Title: Torsion cosmology and beyond

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Abstract: Torsion is a popular ingredient in gravity, yet fraught with quantum and classical pathologies. I develop a novel torsion theory, consistent with power-counting and unitarity. The Friedmann equations emerge (with dark energy and radiation), as do pp waves and the Schwarzschild vacuum, all without an Einstein-Hilbert term. I show that cosmology sees torsion as a non-canonical scalar, revealing a rich phenomenology of conformal or waterfall inflatons, and cuscutons. I finally argue that future work will be driven less by toy-models, and more by computer surveys. I advocate Hamiltonian and effective field theory approaches to non-Riemannian geometry in general, relevant to ultraviolet completion and modified gravity alike. Such methods should be oriented towards the ultimate test of gravity: observed cosmological structure.

Torsion cosmology and beyond

Based on 2003.02690, 2006.03581

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Introducing torsion

Gravitational potentials

- Metric $g_{\mu\nu}$ gives Levi–Civita connection

$$\Gamma^{\mu}{}_{\nu\lambda} = C^{\mu}{}_{\nu\lambda} \equiv \tfrac{1}{2} g^{\mu\sigma}(\partial_\nu g_{\lambda\sigma} + \partial_\lambda g_{\nu\sigma} - \partial_\sigma g_{\nu\lambda})$$

- Tetrad $b^a{}_\mu$ and inverse $h_a{}^\mu$ and spin connection $A^{ab}{}_\mu$

$$g_{\mu\nu} \equiv \eta_{ab} b^a{}_\mu b^b{}_\nu, \quad g^{\mu\nu} \equiv \eta^{ab} h_a{}^\mu h_b{}^\nu$$
$$\Gamma^{\mu}{}_{\nu\lambda} \equiv h_a{}^\mu \partial_\lambda b^a{}_\nu + h_a{}^\mu b^b{}_\nu A^a{}_{b\lambda}$$

- Since $A^{ab}{}_\mu$ is independent we may have contorsion $K^{\mu}{}_{\lambda\nu}$

$$\Gamma^{\mu}{}_{\nu\lambda} = C^{\mu}{}_{\nu\lambda} + K^{\mu}{}_{\nu\lambda}$$

Gravitational fields

- Riemann curvature $R_{\alpha\beta\mu}{}^{\nu}$

$$R_{\alpha\beta\mu}{}^{\nu} \equiv 2\left(\partial_{[\beta}\Gamma^{\nu}{}_{\alpha]\mu} + \Gamma^{\lambda}{}_{[\alpha|\mu}\Gamma^{\nu}{}_{|\beta]\lambda}\right)$$

- Alternatively Riemann–Cartan curvature $\mathcal{R}^{ij}{}_{kl}$

$$\mathcal{R}^{ij}{}_{kl} \equiv 2h_k{}^{\mu}h_l{}^{\nu}\left(\partial_{[\mu}A^{ij}{}_{\nu]} + A^{i}{}_{o[\mu}A^{oj}{}_{\beta]}\right)$$

- Contorsion also gives us torsion $\mathcal{T}_{abc} \equiv 2h_a{}^{\sigma}h_b{}^{\nu}h_c{}^{\lambda}K_{\sigma[\lambda\nu]}$

$$\mathcal{T}^{i}_{kl} \equiv 2h_k{}^{\mu}h_l{}^{\nu}\left(\partial_{[\mu}b^{i}{}_{\nu]} + A^{i}{}_{o[\mu}b^{o}{}_{\nu]}\right)$$

- Overall structure

$$R \sim \partial^2 g + (\partial g)^2, \quad \mathcal{R} \sim \partial A + A^2, \quad \mathcal{T} \sim \partial b + bA$$

Why have we done this?

- Spinors call for transition to tetrads and spin connection

$$g_{\mu\nu} \quad \rightarrow \quad h_a{}^{\mu}, \quad b^a{}_{\mu}, \quad A^{ab}{}_{\mu}$$

- Gauge whole Poincaré group vs diffeomorphisms

$$\mathbb{R}^{1,3} \quad \rightarrow \quad \mathbb{R}^{1,3} \rtimes \mathrm{SO}^+(1,3)$$

- Gravitational fields now adopt familiar Yang–Mills form

$$R \sim \partial^2 g + (\partial g)^2 \quad \rightarrow \quad \mathcal{R} \sim \partial A + A^2, \quad \mathcal{T} \sim \partial b + bA$$

- This is Poincaré gauge theory Ryoyu Utiyama (1956) , D. W. Sciama (1964) , T. W. B. Kibble (1961)
- Torsion not intrinsically desirable, but worth including for completeness

The hard part is choosing a theory

- Traditional building blocks

$$R^{\alpha\beta}{}_{\mu\nu}, \quad R^{\alpha}{}_{\mu} \equiv R^{\alpha\beta}{}_{\mu\beta}, \quad R \equiv R^{\alpha}{}_{\alpha}$$

- Gold standard set by the Einstein–Hilbert theory

$$L_{\mathrm{G}} = -\tfrac{1}{2} m_{\mathrm{p}}{}^{2} R$$

- New building blocks

$$\mathcal{R}^{ij}{}_{kl}, \quad \mathcal{R}^{i}{}_{k} \equiv \mathcal{R}^{ij}{}_{kj}, \quad \mathcal{R} \equiv \mathcal{R}^{i}{}_{i}, \quad \mathcal{T}^{i}{}_{jk}, \quad \mathcal{T}_{j} \equiv \mathcal{T}^{i}{}_{ji}$$

- Einstein–Cartan and teleparallel 'equivalent' alternatives

$$L_{\mathrm{G}} = -\tfrac{1}{2} m_{\mathrm{p}}{}^{2} \mathcal{R} \qquad L_{\mathrm{G}} = m_{\mathrm{p}}{}^{2} \left(\tfrac{1}{8} \mathcal{T}_{abc} \mathcal{T}^{abc} + \tfrac{1}{4} \mathcal{T}_{abc} \mathcal{T}^{bac} - \tfrac{1}{2} \mathcal{T}_{a} \mathcal{T}^{a} \right)$$

The hard part is choosing a theory

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- Einstein–Cartan and teleparallel ‘equivalent’ alternatives

$$L_{\mathrm{G}} = -\tfrac{1}{2}{m_{\mathrm{p}}}^2\mathcal{R} \qquad L_{\mathrm{G}} = {m_{\mathrm{p}}}^2\left(\tfrac{1}{8}\mathcal{T}_{abc}\mathcal{T}^{abc} + \tfrac{1}{4}\mathcal{T}_{abc}\mathcal{T}^{bac} - \tfrac{1}{2}\mathcal{T}_a\mathcal{T}^a\right)$$

The popular Lagrangian

- The general ten-parameter theory is most commonly studied

$$
\begin{aligned}
L_{\mathrm{G}} = &-\tfrac{1}{2}\alpha_0 {m_{\mathrm{p}}}^2\mathcal{R} + \alpha_1\mathcal{R}^2 + \alpha_2\mathcal{R}_{ab}\mathcal{R}^{ab} + \alpha_3\mathcal{R}_{ab}\mathcal{R}^{ba} \\
&+ \alpha_4\mathcal{R}_{abcd}\mathcal{R}^{abcd} + \alpha_5\mathcal{R}_{abcd}\mathcal{R}^{acbd} + \alpha_6\mathcal{R}_{abcd}\mathcal{R}^{cdab} \\
&+ \beta_1 {m_{\mathrm{p}}}^2\mathcal{T}_{abc}\mathcal{T}^{abc} + \beta_2 {m_{\mathrm{p}}}^2\mathcal{T}_{abc}\mathcal{T}^{bac} + \beta_3 {m_{\mathrm{p}}}^2\mathcal{T}_a\mathcal{T}^a
\end{aligned}
$$

- Ostrogradsky's theorem: avoid cubic and higher invariants
- Parity: avoid dual invariants
- Gauss–Bonnet identity: can eliminate one of α_1, α_3 or α_6
- Otherwise ten dimensionless couplings α_i, β_i (I'll always put these in blue!)

