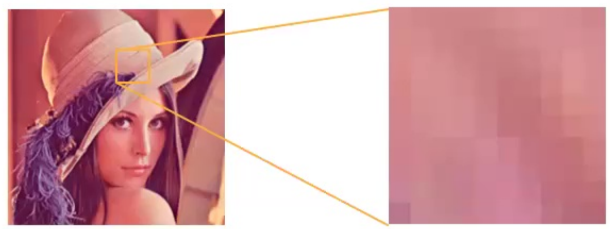
Result 2: [**Bai**, Yang, Chiribella, New Journal of Physics, 2020]



Real Data are Structured

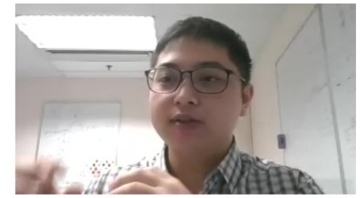
Data that are geometrically closer are usually more correlated



Adjacent pixels have similar colors

Observation: in many physical systems, particles that are geometrically closer are usually more correlated

The correlations give a certain structure of states that could help compression – How to model the structure?



Tensor Networks

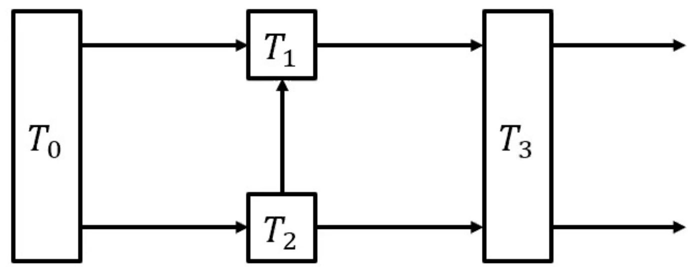
A compact way (less parameters) to express multipartite quantum states

Characterize correlation structures between systems by a graph

Allow efficient numerical simulation of states

Model of locally correlated states

- Cluster states
- Matrix product states (MPS)
- Projected entangled pair states
- Graph states
- Multi-scale entanglement renormalization ansatz



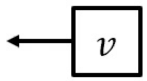
Tensor Network Notations

Directed graph $G = (V, E)$ such that

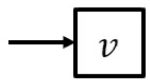
- Each edge $e \in E$ is assigned a $d(e)$-dimensional Hilbert space
- Each vertex $v \in V$ is assigned a tensor

Vectors & operators:

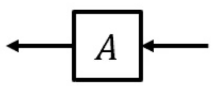
$$|v\rangle = \sum_i v_i |i\rangle =$$



$$\langle \bar{v}| = |v\rangle^T = \sum_i v_i \langle i| =$$

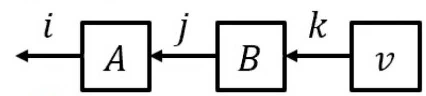


$$A = \sum_{i,j} A_{ij} |i\rangle\langle j| =$$



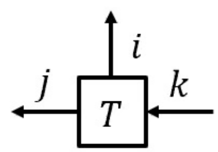
Multiplication forms network:

$$AB|v\rangle = \sum_{i,j,k} A_{ij} B_{jk} v_k |i\rangle =$$

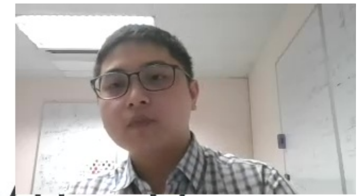


Open index - open edge

Higher-order tensors:



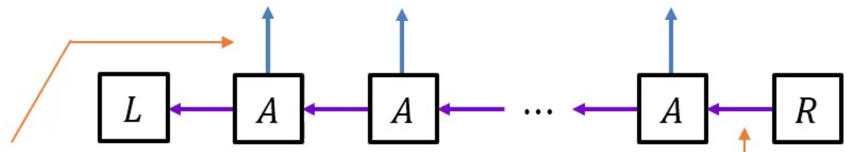
$$= \sum_{i,j,k} T_{ijk} |i\rangle|j\rangle\langle k|$$



Example: Matrix Product States (MPS)

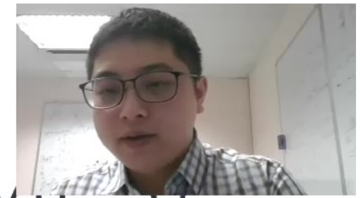
Model of locally correlated states, e.g., AKLT model, 1-d Ising model

A site-independent MPS (SIMPS) with open boundary conditions is specified by



1. n physical systems, each of dimension d_p
2. A correlation system with dimension d_c, called the bond dimension
3. A set of $d_c \times d_c$ matrices $\{A_i\}_{i=1,\dots,d_p}$
4. Two vectors $|L\rangle, |R\rangle$, called the boundary conditions

$$|\Psi_{A,L,R}\rangle \coloneqq \sum_{i_1,\dots,i_n=1}^{d_p} \langle \bar{L}| A_{i_1} A_{i_2} \dots A_{i_n} |R\rangle |i_1, i_2, \dots, i_n\rangle$$



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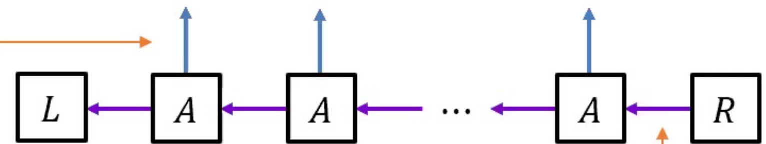

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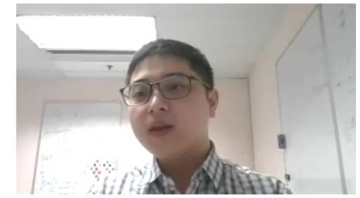
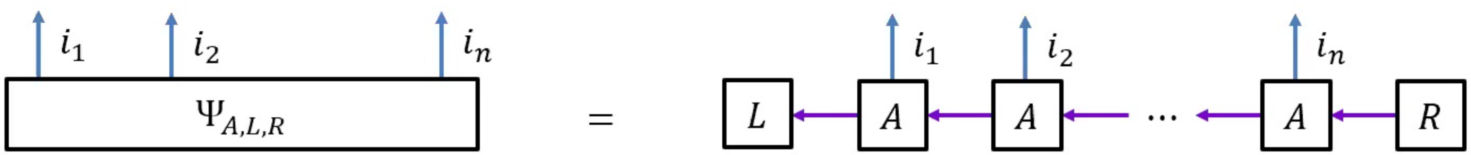
$$|\Psi_{A,L,R}\rangle := \sum_{i_1,\ldots,i_n=1}^{d_p} \langle \bar{L}| A_{i_1} A_{i_2} \ldots A_{i_n} |R\rangle \underbrace{|i_1, i_2, \ldots, i_n\rangle}_{\text{Basis of } n\text{-partite system}}$$



MPS as a Tensor Network

$$|\Psi_{A,L,R}\rangle := \sum_{i_1,\ldots,i_n=1}^{d_p} \langle \bar{L}|A_{i_1}A_{i_2}\ldots A_{i_n}|R\rangle |i_1, i_2, \ldots, i_n\rangle$$

Equivalently, we will regard the set of $d_c \times d_c$ matrices $\{A_i\}_{i=1,\ldots,d_p}$ as one order-3 tensor of dimension $d_p \times d_c \times d_c$, denoted as A

