Title: Self-torque and frame nutation in binary black hole simulations

Speakers: Maria JosÃ© Bustamante

Series: Strong Gravity

Date: December 10, 2020 - 1:00 PM

URL: http://pirsa.org/20120019

Abstract: We investigate the precession of the spin of the smaller black hole in binary black hole simulations. By considering a sequence of binaries at higher mass ratios, we approach the limit of geodetic precession of a test spin. This precession is corrected by the ``self-torque" due to the smaller black hole's own spacetime curvature. We find that the spins undergo spin nutations which are not described in conventional descriptions of spin precession, an effect that has been noticed previously in simulations. These nutations arise because the spins are not measured in a frame where the smaller hole is stationary. We develop a simple model for these frame nutations, extract the instantaneous spin precession rate, and compare our results to PN and extreme-mass-ratio approximations for the self-torque.

Self Torque from Numerical Simulations

(and other projects)

María José Bustamante Rosell
Aaron Zimmerman

Department of Physics
University of Texas at Austin

December 10th, 2020

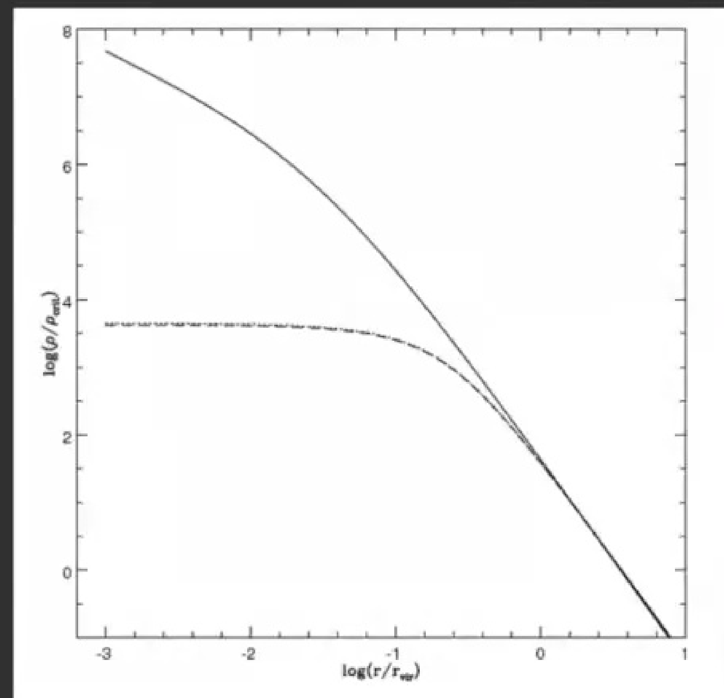
Overview

1. Dynamical analysis of Leo I
2. Self Torque from Numerical Simulations
   - Geodetic Precession and GSF
   - Simulations
   - Results
   - Future Directions
3. Other Projects

Dynamical analysis of Leo I

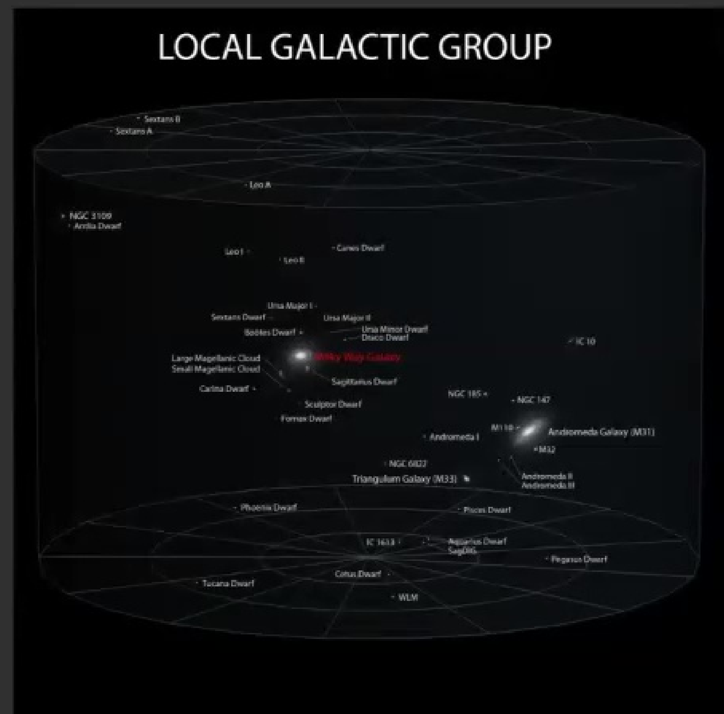
The core-cusp problem

- Central density profile discrepancy
  - ΛCDM (CUSPY): $(\rho \sim r^{-1})$
  - Observations (CORE) $(\rho \sim r^{0})$
- Dwarf Spheroidal Galaxies
  - Dark matter dominated
  - Nearby

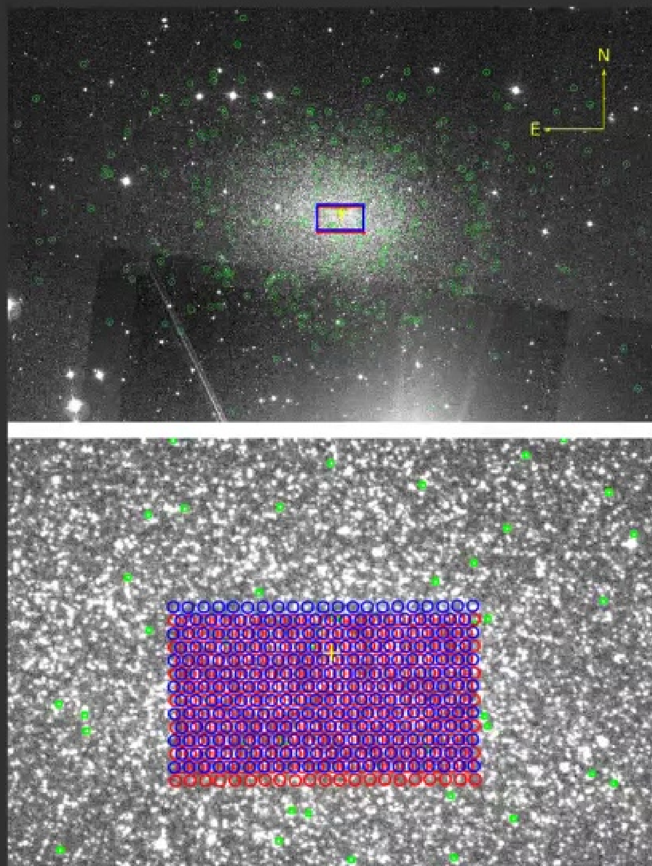


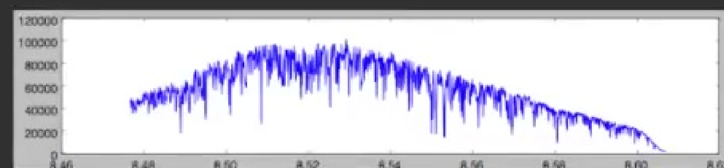
The core-cusp problem

- Central density profile discrepancy
  - ΛCDM (CUSPY): $(\rho \sim r^{-1})$
  - Observations (CORE) $(\rho \sim r^{0})$
- Dwarf Spheroidal Galaxies
  - Dark matter dominated
  - Nearby



Dynamical Analysis

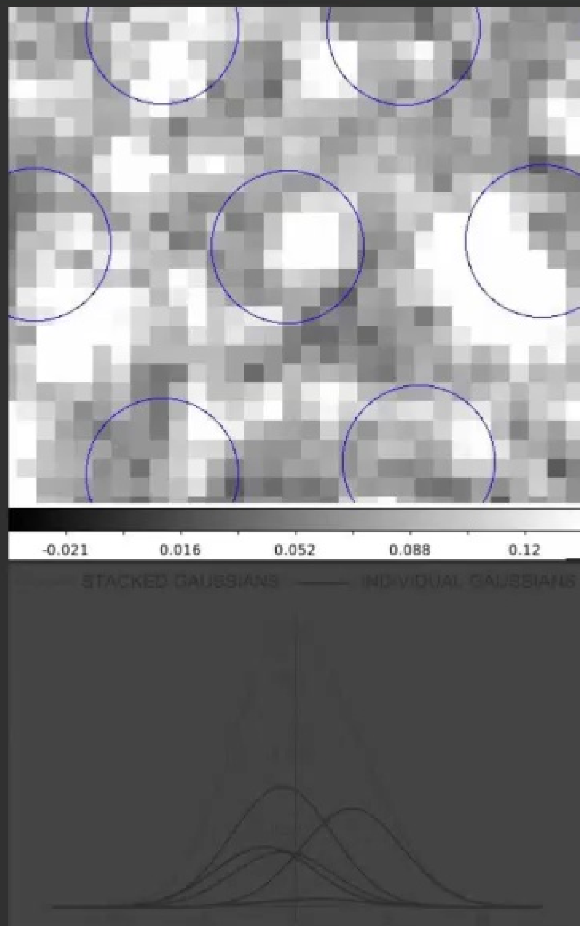


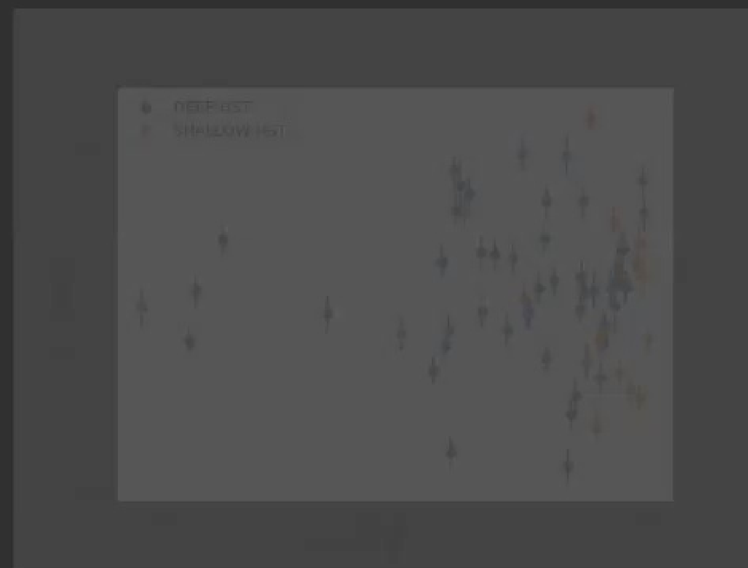


Three improvements

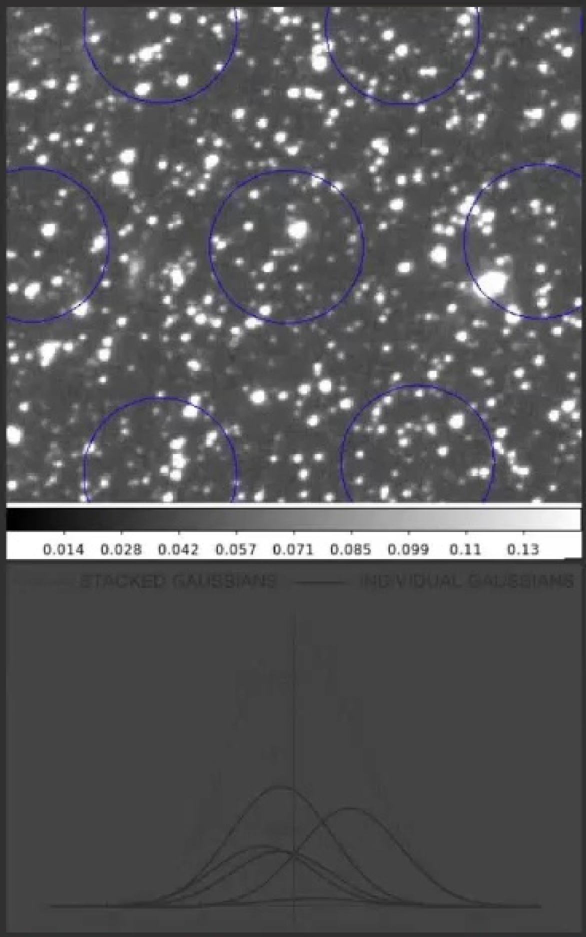
- How tidally disrupted is the galaxy?
  (At what max radius can we probe the potential?)
- Avoid Jeans analysis
  (Schwarzschild orbit superposition method)
- The Crowding Problem