Flat, weak and free

Flat, weak and free

- Linearise on Minkowski background without matter or cosmological constant
- Extra Lagrangian symmetries emerge if the α_i, β_i obey certain equalities: these critical cases must be considered separately
- Lin, Hobson and Lasenby performed an exhaustive survey 1812.02675, 1910.14197, building on earlier work D E. Neville (1980), E. Sezgin et al. (1980)
- 1918 critical cases in total
- 450 of which are free of ghosts and tachyons under further unitarity inequalities on the α_i, β_i
- 58 of which are power-counting renormalisable

Power-counting abhors Einstein–Hilbert

#	criticality equalities	ghost-tachyon exorcism inequalities	0^-	0^+	1^-	1^+	2^-	2^+	d.o.f
1	$l=r_1=t_1=t_3=r_3-2r_4=0$	$0<t_2, r_2<0, r_3(2r_3+r_5)(r_3+2r_5)<0$	•		∘	∘∘		∘	• ∘ ∘
2	$l=r_1=t_1=r_3-2r_4=0$	$0<t_2, r_2<0, r_3(2r_3+r_5)(r_3+2r_5)<0$	•	∘∘	∘∘∘	∘∘		∘	
8	$l=r_2=r_4=t_1=t_2=r_1-r_3=0$	$r_1(r_1+r_5)(2r_1+r_5)<0$		∘∘	∘∘∘	∘	∘		
^{*1}9	$l=r_2=r_4=t_1=t_2=t_3=r_1-r_3=0$	$r_1(r_1+r_5)(2r_1+r_5)<0$			∘	∘	∘		
^{*3}10	$l=r_1=r_2=t_1=t_2=t_3=r_3-2r_4=0$	$r_3(2r_3+r_5)(r_3+2r_5)<0$				∘		∘	
^{*4}11	$l=r_1=t_1=t_2=t_3=r_3-2r_4=0$	$r_3(2r_3+r_5)(r_3+2r_5)<0$	∘		∘	∘		∘	∘ ∘
12	$l=r_1=r_2=t_1=t_3=r_3-2r_4=0$	$r_3(2r_3+r_5)(r_3+2r_5)<0$	∘		∘	∘∘		∘	
^{*2}13	$l=r_2=t_1=t_2=t_3=2r_1-2r_3+r_4=0$	$0<r_1(r_1-2r_3-r_5)(2r_3+r_5)$		∘	∘	∘	∘		
14	$l=r_1=r_2=t_1=t_2=r_3-2r_4=0$	$r_3(2r_3+r_5)(r_3+2r_5)<0$		∘∘	∘∘∘	∘		∘	
15	$l=r_1=r_2=t_1=r_3-2r_4=0$	$r_3(2r_3+r_5)(r_3+2r_5)<0$	∘	∘∘	∘∘∘	∘∘		∘	
16	$l=r_1=t_1=t_2=r_3-2r_4=0$	$r_3(2r_3+r_5)(r_3+2r_5)<0$	∘	∘∘	∘∘∘	∘		∘	
20	$l=r_1=r_3=r_4=r_5=0$	$0<t_2, r_2<0$	•	∘∘	∘∘∘	∘∘	∘	∘∘	
21	$l=r_1=r_3=r_4=r_5=t_1+t_2=0$	$r_2<0, t_1<0$	•	∘∘	∘∘∘	∘∘	∘	∘∘	
22	$l=r_1=r_3=r_4=r_5=t_1+t_3=0$	$0<t_2, r_2<0$	•	∘∘	∘∘∘	∘∘	∘	∘∘	
23	$l=r_1=r_3=r_4=r_5=t_1+t_2=t_1+t_3=0$	$r_2<0, t_1<0$	•	∘∘	∘∘∘	∘∘	∘	∘∘	
24	$l=r_1=r_3=r_4=t_1=0$	$0<t_2, r_2<0$	•	∘∘	∘∘∘	∘∘			
^{*5}25	$l=r_1=r_3=r_4=r_5=t_1=0$	$0<t_2, r_2<0$	•	∘∘	∘∘∘	∘∘			
^{*6}26	$l=r_1=r_3=r_4=r_5=t_1=t_3=0$	$0<t_2, r_2<0$	•			∘∘			
27	$l=r_1=t_1=t_3=r_3-2r_4=r_3+2r_5=0$	$0<t_2, r_2<0$	•			∘∘		∘	
28	$l=r_1=r_3=r_4=t_1=t_3=0$	$0<t_2, r_2<0$	•		∘	∘∘			•
29	$l=r_4=t_1=r_1-r_3=2r_1+r_5=0$	$0<t_2, r_2<0$	•	∘∘	∘∘∘	∘∘	∘		
^{*7}30	$l=r_4=t_1=t_3=r_1-r_3=2r_1+r_5=0$	$0<t_2, r_2<0$	•		∘	∘∘	∘		
^{*8}31	$l=r_1=t_1=t_3=2r_3-r_4=2r_3+r_5=0$	$0<t_2, r_2<0$	•	∘		∘∘			
32	$l=r_1=r_3=r_4=r_5=t_3=0$	$0<t_2, r_2<0$	•		∘∘∘	∘∘	∘	∘∘	
33	$l=r_1=r_3=r_4=r_5=t_3=t_1+t_2=0$	$r_2<0, t_1<0$	•		∘∘∘	∘∘	∘	∘∘	
34	$l=r_1=t_1=t_3=2r_3-r_4=0$	$0<t_2, r_2<0$	•	∘	∘	∘∘			
^{*9}35	$l=r_1=t_1=t_3=r_3-2r_4=2r_3+r_5=0$	$0<t_2, r_2<0$	•		∘	∘∘		∘	
^{*10}36	$l=t_1=t_3=2r_3+r_5=2r_1-2r_3+r_4=0$	$0<t_2, r_2<0$	•	∘	∘	∘∘	∘		
37	$l=r_1=t_1=r_3-2r_4=2r_3+r_5=0$	$0<t_2, r_2<0$	•	∘∘	∘∘∘	∘∘		∘	
38	$l=r_1=t_3=2r_3-r_4=2r_3+r_5=0$	$0<t_2, r_2<0$	•	∘	∘∘∘	∘∘	∘	∘∘	
39	$l=r_1=t_3=2r_3-r_4=2r_3+r_5=t_1+t_2=0$	$r_2<0, t_1<0$	•	∘	∘∘∘	∘∘	∘	∘∘	
40	$l=r_1=t_1=t_3=r_4+r_5=0$	$0<t_2, r_2<0$	•	∘		∘∘		∘	
41	$l=r_1=t_1=r_3-2r_4=r_3+2r_5=0$	$0<t_2, r_2<0$	•	∘∘	∘∘∘	∘∘		∘	

- Note linear shuffling of couplings (sorry!)

$$\alpha_i, \beta_i \rightarrow l, r_i, t_i$$

- First column defines cases
- Second column unitarity conditions
- Final columns particle content

Background cosmology

- Cosmology is a convenient test
- Ansatz for tetrads is flat, open or closed FRW metric

$$ds^2 = dt^2 - \frac{a^2 dr^2}{1 - kr^2/{r_0}^2} - a^2 r^2 (d\vartheta^2 + \sin^2\vartheta d\varphi^2), \quad k \in \{\pm 1, 0\}$$

- Ansatz for torsion is scalar U and pseudoscalar Q

$$\mathcal{T}^a{}_{bc} = (\hat{\mathbf{e}}_t)^d \left(\tfrac{2}{3} U \delta^a_{[c} \eta_{db]} - Q \epsilon^a{}_{dbc} \right)$$

- Cosmological fluids are spinless radiation, matter and dark energy defined by equation-of-state parameter $P_i = w_i \rho_i$

$$w_{\mathrm{r}} = 1/3, \quad w_{\mathrm{m}} = 0, \quad w_{\Lambda} = -1$$

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Cosmological couplings

- Recall the general ten-parameter theory

$$L_{\mathrm{G}} = -\tfrac{1}{2}\alpha_0 {m_{\mathrm{p}}}^2\mathcal{R} + \alpha_1\mathcal{R}^2 + \alpha_2\mathcal{R}_{ab}\mathcal{R}^{ab} + \alpha_3\mathcal{R}_{ab}\mathcal{R}^{ba} + \alpha_4\mathcal{R}_{abcd}\mathcal{R}^{abcd} + \alpha_5\mathcal{R}_{abcd}\mathcal{R}^{acbd} + \alpha_6\mathcal{R}_{abcd}\mathcal{R}^{cdab} + \beta_1 {m_{\mathrm{p}}}^2\mathcal{T}_{abc}\mathcal{T}^{abc} + \beta_2 {m_{\mathrm{p}}}^2\mathcal{T}_{abc}\mathcal{T}^{bac} + \beta_3 {m_{\mathrm{p}}}^2\mathcal{T}_a\mathcal{T}^a$$