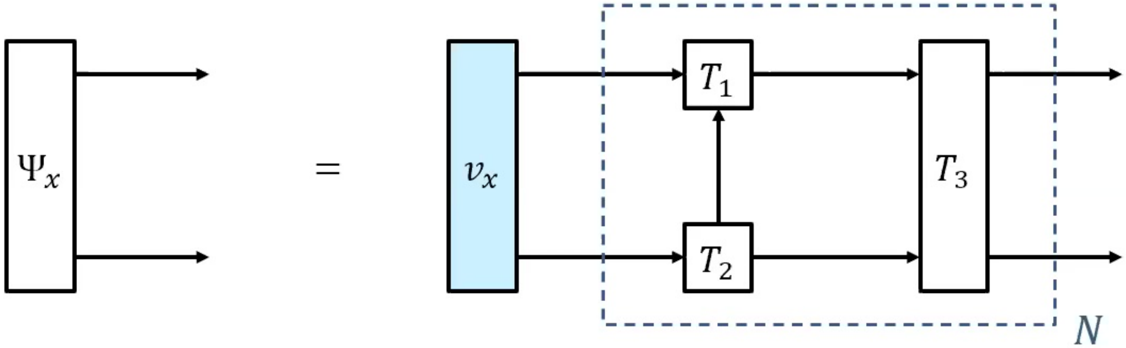


Vertical arrows: physical systems, each has dimension d_p

Horizontal arrows: correlation systems, each has dimension d_c

Tensor Network State Family

A tensor network with variable and constant tensors defines a state family $\{\Psi_x\}_{x \in \mathbf{X}}$



One can always regard all variable tensors as one vector $|v_x\rangle$, and all constant tensors as a linear operator N, and write $|\Psi_x\rangle = N|v_x\rangle$

Theoretical Limit of Compression

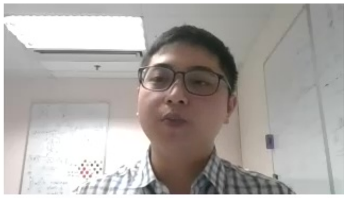
One can always map $\{|\Psi_x\rangle = N|v_x\rangle\}$ into a space $\mathcal{H}_M$ with

$$\dim \mathcal{H}_M = \dim \operatorname{Span}\{|\Psi_x\rangle\}_{x\in\mathbf{X}} \le \operatorname{rank}(N)$$

- $\lceil \log \dim \operatorname{Span}\{|\Psi_x\rangle\}_{x\in\mathbf{X}} \rceil$ is the optimal memory size for exact compression

But computing the linear span or $\operatorname{rank}(N)$ is infeasible

- Lengths of vectors grows exponentially with number of particles



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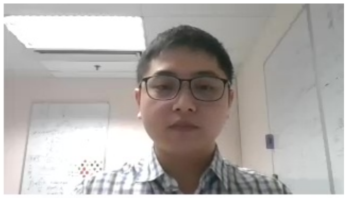
One can exploit the tensor network structure to

- Estimate optimal memory size
- Build efficient compression protocols



Information Flow in Tensor Networks

Tensor Network → Flow Network

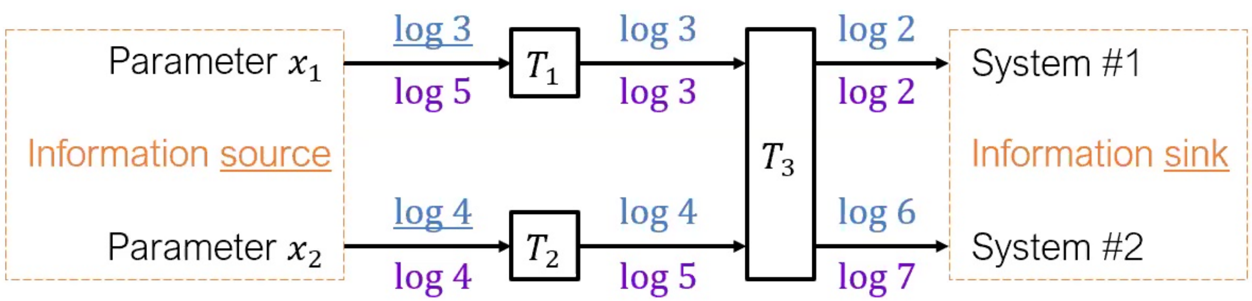
Information "flows" in the network like a fluid from the source of the parameters (variable tensors) to the sink of output physical systems

Define the capacity of an edge e as $c(e) = \log d(e)$

- $d(e)$-dimensional Hilbert space can carry at most $\log d(e)$ qubits of information

A flow assigns a number $f(e)$ to each edge s.t.

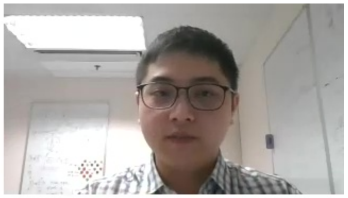
- $0 \leq f(e) \leq c(e)$
- Sum of flow going in = sum of flow going out, for each vertex except source/sink



Legend: $\xrightarrow[c(e)]{f(e)}$

Value of flow in the network
= total flow leaving the source
= $\log 3 + \log 4$

The Cut Bottlenecks the Flow



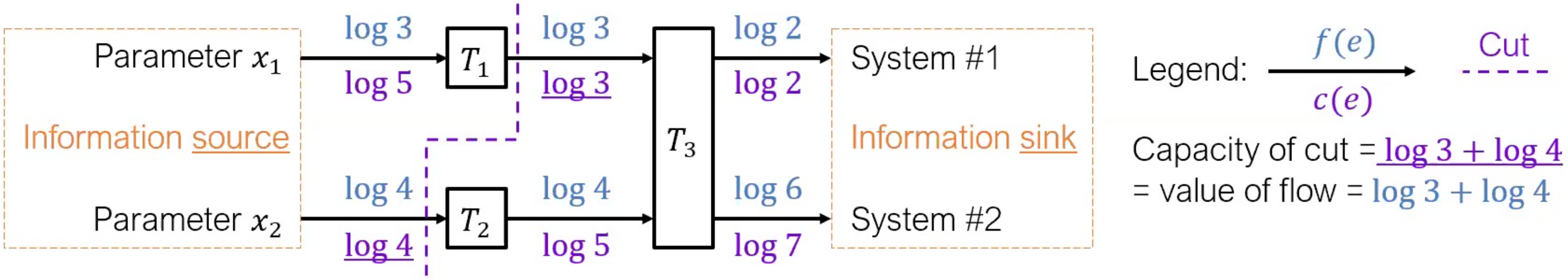
A cut is a bipartition of the network s.t. the source and sink are on different sides

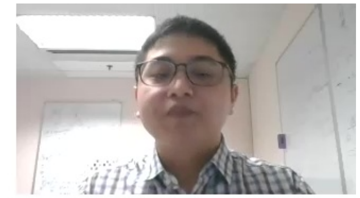
The capacity of a cut is the sum of capacities of edges crossing the cut