The Crowding Problem



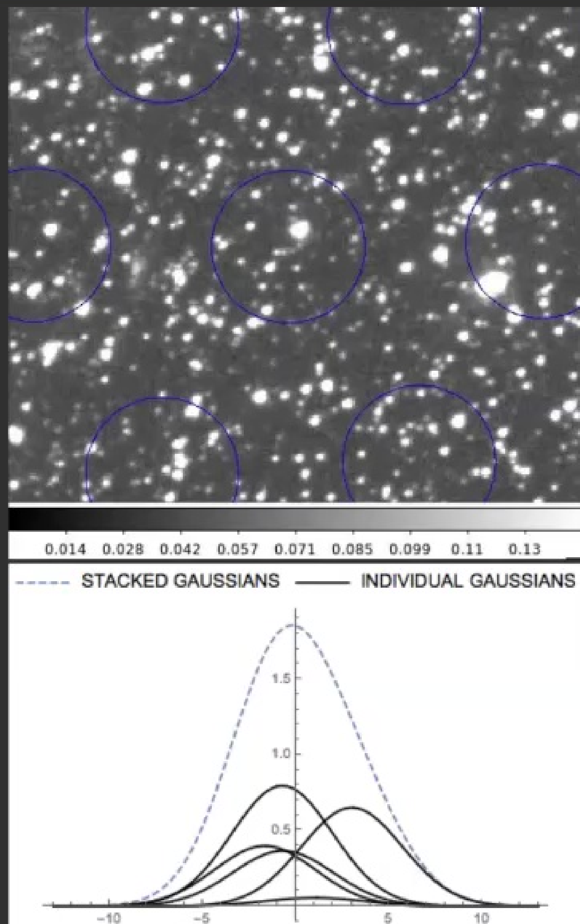


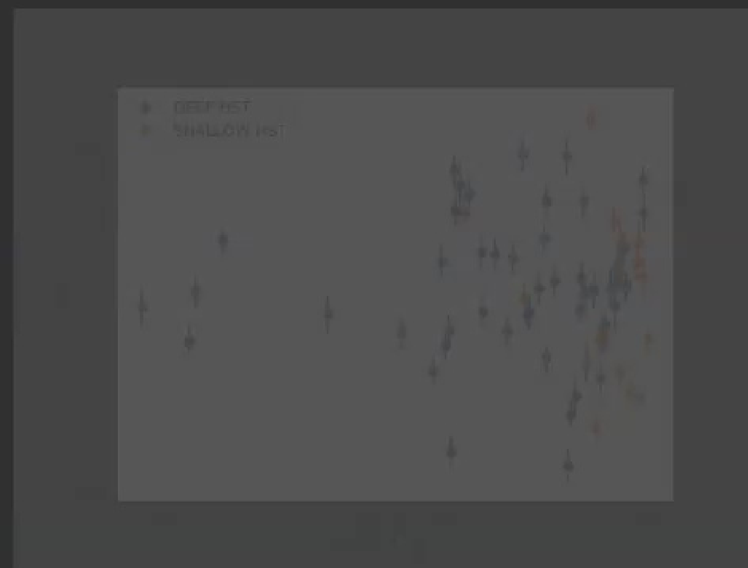
The Crowding Problem



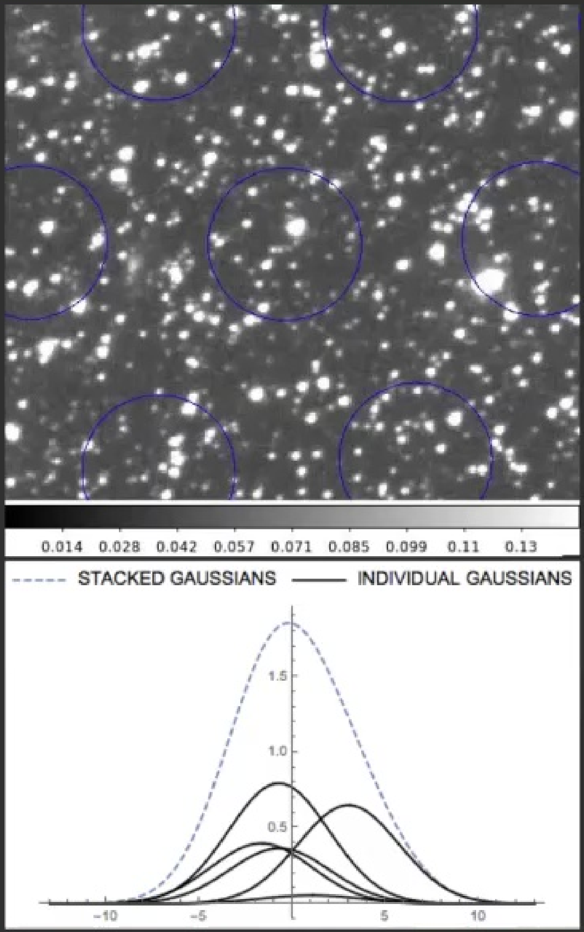
ID: 957 0946 6624 Stop Share

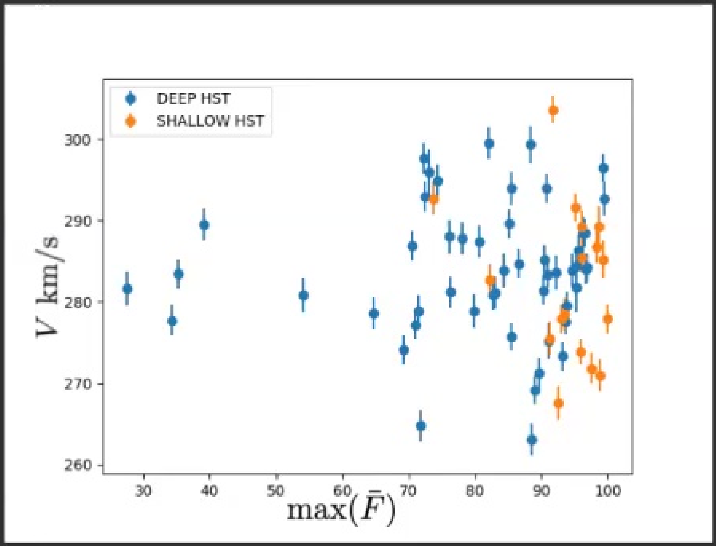
The Crowding Problem





The Crowding Problem





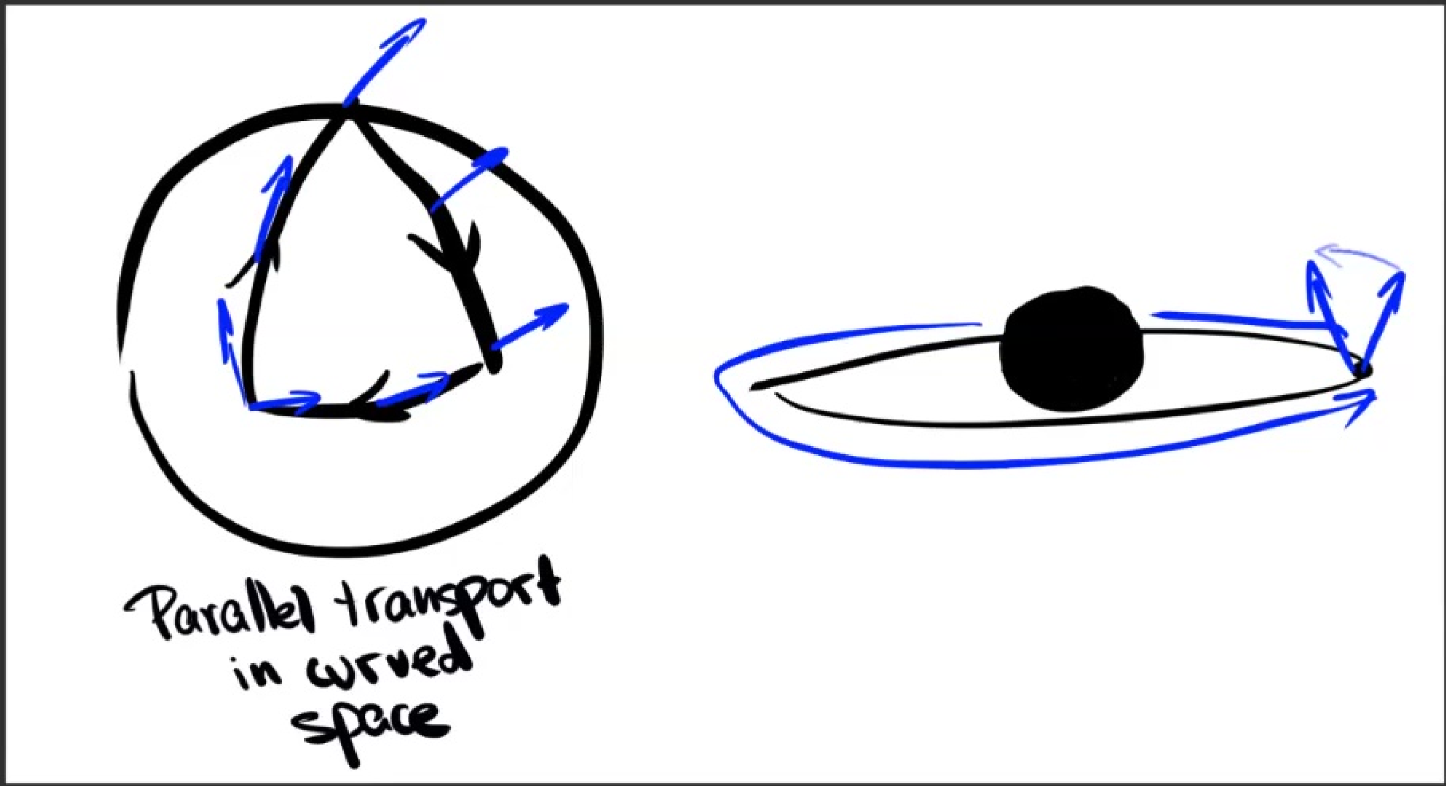
ID: 957 0946 6624 Stop Share

We found a black hole!

$$M_\bullet = 3.2 \pm 1.5 \times 10^6 M_\odot$$

Self Torque from Numerical Simulations

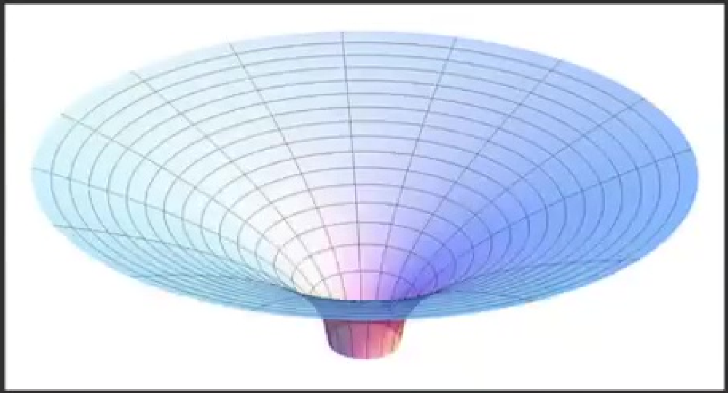
Geodetic Precession



Ω_p in different limits

Weak Field Limit:
$\Omega_p \sim \frac{3}{2}\mathbf{v} \times \boldsymbol{\nabla}\boldsymbol{\Phi}$

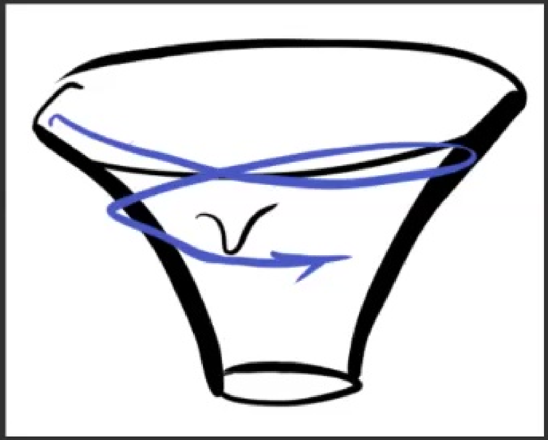
Exact Space-Time:
$\Omega_p = \Omega_\phi(1 - \sqrt{1 - 3M/r})$



- But what happens in perturbed spacetimes?