- Apart from α_0 (i.e. Einstein–Hilbert), five coupling combinations are seen by cosmology

$$\sigma_1 \equiv \tfrac{3}{2}\alpha_1 + \tfrac{1}{4}\alpha_2 + \tfrac{1}{4}\alpha_3 + \tfrac{1}{4}\alpha_5 - \tfrac{1}{2}\alpha_6$$
$$\sigma_2 \equiv \tfrac{3}{2}\alpha_1 + \tfrac{1}{2}\alpha_2 + \tfrac{1}{2}\alpha_3 + \tfrac{3}{2}\alpha_4 - \tfrac{1}{4}\alpha_5 + \tfrac{1}{4}\alpha_6$$
$$\sigma_3 \equiv \tfrac{3}{2}\alpha_1 + \tfrac{1}{2}\alpha_2 + \tfrac{1}{2}\alpha_3 + \tfrac{1}{2}\alpha_4 - \tfrac{1}{4}\alpha_5 + \tfrac{1}{2}\alpha_6$$
$$\upsilon_1 \equiv \beta_2 - 2\beta_1 \quad \upsilon_2 \equiv 2\beta_1 + \beta_2 + 3\beta_3$$

A quick aside: k-screening

- Some cumbersome minisuperspace equations follow (we will soon show a far better way to do this)

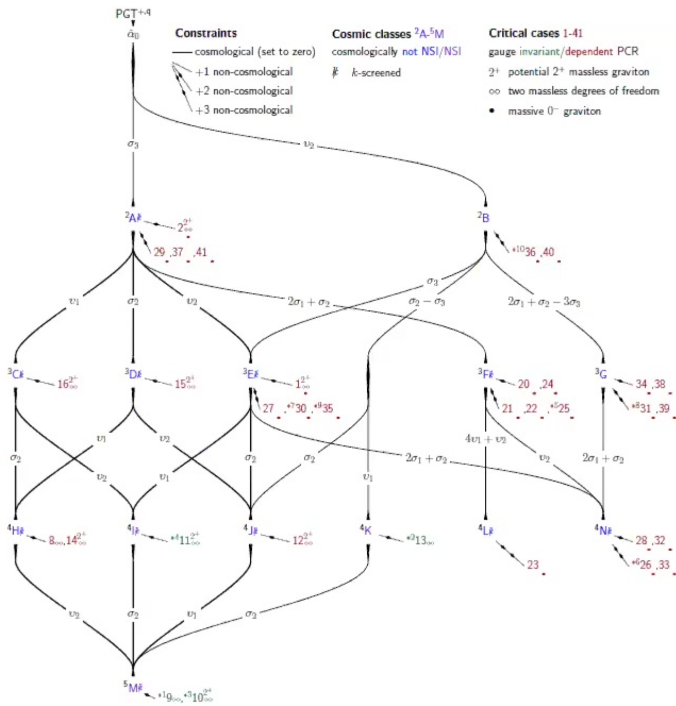
$$
\begin{aligned}
0 &= (\upsilon_2 + \alpha_0)a(aX + \partial_\tau a) - 8{m_\mathrm{p}}^{-2}\sigma_3\partial_\tau^2 X - 4{m_\mathrm{p}}^{-2}\sigma_1 Y\partial_\tau Y - 4{m_\mathrm{p}}^{-2}X(\sigma_2 Y^2 - 4\sigma_3(X^2 + k)),\\
0 &= (4\upsilon_1 - \alpha_0)a^2 Y - 4{m_\mathrm{p}}^{-2}(\sigma_3 - \sigma_2)\partial_\tau^2 Y + 16{m_\mathrm{p}}^{-2}\sigma_1 Y\partial_\tau X + 4{m_\mathrm{p}}^{-2}Y(\sigma_3 Y^2 - 4{m_\mathrm{p}}^{-2}(\sigma_2 X^2 + \sigma_3 k)),\\
0 &= 12\upsilon_2\partial_\tau^2 a + 12(\upsilon_2 + \alpha_0)a(\partial_\tau X - X^2) - 3(4\upsilon_1 - \alpha_0)aY^2 - 12\alpha_0 ka + 2m_\mathrm{p}\varrho_\mathrm{m} + 8\Lambda a^3,\\
0 &= 12\upsilon_2(2a\partial_\tau^2 a - (\partial_\tau a)^2) + 12(\upsilon_2 + \alpha_0)a^2(2\partial_\tau X - X^2) - 3(4\upsilon_1 - \alpha_0)a^2Y^2 - 12\alpha_0 ka^2 + 6{m_\mathrm{p}}^{-2}\sigma_3(16X^2(X^2 + 2k)\\
&\quad + Y^2(Y^2 - 8k) + 16k^2 - 2(\partial_\tau Y)^2 - 16(\partial_\tau X)^2) + 12{m_\mathrm{p}}^{-2}\sigma_2((\partial_\tau Y)^2 - 2X^2Y^2) - 4{m_\mathrm{p}}^{-2}\varrho_\mathrm{r} + 12\Lambda a^4
\end{aligned}
$$

- Main point is that spatial curvature $k \in \{\pm 1, 0\}$ is eliminated from the system (i.e. 'k-screened') if we set

$$\alpha_0 = \sigma_3 = 0$$

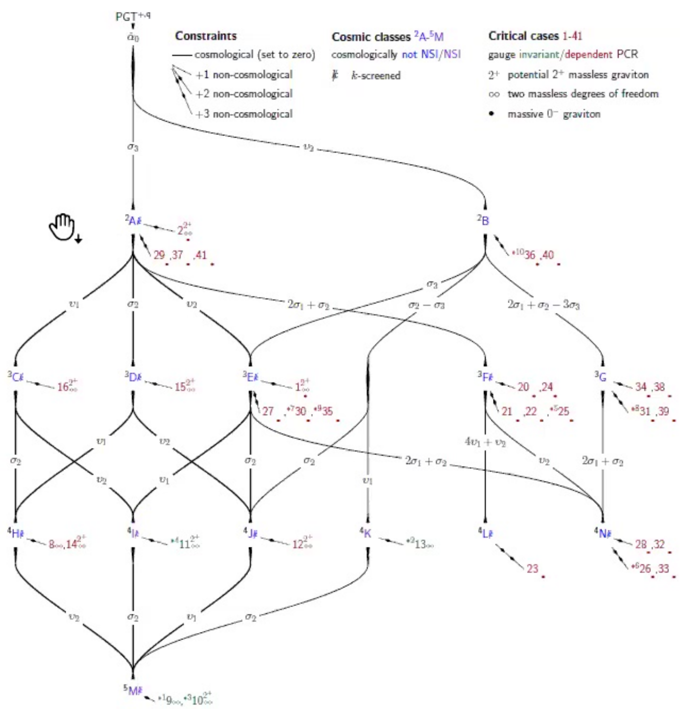
- Recall α_0 couples ${m_\mathrm{p}}^2\mathcal{R}$ and σ_3 couples some $\mathcal{R}^2$ combination

Return to new theories: fourteen cosmologies



- Map new theories by cosmological α_0, σ_i, υ_i
- Power-counting $\alpha_0 = 0$ at the top, always removes ${m_\mathrm{p}}^2\mathcal{R}$
- Some literature on RHS
  A. V. Minkevich (1980),
  1009.5112,
  1105.5001,
  Fei-Hung Ho et al. (2011),
  1512.01202,
  1906.04340

Return to new theories: fourteen cosmologies



- Map new theories by cosmological α_0, σ_i, υ_i
- Power-counting $\alpha_0 = 0$ seen at the top always removes ${m_p}^2\mathcal{R}$
- Highlighted 'cube' on LHS is novel and imposes final k-screening condition $\sigma_3 = 0$ on the $\mathcal{R}^2$ couplings

No $\mathcal{R}$ and no k, can this possibly work?

