

Title: Dynamics and observational traces of cosmological ultra-supercooled phase transitions

Speakers: Ryusuke Jinno

Series: Particle Physics

Date: December 11, 2020 - 11:00 AM

URL: <http://pirsa.org/20120015>

Abstract: In recent years, there has been growing interest in cosmological first-order phase transitions in view of gravitational wave observations with space interferometers such as LISA. However, there is only limited understanding on the bubble dynamics and the gravitational wave signals arising from ultra-supercooled transitions (in which the released energy dominates the plasma energy, i.e., near-vacuum transitions), due to the highly relativistic nature of the transition.

In this talk, I introduce some approaches to understand the dynamics and the gravitational wave signals of ultra-supercooled first-order phase transitions:

- (1) These transitions proceed with the propagation and collision of highly relativistic fluid profiles involving shock waves. I introduce an approach to construct an effective description of the propagation of such relativistic profiles (1905.00899).
- (2) I present an approach to extend the existing model of gravitational wave production and calculate the gravitational wave signals analytically (1707.03111).

Dynamics and observational traces of cosmological ultra-supercooled phase transitions

Ryusuke Jinno (DESY)

12.11.2020 @ Perimeter Institute

1707.03111 ([R. Jinno](#), M. Takimoto)

1905.00899 ([R. Jinno](#), H. Seong, M. Takimoto, C.M. Um)

Self introduction



■ Career

Ph.D. (2016) @ University of Tokyo → KEK (Japan) → IBS-CTPU (Korea) → DESY (Germany)

■ Research interests

Gravitational waves & New physics

{ First-order phase transitions
Spectral deformation [Domcke, Jinno, Rubira '20]
Inflation, (p)reheating
Imprints of light dof

Higgs dynamics in the early Universe

e.g. Unitarity violation in Higgs inflation during preheating [Jinno '16] (thesis) & [Ema, Jinno, Mukaida, Nakayama '16]

Machine learning & Neural networks

e.g. Proposal of the calculation of bounce action as image recognition [Jinno '18]

Tunneling & Gravity

e.g. Infinite # of negative modes in CDL bounce [Jinno & Sato '20]

Outline of the talk

- In the coming decades, we have the opportunity to test first-order phase transitions in the early Universe with gravitational waves (GWs)
- However, in *extremely* strong transitions, it's hard to predict the GW signal (albeit they are both theoretically and observationally interesting)
- I introduce some researches addressing this problem

Gravitational waves: a new probe to the Universe

- Gravitational waves (GWs)

Transverse-traceless part of the metric perturbation

$$ds^2 = -dt^2 + a^2(\delta_{ij} + h_{ij})dx^i dx^j$$

Obeys a wave equation sourced by
the energy-momentum tensor of the system

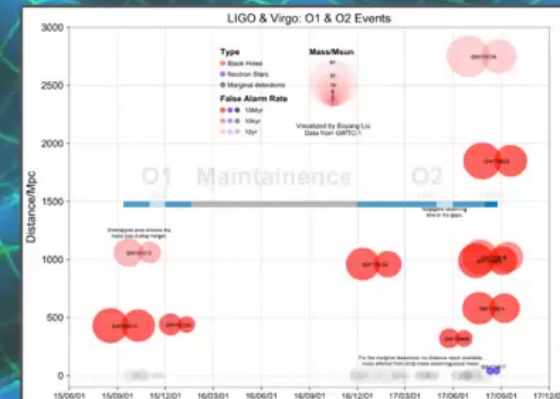
$$\square h_{ij} \sim GT_{ij}$$

- Detections by LIGO & Virgo collaboration
have been exciting us

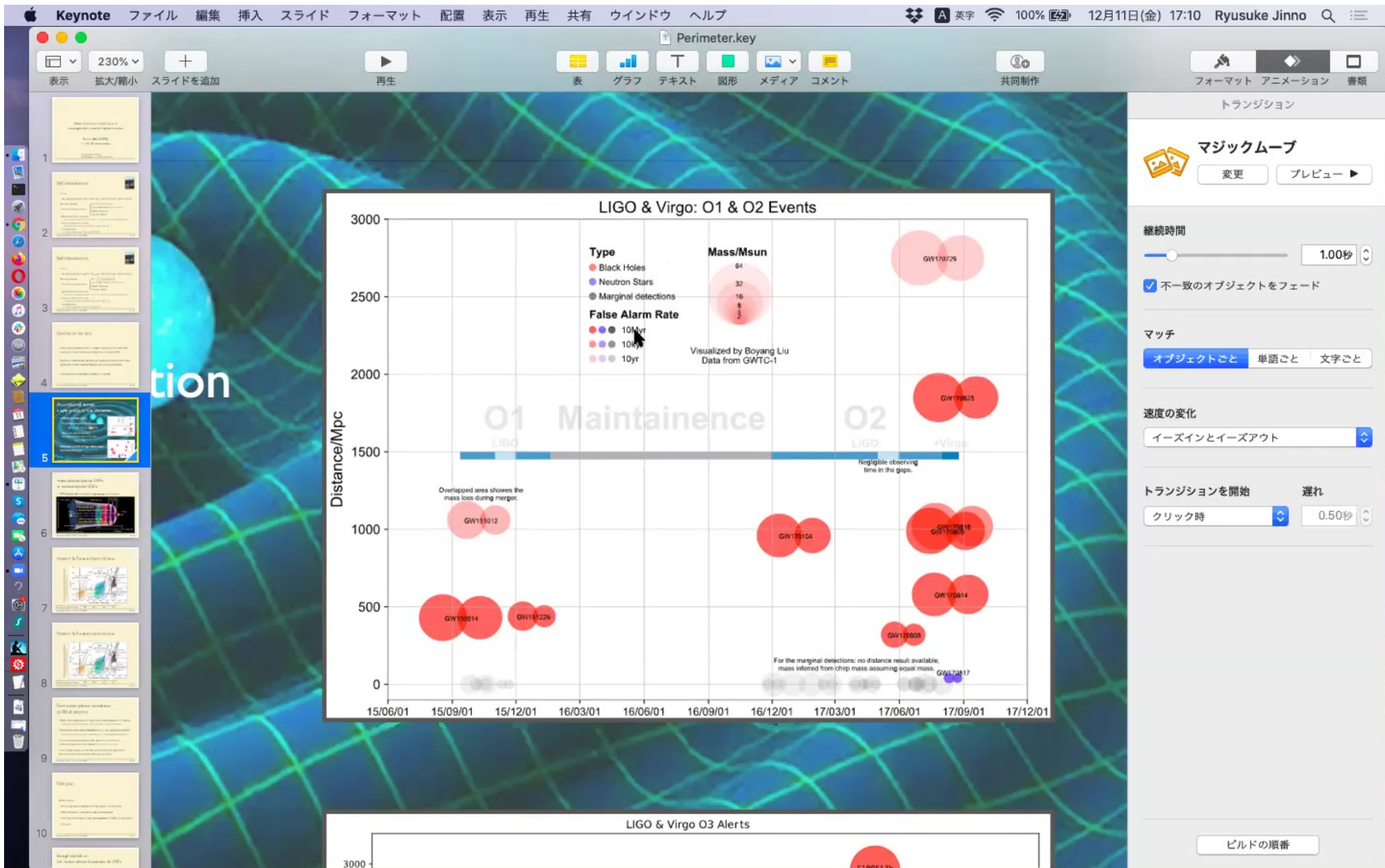
[Wikipedia "List of gravitational wave observations"]

[see also <https://gracedb.ligo.org/superevents/public/O3/>]

