Title: Testing Gauge Gravity duality with Matrix models

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Testing Gauge Gravity duality with Matrix models

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Abstract

I will review the numerical approach to testing gauge/gravity duality using matrix models. This will lead to a summary of recent results from the BFSS, BMN and Berkooz-Douglas matrix models and a strong non-perturbative test of gauge/gravity duality.

The General Framework

Berkooz-Douglas model

The Berkooz-Douglas model [hep-th/9610236]
is $\mathcal{N} = 1$ SUSY in 6-dim, or $\mathcal{N} = 2$ in 4-dim reduced to 1-dim i.e. time. Also see Van Raamsdonk [hep-th/0112081].
The system describes a D0/D4 intersection.

The more general framework involves $Dp/D(p+4)$ systems.

BD-matrix model at finite temperature $T = 1/\beta$

$$S_{BD} = S_{BFSS} + S_{\Phi} + S_{\chi}\,.$$

with X and ϕ matrices of sizes $N \times N$ and $N \times N_f$ and χ fermions.

The Bosonic Euclidean thermal action is

$$S_{\text{bos}} = N \int_0^{\beta} d\tau \Bigg[\mathrm{Tr}\Bigg(\frac{1}{2} D_\tau X^a D_\tau X^a + \frac{1}{2} D_\tau \bar{X}^{\rho\dot{\rho}} D_\tau X_{\rho\dot{\rho}} - \frac{1}{4}[X^a, X^b]^2 + \frac{1}{2}[X^a, \bar{X}^{\rho\dot{\rho}}][X^a, X_{\rho\dot{\rho}}] \Bigg) + \mathrm{tr}\left(D_\tau \bar{\Phi}^\rho D_\tau \Phi_\rho + \bar{\Phi}^\rho (X^a - m^a)^2 \Phi_\rho \right) + \frac{1}{2} \mathrm{Tr} \sum_{A=1}^{3} \mathcal{D}^A \mathcal{D}^A \Bigg]$$

$\mathcal{D}^A = \sigma_{\rho}^{A\,\sigma} \left(\frac{1}{2}[\bar{X}^{\rho\dot{\rho}}, X_{\sigma\dot{\rho}}] - \Phi_\sigma \bar{\Phi}^\rho \right)$ and $a = 1, \ldots, 5$.
The model admits massive deformations (Kim,Yi and Park [hep-th/0207264]).

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E.g. The BFSS to BMN Deformation

X^5 and two Hermitian components of $X_{\rho\dot{\rho}}$ form the X^r. The massive deformation of the BFSS model gives the BMN model

$$S[X,\psi] = N\int_0^\beta d\tau\ \mathrm{Tr}\left[\frac{1}{2}D_\tau X^i D_\tau X^i - \frac{1}{4}\left([X^r,X^s] + \frac{i\mu}{3}\varepsilon^{rst}X_t\right)^2 - \frac{1}{2}[X^r,X^m]^2 - \frac{1}{4}[X^m,X^n]^2 + \frac{1}{2}\left(\frac{\mu}{6}\right)^2 X_m^2 + \frac{1}{2}\psi^T\mathcal{C}\left(D_\tau - \frac{i\mu}{4}\gamma^{567}\right)\psi - \frac{1}{2}\psi^T\mathcal{C}\gamma^i[X^i,\psi]\right]$$

Taking μ to infinity gives a supersymmetric gauge Gaussian model.

The gauge Gaussian model.

A simple sub-model of the mass deformed BD (and BMN) model is the gauge Gaussian model.

$$S_{GG}[X] = N \int_0^\beta d\tau \sum_{a=1}^{D} \frac{1}{2} \mathrm{Tr} \left[D_\tau X^a D_\tau X^a + m^2 X^a X^a \right]$$

This model can be analysed in great detail.

The Hamiltonian formulation involves a system of harmonic oscillators with a Gauss law constraint which insists on $SU(N)$ singlets.

Properties of gauge Gaussian models

We can understand qualitatively what will happen in the model:

- The eigenvalues of X^i have a Wigner semi-circle distribution.
- At $T = 0$, we can gauged A away, while for large T we get a pure matrix model with A one of the matrices.
- The entry of A as an additional matrix in the dynamics signals a phase transition.
- The transition can be observed as centre symmetry breaking in the Polyakov loop.

Bosonic matrix membranes are approximately gauge gaussian models V. Filev and D.O'C. [1506.01366 and 1512.02536].

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Analysing gauge Gaussian models

Integrating out the X^a gives the effective action

$$S_{GG}(\theta) = \frac{D(N^2-1)}{2}\beta m + \frac{D}{2}\sum_{i,j=1}^{N} \ln|1 - \mathrm{e}^{-\beta m + i(\theta_i - \theta_j)}|^2$$

$$- \frac{1}{2}\sum_{i \neq j = 1}^{N} \ln|1 - \mathrm{e}^{i(\theta_i - \theta_j)}| \, .$$

The θ_i are eigenvalues of βA in static gauge.
Expanding the logarithms and with $u_n = \frac{1}{N}\sum_{i=1}^{n} \mathrm{e}^{in\theta_i}$ gives

$$S_{GG}(\theta) = \frac{D(N^2-1)}{2}\beta m + N^2 \sum_{n=0}^{\infty}\{\frac{1 - D\mathrm{e}^{-nm\beta}}{n}|u_n|^2 - \frac{1}{nN}\}$$

The u_n are moments of the distribution $\rho(\theta)$.

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The Hagedorn transition

Examining

$$S_{GG}(\theta) = \frac{D(N^2-1)}{2}\beta m + N^2 \sum_{n=0}^{\infty} \{ \frac{1 - D\mathrm{e}^{-nm\beta}}{n} |u_n|^2 - \frac{1}{nN} \}$$

At low temperature (large β) all u_n have a minimum at 0 and the free energy is given by the zero point energy term. As the temperature is increased u_1 becomes unstable first.