- Remaining couplings σ_1, σ_2 for $\mathcal{R}^2$ and υ_2 for $m_\text{p}{}^2\mathcal{T}$
- Eliminate scalar torsion U for pseudoscalar Q and Hubble H
- Then get a Friedmann-like equation and another for Q

$$\sum_i \Omega_i = g \quad f_1\frac{\ddot{Q}}{Q} + f_2\frac{\dot{Q}^2}{Q^2} + f_3\frac{\dot{Q}}{Q}H + f_4\dot{H} + f_5H^2 = 0$$

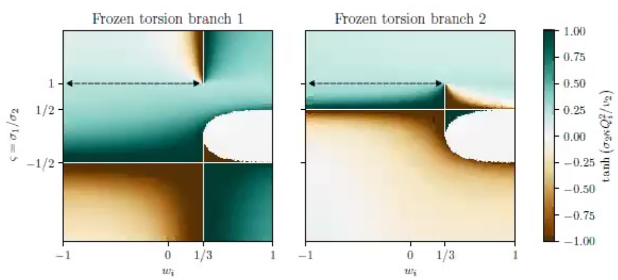
- Note $g = g(Q, H|\sigma_1, \sigma_2, \upsilon_2)$ and $f_i = f_i(Q, H|\sigma_1, \sigma_2, \upsilon_2)$
- When matter $P_i = w_i\rho_i$ dominates, pseudoscalar freezes

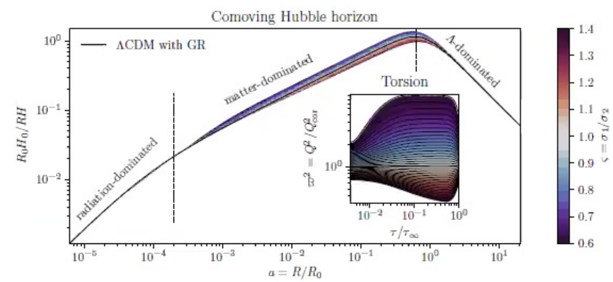
$$g \to g_i, \quad Q \to Q_i$$

- So this is a flat Friedmann solution for H with renormalised gravitational constant G/g_i

Recovering the Friedmann equations

- Dependence of g_i and Q_i on dominant w_i is cumbersome, but can be tuned with ratio of $\mathcal{R}^2$ couplings $\varsigma \equiv \sigma_1/\sigma_2$
- Main point is that for numerically natural choices $\varsigma = 1$ and $\upsilon_2 = -3/4$ you get $g_i \equiv 1$ for all dominant w_i
- So usual flat Friedmann equation $\sum_i \Omega_i = 1$ is recovered



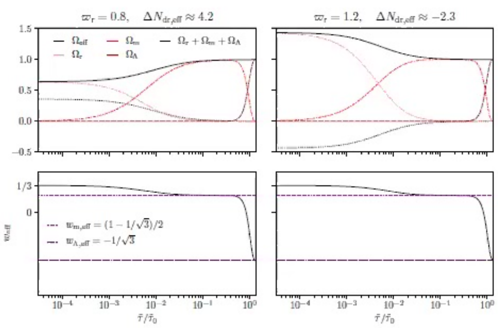
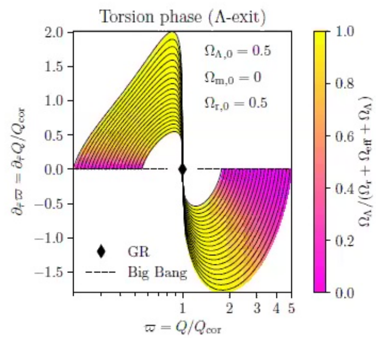


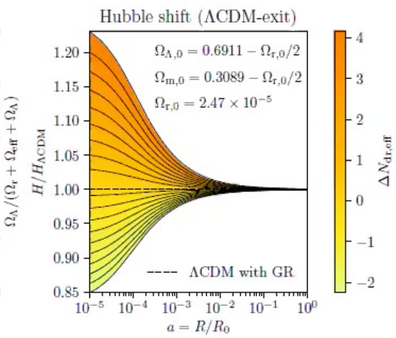
Emergent dark radiation

- Cast perturbation of flat Friedmann solution as 'effective' matter component $\rho \to \rho + \epsilon\rho_{\text{eff}}$
- Effective equation-of-state parameter also depends on dominant fluid $w_{\text{eff}} = w_{\text{eff}}(w_i) \equiv \frac{1}{2}(w_i + 1) - \frac{1}{6}\sqrt{9w_i^2 + 3}$

$$w_{\text{eff}}(1/3) = 1/3 \quad w_{\text{eff}}(0) \approx 0.211 \quad w_{\text{eff}}(-1) \approx -0.577$$

- Perturbation redshifts away at late times, looks like radiation at early times ('dark' radiation)



The Hubble tension

- H_0 tension between low CMB inference and high local observation 1903.07603
- Early-Universe solution seeks to preserve the position of the first CMB multipole peak $l_a = \pi D_A(z_{rec})/r_s$

$$D_A(z_{rec}) = \frac{\sin\left(\sqrt{-\Omega_{k,0}}\int_0^{z_{rec}}\frac{H_0 dz}{H}\right)}{H_0\sqrt{-\Omega_{k,0}}} \quad r_s = \int_0^{t_{rec}}\frac{c_s dt}{a}$$

- New physics increases early H for $z_{rec} < z$, e.g. dark radiation
- Inferred increased H for late $z < z_{rec}$
- Popular 0702343, 1608.01309, 1902.10636 but problematic 1801.07260

Torsion equals scalar-tensor

Torsion-free analogue theory for cosmology

- Return to general ten-parameter torsion theory

$$L_{\mathrm{G}} = -\tfrac{1}{2}\alpha_0 m_{\mathrm{p}}{}^2\mathcal{R} + \alpha_1\mathcal{R}^2 + \alpha_2\mathcal{R}_{ab}\mathcal{R}^{ab} + \alpha_3\mathcal{R}_{ab}\mathcal{R}^{ba} + \alpha_4\mathcal{R}_{abcd}\mathcal{R}^{abcd} + \alpha_5\mathcal{R}_{abcd}\mathcal{R}^{acbd} + \alpha_6\mathcal{R}_{abcd}\mathcal{R}^{cdab} + \beta_1 m_{\mathrm{p}}{}^2\mathcal{T}_{abc}\mathcal{T}^{abc} + \beta_2 m_{\mathrm{p}}{}^2\mathcal{T}_{abc}\mathcal{T}^{bac} + \beta_3 m_{\mathrm{p}}{}^2\mathcal{T}_a\mathcal{T}^a$$

- Restrict ourselves to flat $k=0$ with Hubble $H = \dot{a}/a$

$$ds^2 = dt^2 - a^2 d\boldsymbol{x}^2, \quad \mathcal{T}^a{}_{bc} = (\hat{\boldsymbol{e}}_t)^d\left(\tfrac{2}{3}U\delta^a_{[c}\eta_{db]} - Q\epsilon^a{}_{dbc}\right)$$

- Carefully define scalars ϕ, ψ with the right conformal weight

$$U = 3(\tfrac{1}{2}\phi + H), \quad Q = \psi$$

Quest for a scalar-tensor version

- Recall we have Riemann curvature $R^{\alpha\beta}{}_{\mu\nu}$, Riemann–Cartan curvature $\mathcal{R}^{ij}{}_{kl}$ and torsion $\mathcal{T}^{i}_{jk}$
- Fundamental fields redefined as

$$b^{a}{}_{\mu}, A^{ab}{}_{\mu} \quad \rightarrow \quad g_{\mu\nu}, \phi, \psi$$

- Lagrangian building blocks, $X^{\phi\phi} = \frac{1}{2}\nabla_{\mu}\phi\nabla^{\mu}\phi$

$$\mathcal{R}^{ij}{}_{kl}, \mathcal{R}^{i}{}_{k}, \mathcal{R}, \mathcal{T}^{i}_{jk}, \mathcal{T}_{j} \quad \rightarrow \quad R^{\alpha\beta}{}_{\mu\nu}, R^{\alpha}{}_{\mu}, R, \phi, \psi, X^{\phi\phi}, X^{\phi\psi} \dots$$

- Not as simple as separating out the contorsion: have to match equations of motion

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- Not as simple as separating out the contorsion: have to match equations of motion

Final answer: non-canonical bi-galileon

- Need a neutral vector B^μ and third scalar χ

$$L_{\mathrm{G}} \simeq G_2 + G_4 R + \left(G_6^{\phi} \nabla_\mu \phi + G_6^{\psi} \nabla_\mu \psi\right) B^\mu + m_{\mathrm{p}}\left({m_{\mathrm{p}}}^2 - D_\mu B^\mu\right)\chi$$

- But χ is a multiplier which turns B^μ into a Lorentz-violating vector field 0407149
- Both χ and B^μ are non-dynamical constraints, yielding

$$\begin{aligned} L_{\mathrm{G}} \simeq & \left[\tfrac{1}{2}{m_{\mathrm{p}}}^2 \upsilon_2 + \sigma_3 \phi^2 + \tfrac{1}{2}(\sigma_3 - \sigma_2)\psi^2\right] R \\ & + 12\sigma_3 X^{\phi\phi} + 6(\sigma_3 - \sigma_2) X^{\psi\psi} + \sqrt{|J_\mu J^\mu|} \\ & + \tfrac{3}{4}{m_{\mathrm{p}}}^2(\alpha_0 + \upsilon_2)\phi^2 - \tfrac{3}{4}{m_{\mathrm{p}}}^2(\alpha_0 - 4\upsilon_1)\psi^2 \\ & + \tfrac{3}{2}\sigma_3\phi^4 - 3\sigma_2\phi^2\psi^2 + \tfrac{3}{2}\sigma_3\psi^4 \\ J_\mu \equiv & \; 4\sigma_1 \psi^3 \partial_\mu(\phi/\psi) - {m_{\mathrm{p}}}^2(\alpha_0 + \upsilon_2)\partial_\mu\phi \end{aligned}$$