The Self-Force approximation

- $g_{\mu\nu} = g^0_{\mu\nu} + h^R_{\mu\nu}$
- Finite mass perturbs BH spacetime $\Rightarrow$ "test particle" has non-geodesic motion in $g^0_{\mu\nu}$ (self-force).
- Or, geodesic motion in the full spacetime $g_{\mu\nu}$.



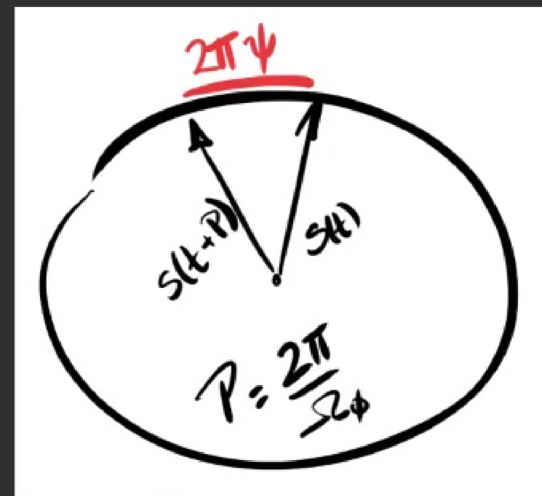
Self-Torque

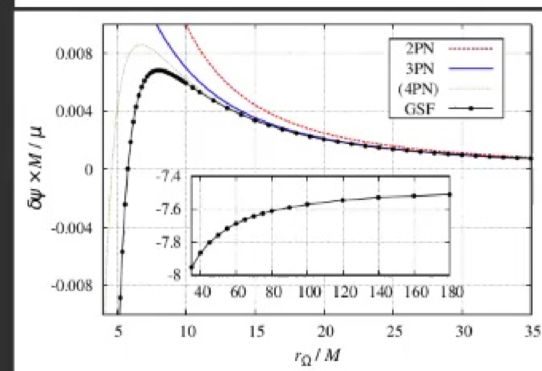
- There's a scalar! $\psi = \Omega_p / \Omega_\phi$
- Precession: transport by regularized metric

$$g_{\mu\nu} = g^0_{\mu\nu} + h^R_{\mu\nu}$$
$$\psi = \psi^0 + q^{-1}\delta\psi^1$$

- Let's study this in NR (Actually solving Einstein Eqs)

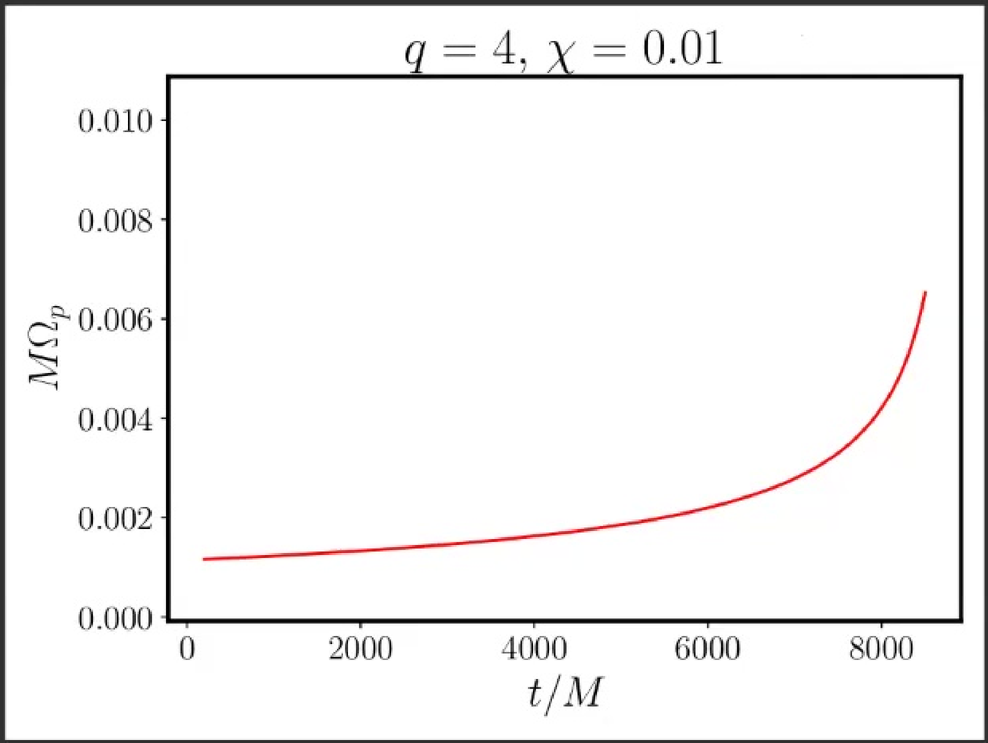
$$\psi = \psi^0 + q^{-1}\delta\psi^1 + q^{-2}\delta\psi^2 + \dots$$





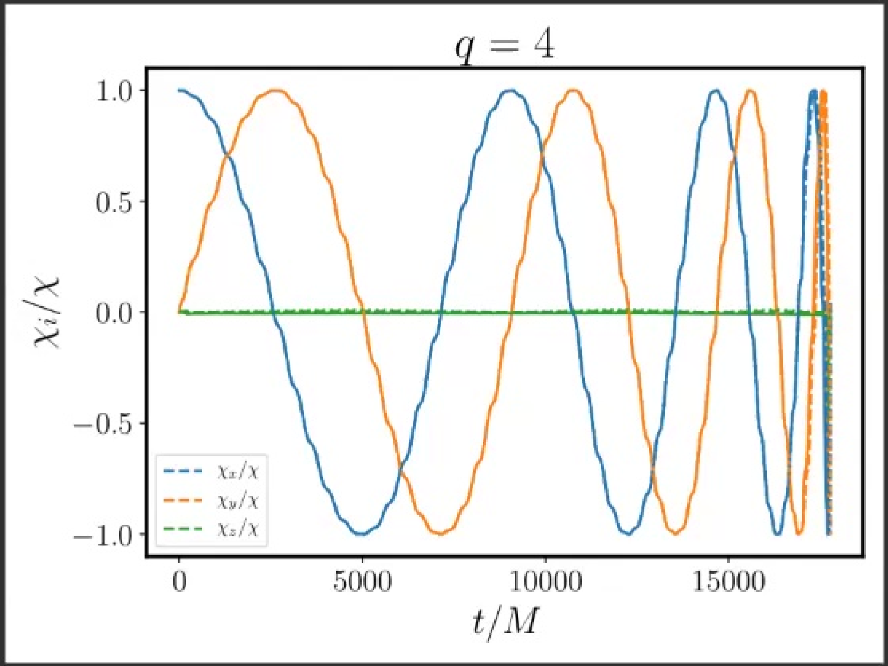
Dolan et al.(2014,2015)

Expectation (from 3PN)



A peek into numerics...

Setup a "test" particle: $q = \{2, 4, 6\}$, $\chi = \{0.01, 0.05, 0.1\}$

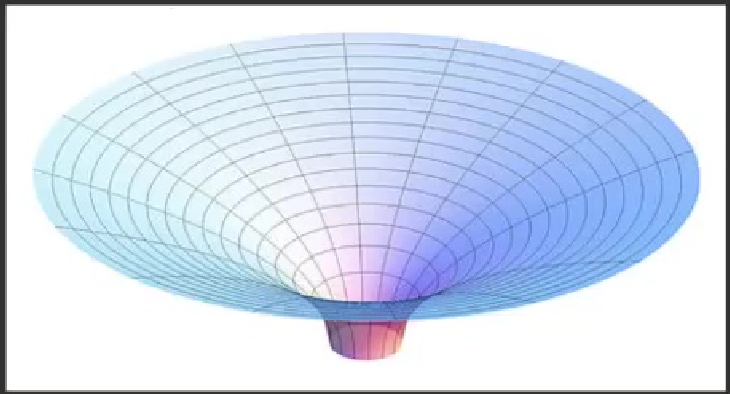


Taking all different (and small) spins, but we see a nutation independent of χ.

Ω_p in different limits

Weak Field Limit:
$\Omega_p \sim \frac{3}{2}\mathbf{v} \times \boldsymbol{\nabla}\boldsymbol{\Phi}$

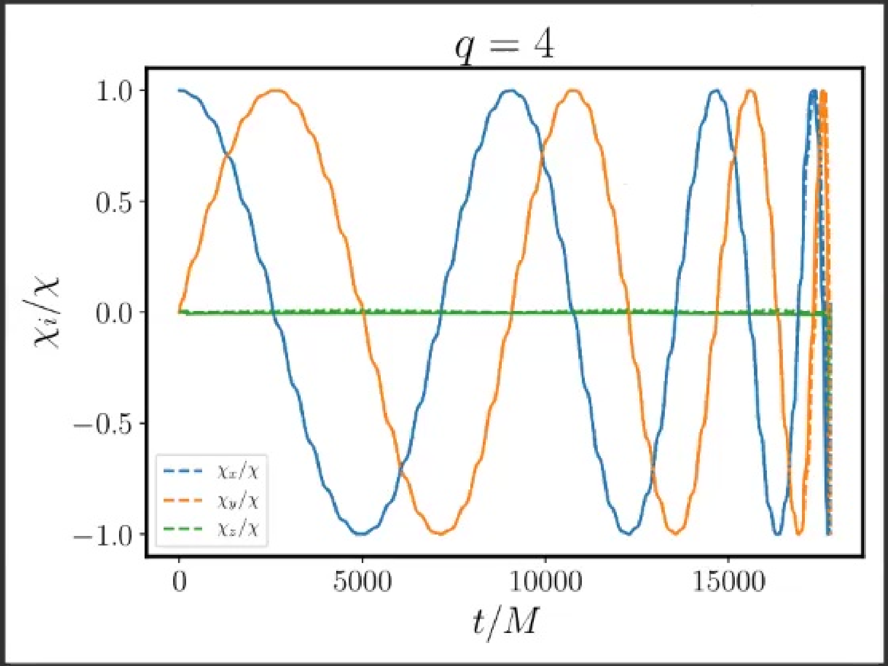
Exact Space-Time:
$\Omega_p = \Omega_\phi(1 - \sqrt{1 - 3M/r})$



- But what happens in perturbed spacetimes?

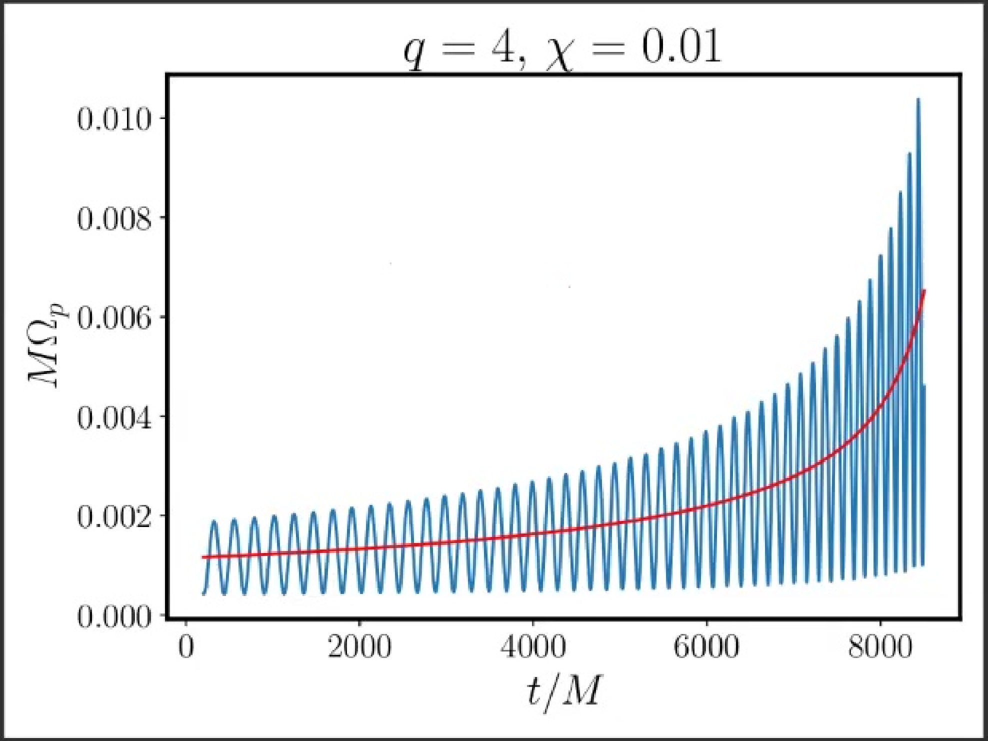
A peek into numerics...