The minimum cut is the "bottleneck" of information flow, so

minimum capacity of cuts ≥ maximum value of flows

- In fact, min-cut = max-flow by Max-Flow Min-Cut Theorem





Quantum Max-Flow Min-Cut

The memory size of the optimal compression scheme is hard to determine

- Naïve way requires exponential computing time due to exponential # parameters

The minimum cut of the network is efficiently computable

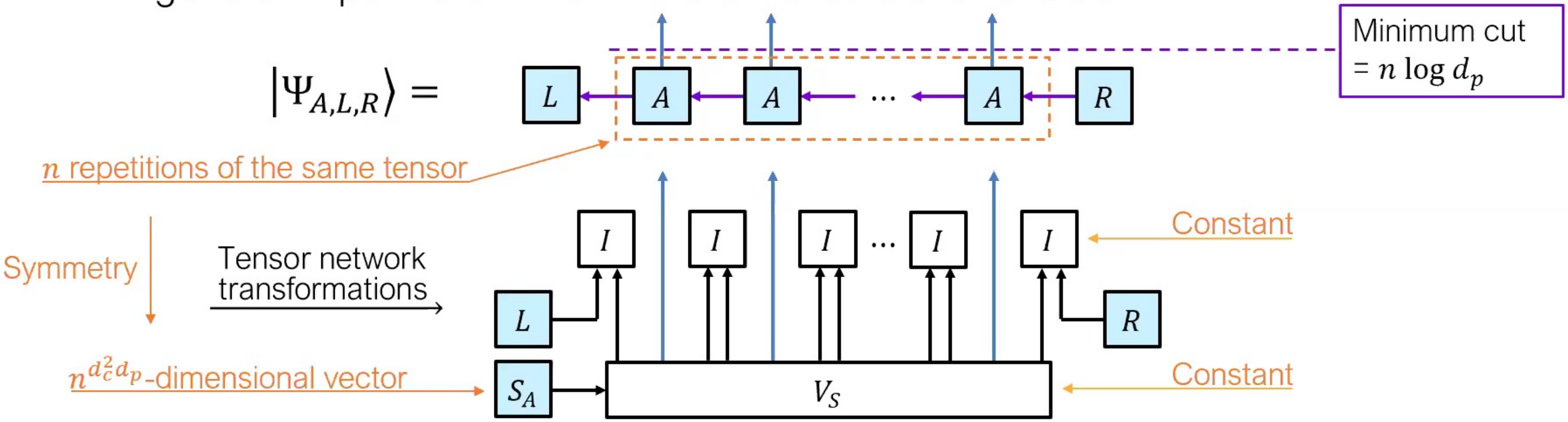
- The optimal memory size to encode the states ~ the "flow of information" from parameters to systems ≤ the minimum cut of the network

Theorem 1 *(optimal memory size)*.
For a tensor network state family $\{|\Psi_x\rangle = N|v_x\rangle\}$,

$$\text{optimal memory size} \leq \text{min-cut}$$

Example: SIMPS

A general n-partite SIMPS where all tensors are variable



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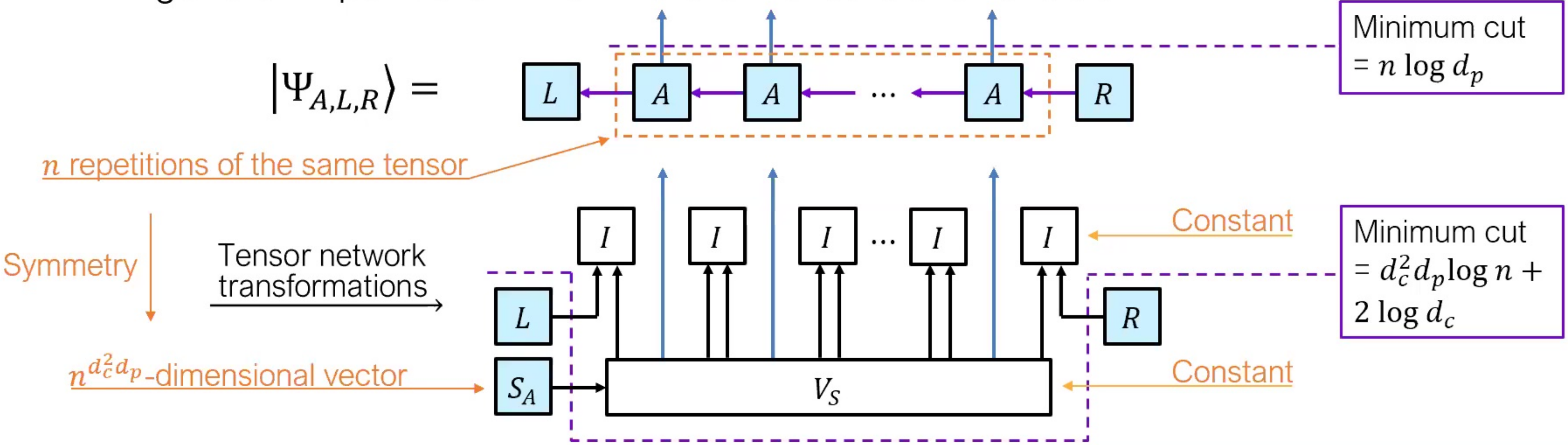
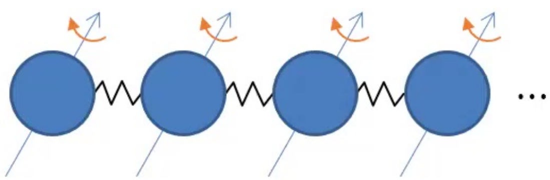


By Theorem 1, $|\Psi_{A,L,R}\rangle$ can be compressed into $\lceil d_c^2 d_p \log n + 2 \log d_c \rceil$ qubits

Summary of Implications

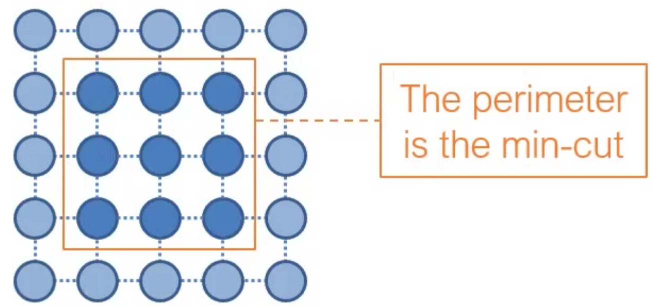
Memory to encode n-partite SIMPS $= O(\log n)$

Generalization of independent identically prepared (i.i.p.) case by allowing entanglement



$O(n)$ qubits $\xrightarrow[\text{memory reduction}]{\text{Exponential}}$ $O(\log n)$ qubits

Rediscovers holographic compression [Wilming & Eisert, 2019]: memory to encode locally interacting particles in a region of a lattice $\propto$ the perimeter of the region



$O(n^2)$ qubits $\xrightarrow[\text{memory reduction}]{\text{Quadratic}}$ $O(n)$ qubits

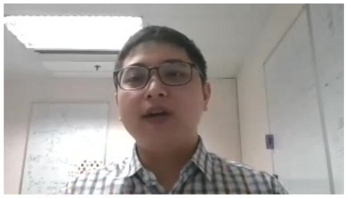
Efficient Compression Algorithm

We propose an efficient compression algorithm based on quantum machine learning & tensor networks

Theorem 2 *(efficient compression algorithm)*.