Anatomy

- Torsion implies a Jordan frame

$$
\begin{aligned}
L_G \simeq &\left[\frac{1}{2}m_p{}^2\upsilon_2 + \sigma_3\phi^2 + \frac{1}{2}(\sigma_3 - \sigma_2)\psi^2\right] R \\
&+ 12\sigma_3 X^{\phi\phi} + 6(\sigma_3 - \sigma_2)X^{\psi\psi} + \sqrt{|J_\mu J^\mu|} \\
&+ \frac{3}{4}m_p{}^2(\alpha_0 + \upsilon_2)\phi^2 - \frac{3}{4}m_p{}^2(\alpha_0 - 4\upsilon_1)\psi^2 \\
&+ \frac{3}{2}\sigma_3\phi^4 - 3\sigma_2\phi^2\psi^2 + \frac{3}{2}\sigma_3\psi^4
\end{aligned}
$$

$$
J_\mu \equiv 4\sigma_1\psi^3\nabla_\mu\left(\frac{\phi}{\psi}\right) - m_p{}^2(\alpha_0 + \upsilon_2)\nabla_\mu\phi
$$

Anatomy

- Non-minimal coupling naturally conformal

$$L_{\mathrm{G}} \simeq \left[\frac{1}{2}m_{\mathrm{p}}{}^2\upsilon_2 + \sigma_3\phi^2 + \frac{1}{2}(\sigma_3 - \sigma_2)\psi^2\right] R$$
$$+ 12\sigma_3 X^{\phi\phi} + 6(\sigma_3 - \sigma_2)X^{\psi\psi} + \sqrt{|J_\mu J^\mu|}$$
$$+ \frac{3}{4}m_{\mathrm{p}}{}^2(\alpha_0 + \upsilon_2)\phi^2 - \frac{3}{4}m_{\mathrm{p}}{}^2(\alpha_0 - 4\upsilon_1)\psi^2$$
$$+ \frac{3}{2}\sigma_3\phi^4 - 3\sigma_2\phi^2\psi^2 + \frac{3}{2}\sigma_3\psi^4$$

$$J_\mu \equiv 4\sigma_1\psi^3\nabla_\mu\left(\frac{\phi}{\psi}\right) - m_{\mathrm{p}}{}^2(\alpha_0 + \upsilon_2)\nabla_\mu\phi$$

Anatomy

- Mass terms (eliminate tachyons)

$$
\begin{aligned}
L_{\rm G} \simeq & \left[\frac{1}{2}m_{\rm p}{}^2\upsilon_2 + \sigma_3\phi^2 + \frac{1}{2}(\sigma_3 - \sigma_2)\psi^2\right] R \\
& + 12\sigma_3 X^{\phi\phi} + 6(\sigma_3 - \sigma_2)X^{\psi\psi} + \sqrt{|J_\mu J^\mu|} \\
& + \frac{3}{4}m_{\rm p}{}^2(\alpha_0 + \upsilon_2)\phi^2 - \frac{3}{4}m_{\rm p}{}^2(\alpha_0 - 4\upsilon_1)\psi^2 \\
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\end{aligned}
$$

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J_\mu \equiv 4\sigma_1\psi^3\nabla_\mu\left(\frac{\phi}{\psi}\right) - m_{\rm p}{}^2(\alpha_0 + \upsilon_2)\nabla_\mu\phi
$$

Example: Einstein–Cartan equals cuscuton

- Recall Einstein–Cartan 'equivalent' to general relativity

$$L_{\mathrm{G}} = -\frac{1}{2} {m_{\mathrm{p}}}^2 \mathcal{R}$$

- Scalar-tensor version doesn't even contain R!

$$L_{\mathrm{G}} \simeq -{m_{\mathrm{p}}}^2 \left(\sqrt{2|X^{\phi\phi}|} - \frac{3}{4}\phi^2 + \frac{3}{4}\psi^2 \right)$$

- Reconcile this with cuscuton on FRW 0702002

$$c_1 {m_{\mathrm{p}}}^2 \sqrt{|X^{\phi\phi}|} - c_2 {m_{\mathrm{p}}}^2 \phi^2 \quad \simeq \quad \frac{3{c_1}^2}{16 c_2} {m_{\mathrm{p}}}^2 R$$

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Example: teleparallel equals Einstein–Hilbert

- Teleparallel theory also 'equivalent'

$$L_{\mathrm{G}} = m_{\mathrm{p}}{}^2 \left(\frac{1}{8} \mathcal{T}_{abc} \mathcal{T}^{abc} + \frac{1}{4} \mathcal{T}_{abc} \mathcal{T}^{bac} - \frac{1}{2} \mathcal{T}_a \mathcal{T}^a \right)$$

- Scalar tensor version contains R this time

$$L_{\mathrm{G}} \simeq -\frac{1}{2} m_{\mathrm{p}}{}^2 R + m_{\mathrm{p}}{}^2 \left(\sqrt{2|X^{\phi\phi}|} - \frac{3}{4}\phi^2 + \frac{3}{4}\psi^2 \right)$$

- But Weitzenböck connection removes $\phi \equiv \psi \equiv 0$

$$L_{\mathrm{G}} \simeq -\frac{1}{2} m_{\mathrm{p}}{}^2 R$$

Example: conformal inflaton

- Set $\alpha_0 = -\upsilon_2 = 1$ and $\sigma_3 = 0$ and define weightless $\zeta \equiv \phi/\psi$

$$L_G \simeq -\frac{1}{2}{m_p}^2 R - \frac{1}{2}\sigma_2\psi^2 R - 6\sigma_2 X^{\psi\psi}$$
$$+4\sigma_1\psi^3\sqrt{2|X^{\zeta\zeta}|} - 3\sigma_2\zeta^2\psi^4 - \frac{3}{4}{m_p}^2(1-4\upsilon_1)\psi^2$$

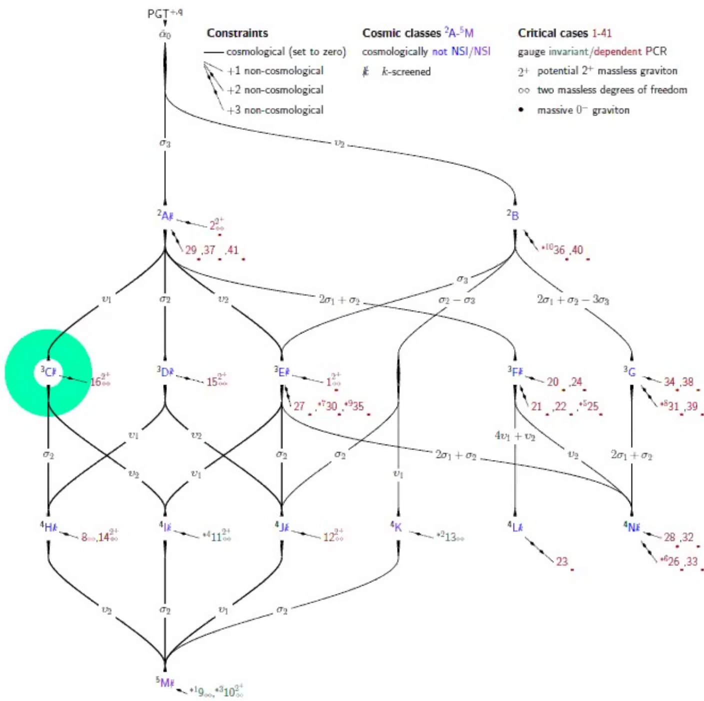
- New conformal cuscuton identity

$$c_1\psi^3\sqrt{2|X^{\zeta\zeta}|} - c_2\zeta^2\psi^4 \quad \simeq \quad \frac{9{c_1}^2}{4c_2}\left(X^{\psi\psi} + \frac{1}{12}\psi^2 R\right)$$

- So end up with conformally coupled massive scalar!

$$L_G \simeq -\frac{1}{2}{m_p}^2 R + \frac{1}{12}\psi^2 R + X^{\psi\psi} - \frac{1}{2}{m_\psi}^2\psi^2, \quad m_\psi \equiv \frac{\sigma_2\sqrt{1-4\upsilon_1}}{2\sqrt{6}(4{\sigma_1}^2-{\sigma_2}^2)}m_p$$