Setup a "test" particle: $q = \{2, 4, 6\}$, $\chi = \{0.01, 0.05, 0.1\}$

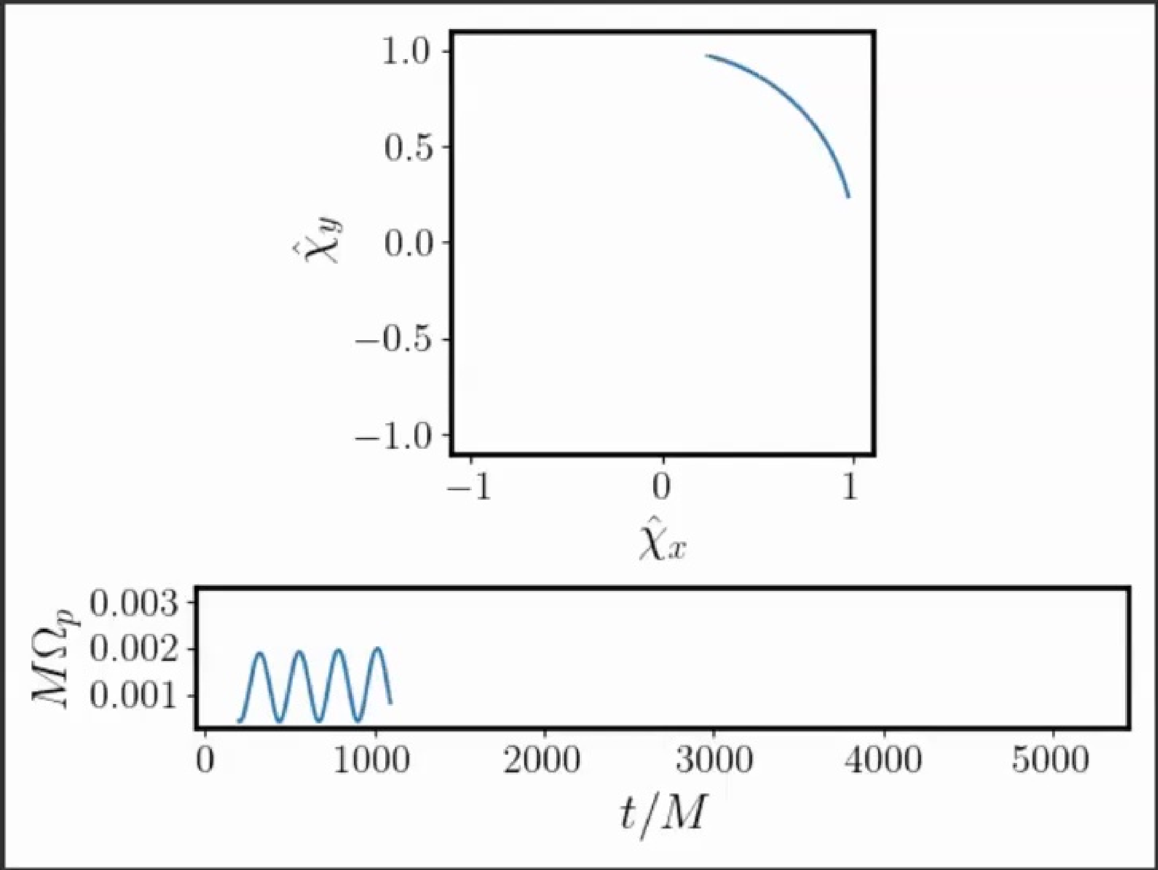


Taking all different (and small) spins, but we see a nutation independent of χ.

A peek into numerics...



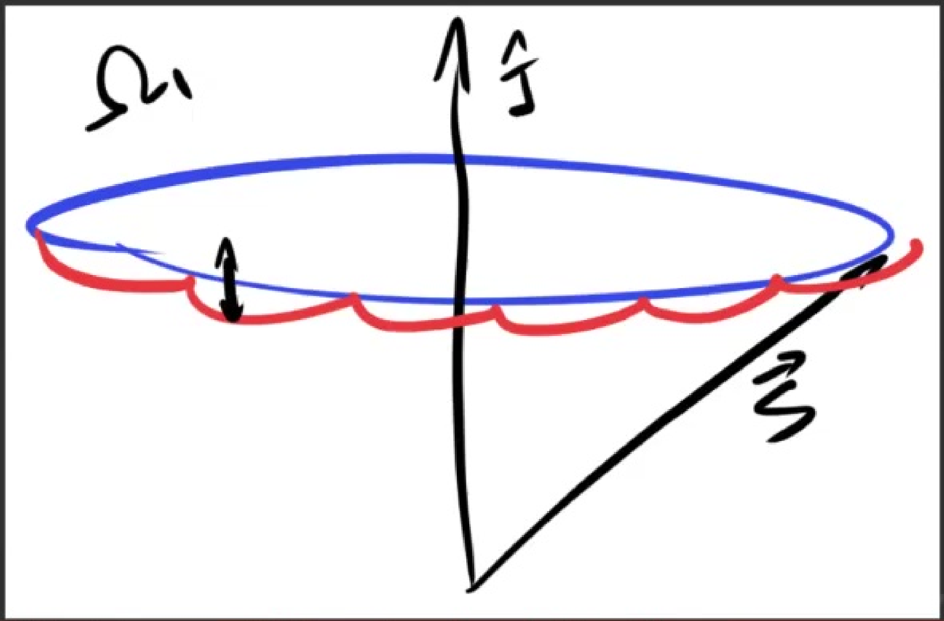
A video of the spin precession



$\mathcal{O}(S^2)$ nutation

- Qualitatively different from nutation from $\mathcal{O}(S^2)$ effects (perpendicular to precession cone)

$$\frac{\mathrm{d}\vec{S}}{\mathrm{d}t} = \frac{\Omega_p}{|\vec{L}-\vec{S}|}\left(\vec{L}-\vec{S}\right)\times\vec{S}$$



What is going on?

- Spin supplementary condition

$$u^\nu S_\nu = 0$$

- Mathisson-Papapetrou equations

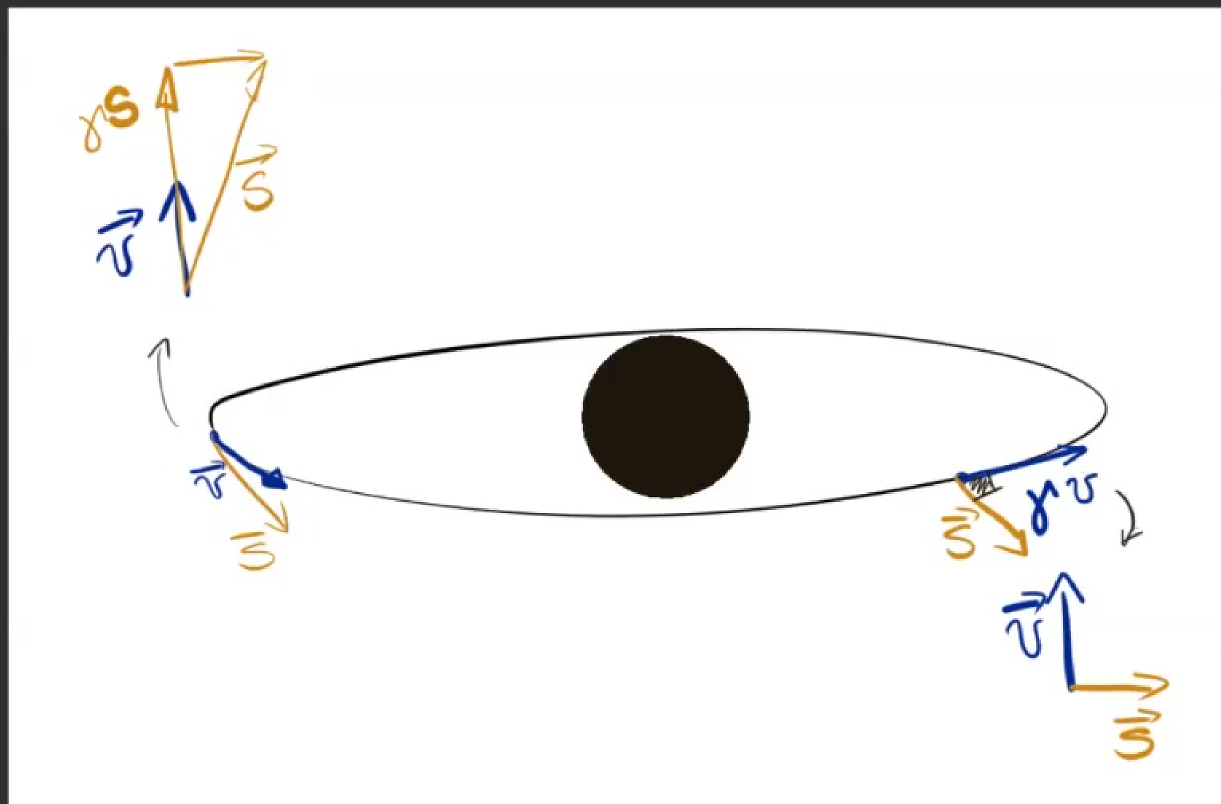
$$u^\mu \nabla_\mu u^\nu = O(S)$$
$$u^\mu \nabla_\mu S^\nu = O(S^2)$$

- Precession equation

$$\frac{\mathrm{d}\vec{S}}{\mathrm{d}t} = \Omega_S \hat{\ell} \times \vec{S}$$

- But in the simulation frame...

The spin is boosted!

