Title: 2D Holography beyond JT

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Abstract: Dilaton-gravity models are integrable in two dimensions and admit a holographic description. In this talk, the holographic description of the Dilaton-gravity in flat spacetime is discussed. Using the gauge theory formulation of the model we obtain the boundary action which under certain boundary conditions is of the Warped-Schwarzian type. We calculate the 1-loop partition function of the model as the coadjoint orbit of the warped Virasoro group.

2D Holography beyond JT

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Outlook

- Introduction and motivation
- JT gravity and the Schwarzian action
- Warped Schwarzian action
- Flat space analogue of JT gravity

Motivation

- How/if Holography works beyond AdS.
- Construct models of quantum gravity in Flat spacetime.
- Low dimensions can help.
- Extend the JT/SYK correspondence to the case of flat.
- What is the flat analogue of the Schwarzian action.

Introduction

- 2D Dilaton-gravity is rich enough to accommodate holographic features. The famous example is the JT/SYK correspondence.
- It can provide simple models for semi-classical BH formation/evaporation. The developments associated to the page curve.
- It is exactly solvable.

2D gravity

- 2D Dilaton-gravity can be considered as dimensional reduction from higher dimensions where the Dilaton plays the role of radial direction $R = \ell e^{-\phi}$;

$$ds_d^2 = ds_2^2 + e^{-2\phi} d\Omega_{d-2}^2$$

- In the near horizon limit

$$R \sim r_H + r\,, \qquad e^{-2\phi} \sim e^{-2\phi_H}(1 + X(r,t))$$

- Expand and keep only $\mathcal{O}(X)$

$$S_{\text{JT}} = S_0 + \frac{r_H^2}{2} \int d^2x \sqrt{-g}\, X(R + r_H^{-2}) \qquad \text{AdS}$$

$$S_{\text{CGHS}} = S_0 + \frac{r_H^2}{2} \int d^2x \sqrt{-g}\, (XR + r_H^{-2}) \qquad \text{Flat}$$

1D Hologram

- The holographic description of the near horizon demands finite temperature in the boundary and a Euclidean time

$$\tau \sim \tau + \beta$$

- All fields of the system living on the circle is in some representation of $Diff(S^1)$. They are arranged as vectors, functions, ... A diffeomorphism f itself is a vector and a scalar field g is a function;

$$f(\tau) \in Vec(S^1)\,, \qquad g(\tau) \in C^\infty(S^1)\,, \quad \cdots$$

- These fields form a centrally extended symmetry group $\hat{G}(S^1)$ on the circle e.g. Virasoro, Warped Virasoro, ...

1D Hologram

- Semi-classical holographic degrees of freedom usually have a description in terms of Goldstone modes of the broken symmetry to a global subgroup G_0.

$$f(\tau), g(\tau), \cdots \in \mathcal{M} = \frac{\hat{G}(S^1)}{G_0(S^1)}$$

- These quotient subspaces are called *coadjoint orbits* of the group and can be constructed and studied systematically. The full semi-classical description of the model is in terms of a partition function which turns out to be one-loop exact; `Stanford-Witten-2017`

$$Z = \int_{\mathcal{M}} Df Dg \cdots \exp\left(\tfrac{i}{\hbar} I[f, g, \cdots]\right) = e^{\frac{i}{\hbar} I_{on-shell}} Z_{one-loop}$$

Schwarzian theory

$$I_{Schw} = \oint \left(-T_0^c f'^2 + \{f;\tau\}\right) , \qquad \{f;\tau\} = \left[\tfrac{f''}{f'}\right]' - \tfrac{1}{2}\left[\tfrac{f''}{f'}\right]^2$$

- In order to perform the 1-loop path integral we need two ingredients; the action and the measure of integration which both can be obtained by the fact that coadgoint orbits are simplectic manifolds. For the case of Virasoro group

$$\hat{G} = Diff(S^1) \times \mathbb{R}$$

- When G_0 is the maximal 3 dim global subgroup $SL(2,R)$ corresponding to symmetries of the the hyperbolic disk

$$T_0^c = -\frac{\pi^2 c}{6\beta^2} , \qquad Z_{one-loop} \sim \frac{1}{\beta^{3/2}}$$

