Title: Quantum black holes without strings

Speakers: Gerard 't Hooft

Series: Quantum Gravity

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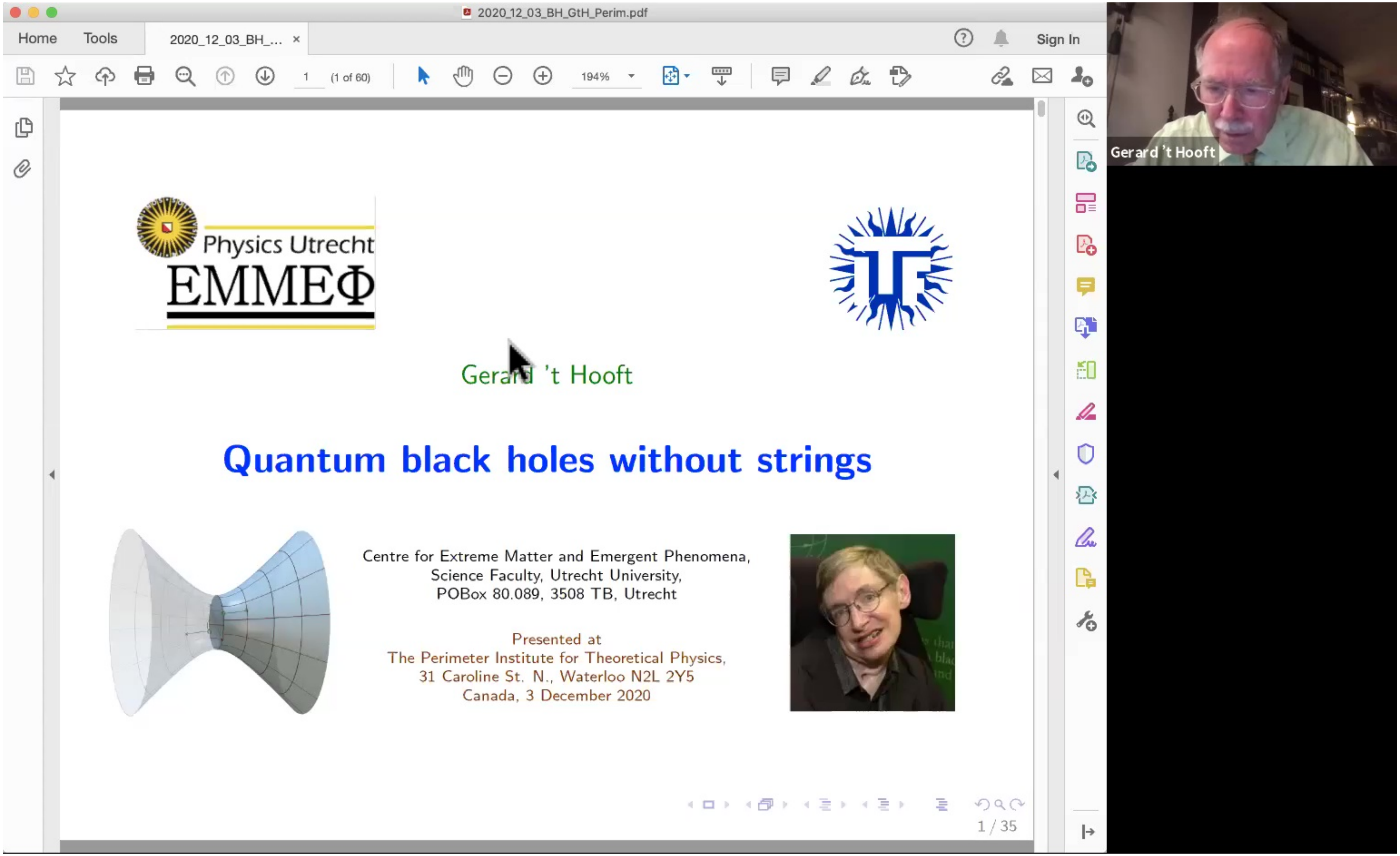
Abstract: The quantum states of matter in the immediate vicinity of a black hole can be studied using no other information than Standard Model physics combined with perturbative gravity. The point is that the relevant energy scale of the most important fields involved is low compared to the Planck scale, provided the black hole is big compared to the Planck scale. Usually this problem is investigated by using the metric that includes the effects of matter that formed the black hole in the distant past and sometimes also matter that is radiated away in the distant future. Arguments are presented however to justify that one should ignore those effects. The metric then becomes invariant under time translation, and this is what we need to get the energy eigen states. It is this scheme that forces us to impose the antipodal identification as a new boundary condition, giving us a beautiful picture of black hole quantum evolution. The logic of choosing this compulsory topological twist in space-time is explained. There are still many very hard questions and I hope to be able to inspire people to look into these.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Gerard 't Hooft
Quantum black holes without strings
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Presented at
The Perimeter Institute for Theoretical Physics,
31 Caroline St. N., Waterloo N2L 2Y5
Canada, 3 December 2020
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Gerard 't Hooft