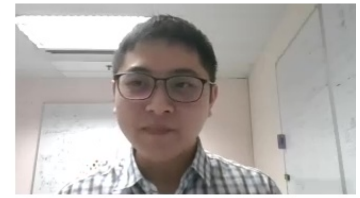
Our algorithm achieves optimal compression and runs on a quantum computer in $\text{poly}(n)$ time for any MPS family of length n compressible within $O(\log(n))$ qubits (which can be checked with Theorem 1)

Efficiency for general case is given by a set of sufficient conditions



Summary

- Quantum memory is expensive
  - ↓ Compression: minimize memory size to carry the information
    - Efficiently computable memory bounds
      - Real data are structured
      - ↓ Model structure of quantum data as a network
      - ↓ Optimal memory size $\leq$ min-cut of network
    - Efficient compression algorithm
      - Reaches optimal memory size
      - Runs in polynomial time for MPS families

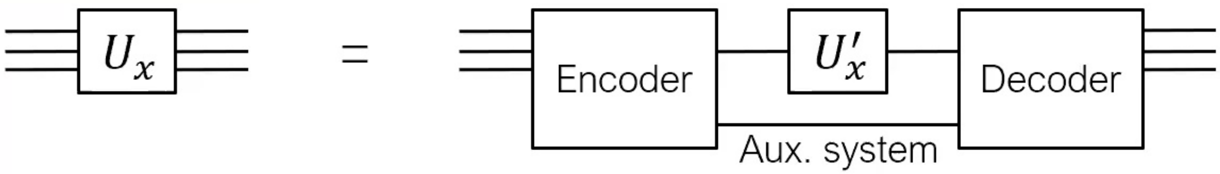


Future Directions

Exact compression ⟶ approximate compression

- For i.i.p. states, allowing a small error saves a lot of memory [Yang, et al. 2016]
- Allowing error grants better efficiency and memory cost for correlated states?

State compression ⟶ gate compression



Where U_x' is a smaller gate than U_x

- Reduces transmission cost for cloud computing
- Solved for "i.i.p." unitary gates $U^{\otimes n}$ [Chiribella et al. 2015]
- Deal with correlated multipartite gates with tensor networks & machine learning?

Quantum Causal Order Discovery

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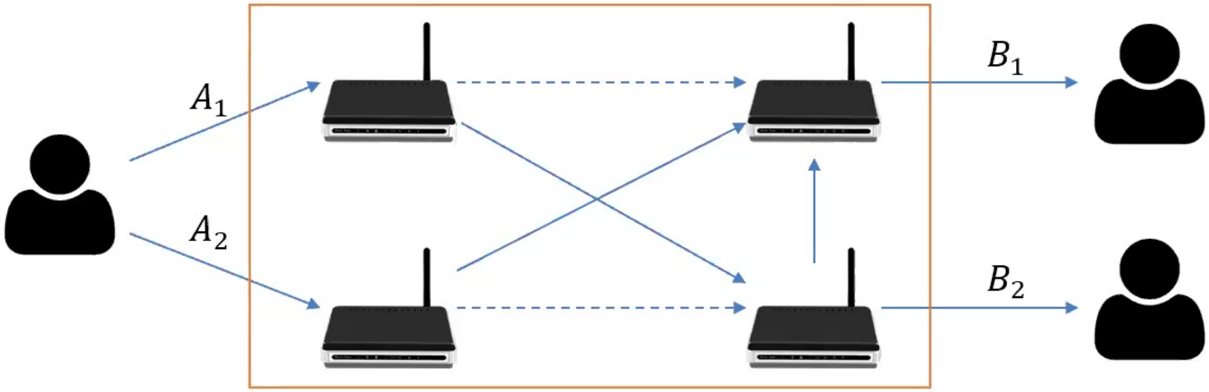
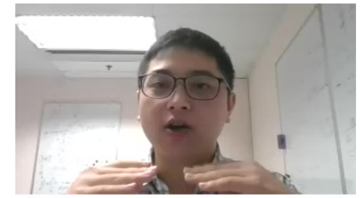
Network Structural Changes

In a quantum network, data are transmitted via multiple intermediate nodes

Change of availability of nodes → Change of network structure

Which paths are accessible? – Discovery of causal structure





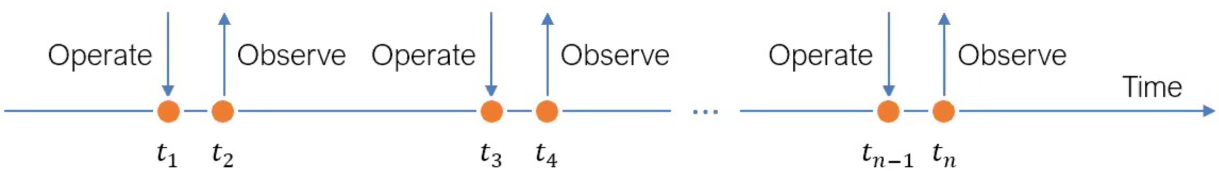
Causal Order Discovery

Each event is an operation or an observation

Operation A is before observation $B \Rightarrow$ A may affect B

Operation A is after observation $B \Rightarrow$ A must not affect B

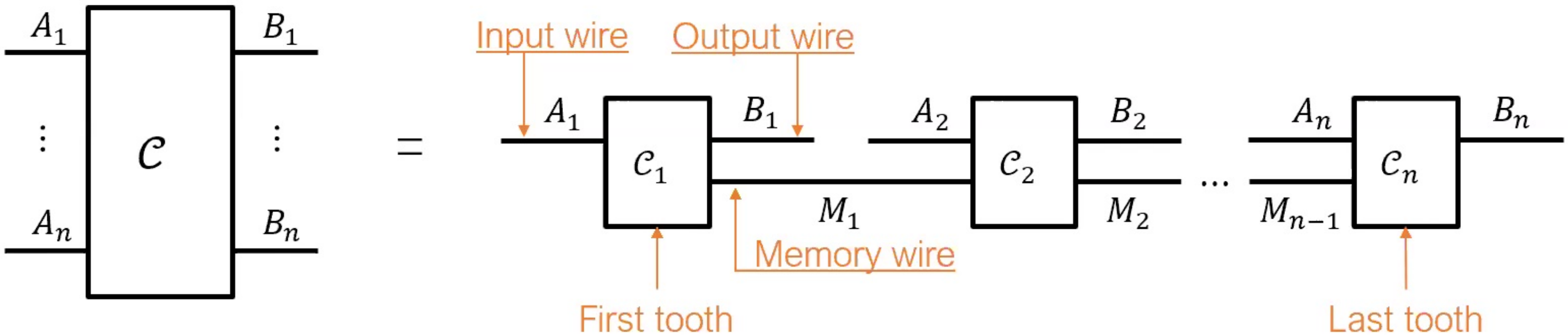
Given access to operations and observations, decide the order of the events ⇒ the path of the particle



Quantum Comb

Any causally ordered quantum process can be represented as a quantum comb, which is a channel written as n channels connected with memory wires [Chiribella, DAriano, P. Perinotti, PRL 2008]

Each of sub-channels $\mathcal{C}_1, \dots, \mathcal{C}_n$ is called a tooth of the comb