The Hagedorn temperature

For $D > 1$ there is a large N phase transition at:

$$\beta_H = \frac{\ln m}{D}$$

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In the large N limit the free energy is

$$\beta F(\rho) \;=\; \frac{Dm\beta}{2} + \frac{D}{2}\int \rho(\alpha)\int \rho(\alpha\prime)\ln|1-\mathrm{e}^{-\beta m + i(\alpha-\alpha\prime)}|^2 d\alpha d\beta$$
$$-\frac{1}{2}P\int \rho(\alpha)\rho(\alpha\prime)\ln|1-\mathrm{e}^{i(\alpha-\alpha\prime)}|d\alpha d\alpha\prime\,.$$

For low temperatures including the transition expanding in $\mathrm{e}^{-m\beta}$ and only retaining the leading exponential is sufficient and equivalent to solving the model

$$Z_{a1} = \int [dU]\mathrm{e}^{a_1 \mathrm{Tr}(U)\mathrm{Tr}(u^{-1})} \quad \text{with } a_1 = D\mathrm{e}^{-m\beta}$$

resulting in

$$\beta F_{a1} = -a_1|u_1|^2 - \frac{1}{2}P\int \rho(\alpha)\rho(\alpha\prime)\ln|1-\mathrm{e}^{i(\alpha-\alpha\prime)}|d\alpha d\alpha\prime\,.$$

The eigenvalue distribution is given by (see Aharony et al [hep-th/0310285])

$$\rho(\theta) = \begin{cases} \frac{1}{2\pi} & \text{for} \quad \beta > \beta_H \\ \frac{1}{\pi s^2}\sqrt{s^2 - \sin^2(\frac{\theta}{2})}\cos(\frac{\theta}{2}) & \text{for} \quad \beta < \beta_H \end{cases}$$

with

$$s^2 \equiv \sin^2(\frac{\theta_0}{2}) = 1 - \sqrt{1 - \frac{1}{a_1}} = 1 - \sqrt{1 - \mathrm{e}^{-m(\beta - \beta_H)}}\,.$$

and

$$\beta F_{GG} = \begin{cases} \frac{Dm\beta}{2} & \text{for} \quad \beta > \beta_H \\ \frac{Dm\beta}{2} + \frac{1}{2} - \frac{1}{2s^2} - \frac{1}{2}\ln s^2 & \text{for} \quad \beta < \beta_H \end{cases}$$

Near the transition we have

$$\beta F_{GG} = \begin{cases} \frac{Dm\beta}{2} & \text{for} \quad \beta > \beta_H \\ \frac{Dm\beta}{2} + \frac{m(\beta-\beta_H)}{4} - \frac{m^{3/2}}{3}(\beta_H - \beta)^{3/2} + \cdots & \text{for} \quad \beta < \beta_H \end{cases}$$

The eigenvalue distribution is given by (see Aharony et al [hep-th/0310285])

$$\rho(\theta) = \begin{cases} \frac{1}{2\pi} & \text{for} \quad \beta > \beta_H \\ \frac{1}{\pi s^2}\sqrt{s^2 - \sin^2(\frac{\theta}{2})}\cos(\frac{\theta}{2}) & \text{for} \quad \beta < \beta_H \end{cases}$$

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and

$$\beta F_{GG} = \begin{cases} \frac{Dm\beta}{2} & \text{for} \quad \beta > \beta_H \\ \frac{Dm\beta}{2} + \frac{1}{2} - \frac{1}{2s^2} - \frac{1}{2}\ln s^2 & \text{for} \quad \beta < \beta_H \end{cases}$$

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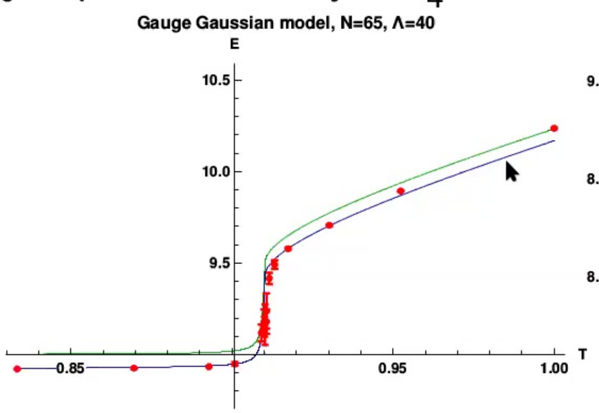
$$\beta F_{GG} = \begin{cases} \frac{Dm\beta}{2} & \text{for} \quad \beta > \beta_H \\ \frac{Dm\beta}{2} + \frac{m(\beta - \beta_H)}{4} - \frac{m^{3/2}}{3}(\beta_H - \beta)^{3/2} + \cdots & \text{for} \quad \beta < \beta_H \end{cases}$$

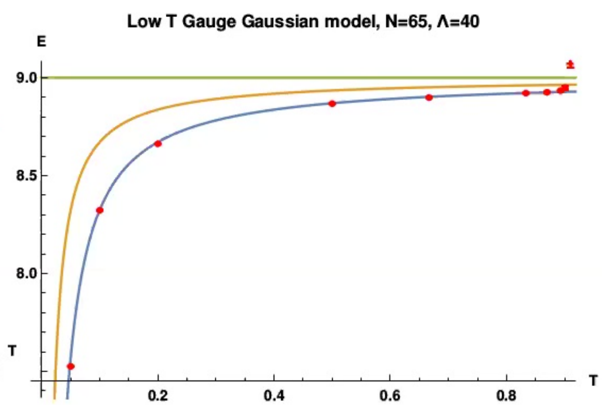
Energy and Lattice Effects

The energy near the Hagedorn temperature is

$$E = \frac{\partial(\beta F)}{\partial \beta} = \begin{cases} \frac{Dm}{2} & \text{for} \quad \beta > \beta_H \\ \frac{Dm}{2} + \frac{m}{4} + \frac{m^{3/2}}{2}\sqrt{\beta_H - \beta} + \cdots & \text{for} \quad \beta < \beta_H \,. \end{cases}$$

The energy is discontinuous across the transition, undergoing a jump discontinuity of $\frac{m}{4}$.