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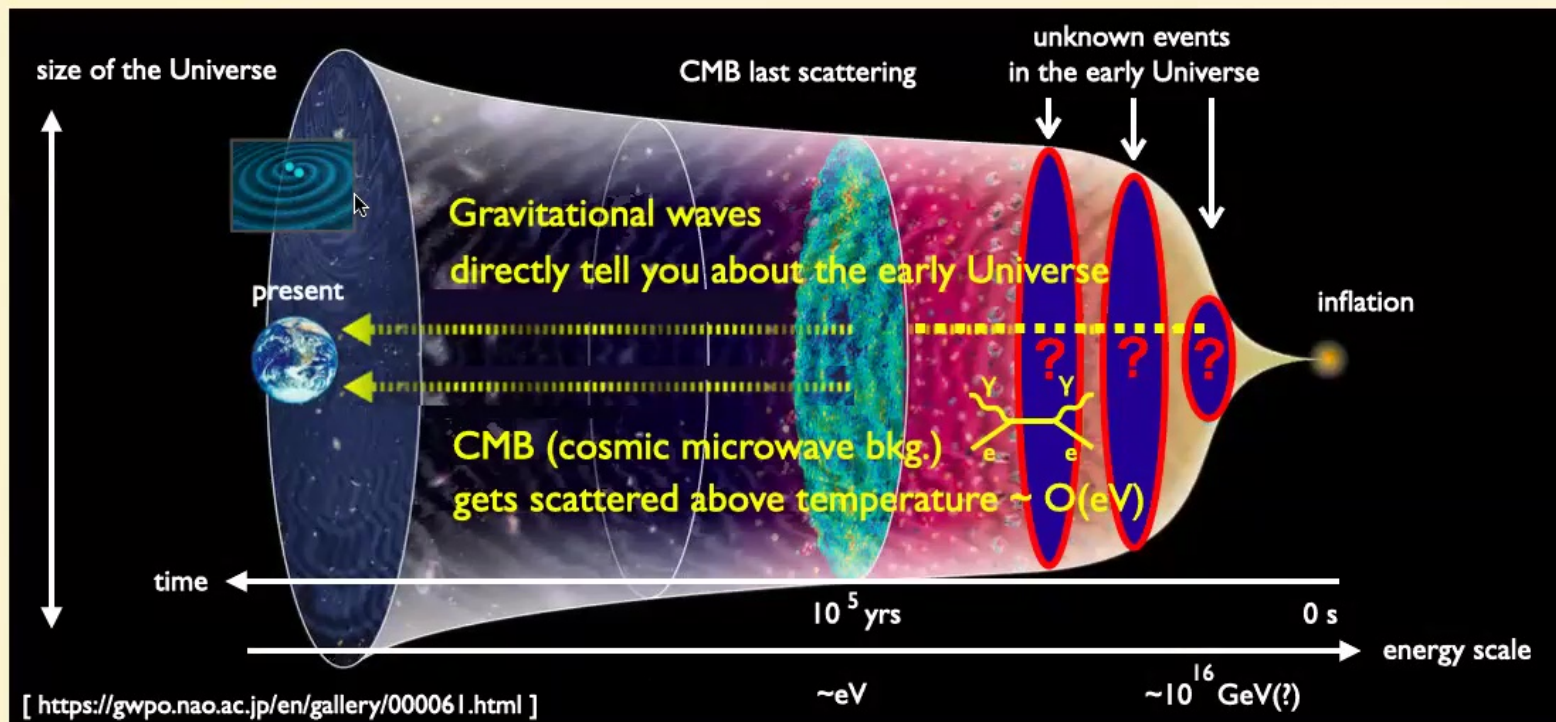


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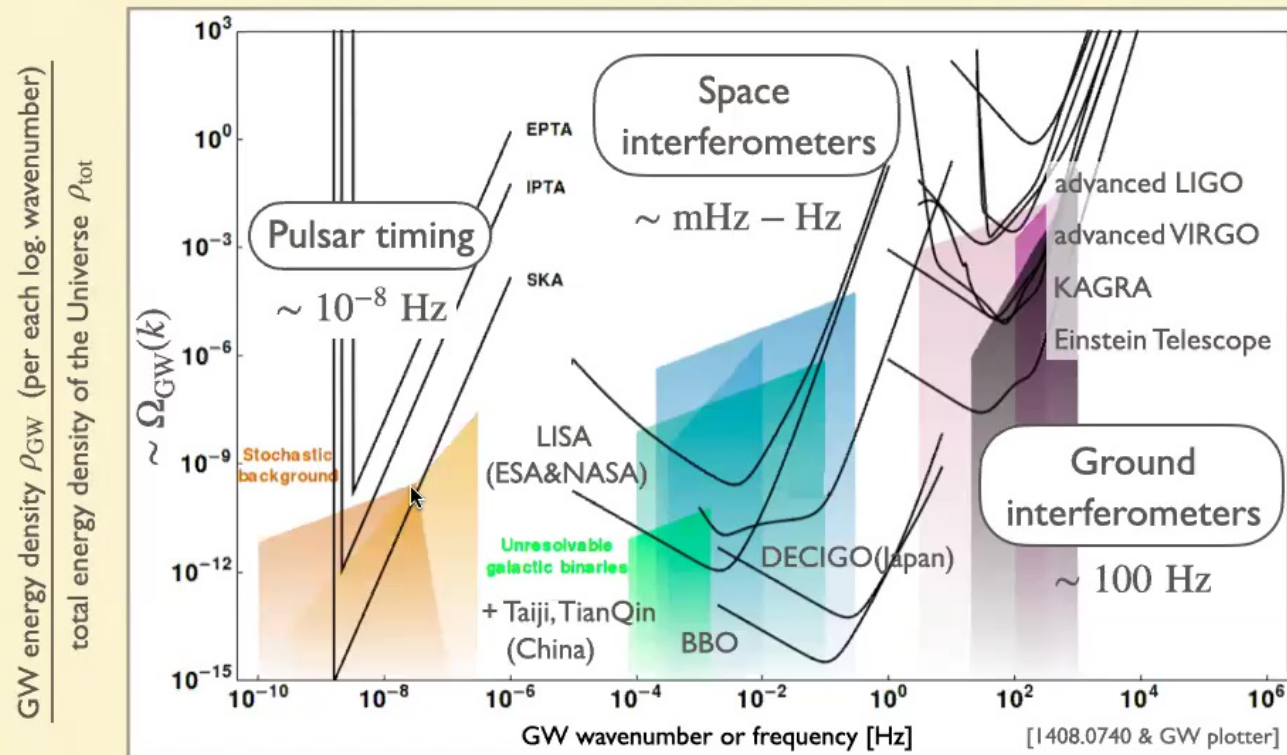


From astrophysical GWs to cosmological GWs

- GWs directly tells us about the high-energy early Universe



Present & Future observations



Temperature of the Universe @ GW prod.
(if GWs are produced with horizon/1000)

MeV

GeV

TeV

PeV

Ryusuke Jinno (DESY) / 1707.03111, 1905.00899

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First-order phase transitions in SM & beyond

- Within the standard model, the electroweak phase transition is a crossover

[Kajantie, Laine, Rummukainen, Shaposhnikov '96] [Gurtler, Ilgenfritz, Schiller '97] [Csikor, Fodor, Heitger '98]...

- However, first-order phase transitions occur in many extensions of the SM

[Giudice '92] [Espinosa, Quiros, Zwirner '93]... [Randall, Servant '06]... [Profumo, Ramsey-Musolf, Shaughnessy '07]...