Return to promising theories

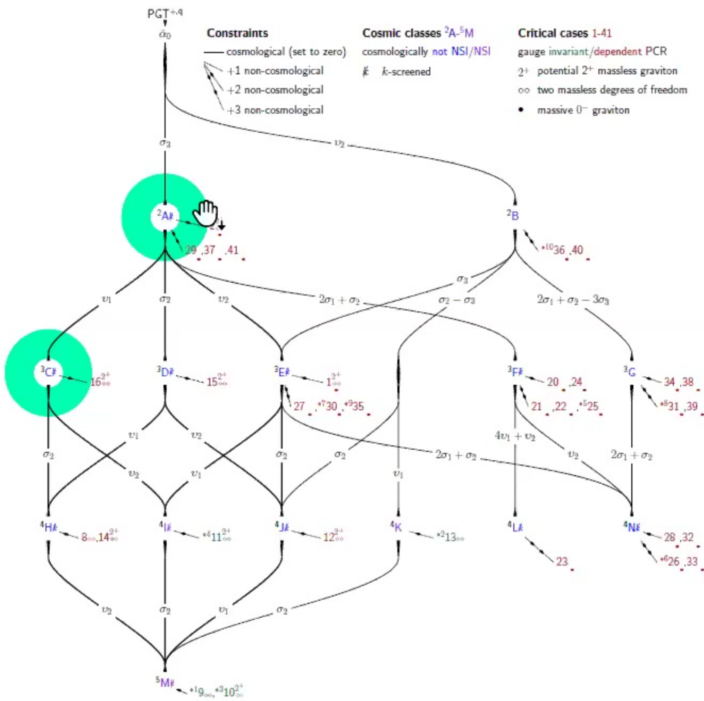


- Unitary and power-counting renormalisable when linearised on Minkowski
- Our lucky theory

$$\alpha_0 = \sigma_3 = \upsilon_1 = 0$$

- Replicates flat Friedmann solutions for any k
- Can add or dark radiation as reheating b.c.

Return to promising theories



- More general version

$$\alpha_0 = \sigma_3 = 0$$

- Propagates two massless modes and a massive pseudoscalar
- Let's see what it can do!

Move into Einstein frame

- Recycle $\sigma_2 = \sigma_1$, $\upsilon_2 = -3/4$, leaving σ_1, υ_1 free, conformal shift to Einstein frame and reparameterise ϕ, ψ to ζ, ξ

$$L_{\text{G}} \simeq -\frac{1}{2}{m_{\text{p}}}^2 R + X^{\xi\xi} - V(\xi) + {m_{\text{p}}}^2\omega(\xi)^3\sqrt{|X^{\zeta\zeta}|} + \frac{3}{4}{m_{\text{p}}}^2\omega(\xi)^4\zeta^2$$

- Note $\omega(\xi)$ regulates both cuscuton ζ, and conformal shift

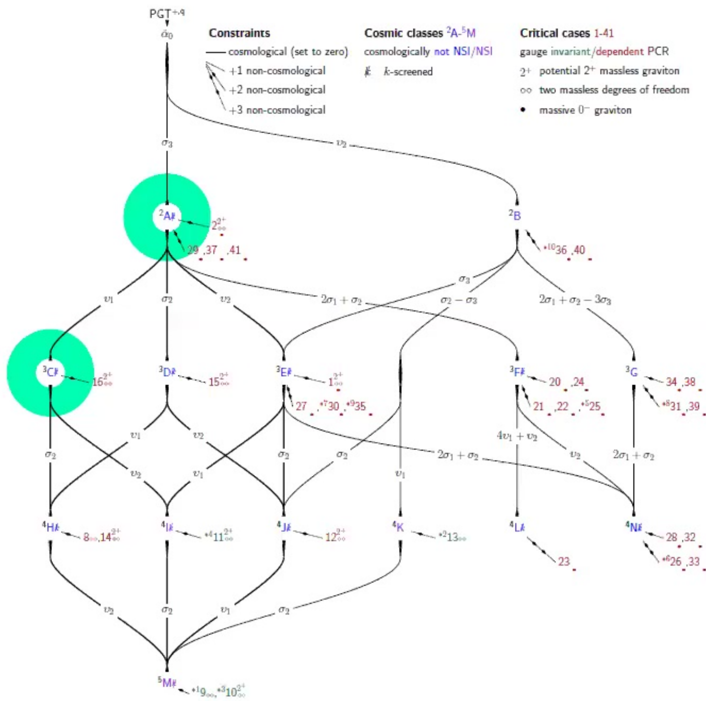
$$g_{\mu\nu} \quad \mapsto \quad \left(1 + \frac{1}{8}\omega(\xi)^2\right) g_{\mu\nu}$$

- Canonical scalar ξ has a potential $V(\xi)$

$$V(\xi) \equiv \frac{\upsilon_1}{\sigma_1}{m_{\text{p}}}^4\left(1 + \frac{1}{8}\omega(\xi)^2\right)\left(1 + \frac{1}{2}\omega(\xi)^2\right)$$

- But unitarity $\sigma_1, \upsilon_1 < 0$ so $V(\xi) > 0$ acts like dark energy!

Return to promising theories



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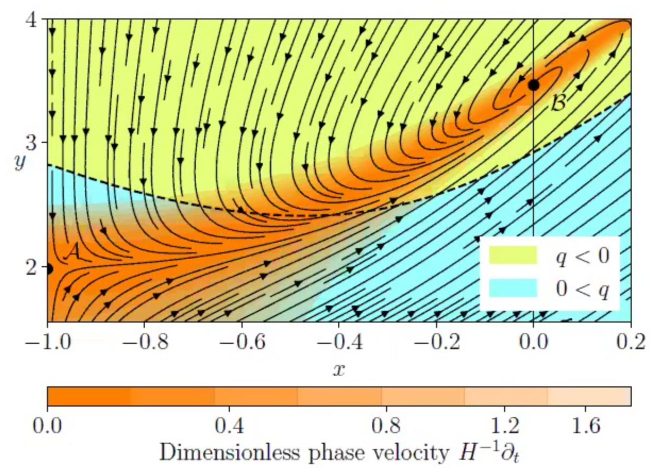
- But unitarity $\sigma_1, \upsilon_1 < 0$ so $V(\xi) > 0$ acts like dark energy!

Inflation from negative vacuum energy

- Add a *negative* bare cosmological constant $\Lambda_{\mathrm{B}} < 0$
- Triggers a de Sitter expansion without reference to $|\Lambda_{\mathrm{B}}|$

$$H^2 = (\upsilon_1/2\sigma_1)m_{\mathrm{p}}{}^2$$

- Easy to see in phase space $x \sim \dot{\xi}/H$ and $y \sim \sqrt{V(\xi)}/H$



Emergent dark energy

- Add a conventional bare cosmological constant $\Lambda_{\mathrm{B}} \geqslant 0$
- Triggers $\omega(\xi) \to 0$, where Einstein and Jordan frames coincide

$$L_{\mathrm{G}} \simeq -\frac{1}{2}{m_{\mathrm{p}}}^2 R + X^{\xi\xi} - V(\xi) + {m_{\mathrm{p}}}^2\omega(\xi)^3\sqrt{|X^{\zeta\zeta}|} + \frac{3}{4}{m_{\mathrm{p}}}^2\omega(\xi)^4\zeta^2$$

- Frozen potential $V(\xi)$ augments Λ_{B}

$$H^2 = \frac{1}{3}\left(\Lambda_{\mathrm{B}} + \frac{\upsilon_1}{\sigma_1}{m_{\mathrm{p}}}^2\right)$$