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Divergent fluctuations

We can furthermore obtain that the specific heat

$$C_{\mathrm{v}} = -\beta^2 \frac{\partial^2(\beta F)}{\partial \beta^2} = \begin{cases} 0 & \text{for} \quad \beta > \beta_H \\ \beta_H^2 \frac{m^{3/2}}{4\sqrt{\beta_H - \beta}} + \cdots & \text{for} \quad \beta < \beta_H \,. \end{cases}$$

The specific heat of the gauge Gaussian model is predicted to diverge with a square root singularity as the Hagedorn temperature is approached from the deconfined high temperature side of the transition.

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Divergent $1/N$ corrections

When the leading $1/N$ corrections in the large N limit are taken into account the partition function in the confined phase becomes

$$Z_{GG}^{Conf} = \mathrm{e}^{-D(N^2-1)m\beta/2} \prod_{n=1}^{\infty} \frac{1}{1-\mathrm{e}^{-nm(\beta-\beta_H)}}$$

which is well approximated by the $n = 1$ term i.e.

$$Z_{GG}^{Conf} \simeq \mathrm{e}^{-D(N^2-1)m\beta/2} \frac{1}{1-\mathrm{e}^{-m(\beta-\beta_H)}}$$

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Near the Hagedorn temperature

$$\beta F = \frac{Dm\beta}{2} + \frac{1}{N^2} \ln\left(m(\beta - \beta_H)\right) + \cdots$$

$$E = \frac{Dm}{2} + \frac{1}{N^2}\frac{1}{\beta - \beta_H} + \cdots \qquad (1)$$

The $1/N^2$ corrections diverge as the Hagedorn temperature is approached. For $T \simeq T_H - \frac{2T_H^2}{N^2 mD}$ the $1/N^2$ corrections can compete with the leading ground state energy contribution.

N.B. Fluctuations are large!

Restricting to words of length $\lessapprox N^2$ removes the divergence and rounds the transition.

A further conclusion, which can be drawn from the deconfined phase of the system, and the corresponding large N limit is that the transition appears NOT to in fact first order. It has a divergent specific heat on either sider of the transition. The stronger divergence appears to be on the low temperature side, but this is coming from subdominant contributions as the limit is approached. The above analysis suggests that the Hagedorn transition in the gauge Gaussian model warrants a closer look.

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The large μ BMN model

A similar analysis for the BMN model gives

$$a_1 = 3e^{-\beta\frac{\mu}{3}} + 6e^{-\beta\frac{\mu}{6}} + 4e^{-\beta\frac{\mu}{4}}$$

The Hagedorn temperature for the BMN model is

$$\beta_H = \frac{12\ln(3)}{\mu}$$

Note: The specific heat of the large μ BMN model also diverges at β_H! The effect of the fermions is to lower the Hagedorn temperature. Without fermions the

$$T_H = \frac{m}{6\ln(3+2\sqrt{3})} \simeq 0.089m$$

whereas with supersymmetry

$$T_H = \frac{m}{12\ln(3)} \simeq 0.076m.$$

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The BMN model

The BMN action

$$S_{BMN} = \quad N\int_0^\beta d\tau\,\mathrm{Tr}\left\{\frac{1}{2}(\mathcal{D}_\tau X^i)^2 + \frac{1}{2}(\frac{\mu}{3})^2(X^i)^2\right.$$
$$+\frac{\mu}{3}i\epsilon_{ijk}X^iX^jX^k - \frac{1}{4}[X^i,X^j]^2$$
$$+\frac{1}{2}\Psi^T D_\tau\Psi + \frac{1}{2}(\frac{\mu}{4})\Psi^T i\gamma^{123}\Psi + \frac{1}{2}\Psi^T\Gamma^i[X^i,\Psi]$$
$$+\frac{1}{2}(\mathcal{D}_\tau X^a)^2 + \frac{1}{2}(\frac{\mu}{6})^2(X^a)^2$$
$$\left.+\frac{1}{2}\Psi^T\Gamma^a[X^a,\Psi] - \frac{1}{2}[X^a,X^j]^2 - \frac{1}{4}[X^a,X^b]^2\right\}\ .$$

The $SO(3)$ X^i shown as red give a matrix model with a transition to between a thermal and fuzzy sphere phase.

At low temperature it has non-trivial fuzzy sphere vacua

$X^i = -\frac{\mu}{3}L^i$, with L^i $su(2)$ generators.

Testing Gauge Gravity duality with Matrix models

Lattice Formulation

$$\Delta_{Bose} = \Delta + r_b a^2 \Delta^2 \quad \text{where} \quad \Delta = \frac{2 - e^{aD_\tau} - e^{-aD_\tau}}{a^2}.$$

The lattice Dirac operator must be anti symmetric

$$D_{Lat} = K_a \mathbf{1}_{16} + i\frac{\mu}{4}\gamma^{567} + \Sigma^{123} K_w\,, \text{ where } \Sigma^{123} = i\Gamma^{123}$$

$$K_a = (1-r)\frac{e^{aD_\tau} - e^{-aD_\tau}}{a} + r\frac{e^{2aD_\tau} - e^{-2aD_\tau}}{4a} \quad \text{anti-symmetric}$$

$$K_w = r_{1f}\, a\Delta + r_{2f}\, a^3\Delta^2 \quad \text{symmetric}$$

The Clifford Algebra

$\mathcal{C}$ and $\mathcal{C}\gamma^i$ are symmetric while $\mathcal{C}\gamma^{ij}$ and $\mathcal{C}\gamma^{ijk}$ are anti-symmetric.