- First-order phase transitions provide a possible explanation for the baryon asymmetry of the Universe [Kuzmin, Rubakov, Shaposhnikov '85]

- In the coming decades, we have chances to observe GW signals from this process with space interferometers such as LISA

Talk plan

✓ 1. Introduction

2. First-order phase transitions & GW production: A brief review

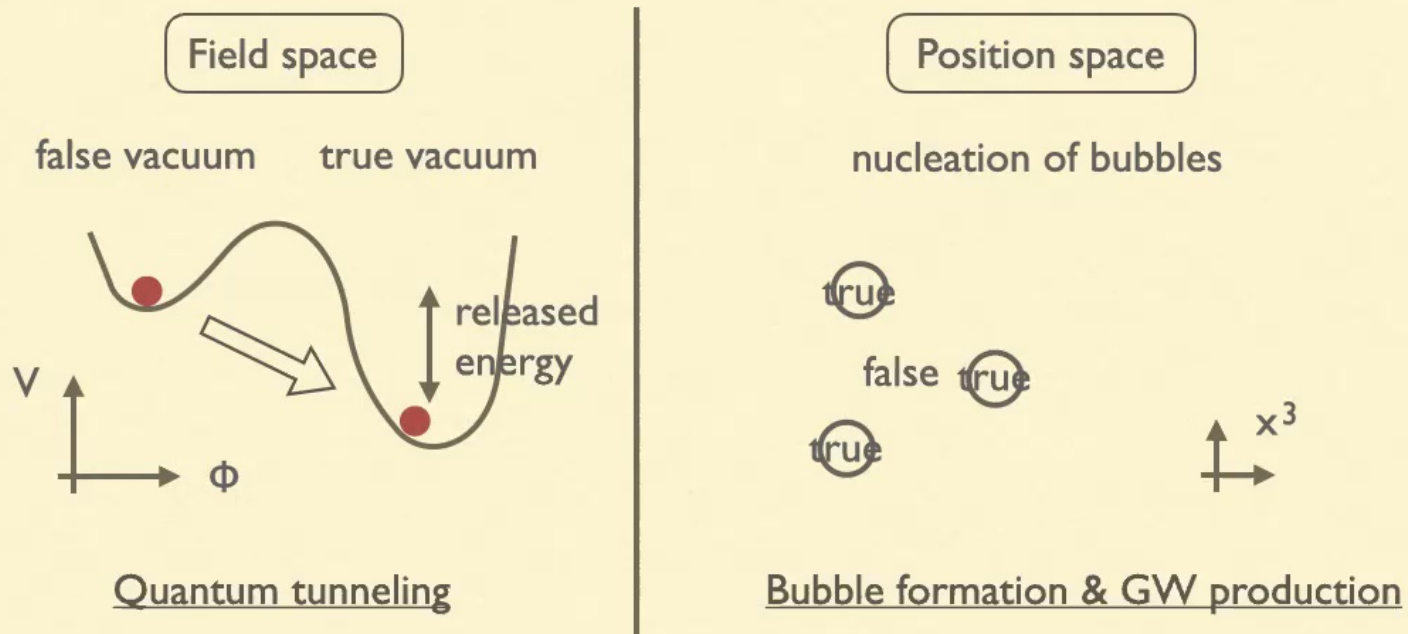
3. Bubble dynamics in extremely strong phase transitions

4. GW signal in extremely strong phase transitions: Possible IR enhancement

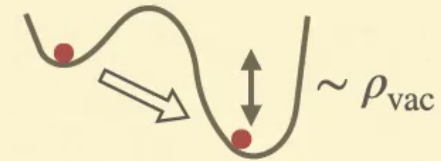
5. Summary

Rough sketch of 1st-order phase transition & GWs

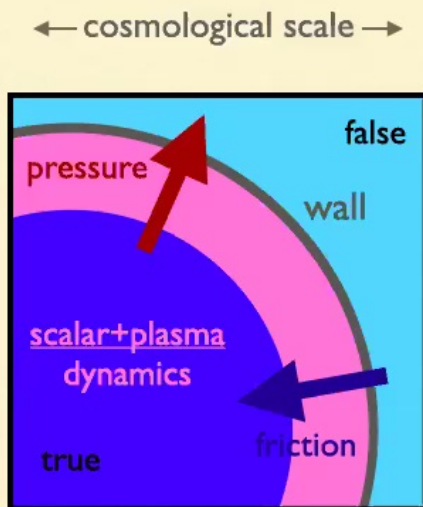
- Bubbles nucleate, expand, collide and disappear, involving fluid dynamics



Bubble dynamics before collision



- "Pressure vs. friction" determines behavior of bubble walls



Pressure: released energy pushes the wall outwards

Parametrized by $\alpha \equiv \frac{\rho_{\text{vac}}}{\rho_{\text{plasma}}}$

[see e.g.
Espinosa et al. '10
Hindmarsh et al. '15
Giese et al. '20
for various definitions]

Friction: plasma particles push back the wall

(note: plasma particles exist everywhere)

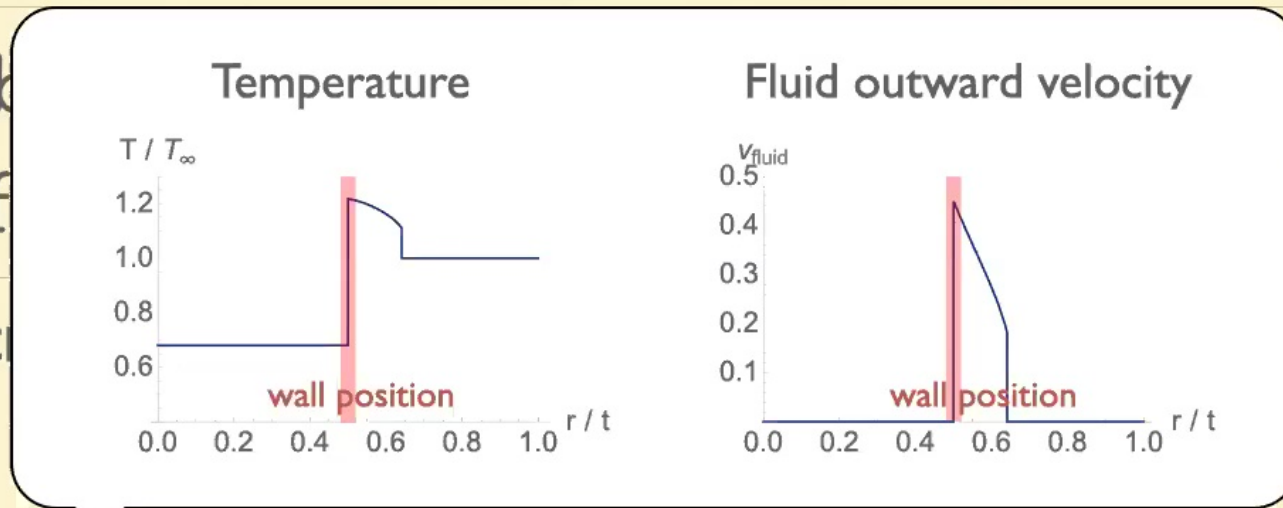
Parametrized by coupling η btwn. scalar and plasma

Let's see how bubbles behave for different α

(with fixed coupling η)

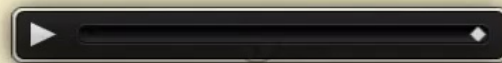
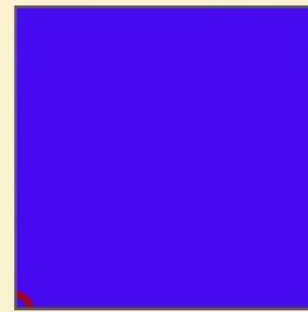
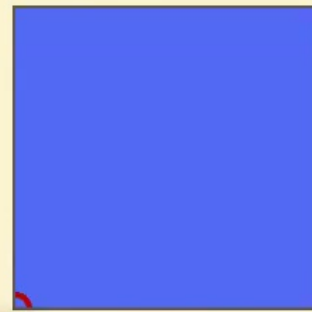
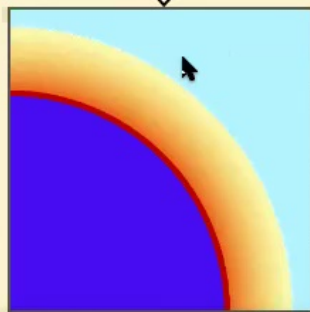
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la
Servant '10]
de Vis '20]