There is general agreement concerning the need for a quantum theory of black holes:

1. If you want a theory that unifies all forces and includes the behavior of space and time themselves, it must include gravity, i.e. General Relativity should be part of such a theory.

2. The best way to study gravitational forces is by considering the strongest possible gravitational fields – or gravitational potentials – under given conditions, and realise that

3. the strongest gravitational force fields are near the horizon of a black hole. Therefore, go study quantum black holes first.
Make sure that your theory is logically coherent and self-consistent.

See what you can conclude about the more general theory.

Many researchers must have heard about my approach towards the problem of quantizing the black hole.
Yet few seem to agree with me that this is the way to go:

The prototype is not the "extreme black hole" but the pure Schwarzschild case, surrounded not by AdS but by flat Minkowski space-time, and physics far from the Planck scale should be all we need as a starting point, if the black hole is sufficiently big.

This talk is an advertisement of an approach that I consider very promising.
I need no string theory, no AdS/CFT, no stacks of D-branes — these might come later but I don't see the need as yet.

Firewall problems are resolved, and entanglement issues do not arise in this framework .

One must consider exactly *all* of Hilbert space generated by the Standard Model, augmented with *low energy* gravitons (i.e. *perturbative* gravity).

In describing the "background space-time for a black hole", other approaches include the imploding material, and sometimes also the cloud of emitted Hawking particles, in their description of a black hole space-time.

This is wrong. Why?

The BH time unit is $2GM/c^3$, let's call that a 'nanosecond'
(although for interesting black holes it would be much smaller)

A black hole formed long ago in BH time units, would be built of matter that a local observer sees Lorentz boosted by an amount

$\gamma \approx e^{\frac{(1\,\mathrm{year})}{(1\,\mathrm{nanosecond})}} = e^{30.000.000.000.000.000}$, a number totally off scale.

Same for Hawking particles (in the distant future).
So *don't* think of these particles, this would not make sense . . .

but see later . . .

Note: *restricting ourselves to only the Hilbert space of the SM + pert. grav. will not go atomatically, it will require extra work.*

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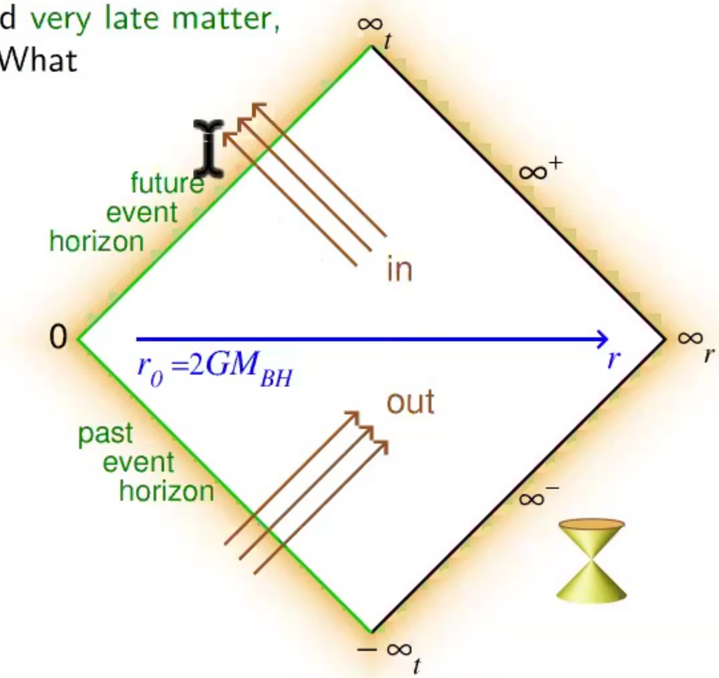
Easiest case: consider a stationary black hole. Every now and then a light particle goes in or out. No significant mass changes.