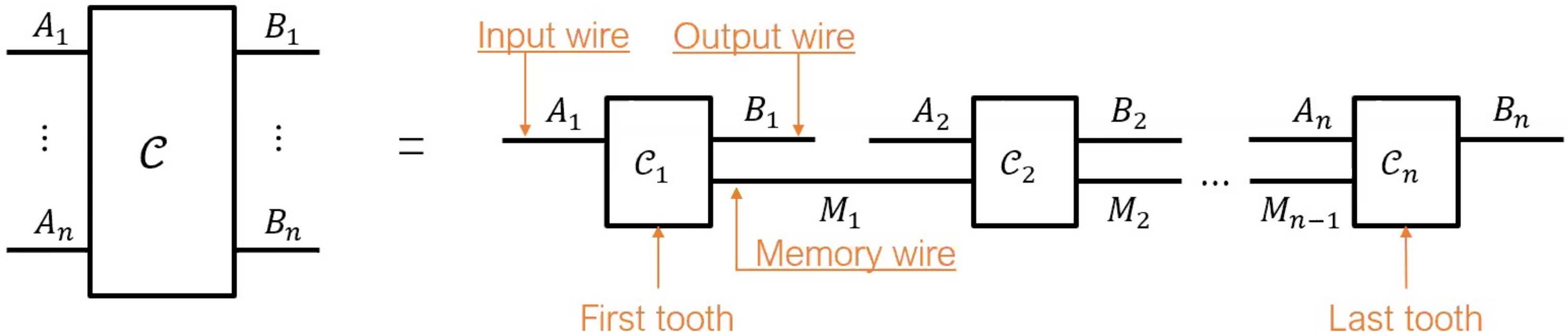


It has a causal order: A_i only affects outputs B_j with $i \leq j$

Quantum Comb

Any causally ordered quantum process can be represented as a quantum comb, which is a channel written as n channels connected with memory wires [Chiribella, DAriano, P. Perinotti, PRL 2008]

Each of sub-channels $\mathcal{C}_1, \dots, \mathcal{C}_n$ is called a tooth of the comb



It has a causal order: A_i only affects outputs B_j with $i \leq j$

$\mathcal{C} \in \text{Comb}[(A_1, B_1), \dots, (A_n, B_n)] \Leftrightarrow \mathcal{C}$ is compatible with causal order $(A_1, B_1), \dots, (A_n, B_n)$

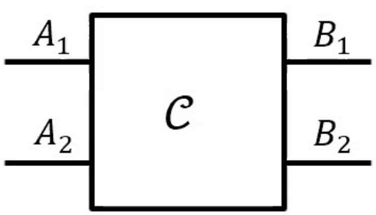
Problem Formulation - Informal

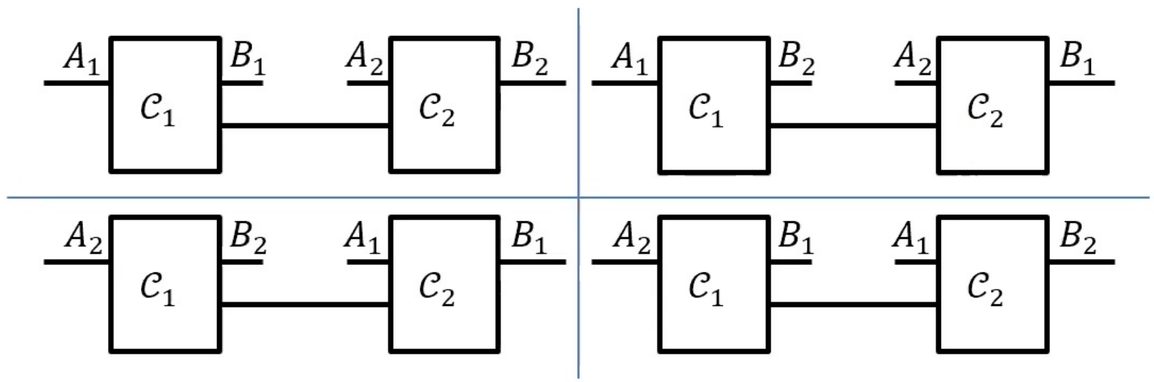
Given black-box access to a channel $\mathcal{C}$ with n input wires and n output wires

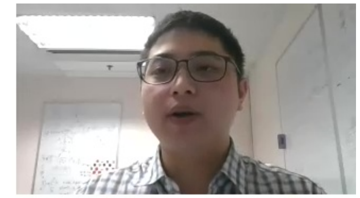
Goal: Determine which causal structure it is compatible with

$\mathcal{C}$ may have one of many possible causal structures

- At most $(n!)^2$ for all possible permutations of inputs and outputs



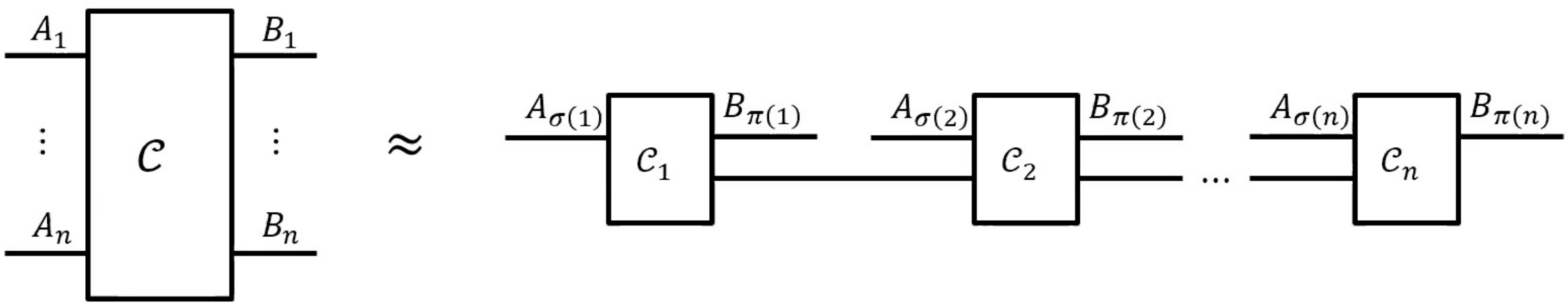




Problem Formulation

Input: black-box access to a quantum channel $\mathcal{C}$ with input wires $A_1, \ldots, A_n$ and output wires $B_1, \ldots, B_n$ satisfying $\mathcal{C} \in \text{Comb}\big[\big(A_{\sigma'(1)}, B_{\pi'(1)}\big), \ldots, \big(A_{\sigma'(n)}, B_{\pi'(n)}\big)\big]$, where σ' and π' are unknown permutations.

Output: permutations σ and π such that $\mathcal{C}$ is (approximately) equal to a quantum comb $\mathcal{D} \in \text{Comb}\big[\big(A_{\sigma(1)}, B_{\pi(1)}\big), \ldots, \big(A_{\sigma(n)}, B_{\pi(n)}\big)\big]$. σ (π) and σ' (π') do not have to be the same.