- Dark energy and dark radiation options for flat Friedmann solutions

Inflation from negative vacuum energy

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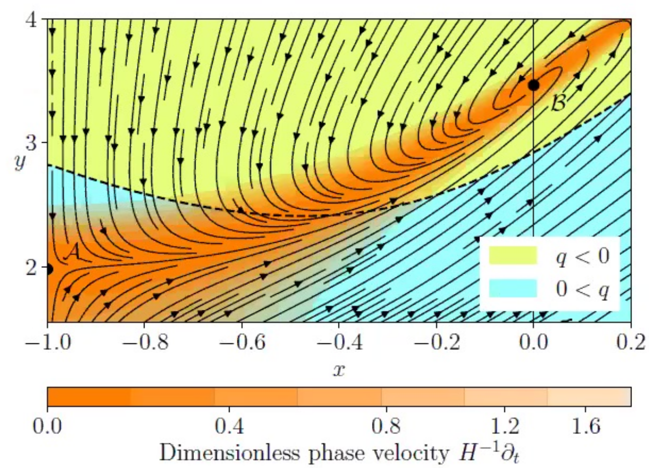
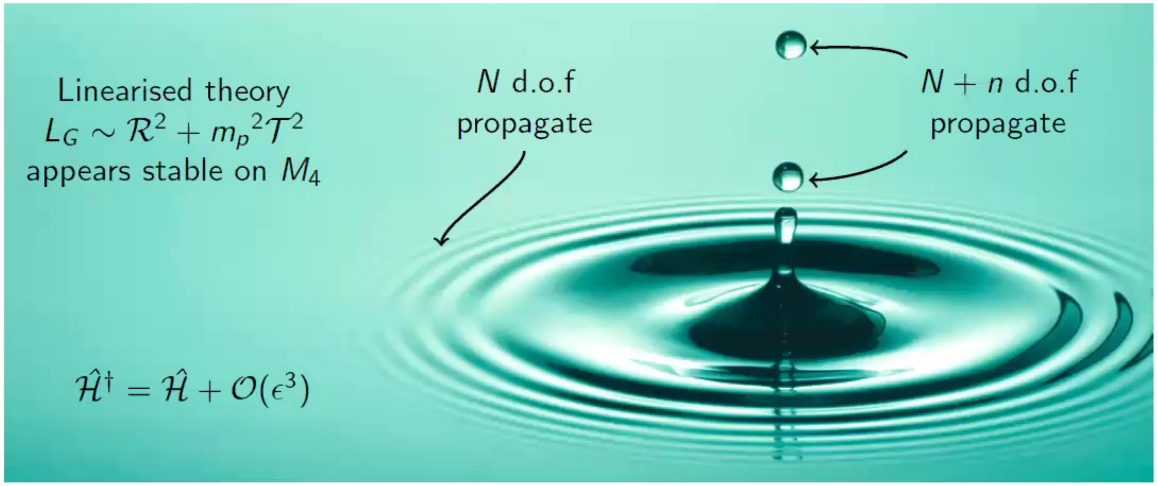


Questionable assumptions

- Ghost, tachyon free power-counting for 'flat, weak and free'
- Linear renormalisability never very important...
- Yet linear unitarity almost certainly insufficient!
- Need to transition to nonlinear Hamiltonian analysis



Propagating degrees of freedom

- Hamiltonian structure M. Blagojević et al. (1983), 9902032, 0112030, 1804.05556
- Torsion implies 40 field d.o.f

$$16[h_a^{\mu}] + 24[A^{ab}{}_{\mu}] = 40$$

- Poincaré symmetry leaves 20 potentially propagating d.o.f

$$40 - 2[\text{gauge}] \times 10[\mathbb{R}^{1,3} \rtimes SO^{+}(1,3)] = 20$$

- These are 2^{+} massless graviton and $0^{\pm}$, $1^{\pm}$, $2^{\pm}$ torsion particles (called *tordions* or *rotons*), usually massive

Hamiltonian building blocks

- Among the canonical variables there are 30 d.o.f which are accounted for by $O(3)$ irreps under the $3+1$ decomposition:

$$30 = 1[0^+] + 3[1^+] + 3[1^-] + 5[2^+] + 1[0^+] + 1[0^-] + 3[1^+] + 3[1^-] + 5[2^+] + 5[2^-]$$

- We label these basic moving parts

$$\varphi[0^+], \quad \hat{\varphi}_{\overline{kl}}[1^+], \quad \varphi_{\perp\overline{k}}[1^-], \quad \tilde{\varphi}_{\overline{kl}}[2^+], \quad \varphi_\perp[0^+], \quad {}^{\mathrm{P}}\varphi[0^-], \quad \hat{\varphi}_{\perp\overline{kl}}[1^+], \quad \vec{\varphi}_{\overline{k}}[1^-], \quad \tilde{\varphi}_{\perp\overline{kl}}[2^+], \quad {}^{\mathrm{T}}\varphi_{\overline{klo}}[2^-]$$

- Main point to remember is φ are linear in momenta π

$$\varphi \sim \pi + \mathcal{R} \quad \text{or} \quad \varphi \sim \pi + \mathcal{T}$$

Primary constraints

- Standard definition of canonical momenta

$$\pi_i{}^{\mu} \equiv \frac{\partial b L_{\mathrm{G}}}{\partial \partial_0 b^i{}_{\mu}}, \quad \pi_{ij}{}^{\mu} \equiv \frac{\partial b L_{\mathrm{G}}}{\partial \partial_0 A^{ij}{}_{\mu}}$$

- For certain α_i, β_i these definitions may imply one or more of the φ become *primary* constraints

$$\varphi \approx 0$$

- Total Hamiltonian enforces these with multipliers u

$$\mathcal{H}_{\mathrm{T}} \equiv \mathcal{H}_{\mathrm{C}} - u \cdot \varphi$$

Dirac–Bergmann and Castellani algorithms

- But if $\varphi \approx 0$ then you need $\dot{\varphi} \approx 0$

$$\dot{\varphi} \equiv \int d^3x' \{\varphi, \mathcal{H}'_{\mathrm{T}}\} \equiv \int d^3x' \left(\{\varphi, \mathcal{H}'_{\mathrm{C}}\} - u \cdot \{\varphi, \varphi\}\right)$$

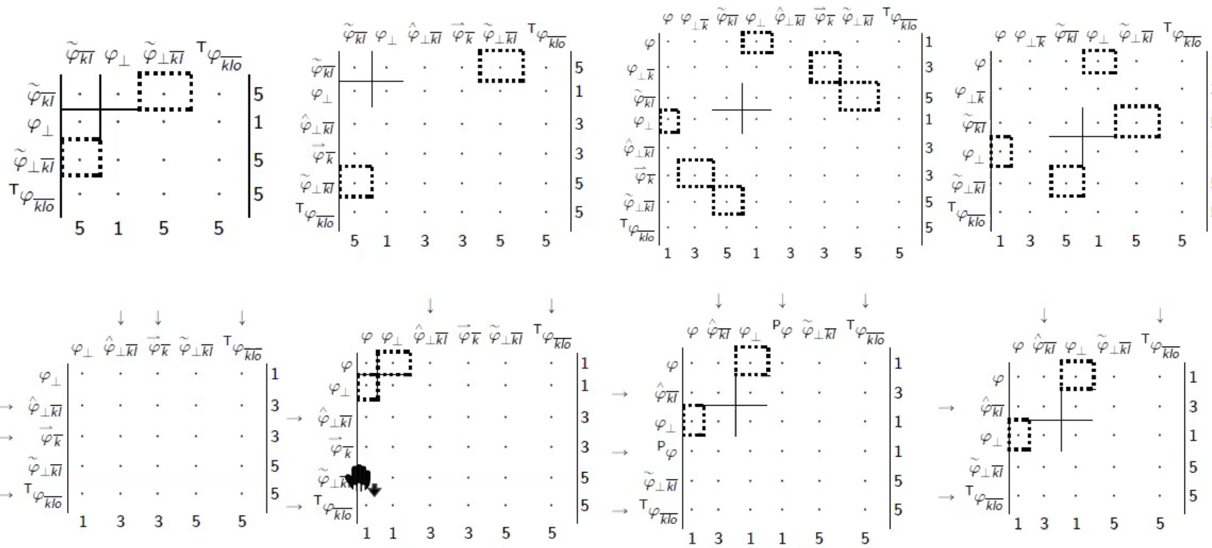
- If $\{\varphi, \varphi\} \not\approx 0$ you can satisfy $\dot{\varphi} \approx 0$ with some u
- If $\{\varphi, \varphi\} \approx 0$ you get *secondary* constraint $\chi \equiv \{\varphi, \mathcal{H}'_{\mathrm{C}}\}$
- Process continues to satisfy $\chi \approx \dot{\chi} \approx 0$ and so on...
- Once all consistent, you can count the unconstrained d.o.f!

Primary Poisson matrices

- Matrix $\{\varphi, \varphi\}$ between all primaries φ is significant
- Call this *Primary Poisson matrix*
- Linear matrix structure should be preserved in nonlinear theory
- Change in rank may activate a ghost (strong coupling)
- Rank dependent on position in phase space associated with acausal modes Hsin Chen et al. (1998), 9902032, 0112030

Eight allegedly viable theories

- Linearised...