As we Leave out very early matter and very late matter, these are *not* the ordinary horizons. What sits behind these "new" horizons will be something non-trivial.

Black Hole



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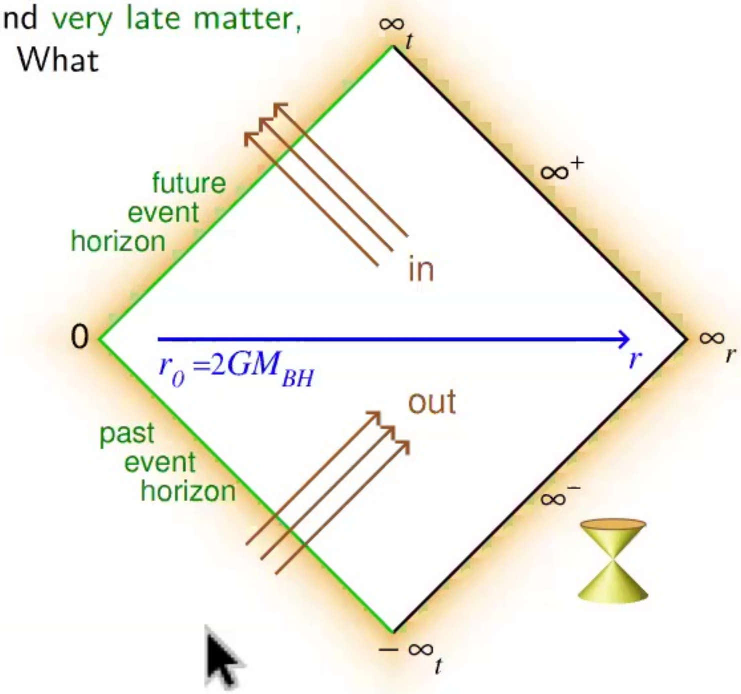

Out-particles seem to be earlier than in-particles,
does this not violate causality?!
Not necessarily!

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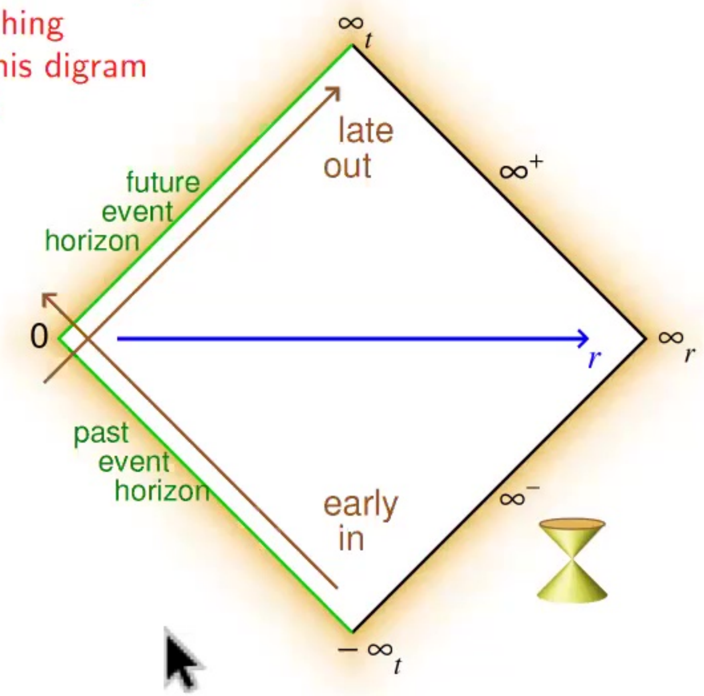
Somewhat later out-particles, may be affected by *somewhat earlier* in-particles, by something that happens very near the center of this digram (the Planckian regime?) Only modest Lorentz transformations suffice.