le plasma



deflagration



②
detonation

~ 1

③
strong detonation

$\gg 1$

④
runaway

α

The GW spectrum (may) differ qualitatively, not quantitatively

Bubble dynamics before collision

$$\alpha \equiv \frac{\rho_{\text{vac}}}{\rho_{\text{plasma}}}$$

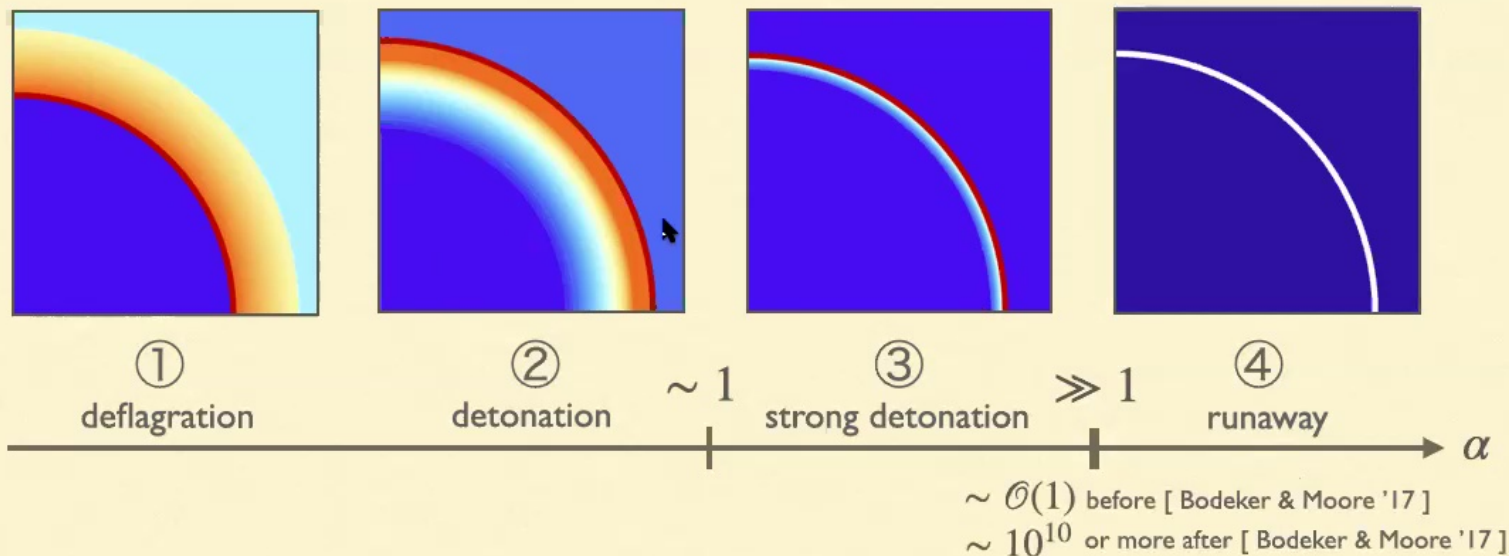
[Espinosa, Konstandin, No, Servant '10]

[Giese, Konstandin, van de Vis '20]

■ Classification of bubble expansion modes

Walls reach **terminal velocity**
due to the balance btwn. pressure & friction

Walls **runaway**
without caring about the plasma

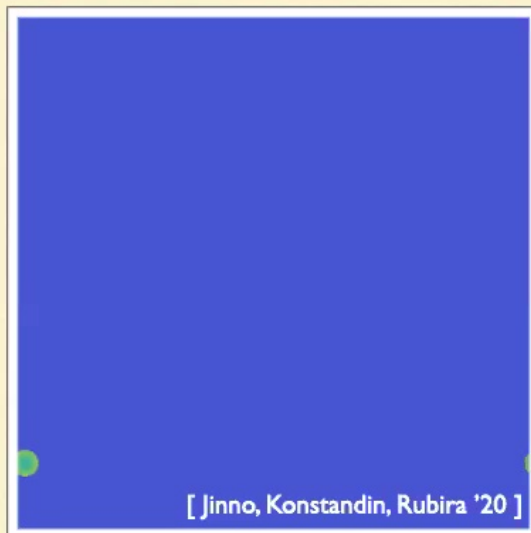


The GW spectrum (may) differ qualitatively, not quantitatively

GW production in weak transitions



- So far, just bubble dynamics before collision has been discussed.
But GW signal comes from bubble dynamics after collision.
- For relatively weak transitions (①&②), the dynamics is more or less known:



- Fluid velocity field overlaps (linearly) everywhere
- The overlap effect works as a long-lasting source of GWs

[Hindmarsh, Huber, Rummukainen, Weir '13, '15, '17]

[Hindmarsh '16, Hindmarsh & Hijazi '19]

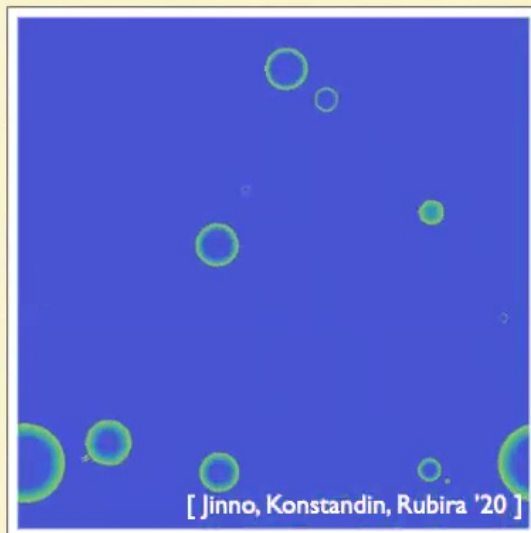
- We proposed a new efficient scheme to calculate the GW spectrum
(Talk yesterday by Henrique)



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[Jinno, Konstandin, Rubira '20]

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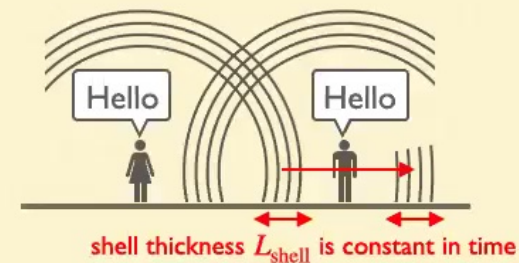
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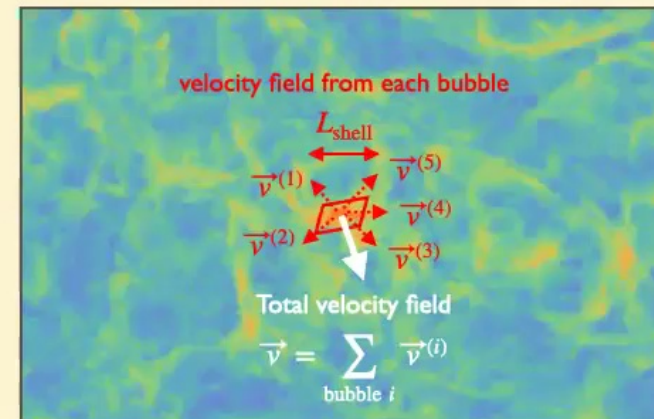
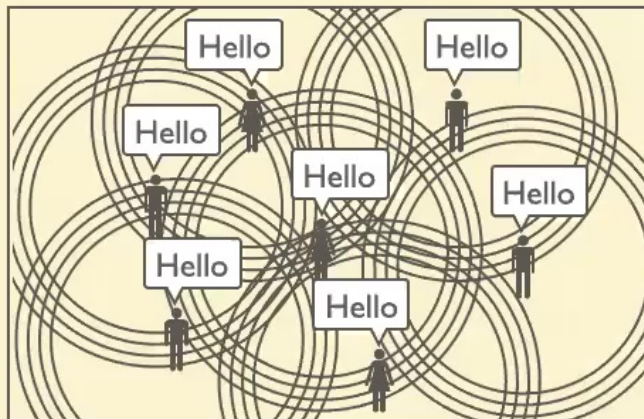
GW enhancement from overlapping shells



- Sound shells propagate inside other bubbles



- Shells overlap, resulting in continuous GW production at wavelength $\lambda \sim L_{\text{shell}}^{-1}$