$\Longrightarrow$ **An alternative to Σ^{123} would be $\Sigma^{12} = i\gamma^{12}$.**

Σ^{12} was used in Anagnostopoulos et al [arXiv:0707.4454], Catterall et al [arXiv:1003.4952], Hanada's code and Berkowitz et al [arXiv:1606.04951].

The BMN model

The BMN action

$$
\begin{aligned}
S_{BMN} = \quad & N \int_0^\beta d\tau \,\mathrm{Tr}\Big\{ \frac{1}{2}(\mathcal{D}_\tau X^i)^2 + \frac{1}{2}(\frac{\mu}{3})^2 (X^i)^2 \\
& + \frac{\mu}{3} i\epsilon_{ijk} X^i X^j X^k - \frac{1}{4}[X^i, X^j]^2 \\
& + \frac{1}{2}\Psi^T D_\tau \Psi + \frac{1}{2}(\frac{\mu}{4})\Psi^T i\gamma^{123}\Psi + \frac{1}{2}\Psi^T \Gamma^i [X^i, \Psi] \\
& + \frac{1}{2}(\mathcal{D}_\tau X^a)^2 + \frac{1}{2}(\frac{\mu}{6})^2 (X^a)^2 \\
& + \frac{1}{2}\Psi^T \Gamma^a [X^a, \Psi] - \frac{1}{2}[X^a, X^j]^2 - \frac{1}{4}[X^a, X^b]^2 \Big\} \ .
\end{aligned}
$$

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At low temperature it has non-trivial fuzzy sphere vacua

$X^i = -\frac{\mu}{3} L^i$, with L^i $su(2)$ generators.

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$$D_{Lat} = K_a \mathbf{1}_{16} + i\frac{\mu}{4}\gamma^{567} + \Sigma^{123} K_w \, , \text{ where } \Sigma^{123} = i\Gamma^{123}$$

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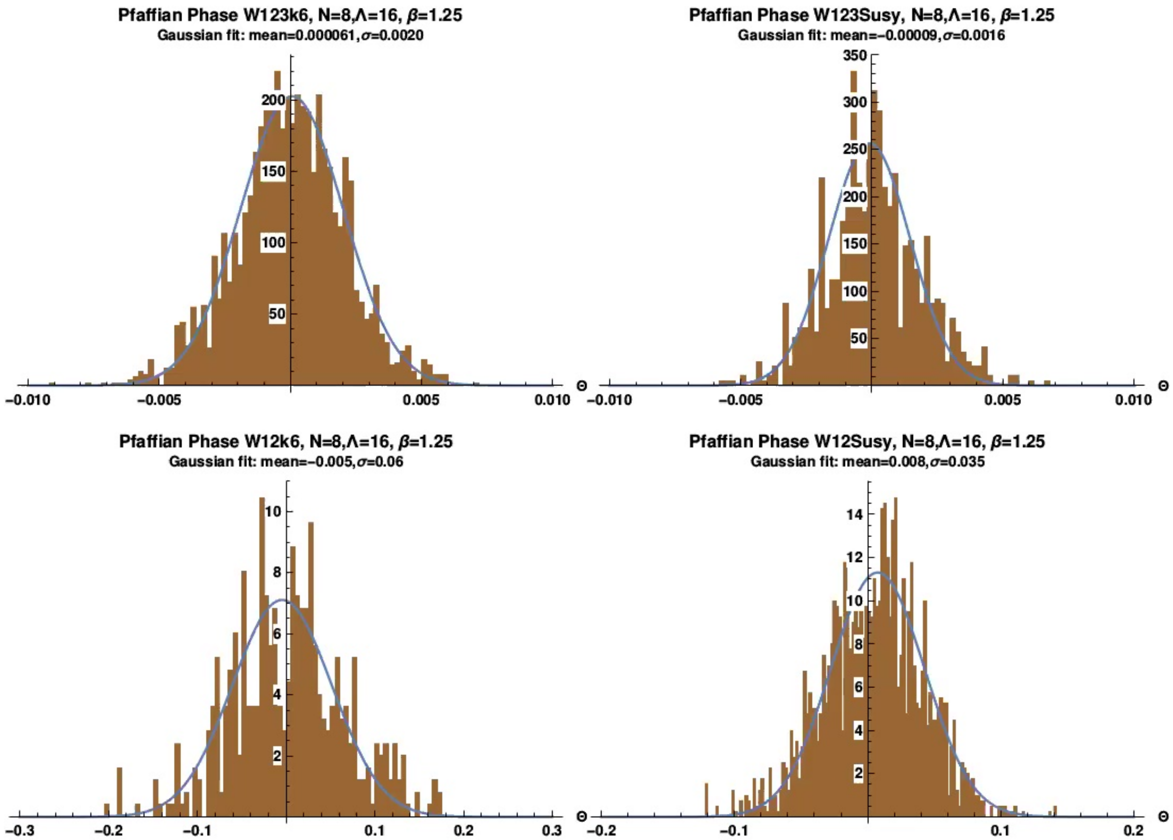
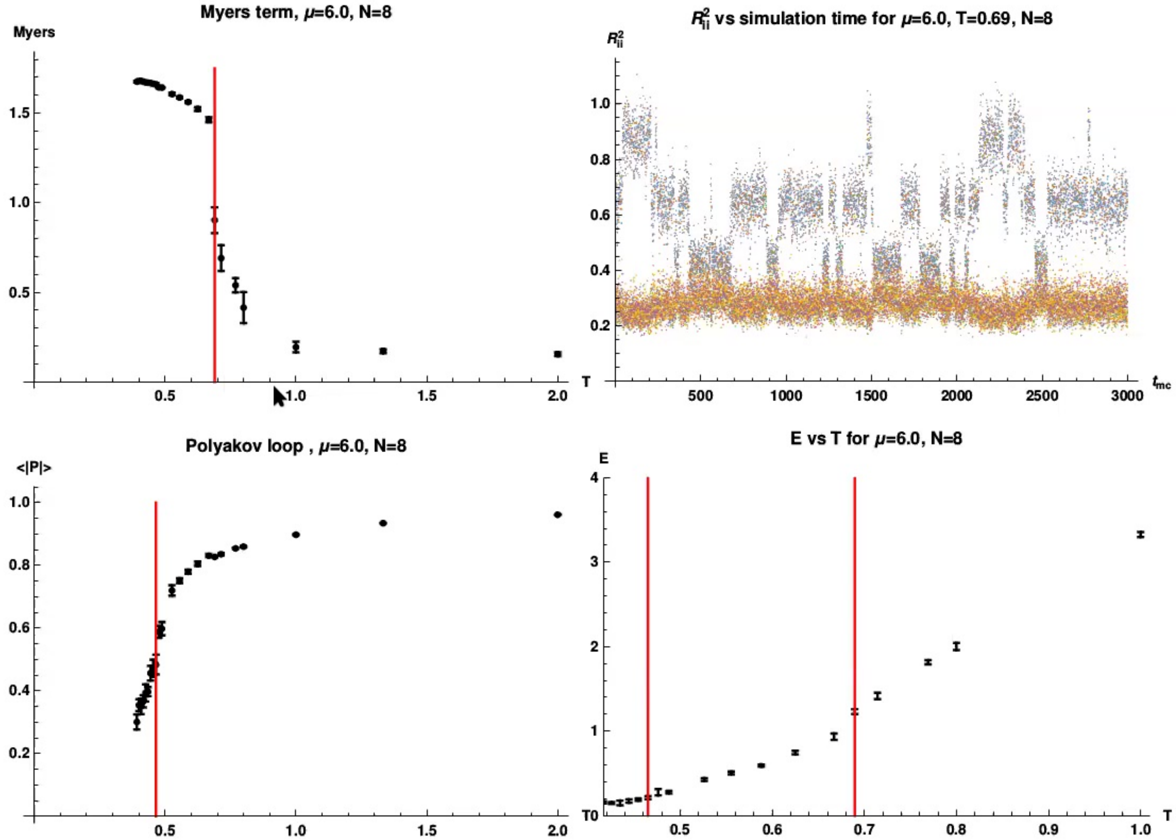
Testing Gauge Gravity duality with Matrix models