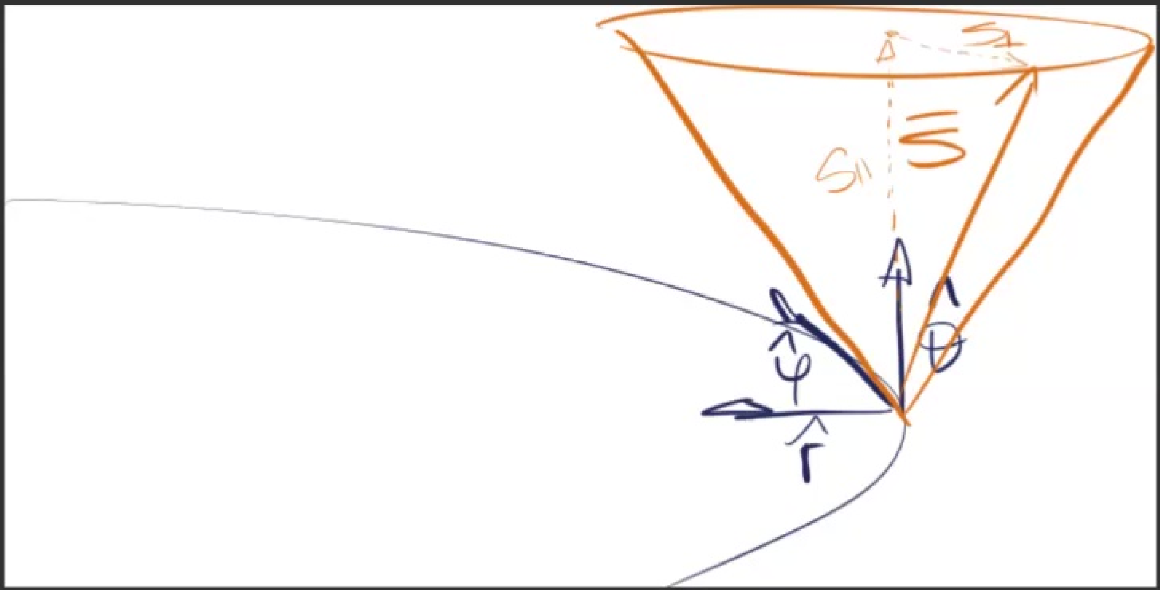


A toy model closer to self-force

- 4-velocity in Schwarzschild coordinates

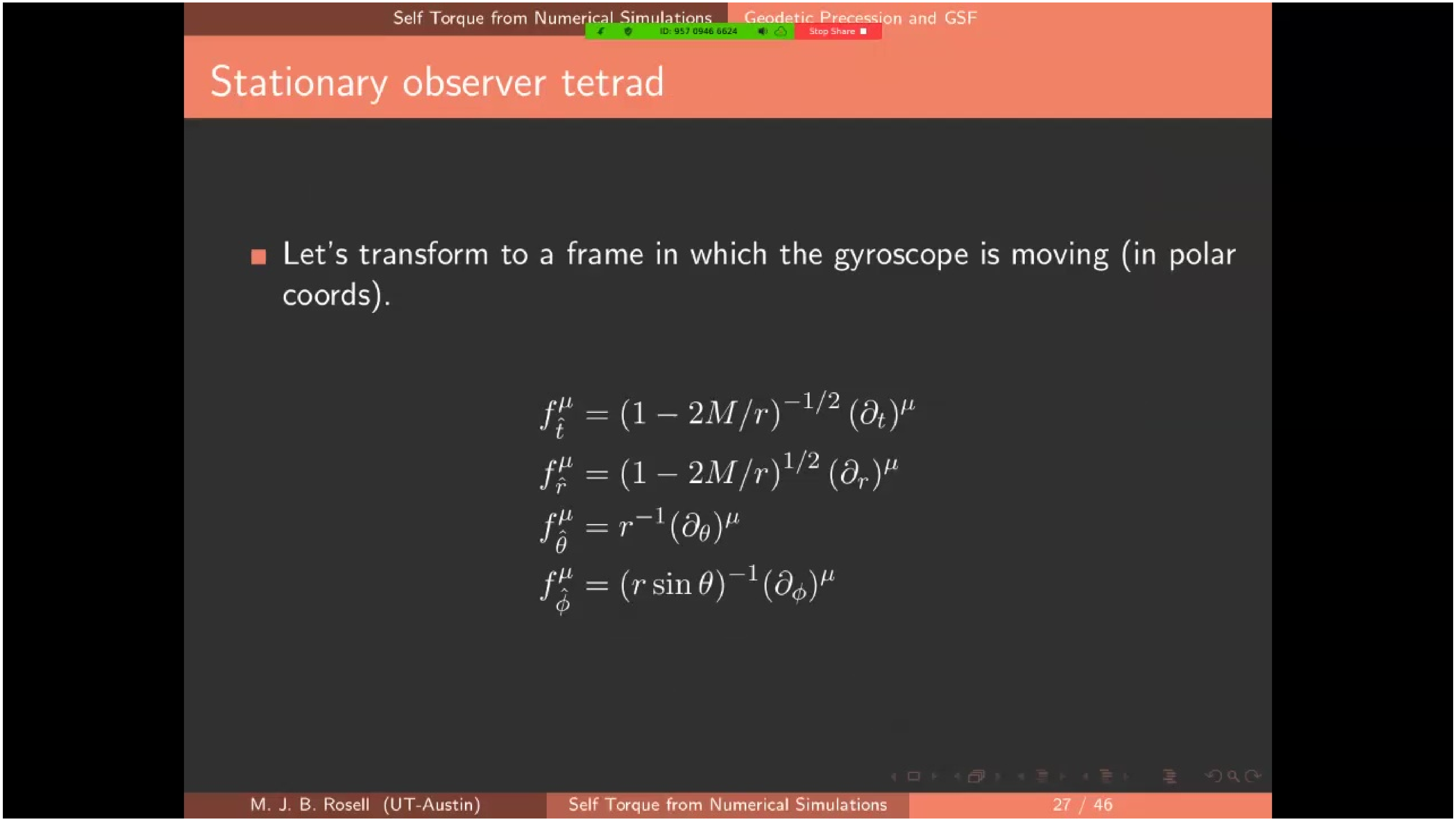
$$u^\mu = \{u^t, 0, 0, u^\phi\}$$
$$u^t = \sqrt{1 - \frac{3M}{r}}$$
$$u^\phi = \Omega_\phi u^t$$
$$\Omega_\phi = \sqrt{\frac{M}{r^3}}$$

Spin in the comoving frame



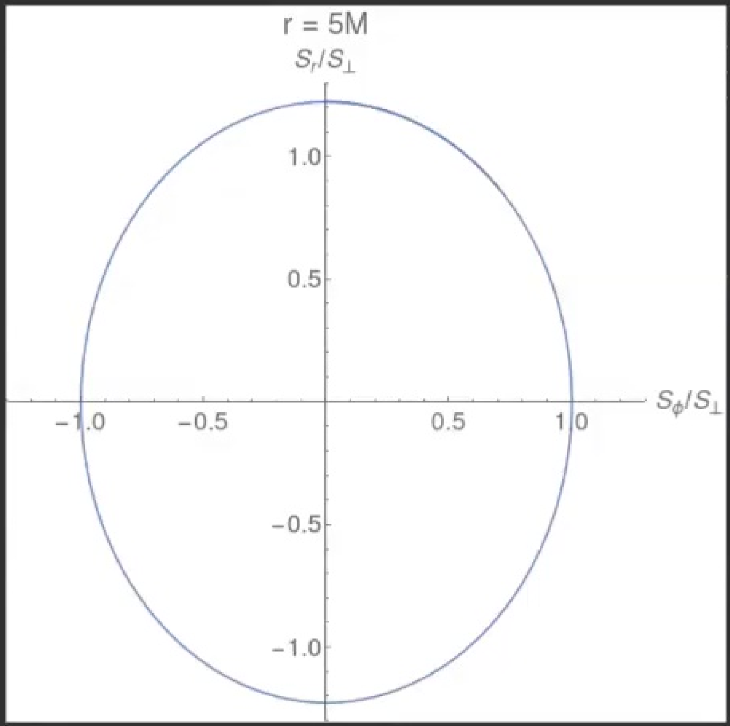
$e^{\mu}_{\hat{i}}$ with $e^{\mu}_{\hat{t}} = u^{\mu}$

$$\Omega_S = \Omega_\phi \sqrt{1 - \frac{3M}{r}}$$


Self Torque from Numerical Simulations
Geodetic Precession and GSF
Stationary observer tetrad
Let's transform to a frame in which the gyroscope is moving (in polar coords).
$f^{\mu}_{\hat{t}} = (1 - 2M/r)^{-1/2} (\partial_t)^{\mu}$
$f^{\mu}_{\hat{r}} = (1 - 2M/r)^{1/2} (\partial_r)^{\mu}$
$f^{\mu}_{\hat{\theta}} = r^{-1} (\partial_{\theta})^{\mu}$
$f^{\mu}_{\hat{\phi}} = (r \sin\theta)^{-1} (\partial_{\phi})^{\mu}$
M. J. B. Rosell (UT-Austin)
Self Torque from Numerical Simulations
27 / 46

Spin vector in the stationary tetrad

$$S_{\hat{t}} = -S_\perp \sqrt{\frac{M}{r-3M}} \cos(\Omega_S t + \varphi_0)$$
$$S_{\hat{r}} = S_\perp \sin(\Omega_S t + \varphi_0)$$
$$S_{\hat{\phi}} = S_\perp \sqrt{\frac{r-2M}{r-3M}} \cos(\Omega_S t + \varphi_0)$$



Boost in ϕ direction

$\Downarrow$

- Spin orbit resembles ellipse.
- Magnitude not constant.

Cartesian spin components

$$S_{\hat{x}} = S_\perp \left[\sin(\Omega_S t + \varphi_0)\cos(\Omega_\phi t) - \sqrt{\frac{r-2M}{r-3M}}\cos(\Omega_S t + \varphi_0)\sin(\Omega_\phi t) \right]$$

$$S_{\hat{y}} = S_\perp \left[\cos(\Omega_S t + \varphi_0)\sin(\Omega_\phi t) - \sqrt{\frac{r-2M}{r-3M}}\sin(\Omega_S t + \varphi_0)\cos(\Omega_\phi t) \right]$$

- Precession with frequency components at both $\Omega_S + \Omega_\phi$ and $\Omega_S - \Omega_\phi$.
- We call this *frame nutation* – the precession stays on the precession cone, but the rate varies.
- Note that this is still the test particle limit.