GW production in extremely strong transitions



- In extremely strong transition (③), bubble dynamics after collision is less known

because $\left\{ \begin{array}{l} \text{high relativisticity of the fluid} \\ \text{shock waves} \\ \text{hierarchy in scales} \end{array} \right\}$ make numerical simulations difficult

even though they are observationally & theoretically interesting

- Central question is: timescale for the energetic shells to break up



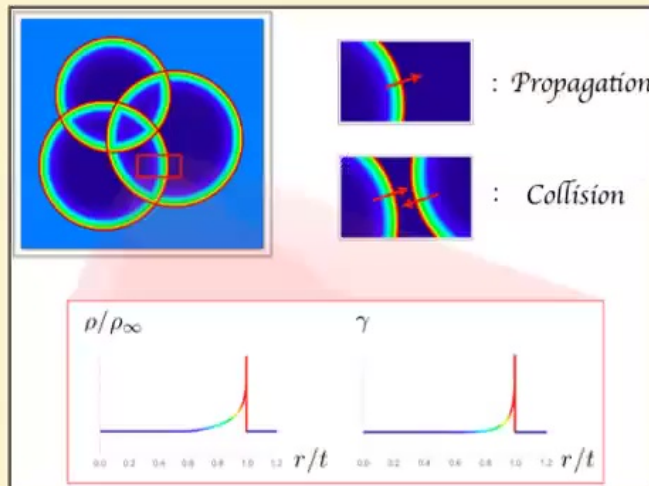
because this can change the GWs produced at $\lambda \sim L_{\text{shell}}^{-1}$ by orders of magnitude

Talk plan

- ✓1. Introduction
- ✓2. First-order phase transitions & GW production: A brief review
3. Bubble dynamics in extremely strong phase transitions
4. GW signal in extremely strong phase transitions: Possible IR enhancement
5. Summary

Reducing the problem

- Let's divide the problem into smaller pieces:



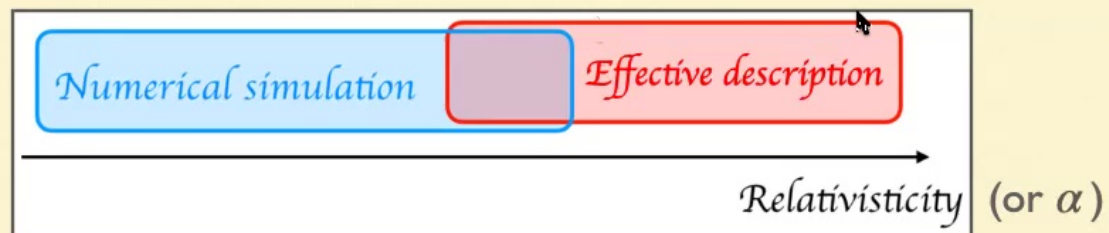
After collision, the relativistic fluid shells
(1) propagate inside other bubbles
(2) collide with other shells

- We study (1) propagation effect, leaving (2) as future work, because propagation alone is already nontrivial due to the nonlinearity in fluid equation

Main idea

- Effective description with a few relevant variables

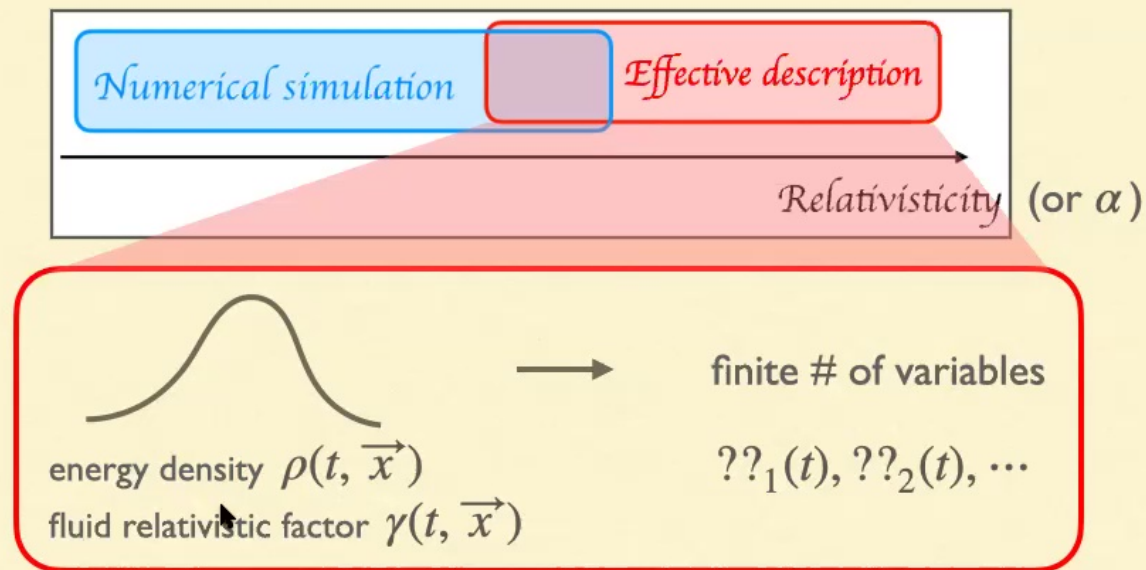
We construct an effective description of fluid propagation with a few variables which is valid in highly relativistic limit (i.e. strong limit of the transition)



Main idea

- Effective description with a few relevant variables

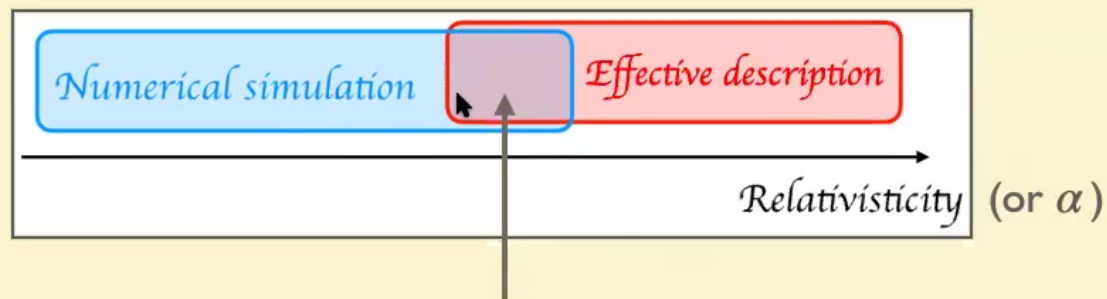
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Main idea

- Effective description with a few relevant variables

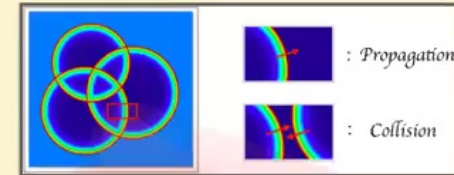
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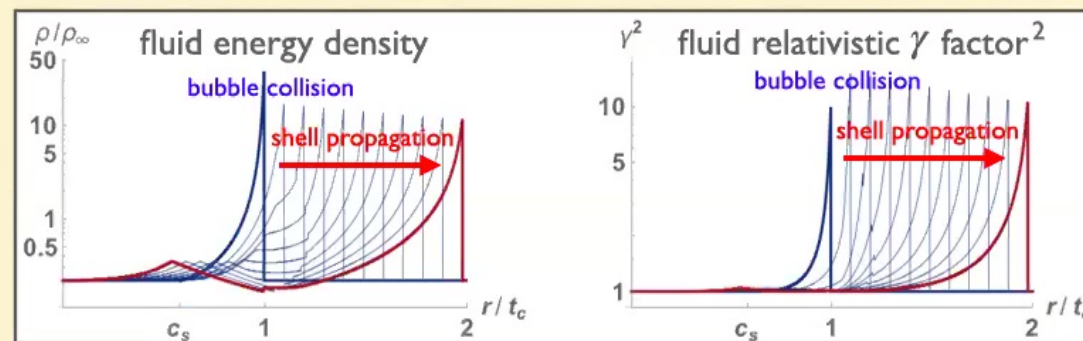
and check its validity against numerical simulation in mildly-relativistic regime