The Phase of the Fermionic Pfaffian for Different Lattices

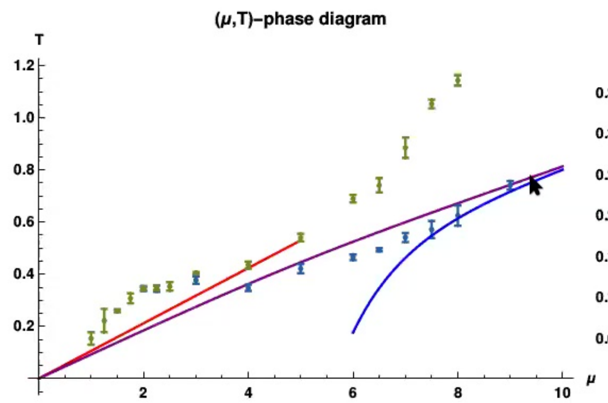


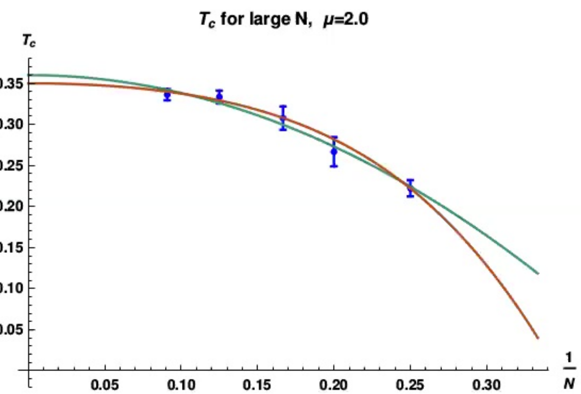
BMN Observables



Testing Gauge Gravity duality with Matrix models

The Phase Diagram of the BMN model





The gravity dual and its geometry

Gauge/gravity duality predicts that the strong coupling regime of the theory is described by II_A supergravity, which lifts to 11-dimensional supergravity.

The bosonic action for eleven-dimensional supergravity is given by

$$S_{11D} = \frac{1}{2\kappa_{11}^2} \int [\sqrt{-g}R - \frac{1}{2} F_4 \wedge *F_4 - \frac{1}{6} A_3 \wedge F_4 \wedge F_4]$$

where $2\kappa_{11}^2 = 16\pi G_N^{11} = \frac{(2\pi l_p)^9}{2\pi}$.

Testing Gauge Gravity duality with Matrix models

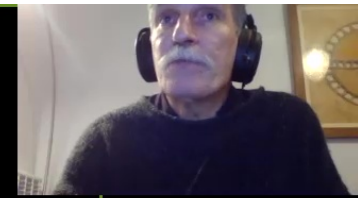
The relevant solution to eleven dimensional supergravity for the dual geometry to the BFSS model corresponds to N coincident $D0$ branes in the IIA theory. It is given by

$$ds^2 = -H^{-1}dt^2 + dr^2 + r^2 d\Omega_8^2 + H(dx_{10} - Cdt)^2$$

with $A_3 = 0$

The one-form is given by $C = H^{-1} - 1$ and $H = 1 + \frac{\alpha_0 N}{r^7}$ where $\alpha_0 = (2\pi)^2 14\pi g_s l_s^7$.