Related Works

Non-scalable special cases of causal order discovery:
- Two optical modes [Ried et al. 2015]
- Two candidate paths [Chiribella & Ebler, 2019]

Causal order discovery given classical description of the process [Giarmatzi and Costa, 2018]
- Classical description can be obtained via e.g., process tomography - inefficient

Our work: general case, black-box, efficient

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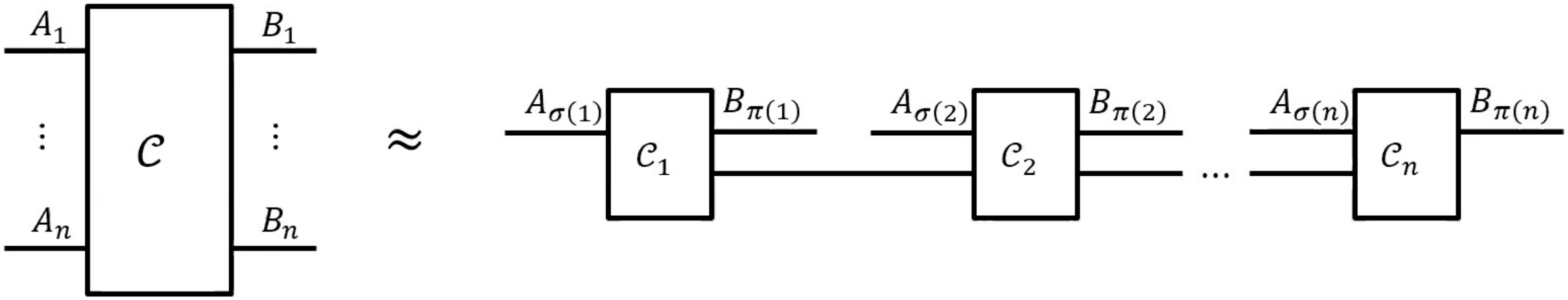

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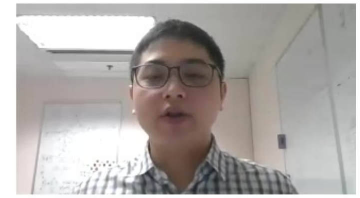
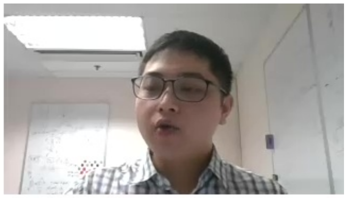


Results

We devise the first efficient algorithm for quantum causal order discovery

Query complexity (number of black-box accesses to $\mathcal{C}$) = $\mathbf{poly}(n)$

1. Algorithm 1: general causal orders
   - A quantum way to observe multiple systems simultaneously
   - Efficient for combs composed of fixed-sized unitary gates
     - Unitary interactions with a fixed-sized quantum system
2. Algorithms 2 & 3: special cases
   - Low query complexity – as low as $O(\log(n))$
   - Use local state preparation and local measurements



Algorithm 1: General Causal Orders

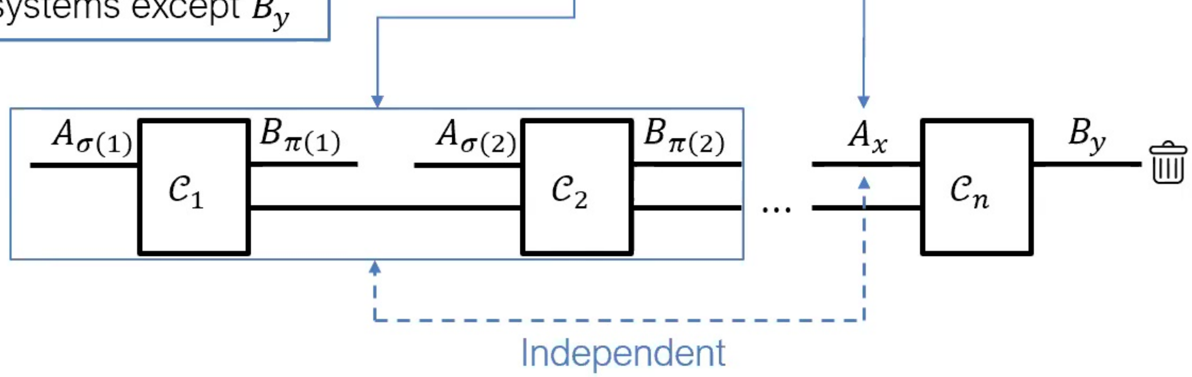
Criteria for Quantum Comb

If A_x is the last input, then it can affect nothing except the last output

Proposition 1 [Chiribella, DAriano, P. Perinotti, PRL 2008]. *Let C be the Choi state of a channel $\mathcal{C}$. (A_x, B_y) is the last tooth if and only if*

$$C_{A_{1\sim n},B_{\neq y}} = C_{A_{\neq x},B_{\neq y}} \otimes I_{A_x}/d_{A_x}$$

Marginal state on systems except B_y

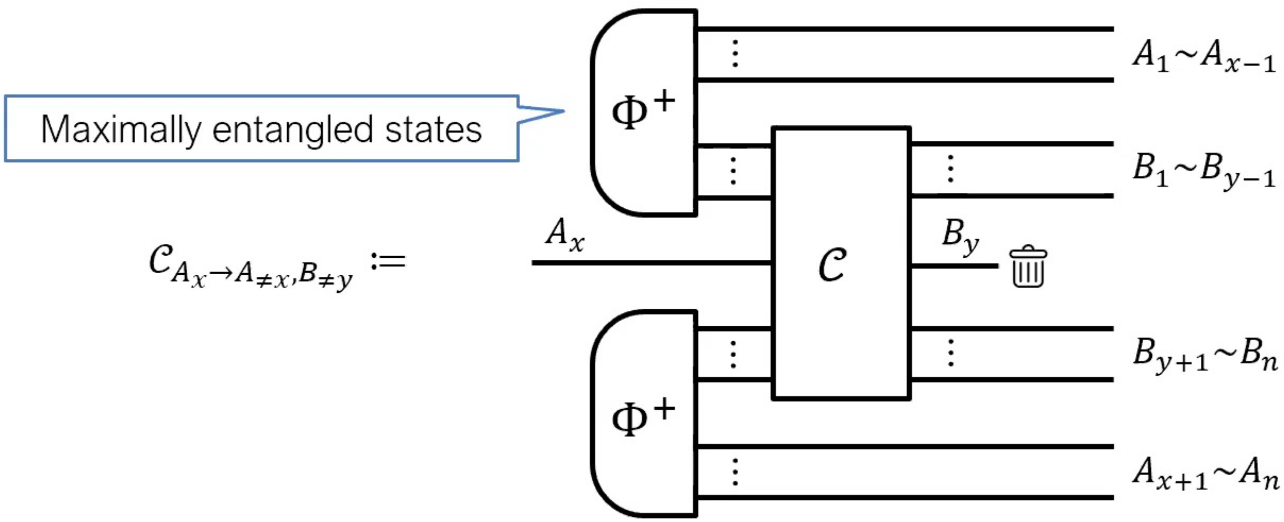


Testing the Last Tooth

Last Tooth → Constant Channel

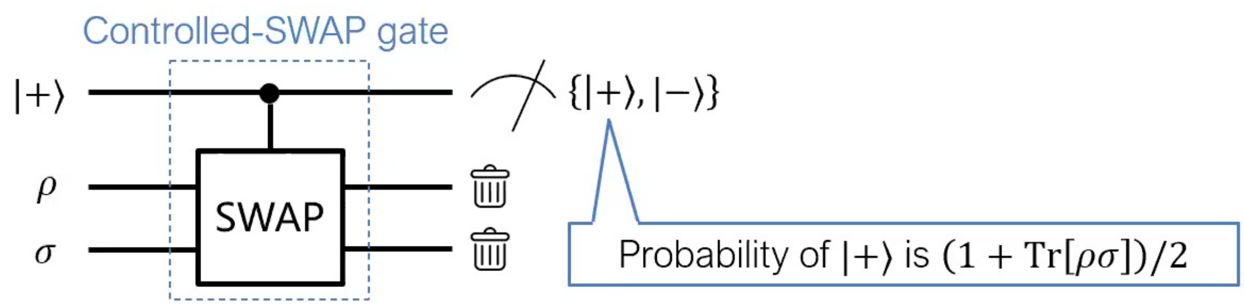
(A_x, B_y) is the last tooth $\Leftrightarrow$ A_x affects nothing except the last output B_y