- I show the result of numerical simulation first, to show what's going on

Effective description of relativistic fluid propagation



■ Numerical simulation



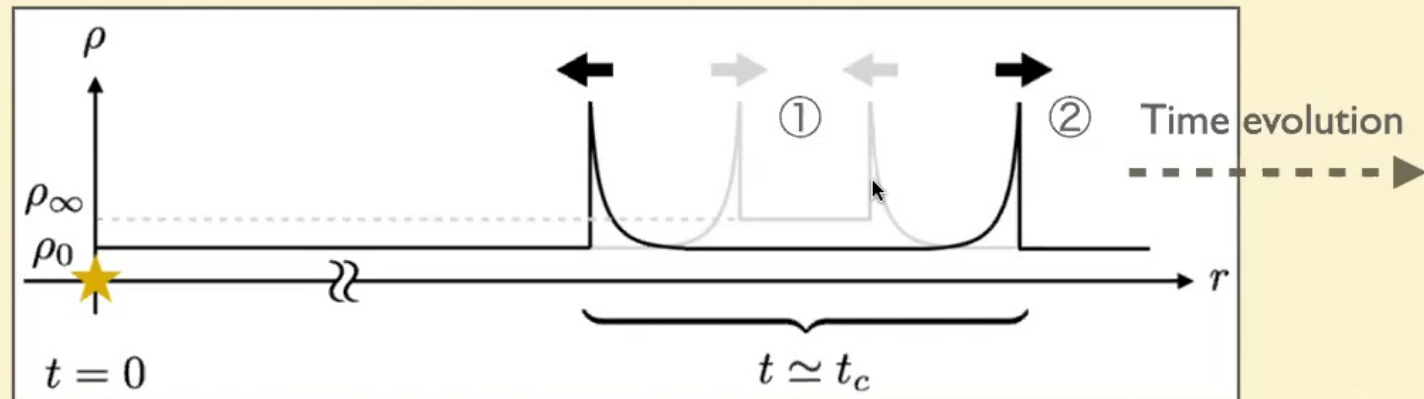
Assumption: perfect fluid $T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p\eta_{\mu\nu}$ & relativistic eos $\rho = 3p$

■ What would be the relevant variables?

- From the viewpoint of GW production, we are interested only in the peak since it dominates the energy
- So, the candidates are quantities characterizing the peak

Initial condition

- The setup we study

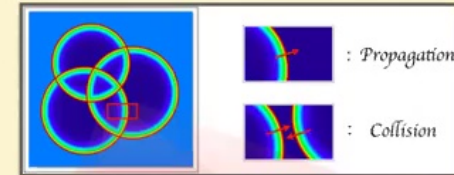


① Fluid profile just before collision: calculated from [Espinosa, Konstandin, No, Servant '10]

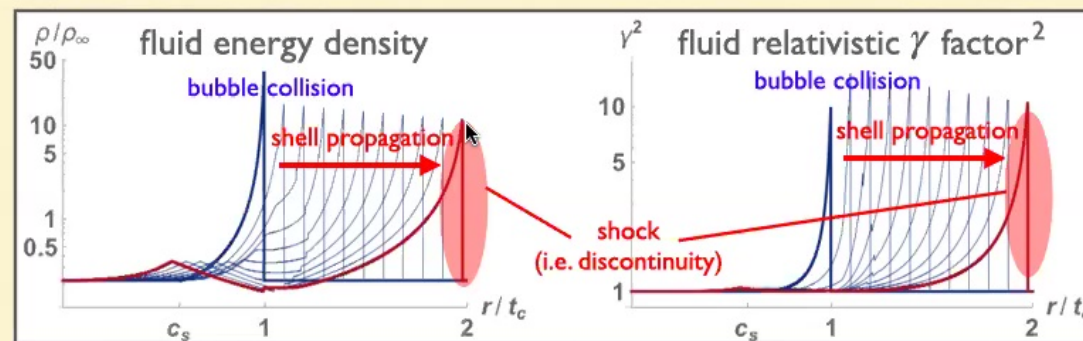
↓ Assumption: the first fluid collision does not change the profile significantly

② Fluid profile just after collision: our interest is in the time evolution from here

Effective description of relativistic fluid propagation



■ Numerical simulation

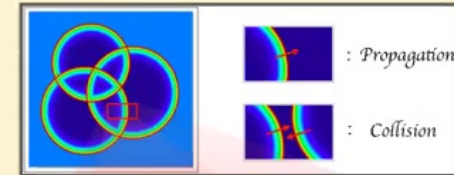


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■ What would be the relevant variables?

- From the viewpoint of GW production, we are interested only in the peak since it dominates the energy
- So, the candidates are quantities characterizing the peak

Effective description of relativistic fluid propagation



■ 5 variables characterizing the peak shape

1) Shock velocity: $v_{\text{shock}}(t)$ (equivalently $\gamma_{\text{shock}}^2(t)$)

2) Peak values: $\rho_{\text{peak}}(t)$, $\gamma_{\text{peak}}^2(t)$

3) Derivatives at the peak: $\rho'_{\text{peak}}(t) \equiv \left. \frac{\partial \rho}{\partial r}(t, r) \right|_{r=r_{\text{peak}}}$, $\gamma'^2_{\text{peak}}(t) \equiv \left. \frac{\partial \gamma^2}{\partial r}(t, r) \right|_{r=r_{\text{peak}}}$



■ 4 equations are easily found

a) Rankine-Hugoniot conditions across the shock : 2 constraints

b) Time evolution equations : 2 evolution equations

Effective description of relativistic fluid propagation



- For completeness, the **4** equations are:

a) Rankine-Hugoniot conditions across the shock : **2** constraints

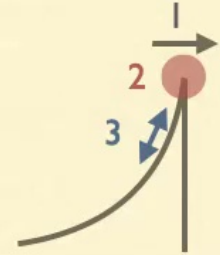
$$p_{\text{peak}} = \frac{p_0 + \rho_0 v_{\text{peak}} v_s}{1 - v_{\text{peak}} v_s}, \quad v_s = \frac{(p_{\text{peak}} + \rho_{\text{peak}}) v_{\text{peak}}}{p_{\text{peak}} v_{\text{peak}}^2 + \rho_{\text{peak}} - \rho_0 (1 - v_{\text{peak}}^2)}$$

b) Time evolution equations : **2** evolution equations

$$\frac{\sqrt{3}}{2} \partial_t \ln \rho_{\text{peak}} + \partial_t \ln \gamma_{\text{peak}}^2 = -\frac{2\sqrt{3}-3}{4} \frac{1}{\gamma_{\text{peak}}^2} \left[\frac{\sqrt{3}}{2} \ln \rho' + \ln \gamma^{2'} \right] - \frac{(\sqrt{3}-1)(d-1)}{t}$$

$$-\frac{\sqrt{3}}{2} \partial_t \ln \rho_{\text{peak}} + \partial_t \ln \gamma_{\text{peak}}^2 = \frac{2\sqrt{3}+3}{4} \frac{1}{\gamma_{\text{peak}}^2} \left[-\frac{\sqrt{3}}{2} \ln \rho' + \ln \gamma^{2'} \right] + \frac{(\sqrt{3}+1)(d-1)}{t}$$

How to construct a closed system



- The last equation?

- So far, we have less equations (**4** eqs.) than the number of quantities (**5** quantities)

- This is natural:

- the original system has infinite # of dof (i.e. # of spatial grids \vec{x}),
 - so the system cannot be described strictly by finite # of dof

- So, the last equation to close the system should be APPROXIMATE at best.

- It will be an equation characterizing our system.