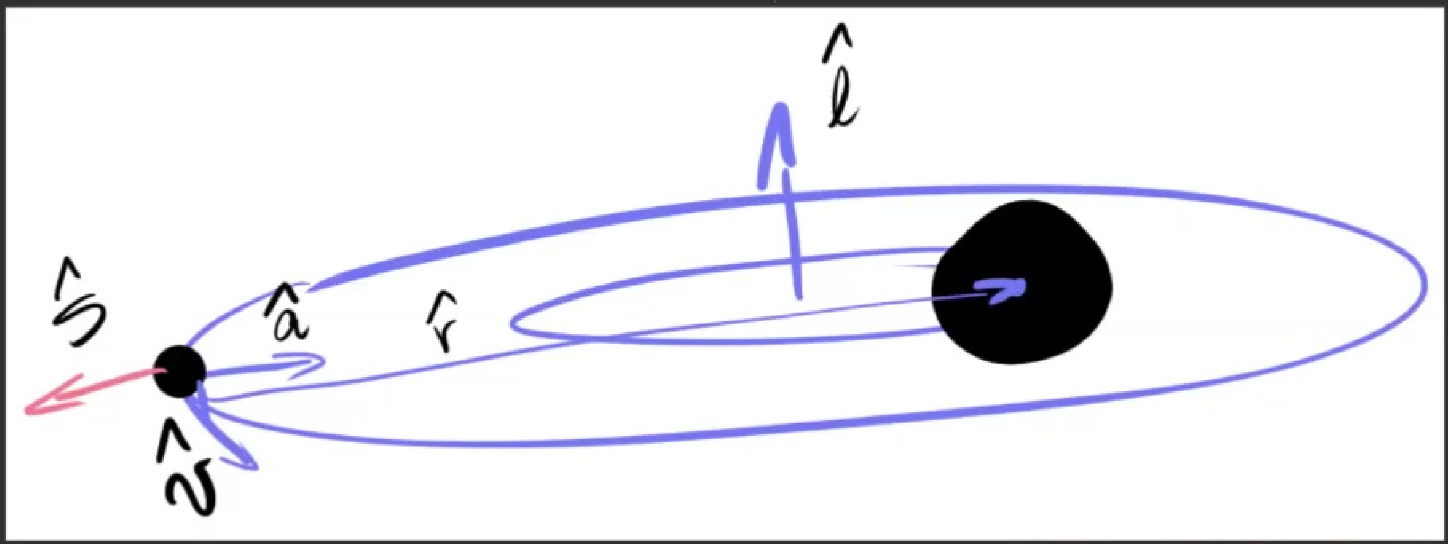
Frame Nutation Model

- In a simulation, take

$$\hat{\mathbf{v}} \equiv \text{velocity unit vector}$$

$$\hat{\ell} \equiv \text{normal to the orbital plane}$$

$$\hat{\mathbf{a}} \equiv \hat{\ell} \times \hat{\mathbf{v}}$$

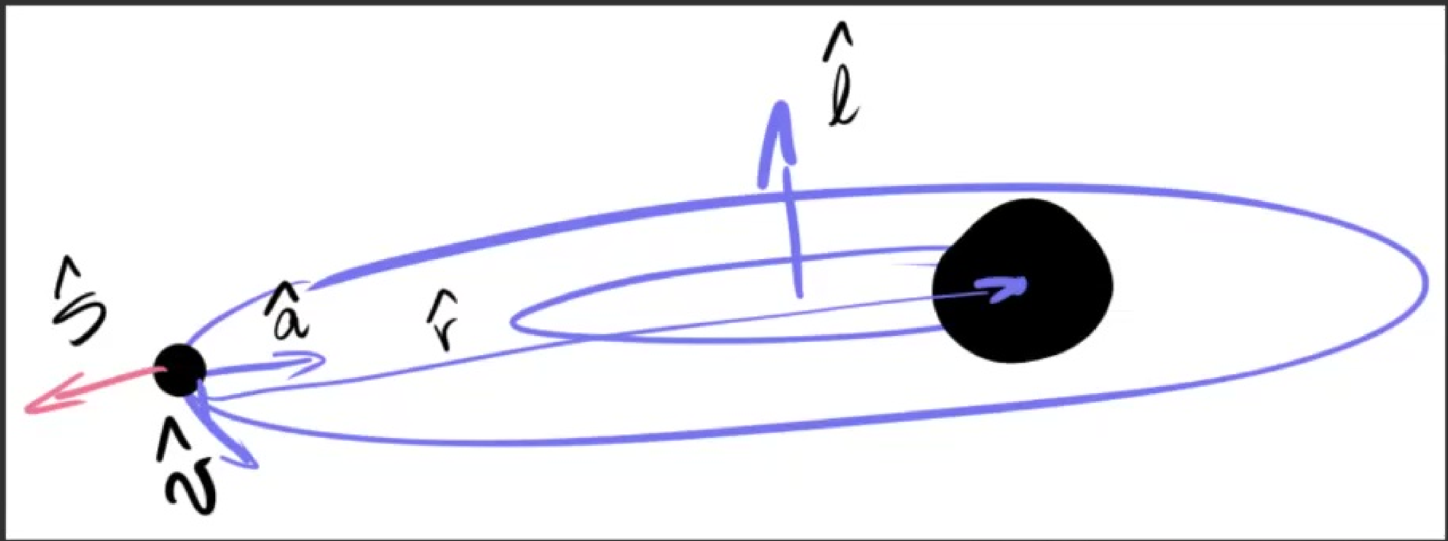


Frame Nutation Model

- Then the spatial components of the spin must take the form

$$\mathbf{S} = S_\perp \sin\left[\varphi(t) + \varphi_0\right] \hat{\mathbf{a}} + S_\perp \gamma \cos\left[\varphi(t) + \varphi_0\right] \hat{\mathbf{v}} + S_\parallel \hat{\boldsymbol{\ell}}$$

$$\varphi(t) = \int_{t_0}^{t} \Omega_S(t')\,\mathrm{d}t'$$



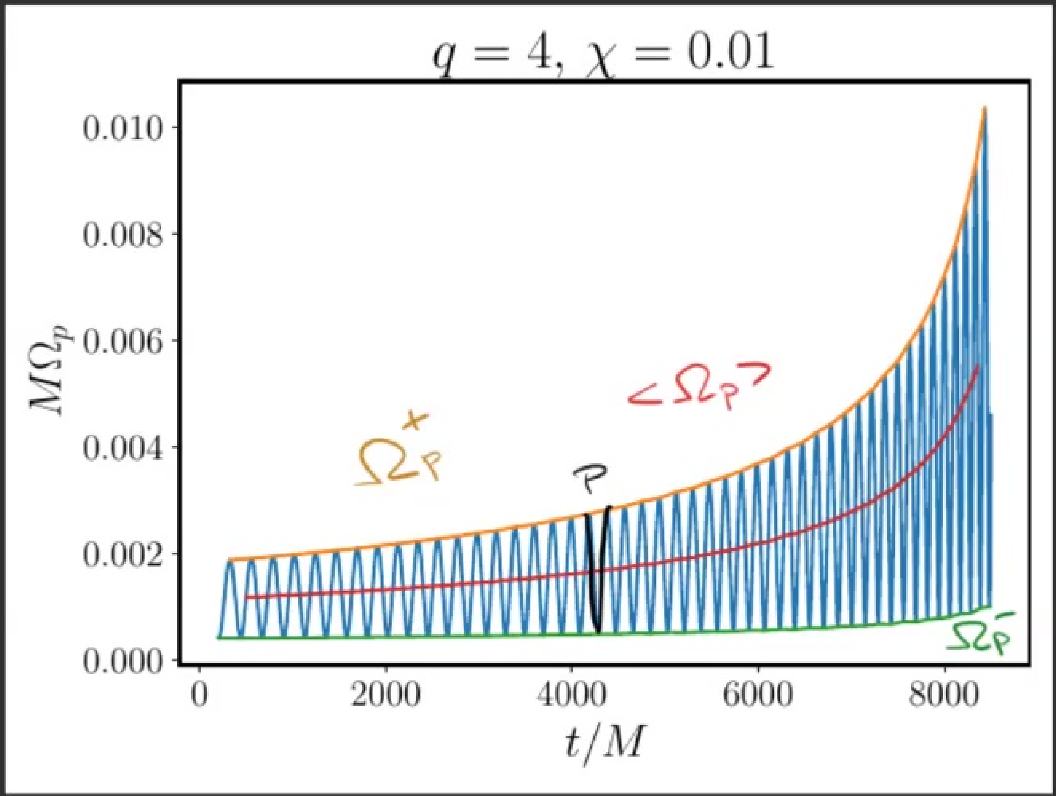
- SpEC however, only knows the direction of the spin $\hat{\chi}$

$$\Omega_p \equiv \left| \hat{\chi}_\perp \times \frac{d\hat{\chi}_\perp}{dt} \right|$$

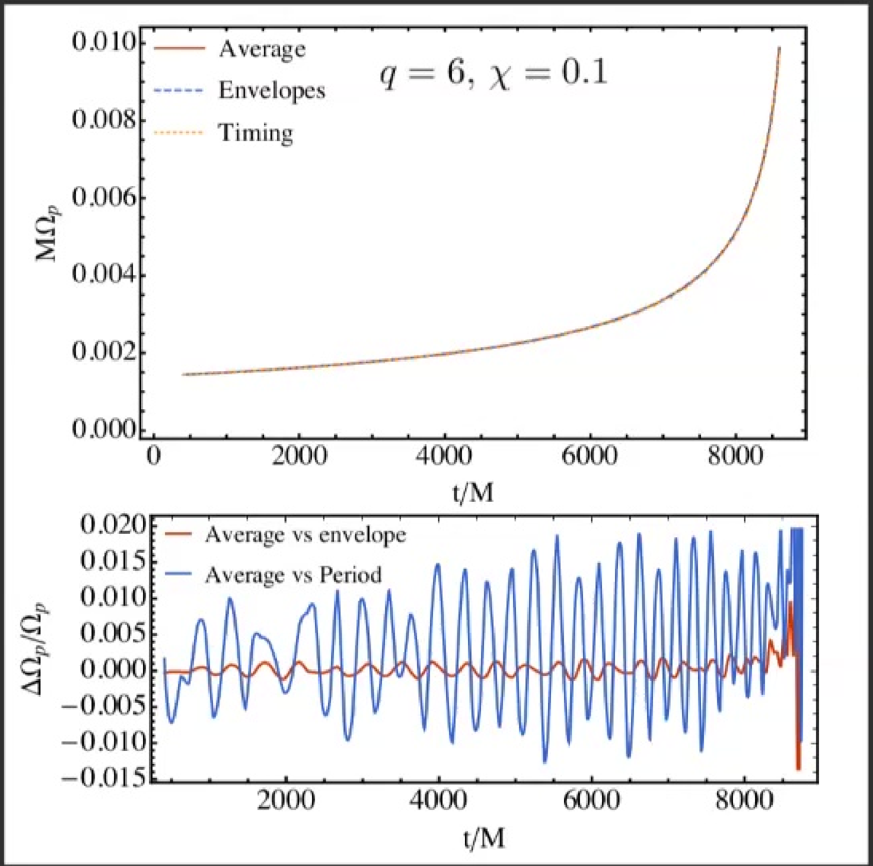
- This results in an (somewhat complicated) expression that implies several ways to compute Ω_p from the data

$$\langle \Omega_p \rangle = \Omega_\phi - \frac{\pi}{P} = \Omega_\phi - \sqrt{(\Omega_\phi - \Omega_p^+)(\Omega_\phi - \Omega_p^-)}$$

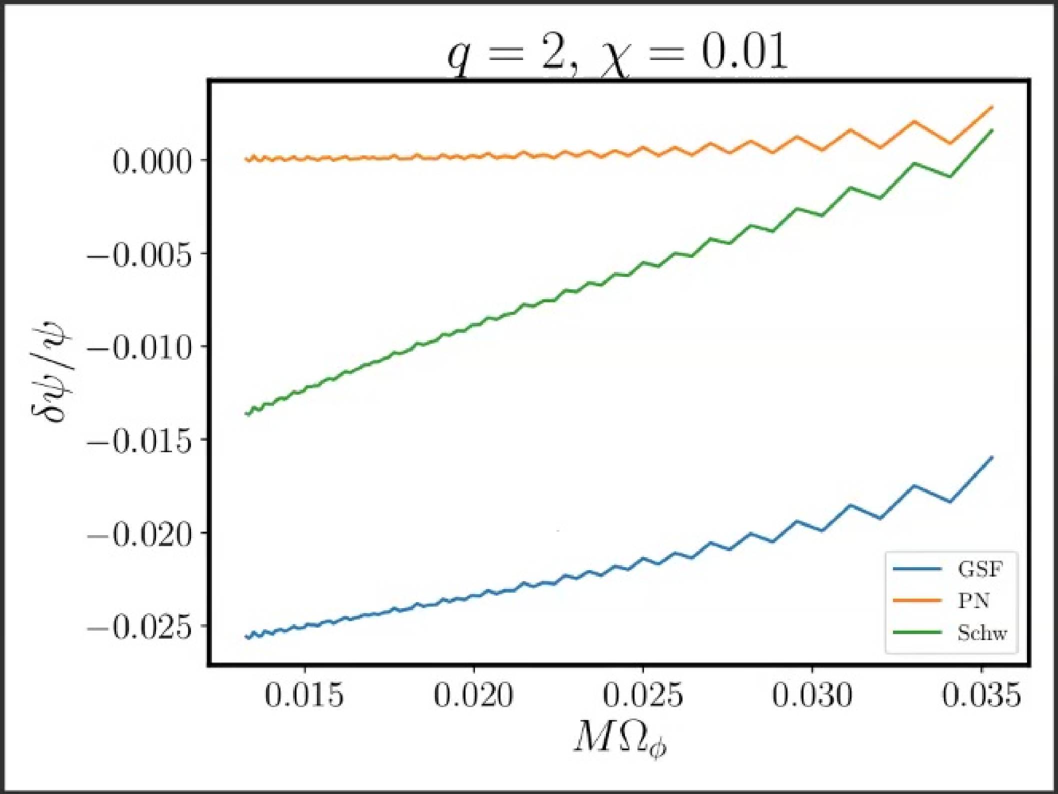
Ω_p


$q = 4,\ \chi = 0.01$
$M\Omega_p$
0.010
0.008
0.006
0.004
0.002
0.000
0
2000
4000
6000
8000
t/M