$\Leftrightarrow$ The following is a constant channel



Testing the Last Tooth
SWAP Test [Buhrman, et al. PRL 2001]

A quantum circuit that estimates the overlap $\mathrm{Tr}[\rho\sigma]$ given copies of ρ and σ



Run this circuit N times, and approximate $\mathrm{Tr}[\rho\sigma]$ with $\frac{2N_+ - N}{N}$, where N_+ is number of runs with outcome $|+\rangle$

Testing the Last Tooth

Test of Constant Channel

To decide whether $\mathcal{C}(\rho)$ is a constant channel, we check whether the outputs are different on different inputs

1. Pick an "informationally complete" set of input states $\{\psi_\alpha\}$
2. Apply $\mathcal{C}$ on each of them, let $\rho_\alpha := \mathcal{C}(\psi_\alpha)$ be the output
3. For each α, compare ρ_α with ρ_1 - use SWAP tests
   - $||\rho_\alpha - \rho_1||_2^2 = \mathrm{Tr}[\rho_\alpha\rho_\alpha] + \mathrm{Tr}[\rho_1\rho_1] - 2\mathrm{Tr}[\rho_\alpha\rho_1]$

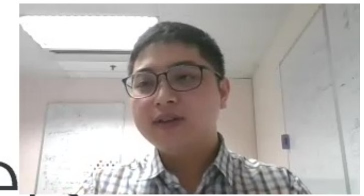
Hilbert-Schmidt distance $||X||_2 := \sqrt{\mathrm{Tr}[X^\dagger X]}$

Estimate each term with SWAP test

4. Decide $\mathcal{C}$ to be a constant channel if all distances are small

$\{\psi_\alpha\}$ is *informationally complete* if every operator X can be written as $X = \sum_\alpha c_\alpha \psi_\alpha$ for some complex numbers $\{c_\alpha\}$

Recall: constant channel ⇔ last tooth

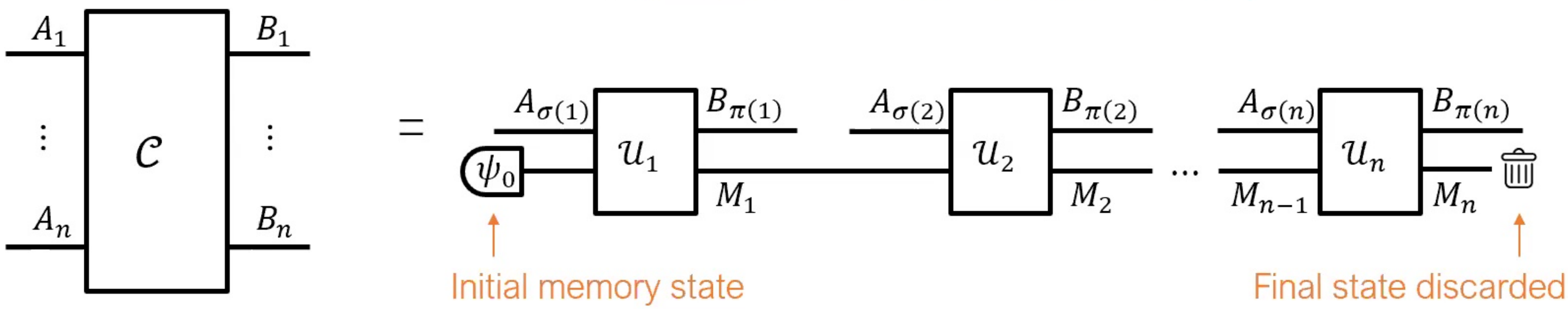


Assumption: Unitary Sub-Channels

Assumption 1. $\mathcal{C}$ consists of unitary interactions with a fixed-sized system:

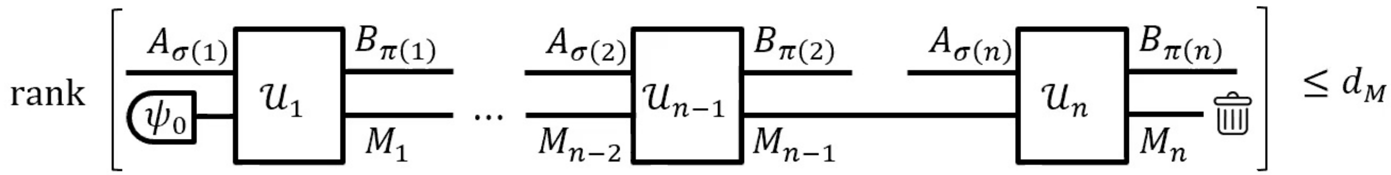
- ψ_0 is a pure state, $\mathcal{U}_1, \mathcal{U}_2, \ldots, \mathcal{U}_n$ are unitary gates
- All dimensions of input and output wires are the same $d_{A_1} = \cdots = d_{A_n} = d_{B_1} = \cdots = d_{B_n} =: d_A$
- All dimensions of the memory wires are the same $d_{M_1} = \cdots = d_{M_n} =: d_M$

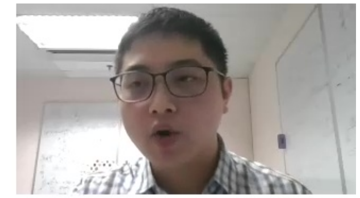
Does not break generality: any quantum comb can be written as concatenation of unitary gates with large enough d_M [Barrett, Lorenz, Oreshkov 2019]



Low-Rank Ensures Efficiency

Assumption 1 ensures that the comb has Kraus rank





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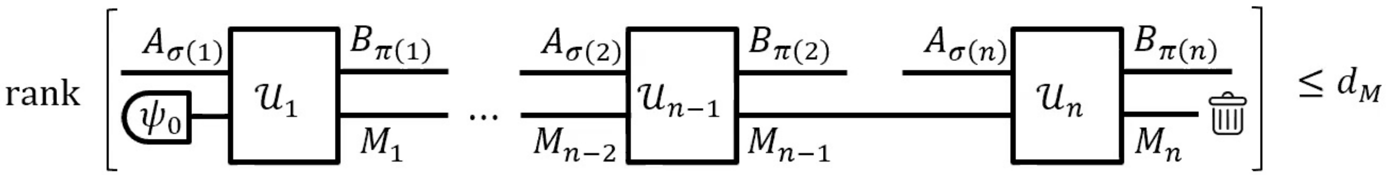
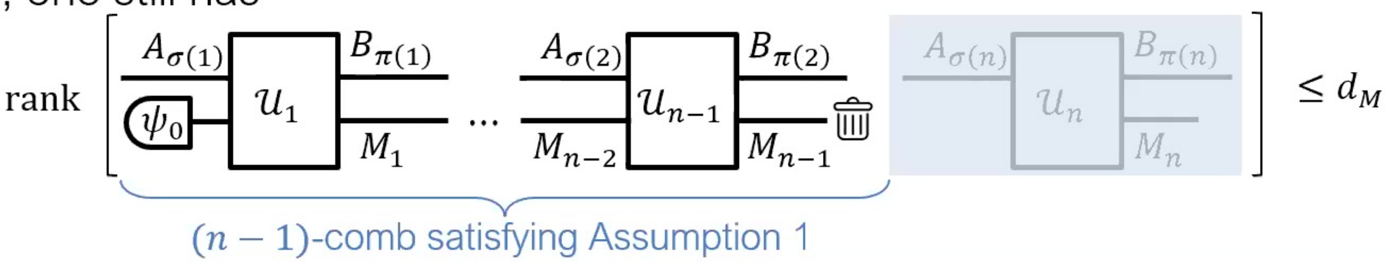
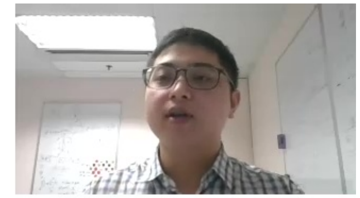