Identifying ghosts

- Focus on kinetic part of canonical Hamiltonian

$$\mathcal{H}_{\mathrm{C}} \equiv N\mathcal{H}_{\perp} + N^{\alpha}\mathcal{H}_{\alpha} - \frac{1}{2}A^{ij}{}_{0}\mathcal{H}_{ij} + \partial_{\alpha}D^{\alpha},$$

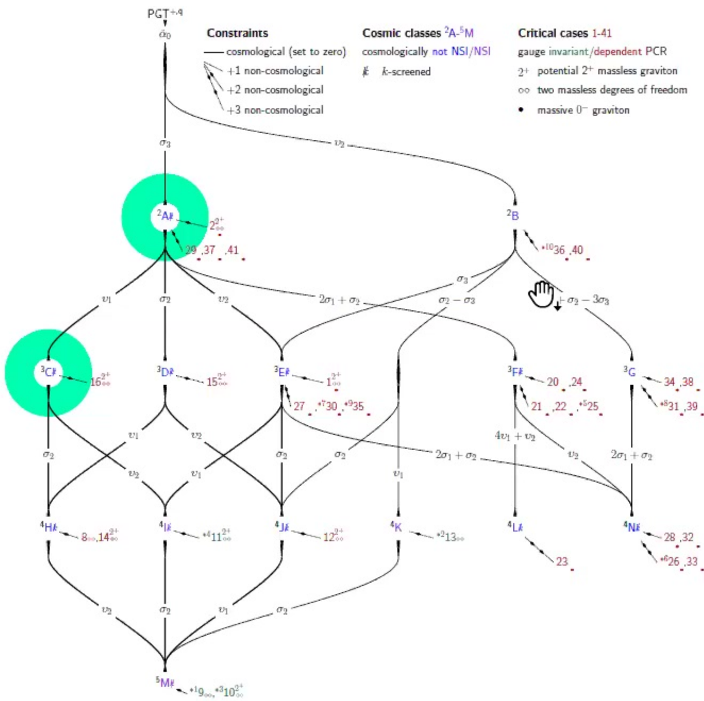
- This can be expressed in terms of unconstrained φ functions

$$\varphi \sim \pi + \mathcal{R} \quad \text{or} \quad \varphi \sim \pi + \mathcal{T}$$

- Signs of quadratic terms then reveal any ghosts

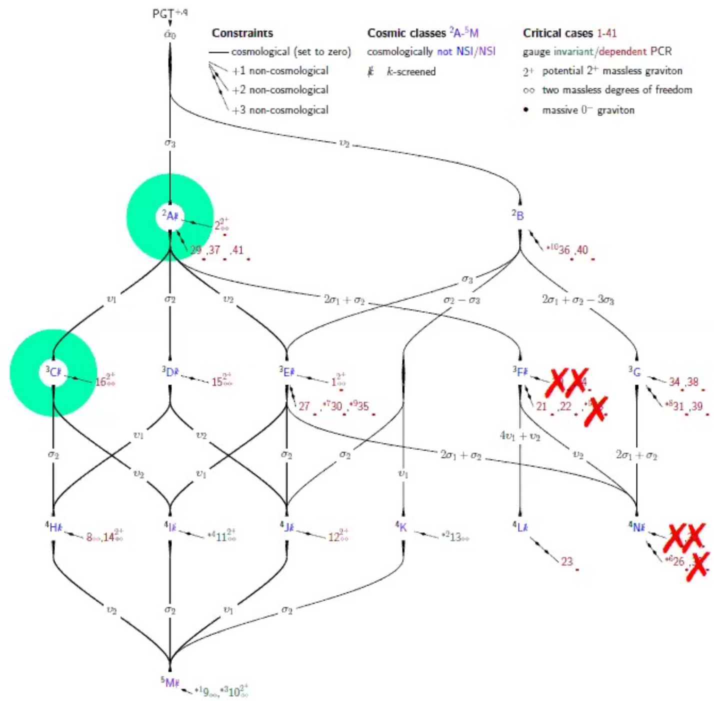
$$\begin{aligned}\mathcal{H}_{\perp} = \frac{J}{16}\Bigg[&\frac{2\varphi^{2}}{3\beta_2 m_{\mathrm{p}}{}^{2}} + \frac{6\hat{\varphi}_{\overline{kl}}\hat{\varphi}^{\overline{kl}}}{(\beta_1+2\beta_3)m_{\mathrm{p}}{}^{2}} + \frac{6\varphi_{\perp\overline{k}}\varphi^{\perp\overline{k}}}{(2\beta_1+\beta_2)m_{\mathrm{p}}{}^{2}} + \frac{2\tilde{\varphi}_{\overline{kl}}\tilde{\varphi}^{\overline{kl}}}{\beta_1 m_{\mathrm{p}}{}^{2}} + \frac{2\varphi_{\perp}{}^{2}}{3(\alpha_4+\alpha_6)} + \frac{{}^{\mathrm{P}}\varphi^{2}}{6(\alpha_2+\alpha_3)} + \frac{2\hat{\varphi}_{\perp\overline{kl}}\hat{\varphi}^{\perp\overline{kl}}}{\alpha_2+\alpha_5}\\ &+ \frac{\vec{\varphi}_{\overline{k}}\vec{\varphi}^{\overline{k}}}{\alpha_4+\alpha_5} + \frac{2\tilde{\varphi}_{\perp\overline{kl}}\tilde{\varphi}^{\perp\overline{kl}}}{\alpha_1+\alpha_4} + \frac{16\,{}^{\mathrm{T}}\varphi_{\overline{klo}}\,{}^{\mathrm{T}}\varphi^{\overline{klo}}}{9(\alpha_1+\alpha_2)}\Bigg] + J\Bigg[\frac{1}{3}(2\beta_1+\beta_3)m_{\mathrm{p}}{}^{2}\mathcal{T}_{\perp\overline{kl}}\mathcal{T}^{\perp\overline{kl}} + \frac{1}{3}(\beta_1+2\beta_2)m_{\mathrm{p}}{}^{2}\vec{\mathcal{T}}_{\overline{k}}\vec{\mathcal{T}}^{\overline{k}}\\ &- \frac{1}{6}\beta_3 m_{\mathrm{p}}{}^{2}\,{}^{\mathrm{P}}\mathcal{T}^{2} + \frac{16}{9}\beta_1 m_{\mathrm{p}}{}^{2}\,{}^{\mathrm{T}}\mathcal{T}_{\overline{klo}}\,{}^{\mathrm{T}}\mathcal{T}^{\overline{klo}} + \frac{1}{6}(\alpha_4+\alpha_6)\underline{\mathcal{R}}^{2} - \frac{1}{6}(\alpha_2+\alpha_3)\,{}^{\mathrm{P}}\mathcal{R}_{\perp\circ}{}^{2} + 2(\alpha_2+\alpha_5)\underline{\mathcal{R}}_{[\overline{kl}]}\underline{\mathcal{R}}^{[\overline{kl}]}\\ &+ (\alpha_4+\alpha_5)\mathcal{R}_{\perp\overline{k}}\mathcal{R}^{\perp\overline{k}} + 2(\alpha_1+\alpha_4)\underline{\mathcal{R}}_{\langle\overline{kl}\rangle}\underline{\mathcal{R}}^{\langle\overline{kl}\rangle} + \frac{16}{9}(\alpha_1+\alpha_2)\,{}^{\mathrm{T}}\mathcal{R}_{\perp\overline{klo}}\,{}^{\mathrm{T}}\mathcal{R}^{\perp\overline{klo}}\Bigg] - n^{k}D_{\alpha}\pi_{k}{}^{\alpha}.\end{aligned}$$

Current status



- Survey is still ongoing!
- So far ruled out six theories on the map
- Since success ratio is so low, might be better to *start* with nonlinear Hamiltonian

Current status



- HiGGS (Hamiltonlan Gauge Gravity Surveyor)/Wolfram
- Primary Poisson Matrix in seconds
- Linear constraint chains in minutes
- Nonlinear constraint chains with human assistance

Summary

- Beginning with linearised unitarity and power-counting renormalisability on Minkowski, there is a catalogue of theories
- Power-counting always eliminates Einstein–Hilbert term
- Power-counting often screens spatial curvature
- Yet viable cosmological background emerges
- Options to add dark radiation/energy
- Schwarzschild and pp waves (*in prep*)
- The torsion background is a non-canonical scalar-tensor
- Hamiltonian analysis more stringent and can be automated

Flat, weak and free

- Linearise on Minkowski background without matter or cosmological constant
- Extra Lagrangian symmetries emerge if the α_i, β_i obey certain equalities: these critical cases must be considered separately
- Lin, Hobson and Lasenby performed an exhaustive survey 1812.02675, 1910.14197, building on earlier work D E. Neville (1980), E. Sezgin et al. (1980)
- 1918 critical cases in total
- 450 of which are free of ghosts and tachyons under further unitarity inequalities on the α_i, β_i
- 58 of which are power-counting renormalisable