How to construct a closed system



- The last equation: energy domination by the peak
 - Any relation like “(peak T^{00}) \times (thickness of the profile) = const.” will work
 - Approximating $\rho(t, r)$ and $\gamma^2(t, r)$ to be exponential in r , we have

$$\sigma \simeq \begin{cases} 1 \\ t \\ t^2 \end{cases} \times \int dr \frac{4}{3} \rho \gamma^2 = \begin{cases} 1 \\ t \\ t^2 \end{cases} \times \frac{4}{3} \frac{\rho_{\text{peak}} \gamma_{\text{peak}}^2}{\ln \rho' + \ln \gamma'^2} \quad \text{for} \quad \begin{cases} d=1 & \text{planar} \\ d=2 & \text{cylindrical} \\ d=3 & \text{spherical} \end{cases}$$

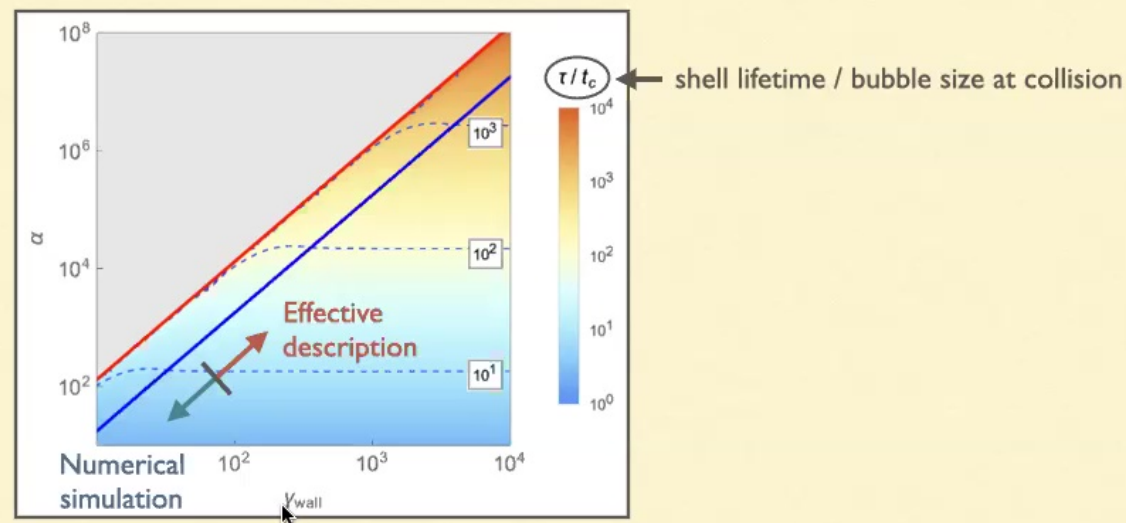
- The resulting system can be analytically solved (with $\delta = 10/13$)

e.g.
$$\frac{1}{\gamma_s^2(t)} = \frac{8}{87} \left(\frac{\rho_0}{\sigma} \right) \left[t^3 - \left(\frac{t}{t_c} \right)^\delta t_c^3 \right] + \frac{1}{\gamma_s^2(t_c)} \left(\frac{t}{t_c} \right)^\delta$$

How to construct a closed system



- Implications: the fluid profile remains to be thin until late times



- This is not yet conclusive, since shell collision is not yet included

GW production from thin sources

- Traditionally, GW signal from thin bubbles had been numerically simulated with so-called thin & envelope approximations



Thin:

Released energy is localized around the thin surface

$$T_{ij} \text{ grows like } \propto \frac{(\text{released energy})}{(\text{surface area})} \propto (\text{radius})$$

Envelope:

The surface disappears as soon as it collides

[Kosowsky, Turner, Watkins '92]

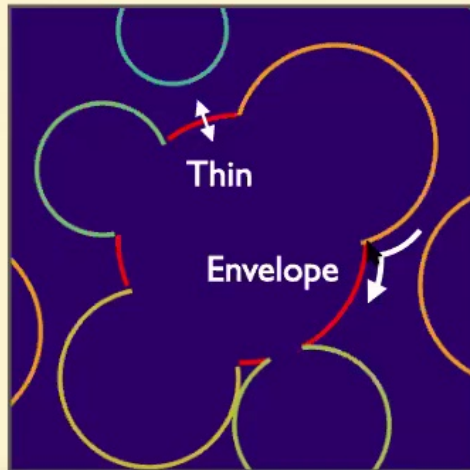
[Huber & Konstandin '08]

[Jinno & Takimoto '16]

- We derived the GW spectrum in this system analytically, and extended the result to a more realistic system [Jinno & Takimoto '17]

GW production from thin sources

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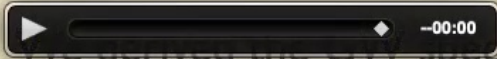
Thin:

Released energy is localized around the thin surface

$$T_{ij} \text{ grows like } \propto \frac{(\text{released energy})}{(\text{surface area})} \propto (\text{radius})$$

Envelope:

The surface disappears as soon as it collides

-  We derived the GW spectrum in this system analytically, and extended the result to a more realistic system [Jinno & Takimoto '17]

GW production from thin sources w/ envelope approximation

- GW spectrum with thin & envelope approximations

- Formal solution to the GW equation of motion

$$\square h_{ij} \sim G(PT)_{ij} \rightarrow h_{ij}(t_{\text{end}}, \vec{k}) \sim \int_{\text{sourcing start}}^{\text{sourcing end}} dt' \text{Green}(t_{\text{end}}, t', k) \times G(PT)_{ij}(t', \vec{k})$$

- GW power spectrum \sim 2-point correlator of $h_{ij} \sim$ 2-point correlator of T_{ij}

$$\Omega_{\text{GW}}(k) \sim \frac{1}{G} \left\langle \dot{h}_{ij}(t_{\text{end}}, \vec{k}) \dot{h}_{ij}^*(t_{\text{end}}, \vec{k}) \right\rangle_{\text{ens}} \propto \int_{\text{sourcing start}}^{\text{sourcing end}} dt_1 \int dt_2 P_{ijkl} \left\langle T_{ij} T_{kl} \right\rangle_{\text{ens}}(t_1, t_2, k) \times \cos(k(t_1 - t_2))$$

- 2-point correlator $\left\langle T_{ij} T_{kl} \right\rangle_{\text{ens}}$ can be calculated analytically from considerations on light cones [Jinno & Takimoto '16]

GW production from thin sources w/ envelope approximation

[Jinno & Takimoto '16]

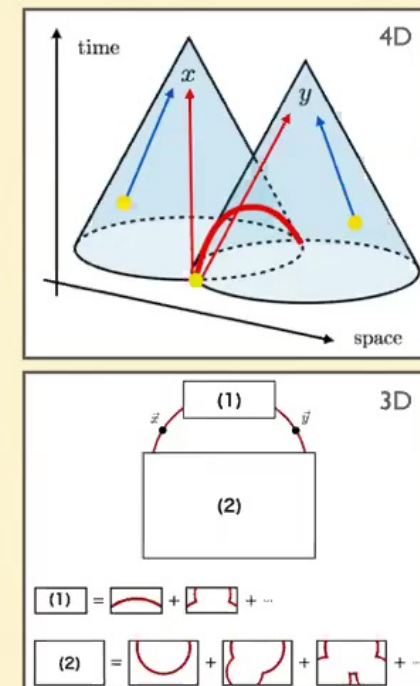
- Calculation of 2-point correlator $\left\langle T_{ij}(t_x, \vec{x}) T_{kl}(t_y, \vec{y}) \right\rangle_{\text{ens}}$ from light cones

1) Fix $x \equiv (t_x, \vec{x})$ and $y \equiv (t_y, \vec{y})$

2) List all possible shell configurations,
and for each configuration calculate

$T_{ij}(t_x, \vec{x}) T_{kl}(t_y, \vec{y})$ and the probability for it to happen

3) Sum up $T_{ij}(t_x, \vec{x}) T_{kl}(t_y, \vec{y}) \times (\text{probability})$



GW production from thin sources w/ envelope approximation

[Jinno & Takimoto '16]

■ Analytical result

GW energy fraction per each log. wavenumber

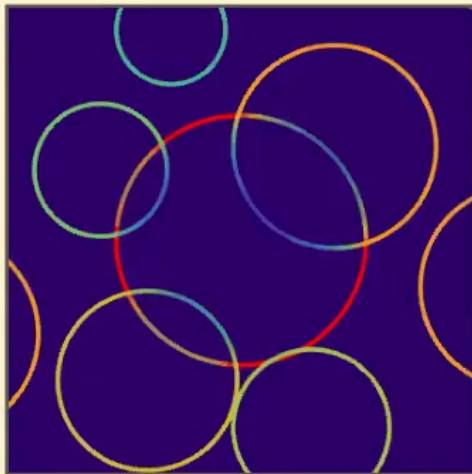
$$\Omega_{\text{GW}}(k) = \Omega_{\text{GW}}^{(s)}(k) + \Omega_{\text{GW}}^{(d)}(k)$$

$$\begin{aligned} \Omega_{\text{GW}}^{(s)} &\propto k^3 \int_{-\infty}^{\infty} dt \int_{|t|}^{\infty} dr \frac{e^{-\beta r/2}}{e^{\beta t/2} + e^{-\beta t/2} + \frac{\beta^2 t^2 - (\beta^2 r^2 + 4\beta r)}{4\beta r} e^{-\beta r/2}} \times \left[\overset{\text{spherical Bessel of order 0}}{j_0(kr)S_0(t, r)} + \overset{\text{order 1}}{\frac{j_1(kr)}{kr}S_1(t, r)} + \overset{\text{order 2}}{\frac{j_2(kr)}{k^2 r^2}S_2(t, r)} \right] \cos(kt) \\ \Omega_{\text{GW}}^{(d)} &\propto k^3 \int_{-\infty}^{\infty} dt \int_{|t|}^{\infty} dr \frac{e^{-\beta r/2}}{\left[e^{\beta t/2} + e^{-\beta t/2} + \frac{\beta^2 t^2 - (\beta^2 r^2 + 4\beta r)}{4\beta r} e^{-\beta r/2} \right]^2} \times \left[\frac{j_2(kr)}{k^2 r^2} D(t, r) D(-t, r) \right] \cos(kt) \end{aligned}$$