Ignoring last tooth (feed the input with maximally mixed state, and discard the output), one still has





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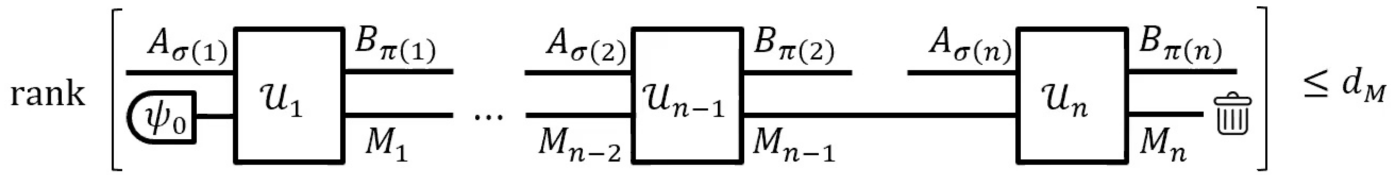

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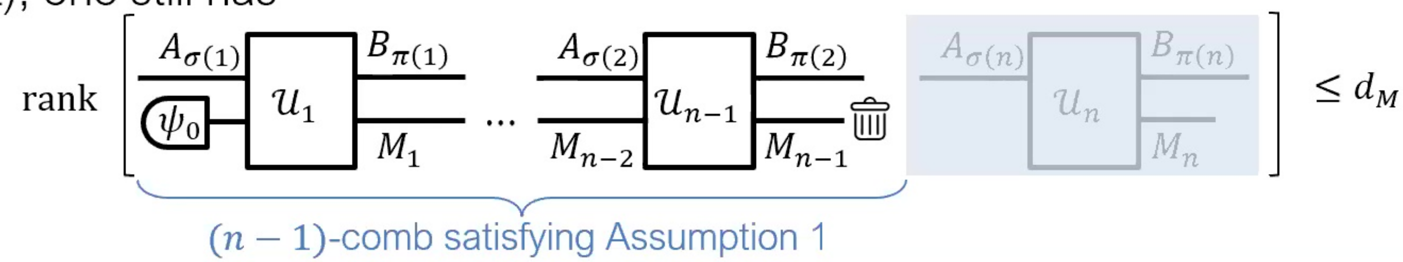


The efficiency of testing constant channels depends on the rank of the channel
The Kraus rank not growing with the execution of algorithm ensures efficiency

Main Algorithm

Recursively Find the Last Tooth

Once we find the last tooth, we ignore it, and the rest is a $(n-1)$-tooth comb

Recursively find the last tooth of the $(n-1)$-tooth comb

[Algorithm 1](). `findlast`$(\mathcal{C})$

1. If $\mathcal{C}$ has only one tooth, output the tooth and exit
2. Enumerate all possible (A_x, B_y) until $\mathcal{C}_{A_x \to A_{\neq x}, B_{\neq y}}$ is a constant channel — Namely (A_x, B_y) is the last tooth
3. Run `findlast`$(\mathcal{C}_{A_{\neq x} \to B_{\neq y}})$ — The channel by ignoring A_x and B_y
4. Output (A_x, B_y) — Append (A_x, B_y) to the end of the output

Main Algorithm

Accuracy and Complexity

Theorem. Under Assumption 1, with probability $1-\kappa_0$, Algorithm 1 outputs a causal order $\left(A_{\sigma(1)}, B_{\pi(1)}\right), \ldots, \left(A_{\sigma(n)}, B_{\pi(n)}\right)$ such that

$$\exists \mathcal{D} \in \mathrm{Comb}\left[\left(A_{\sigma(1)}, B_{\pi(1)}\right), \ldots, \left(A_{\sigma(n)}, B_{\pi(n)}\right)\right], \qquad \|C - D\|_1 \leq \varepsilon_0$$

Trace distance between Choi states

with number of queries to $\mathcal{C}$ in the order of

$$N = O\left(n^{11} d_A^{12} d_M^2 \varepsilon_0^{-8} \log\left(n d_A \kappa_0^{-1}\right)\right)$$

Algorithms with Local Observations

We devise a subroutine that computes a Boolean matrix ind_{ij} s.t.

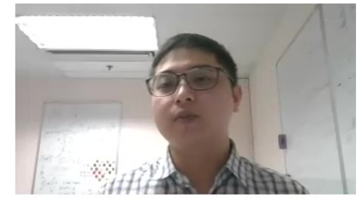
$$ind_{ij} = \text{true} \Leftrightarrow \text{input wire } A_i \text{ and output wire } B_j \text{ are (approximately) independent}$$

It uses only local state preparation and local measurements with query complexity

$$N = O\left(d_A^6 d_B^6 \chi_{\min}^{-2} \underline{\log(n d_A d_B \kappa^{-1})}\right)$$

Logarithmic in n

- d_A, d_B — dimension of each input/output wire
- n — number of teeth
- $1-\kappa$ — success probability
- $\chi_{\min}$ — correlation threshold, below this is considered independent



Summary

The first efficient algorithm that discovers general quantum causal orders

- Discover data transmission paths in quantum networks / trajectories of particles
- Check the input-output correlations of quantum circuits, as a verification technique
- Discover the latent structure of multipartite quantum systems
  - E.g., for efficient compression [Bai et al. NJP 2020] and efficient tomography [Cramer et al. Nat. Comm. 2010] of them

Algorithm 1: general causal order

- Efficient for combs consisting of fixed-sized unitary gates

Algorithms 2 & 3: special cases

- Easier to implement: use local state preparation and local measurements
- More efficient: query complexity logarithmic in n

Future Directions

A more informative causal structure than the comb?

- E.g., a directed acyclic graph to describe causal structure (causal graph)

Our algorithms do not efficiently solve all possible cases

- E.g., combs with an exponentially large memory
- The most general case is also difficult for classical causal order discovery
- New problem formulation needed

A probabilistically approximately correct (PAC) algorithm?

- Answer is “correct” as long as a limited-power verifier cannot disprove it
- Quantum PAC learning of causal structure?

Thank you!

G Bai, Y Yang, and G Chiribella. "Quantum Compression of Tensor Network States." New Journal of Physics 22.4 (2020): 043015. **arXiv: 1904.06772**

G Bai, YD Wu, Y Zhu, M Hayashi, G Chiribella, "Efficient Algorithms for Causal Order Discovery in Quantum Networks." **arXiv:2012.01731**