Warped Schwarzian theory Afshar-2019

$$I_{WSchw} = I_{Schw} + \oint \left[T_0^\kappa f'^2 + \left(\frac{f''}{f'} + P_0^\kappa f'\right) g' - \kappa g''\right] + \oint (k\, g'^2 + P_0^k g' f' + T_0^k f'^2)$$

- The Warped-Virasoro group is the group of diffeomorphism of the circle acting naturally on smooth functions of the circle

$$\hat{G} = Diff(S^1) \ltimes C^\infty(S^1) \times \mathbb{R}^3$$

- when G_0 is the maximal 4 dim global subgroup either $SL(2,R) \times U(1)$ or $ISO(2)_c$ we get

$$T_0^\kappa = \alpha P_0^\kappa = -\frac{2\pi i \kappa \alpha}{\beta}, \qquad T_0^k = 2\alpha P_0^k = \frac{\alpha^2 k}{4}, \qquad Z_{one-loop} \sim \frac{1}{\beta^2}$$

Example: Jackiw-Teitelboim gravity

In the first order form JT-model can be written as a BF theory of SL(2,$\mathbb{R}$).

$$S_{JT} = \int [X_a(de^a + \epsilon^a{}_b \omega e^b) + X(d\omega - \tfrac{1}{2}\lambda\, \epsilon^{ab} e_a e_b)] + I_B = \int \langle B, F\rangle + I_B$$

where $A = e^a P_a + \omega J$ and $B = X^a P_a + XJ$ and

$$[P_a, J] = \epsilon_a{}^b P_b\,, \quad [P_a, P_b] = -\epsilon_{ab} J$$

with the Killing form $\langle P_a, P_b\rangle = \eta_{ab}$ and $\langle J, J\rangle = 1$.

- The bulk term is zero and the boundary term is determined by imposing AdS_2 boundary conditions and requiring having a well-defined variational principle. The boundary term is the 1D Schwarzian action.

Cangemi-Jackiw construction

In the first order form we can write a BF theory for centrally extended Poincaré algebra.

$$S_{\widehat{CGHS}} = \int \left[X_a(de^a + \epsilon^a{}_b \omega e^b) + X d\omega - Y(dC + \tfrac{1}{2}\epsilon_{ab}e^a e^b)\right] = \int \langle B, F\rangle$$

where $A = e^a P_a + \omega J + CZ$ and $B = X^a P_a + YJ + XZ$ and

$$[P_a, J] = \epsilon_a{}^b P_b\,, \quad [P_a, P_b] = \epsilon_{ab} Z$$

with the bilinear form $\langle P_a, P_b\rangle = \eta_{ab}$ and $\langle J, Z\rangle = -1$. On-shell $\widehat{CGHS}$ and $CGHS$ are equivalent

$$S_{\widehat{CGHS}} = \frac{1}{2}\int \sqrt{-g}(XR - 2Y) + \int Y dC$$

Asymptotic symmetries in 2D Flat

- A general solution

$$X = x_1(u)r + x_0(u)\,, \qquad C = r du\,, \qquad ds^2 = -2(r\,P(u) - T(u))du^2 - 2du dr$$

- These solutions are *preserved* by the diff

$$\xi = \epsilon(u)\,\partial_u - \left(\epsilon'(u)r + \sigma'(u)\right)\partial_r$$

The (P, T) transform infinitesimally as;

$$\delta_\xi P = (\epsilon P)' - \epsilon''\,, \qquad \delta_\xi T = \epsilon T' + 2\epsilon' T + \sigma'' + \sigma' P$$

- What is the group of these symmetries?