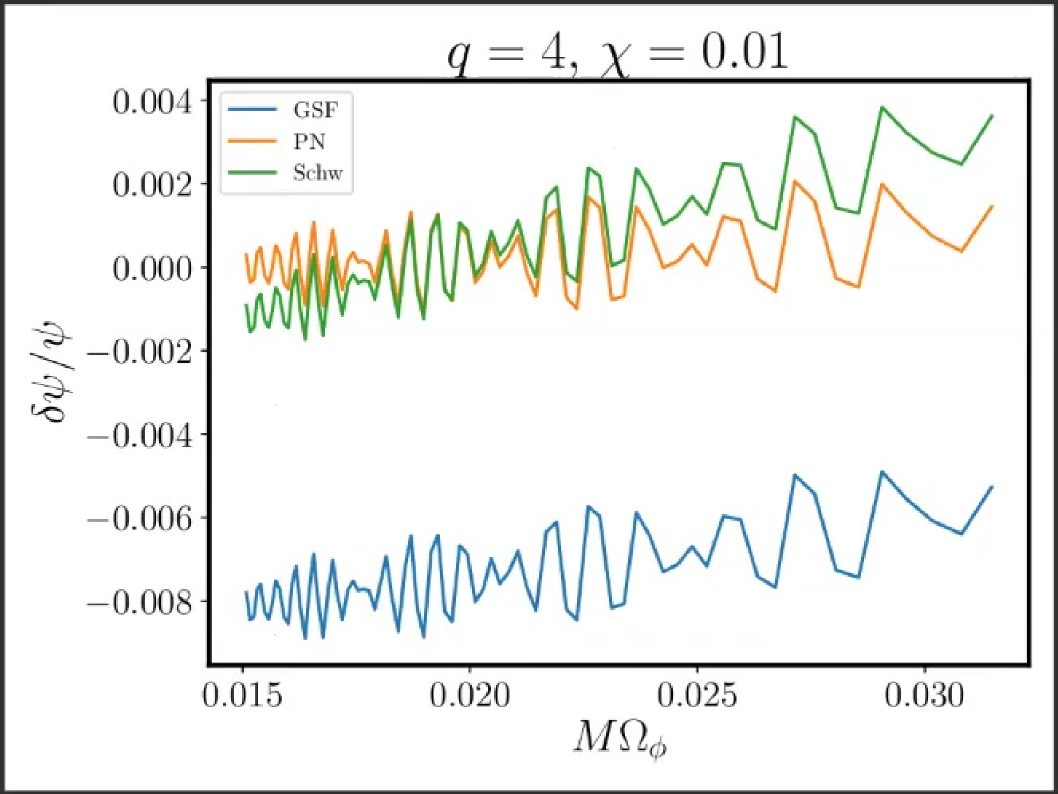
The different Ω_p expressions agree!



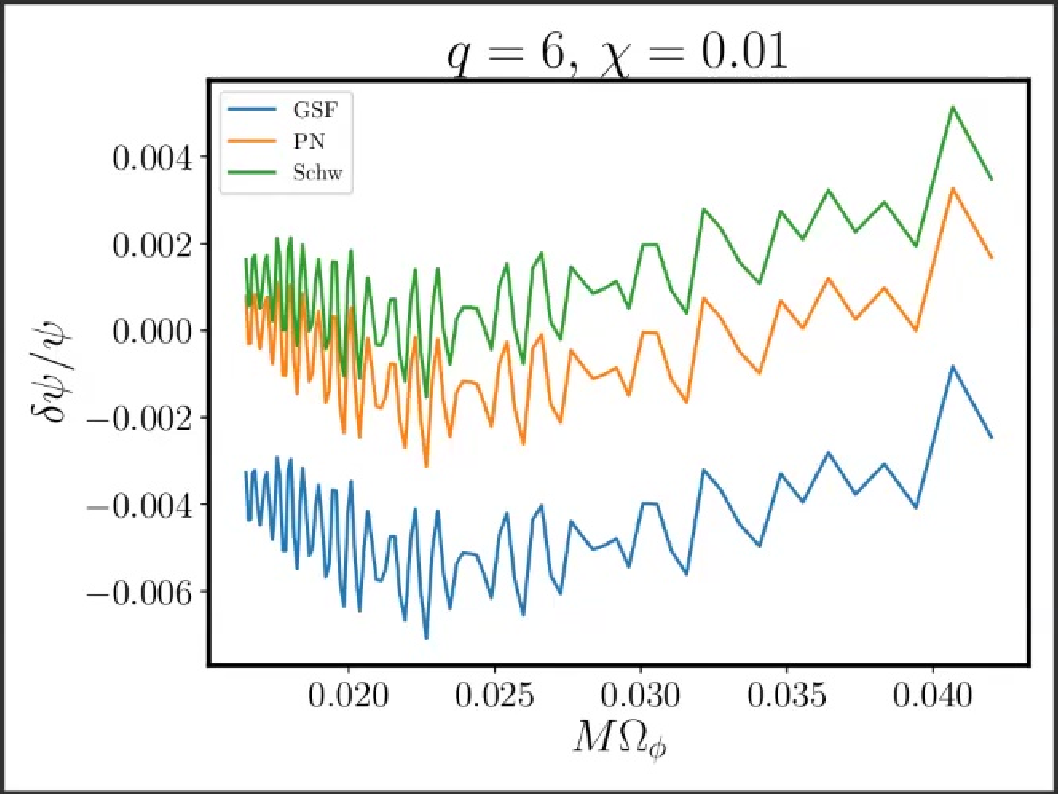
Comparing NR ψ to PN and SF



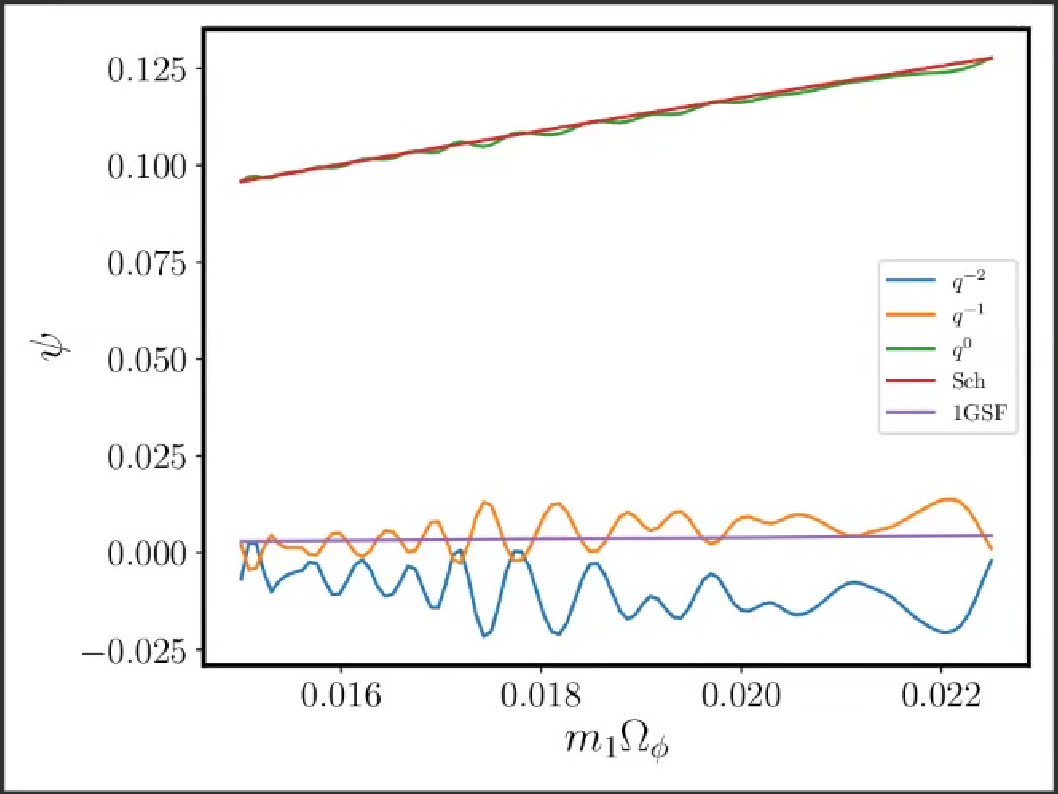
Comparing NR ψ to PN and SF



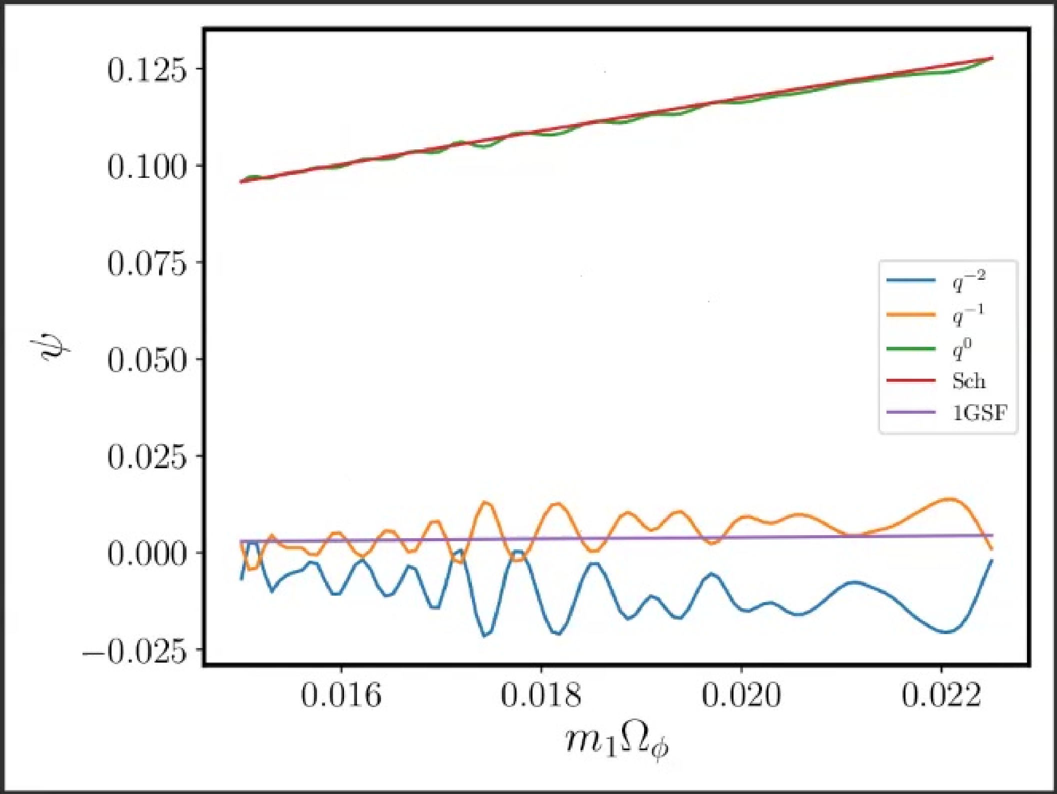
Comparing NR ψ to PN and SF



Plot of inferred $\psi(q)$



Plot of inferred $\psi(q)$



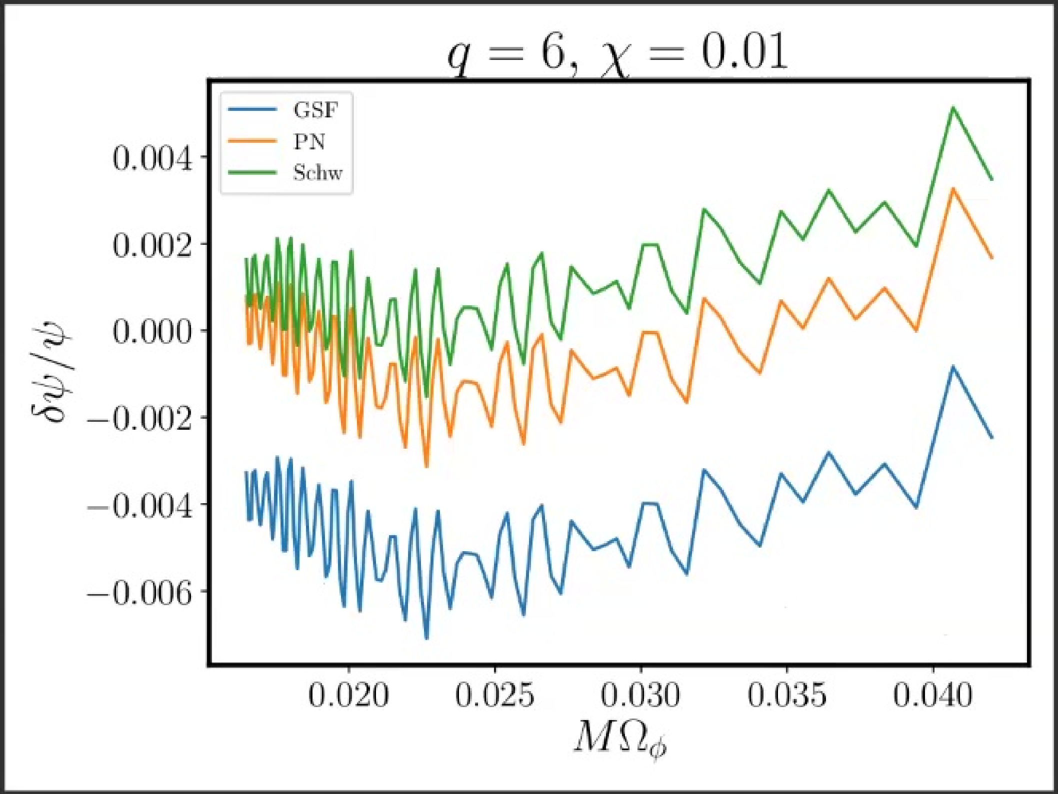
Future Directions

- Investigate residual oscillations
- Is γ useful?