GW production from thin sources: w/o envelope approximation

[Jinno & Takimoto '17]

- In realistic systems, the shells do not vanish just after they collide
- Therefore, we need to modify the thin & envelope modeling



Thin:

Before collision:

$$T_{ij} \text{ grows like } \propto \frac{(\text{released energy})}{(\text{surface area})} \propto (\text{radius})$$

After collision:

$$T_{ij} \text{ decreases like } \propto \frac{1}{(\text{surface area})} \propto (\text{radius})^{-2}$$

but NOT envelope

- GW spectrum is calculable in a similar way as before

GW production from thin sources: w/o envelope approximation

[Jinno & Takimoto '17]

■ Analytical result

GW energy fraction per each log. wavenumber

$$\Omega_{\text{GW}}(k) = \Omega_{\text{GW}}^{(s)}(k) + \Omega_{\text{GW}}^{(d)}(k)$$

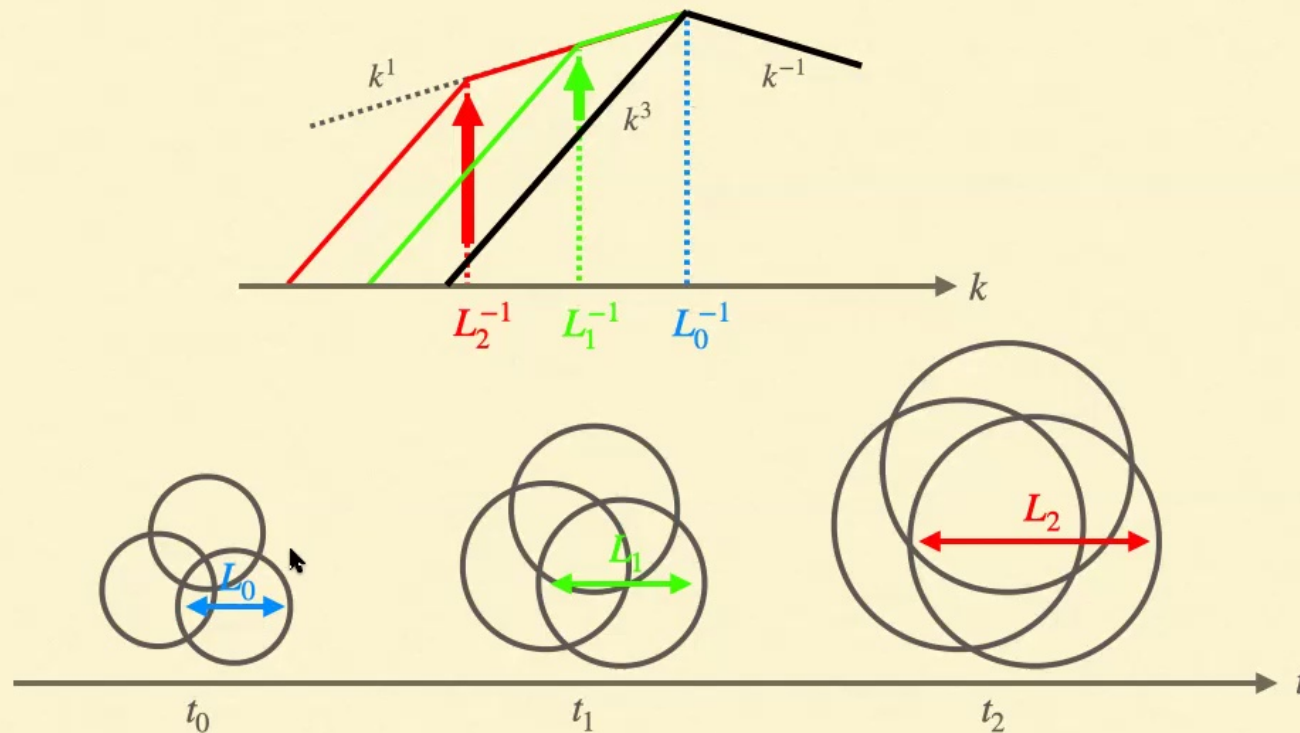
$$\Delta^{(s)} = \int_{-\infty}^{\infty} dt_x \int_{-\infty}^{\infty} dt_y \int_{v|t_x, y|}^{\infty} dr \int_{-\infty}^{t_{\max}} dt_n \int_{t_n}^{t_x} dt_{xi} \int_{t_n}^{t_y} dt_{yi} \left[\frac{k^3}{3} \left[e^{-I(x_i, y_i)} \Gamma(t_n) \frac{r}{r_{xn}^{(s)} r_{yn}^{(s)}} \times \left[j_0(kr) \mathcal{K}_0(n_{xn}, n_{yn}) + \frac{j_1(kr)}{kr} \mathcal{K}_1(n_{xn}, n_{yn}) + \frac{j_2(kr)}{(kr)^2} \mathcal{K}_2(n_{xn}, n_{yn}) \right] \times \partial_{t_{xi}} [r_B(t_{xi}, t_n)^3 D(t_x, t_{xi})] \partial_{t_{yi}} [r_B(t_{yi}, t_n)^3 D(t_y, t_{yi})] \cos(kt_{x,y}) \right] \right]$$

$$\Delta^{(d)} = \int_{-\infty}^{\infty} dt_x \int_{-\infty}^{\infty} dt_y \int_0^{\infty} dr \int_{-\infty}^{t_x} dt_{xn} \int_{-\infty}^{t_y} dt_{yn} \int_{t_{xn}}^{t_x} dt_{xi} \int_{t_{yn}}^{t_y} dt_{yi} \int_{-1}^1 dc_{xn} \int_{-1}^1 dc_{yn} \int_0^{2\pi} d\phi_{xn, yn} \left[\frac{k^3}{3} \left[\Theta_{sp}(x_i, y_n) \Theta_{sp}(x_n, y_i) e^{-I(x_i, y_i)} \Gamma(t_{xn}) \Gamma(t_{yn}) \times r^2 \left[j_0(kr) \mathcal{K}_0(n_{xn}, n_{yn}) + \frac{j_1(kr)}{kr} \mathcal{K}_1(n_{xn}, n_{yn}) + \frac{j_2(kr)}{(kr)^2} \mathcal{K}_2(n_{xn}, n_{yn}) \right] \times \partial_{t_{xi}} [r_B(t_{xi}, t_{xn})^3 D(t_x, t_{xi})] \partial_{t_{yi}} [r_B(t_{yi}, t_{yn})^3 D(t_y, t_{yi})] \cos(kt_{x,y}) \right] \right]$$

GW production from thin sources: w/o envelope approximation

[Jinno & Takimoto '17]

- Physical interpretation



Summary

- Gravitational waves provide us with the opportunity to test first-order phase transitions in the early Universe
- In *extremely* strong first-order phase transitions, it's hard to predict the GW signal though they are both theoretically and observationally interesting
- In such transitions, the fluid shells may remain to be thin after the transition, and the GW enhancement by sound waves may not occur
- However, IR enhancement of the GW signal is expected from long-lived shells, which enhances observational prospects