Warped Virasoro algebra

The Fourier mode generators on the circle

$$L_n = \oint T(\tau)\, e^{\frac{2\pi}{\beta} i n \tau}\,, \qquad P_n = \oint P(\tau)\, e^{\frac{2\pi}{\beta} i n \tau}$$

satisfy the twisted warped-Witt algebra;

$$\begin{aligned}
[L_n, L_m] &= (n-m) L_{n+m}\,, \\
[L_n, P_m] &= -m P_{n+m} - i\kappa (n^2 - n)\delta_{n+m,0}\,, \\
[P_n, P_m] &= 0\,.
\end{aligned}$$

The $\widehat{CGHS}$ model with appropriate boundary conditions is a Warped-Schwarzian theory.

Warped Conformal symmetry *Detournay Hartman Hofman 2012*

From the 2D point of view they are symmetries of a non-relativistic conformal field theory. In general it admits three non-trivial cocycles. *Afshar Grumiller Detournay Oblak 2015*

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}(n^3 - n)\delta_{n+m,0}\,,$$
$$[L_n, P_m] = -mP_{n+m} - i\kappa(n^2 - n)\delta_{n+m,0}\,,$$
$$[P_n, P_m] = kn\delta_{n+m,0}\,.$$

Warped Virasoro group

$$\delta_\xi P = (\epsilon P)' - \epsilon'' , \qquad \delta_\xi T = \epsilon T' + 2\epsilon' T + \sigma'' + \sigma' P$$

What is the finite form of these transformations? The following finite transformations work;

$$\tilde{P}(f(\tau)) = \frac{1}{f'(\tau)} \left[P(\tau) + \frac{f''(\tau)}{f'(\tau)} \right]$$

$$\tilde{T}(f(\tau)) = \frac{1}{f'(\tau)^2} \left[T(\tau) - P(\tau) g'(\tau) - g''(\tau) \right]$$

where $f(\tau) \simeq \tau + \epsilon(\tau)$ and $g(\tau) \simeq \sigma \circ f(\tau) \simeq \sigma(\tau)$.

Boundary action

We derive the boundary action in the BF formulation.

- Translating boundary conditions in the BF-gauge theory up to a gauge transformation

$$A_r = 0$$
$$A_\tau = P_- + T(\tau)P_+ + P(\tau)J$$
$$B = [x_0' + Tx_1]P_+ + [x_1' + Px_1]J + x_1 P_- + x_0 Z$$

- We aim to cancel the boundary term which remains after variation of a boundary action. The variation of the BF-action is

$$\delta I_{BF} = -\oint \langle B, \delta A \rangle = -\oint (x_1(\tau)T(\tau) - x_0(\tau)P(\tau))$$

Boundary action

$$\delta I_{BF} = -\oint \langle B, \delta A \rangle = -\oint \left(x_1(\tau) T(\tau) - x_0(\tau) P(\tau) \right)$$

Using the field equations can write this boundary term in terms of linear and bilinear Casimirs

$$\oint \left(\delta \frac{C_1}{x_1} + C_1 \delta \frac{1}{x_1} - C_0 \delta \frac{x_0}{x_1} + \delta x_0' - [x_0 \delta \ln x_x]' \right)$$

where

$$C_0 = Y = x_1' + P x_1 \,, \qquad C_1 = \frac{1}{2} \langle B, B \rangle = x_0 C_0 - (x_0' + T x_1) x_1$$

Boundary action

$$\oint \left(\delta \frac{C_1}{x_1} + C_1 \delta \frac{1}{x_1} - C_0 \delta \frac{x_0}{x_1} + \delta x_0' - [x_0 \delta \ln x_x]' \right)$$

This boundary term is integrable provided that the following zero modes are fixed

$$f = \oint \frac{1}{x_1} \qquad g = \oint \frac{x_0}{x_1}$$

These (quasi)-periodic functions should have fixed periodicity. Plugging in we have

$$I_B = -\oint df\, C_1$$

which reproduces the Warped-Schwarzian action at level zero.

$$I_{WSchw} = \cancel{I_{Schw}} + \oint \left[T_0^{\kappa} f'^2 + \left(\frac{f''}{f'} + P_0^{\kappa} f' \right) g' - \kappa g'' \right]$$

Thank you for your time. Questions?