Testing Gauge Gravity duality with Matrix models



Including temperature

The idea is to include a **black hole** in the gravitational system.

The Hawking termperature provides the temperature of the system.

Hawking radiation

We expect difficulties at low temperatures, as the system should Hawking radiate. It has been argued that this is related to the flat directions and the propensity of the system to leak into these regions.
Hanada et al Science 23 (2014) Vol. 344 p882

The black hole geometry

$$ds_{11}^2 = -H^{-1}Fdt^2 + F^{-1}dr^2 + r^2 d\Omega_8^2 + H(dx_{10} - Cdt)^2$$

Set $U = r/\alpha'$ and we are interested in $\alpha' \to \infty$
$H(U) = \frac{240\pi^5\lambda}{U^7}$ and the black hole time dilation factor
$F(U) = 1 - \frac{U_0^7}{U^7}$ with $U_0 = 240\pi^5\alpha'^5\lambda$. The temperature

$$\frac{T}{\lambda^{1/3}} = \frac{1}{4\pi\lambda^{1/3}} H^{-1/2} F'(U_0) = \frac{7}{2^4 15^{1/2} \pi^{7/2}} \left(\frac{U_0}{\lambda^{1/3}}\right)^{5/2}.$$

From black hole entropy we obtain the prediction for the Energy

$$S = \frac{A}{4G_N} \sim \left(\frac{T}{\lambda^{1/3}}\right)^{9/2} \implies \frac{E}{\lambda N^2} \sim \left(\frac{T}{\lambda^{1/3}}\right)^{14/5}$$

The observables that we focus on

$$E/N^2 = \left\langle \frac{1}{N\beta} \int_0^\beta dt \, \mathrm{Tr} \left(-\frac{3}{4}[X^i, X^j]^2 + \frac{3}{4}\Psi^T C_{10}\gamma^i [X^i, \Psi] \right) \right\rangle \,,$$

$$\langle R^2 \rangle = \left\langle \frac{1}{N\beta} \int_0^\beta dt \, \mathrm{Tr} \left(X^i \right)^2 \right\rangle \,,$$

$$< |P| > = \langle | \frac{1}{N} \mathrm{Tr} U \, | \rangle,$$

$$U \equiv \mathcal{P} \exp \left(i \int_0^\beta dt \, A_0(t) \right) \,.$$

The best current results (Berkowitz et al 2016) consistent with gauge gravity give

$$\frac{1}{N^2}\frac{E}{\lambda^{1/3}} = 7.41\left(\frac{T}{\lambda^{1/3}}\right)^{\frac{14}{5}} - (10.0 \pm 0.4)\left(\frac{T}{\lambda^{1/3}}\right)^{\frac{23}{5}} + (5.8 \pm 0.5)\,T^{\frac{29}{5}} + ..$$

$$-\frac{5.77\,T^{\frac{2}{5}} + (3.5 \pm 2.0)\,T^{\frac{11}{5}}}{N^2} + \dots$$

Probing the dual geometry.

We need a suitable probe to test the dual geometry further.

Testing Gauge Gravity duality with Matrix models

We use D_4 branes. These adds new fundamental matter fields to the Hamiltonian.

M. Berkooz and M. R. Douglas, "Five-branes in M(atrix) theory," [hep-th/9610236].
In IIA string theory this describes a $D0 - D4$ system.

The more general framework involves $Dp - D(p+4)$ systems.

Add new bosonic degrees of freedom Φ_α as two complex $N \times N_f$ matrices

$$\mathcal{S}^{\mathrm{E}}_{\Phi} = \frac{1}{g^2} \int_0^\beta d\tau \, \mathrm{tr} \Big(D_\tau \bar{\Phi}^\rho D_\tau \Phi_\rho - \bar{\Phi}^\alpha (\sigma^A)_\alpha^{\ \beta} J^A_{ab} [X^a, X^b] \Phi_\beta$$

$$+ \bar{\Phi}^\rho (X^i)^2 \Phi_\rho - \frac{1}{2} \bar{\Phi}^\alpha \Phi_\beta \bar{\Phi}^\beta \Phi_\alpha + \bar{\Phi}^\alpha \Phi_\alpha \bar{\Phi}^\beta \Phi_\beta \Big) \; .$$

J^A and K^A are the $SO(3)$ generators

$$J^A_{ab} = \frac{1}{2} (L_{A4})_{ab} + \frac{1}{4} \varepsilon^{ABC} (L_{BC})_{ab} \; , \; K^A_{ab} = -\frac{1}{2} (L_{A4})_{ab} + \frac{1}{4} \varepsilon^{ABC} (L_{BC})_{ab}$$

equivalent to the

$$SO(4) \text{ generators} \quad (L_{ab})_{cd} = i(\delta_{ad}\delta_{bc} - \delta_{ac}\delta_{bd})$$

Testing Gauge Gravity duality with Matrix models

$$S_\chi = \frac{1}{g^2}\int \mathrm{tr}\Big(i\chi^\dagger D_0\chi + \bar{\chi}\gamma^a X^a\chi$$
$$+\sqrt{2}\,i\,\varepsilon_{\alpha\beta}\,\bar{\chi}\lambda_\alpha\Phi_\beta - \sqrt{2}\,i\,\varepsilon_{\alpha\beta}\,\bar{\Phi}^\alpha\bar{\lambda}_\beta\chi\Big)\ .$$

where $\lambda_\alpha = P^\iota_\alpha\psi_\iota$.