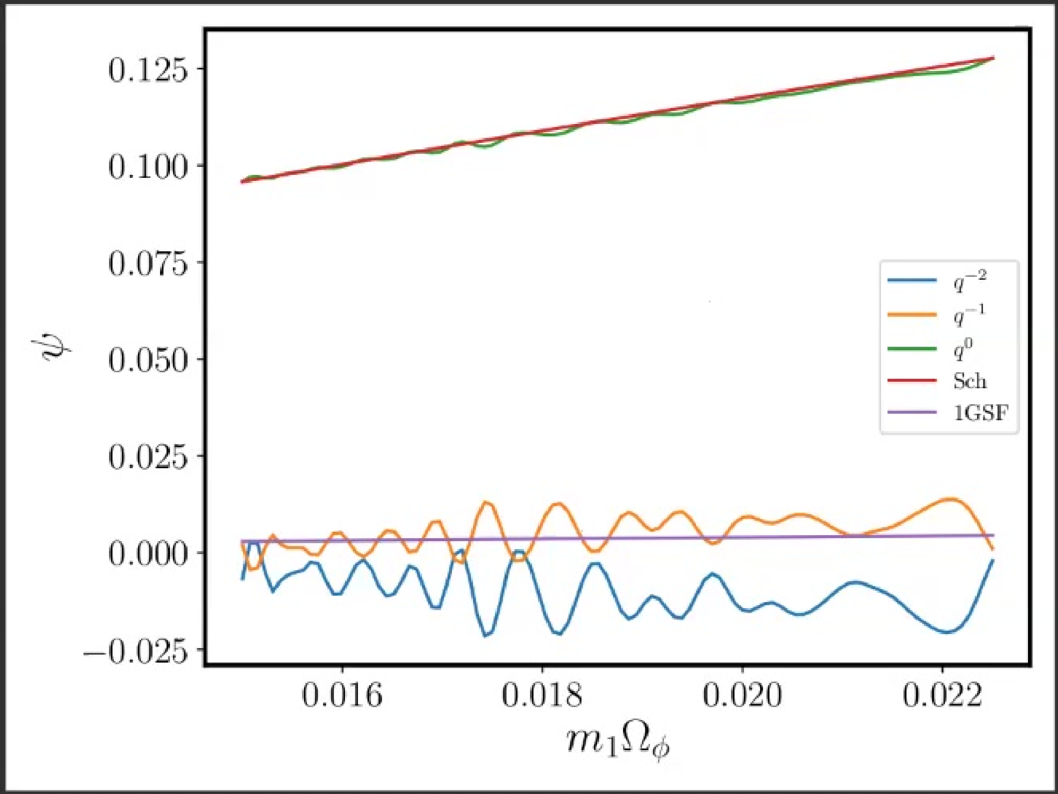
$$\gamma = \sqrt{\frac{\Omega_\phi - \Omega_p^-}{\Omega_\phi - \Omega_\phi^+}}$$

- Simulate more mass ratios (would allow a verification/refinement of our second-order ψ_{GSF} result)
- Comparison to other approaches (Owen et al. 2017)

Comparing NR ψ to PN and SF



Plot of inferred $\psi(q)$



Future Directions

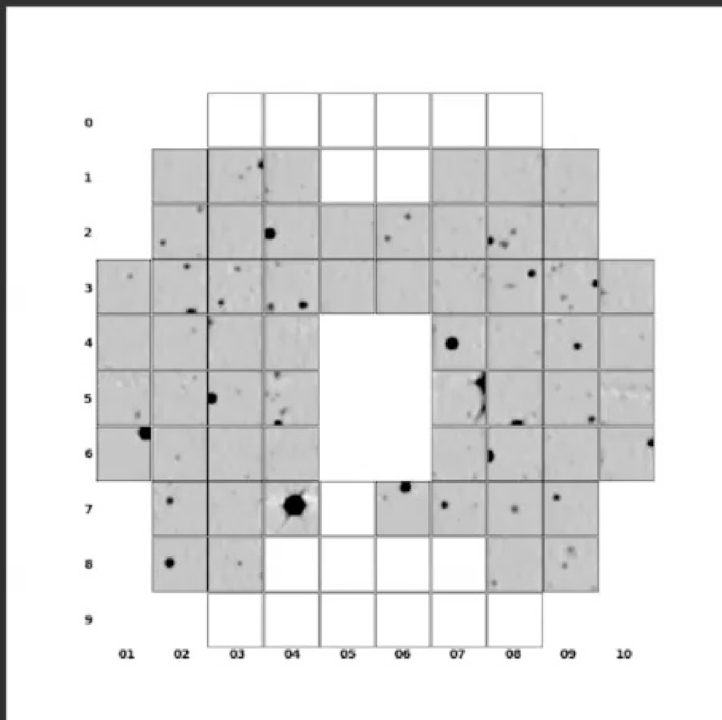
- Investigate residual oscillations
- Is γ useful?

$$\gamma = \sqrt{\frac{\Omega_\phi - \Omega_p^-}{\Omega_\phi - \Omega_\phi^+}}$$

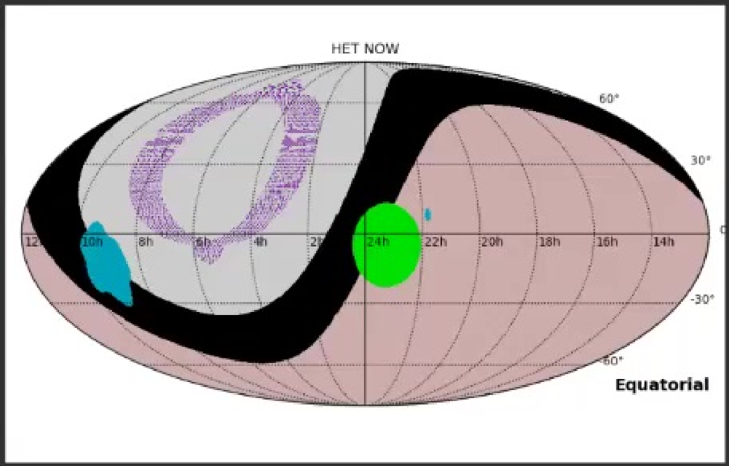
- Simulate more mass ratios (would allow a verification/refinement of our second-order $\psi_{\rm GSF}$ result)
- Comparison to other approaches (Owen et al. 2017)

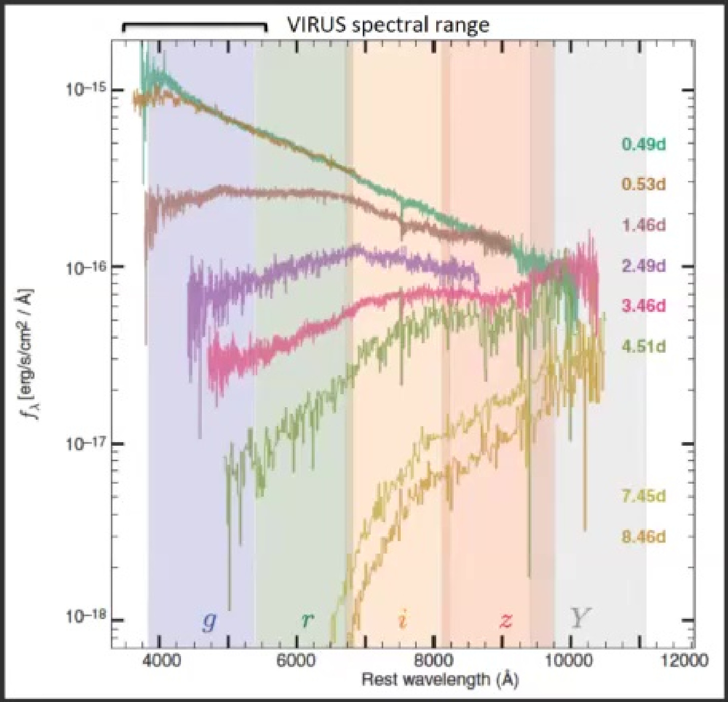
Other Projects

LIGHETR - EM-followup

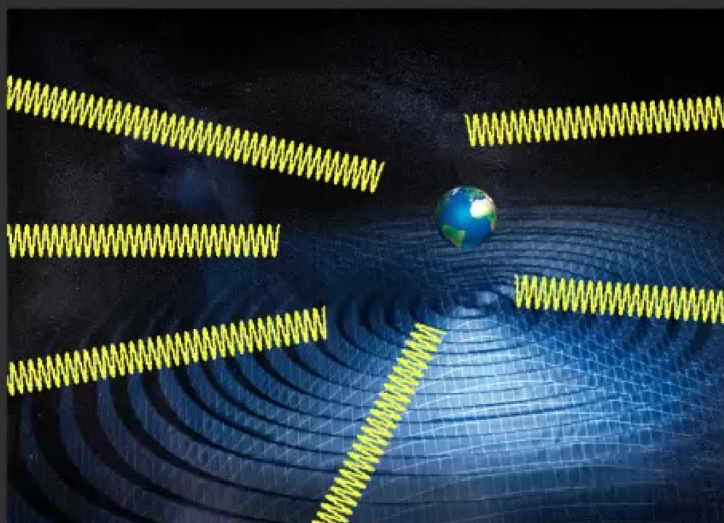


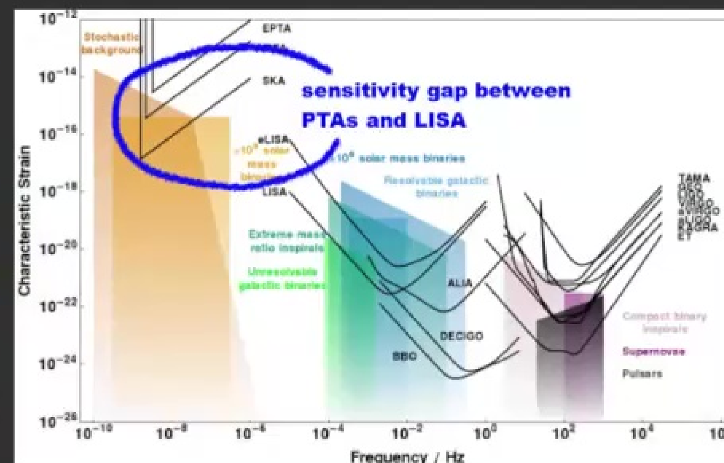
LIGHETR - EM-followup





GWTA - Measuring GWs with GWs





Thanks!

Thanks! Any questions?

PN expansion

Let $y = (M\Omega)^{2/3}$, then

$$\begin{aligned}\psi_{\rm PN} = &\frac{3}{2}y + \frac{9}{8}y^2 + \frac{27}{16}y^3 \\ &+ \frac{1}{q}\left(y^2 - 3y^3\right) \\ &+ \frac{1}{q^2}\left(-\frac{y}{3} + \frac{2}{3}y^2 + \frac{53}{8}y^3\right) \\ &+ \mathcal{O}(y^4, q^{-3})\end{aligned}$$