The full model is

$$S_{BD} = S_{BFSS} + S_\Phi + S_\chi\,.$$

The lattice discretisation is again delicate but works!

The Bosonic model (and ADHM Data)

$$S_{\text{bos}} = N \int_0^\beta d\tau \Bigg[\mathrm{Tr} \left(\frac{1}{2} D_\tau X^a D_\tau X^a + \frac{1}{2} D_\tau \bar{X}^{\rho\dot{\rho}} D_\tau X_{\rho\dot{\rho}} - \frac{1}{4}[X^a, X^b]^2 + \frac{1}{2}[X^a, \bar{X}^{\rho\dot{\rho}}][X^a, X_{\rho\dot{\rho}}] \right) + \mathrm{tr}\left(D_\tau \bar{\Phi}^\rho D_\tau \Phi_\rho + \bar{\Phi}^\rho (X^a - m^a)^2 \Phi_\rho \right) + \frac{1}{2} \mathrm{Tr} \sum_{A=1}^{3} \mathcal{D}^A \mathcal{D}^A \Bigg] .$$

where $X^a = 0$ with

$$\mathcal{D}^A = \sigma_\rho^{A\,\sigma} \left(\frac{1}{2} [\bar{X}^{\rho\dot{\rho}}, X_{\sigma\dot{\rho}}] - \Phi_\sigma \bar{\Phi}^\rho \right) = 0$$

specify ADHM data for Yang-Mills instantons on $\mathbb{R}^4$.

Results for the flavoured bosonic model

With $N_f << N$ the fundamental fields act as probes on the adjoint background. The $SO(9)$ symmetry has been broken to $SO(5) \times SO(4)$. In the low temperature phase the system is well described by a gaussian model with three masses $m_A^t = 1.964 \pm 0.003$, $m_A^l = 2.001 \pm 0.003$ and $m_f = 1.463 \pm 0.001$, the adjoint longitudinal and transverse masses and the mass of the fundamental fields respectively.

Yuhma Asano, Veselin G. Filev, Samuel Kováčik and D. O'C. arXiv 1605.05597

Two new observables

$$r^2 = \frac{1}{\beta N_f} \int_0^\beta d\tau \ \mathrm{tr}\, \bar{\Phi}^\rho \Phi_\rho$$

and the condensate defined as

$$c^a(m) = \frac{\partial}{\partial m^a} \left(-\frac{1}{N\beta} \log Z \right)$$

The Bosonic model (and ADHM Data)

$$S_{\rm bos} = N \int_0^\beta d\tau \Bigg[{\rm Tr} \Bigg(\frac{1}{2} D_\tau X^a D_\tau X^a + \frac{1}{2} D_\tau \bar{X}^{\rho\dot\rho} D_\tau X_{\rho\dot\rho}$$

$$- \frac{1}{4} [X^a, X^b]^2 + \frac{1}{2} [X^a, \bar{X}^{\rho\dot\rho}][X^a, X_{\rho\dot\rho}] \Bigg)$$

$$+ \, {\rm tr} \left(D_\tau \bar\Phi^\rho D_\tau \Phi_\rho + \bar\Phi^\rho (X^a - m^a)^2 \Phi_\rho \right)$$

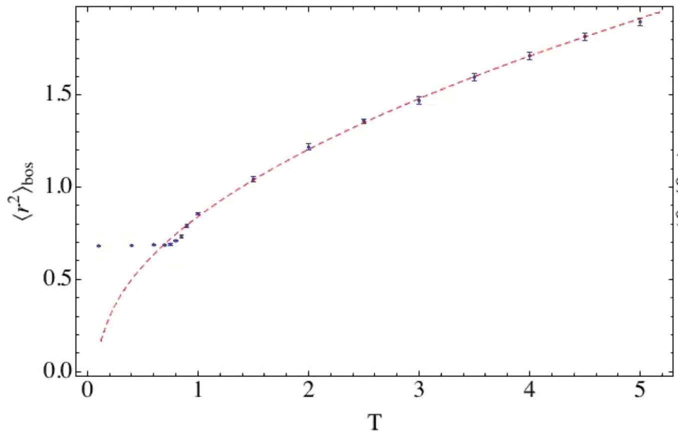
$$+ \frac{1}{2} {\rm Tr} \sum_{A=1}^{3} \mathcal{D}^A \mathcal{D}^A \Bigg] .$$

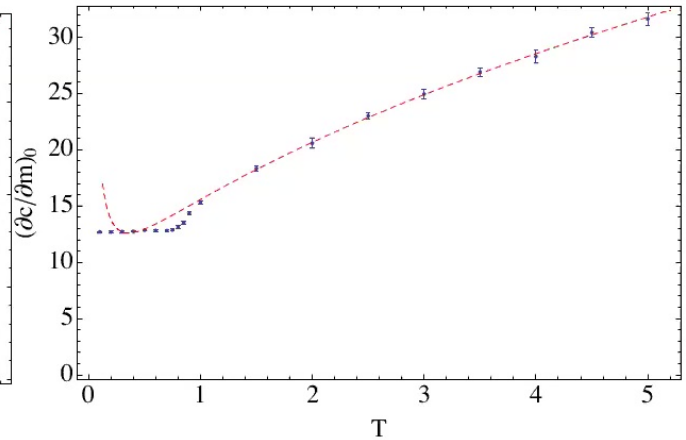
where $X^a = 0$ with

$$\mathcal{D}^A = \sigma_\rho^{A\,\sigma} \left(\frac{1}{2} [\bar{X}^{\rho\dot\rho}, X_{\sigma\dot\rho}] - \Phi_\sigma \bar\Phi^\rho \right) = 0$$

specify ADHM data for Yang-Mills instantons on $\mathbb{R}^4$.

$N_f = 1$ and $N = 10$:





With $X^a \to X^a - m^a$ and for $m^a = 0$ we can look at the condensate susceptibility:

$$\left(\frac{\partial c}{\partial m}\right)_0 = \frac{2}{\beta}\int_0^\beta d\tau\, \mathrm{tr}\, \bar{\Phi}^\rho \Phi_\rho - \frac{N}{5\beta}\left(\int_0^\beta d\tau\, \mathrm{tr}\, 2\bar{\Phi}^\rho X^a \Phi_\rho\right)^2 .$$

The D4-brane as a probe of the geometry.

The dual adds N_f D4 probe branes. In the probe approximation $N_f \ll N_c$, their dynamics is governed by the Dirac-Born-Infeld action:

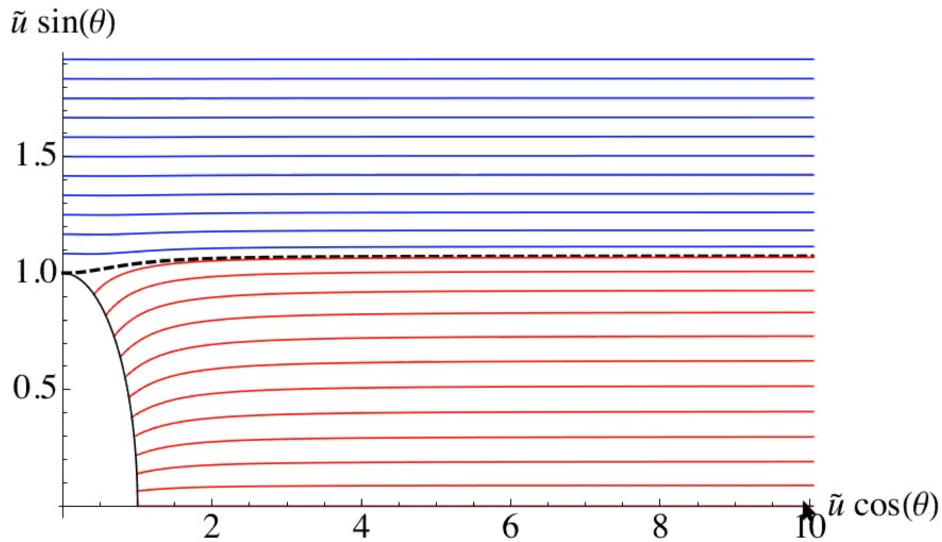
$$S_{\mathrm{DBI}} = -\frac{N_f}{(2\pi)^4\,\alpha'^{5/2}\,g_s}\int d^4\xi\, e^{-\Phi}\sqrt{-\mathrm{det}||G_{\alpha\beta} + (2\pi\alpha')F_{\alpha\beta}||}\ ,$$

where $G_{\alpha\beta}$ is the induced metric and $F_{\alpha\beta}$ is the $U(1)$ gauge field of the D4-brane. For us $F_{\alpha\beta} = 0$.

$$d\Omega_8^2 = d\theta^2 + \cos^2\theta\, d\Omega_3^2 + \sin^2\theta\, d\Omega_4^2$$

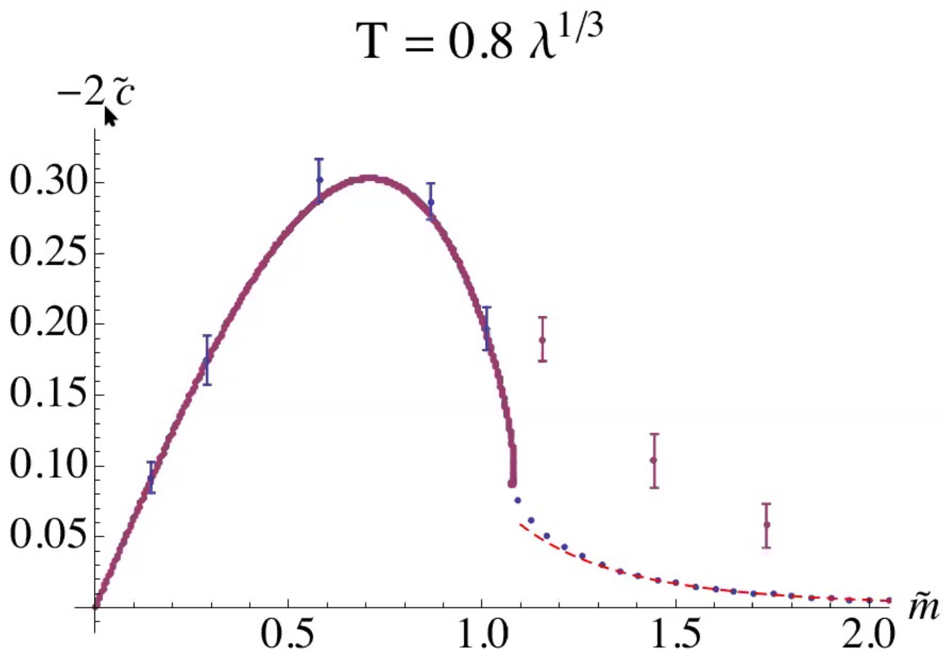
and taking a D4-brane embedding extended along: t, u, Ω_3 with a non-trivial profile $\theta(u)$.

Embeddings



$$\tilde{u} \sin\theta = m + \frac{\tilde{c}}{\tilde{u}^2} + \ldots .$$

The condensate and the dual prediction



V. Filev and D. O'C. arXiv 1512.02536.

The data overlaps surprisingly well with the gravity prediction in the region where the $D4$ brane ends in the black hole.

Conclusions

- The BD model provides a useful probe of the geometry.
- The simulations agree well with predictions and provide a strong test of aspects of the geometry.