Black Hole



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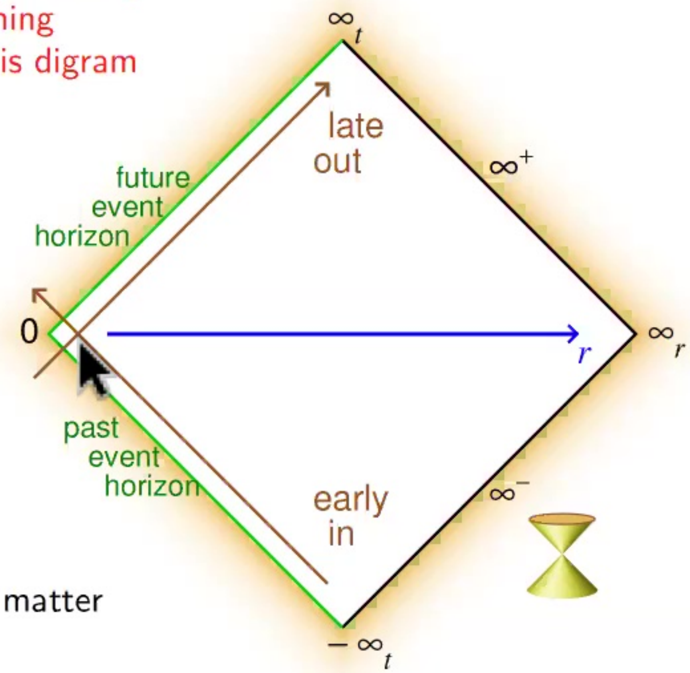

Having no matter will suggest that the complete BH Penrose diagram without matter will be lurking behind these horizons.

What are these universes? See later . . .

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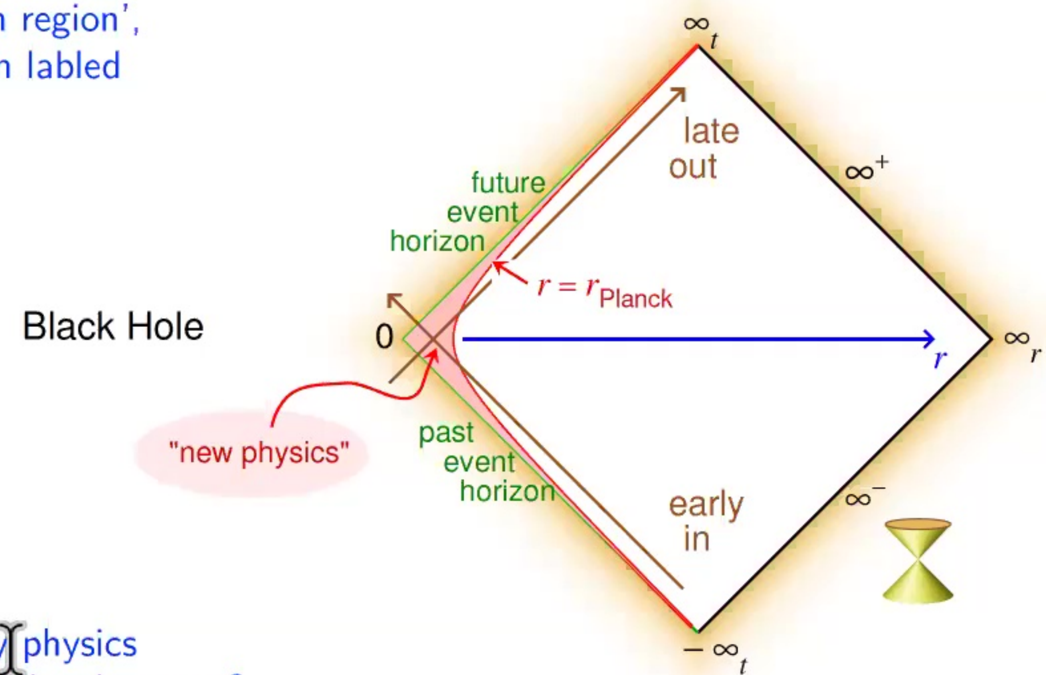
The general problem:
Formulate what happens in
the ‘interaction region’,
the pink donain labled

“new physics”



Can low energy physics
reveal (partly) what happens?

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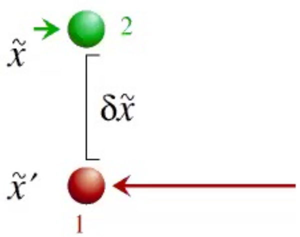
To understand what happens at the Planck scale, we need the gravitational backreaction:

Lorentz boosting the light (or massless) particle gives the *Shapiro time delay* caused by its grav. field:



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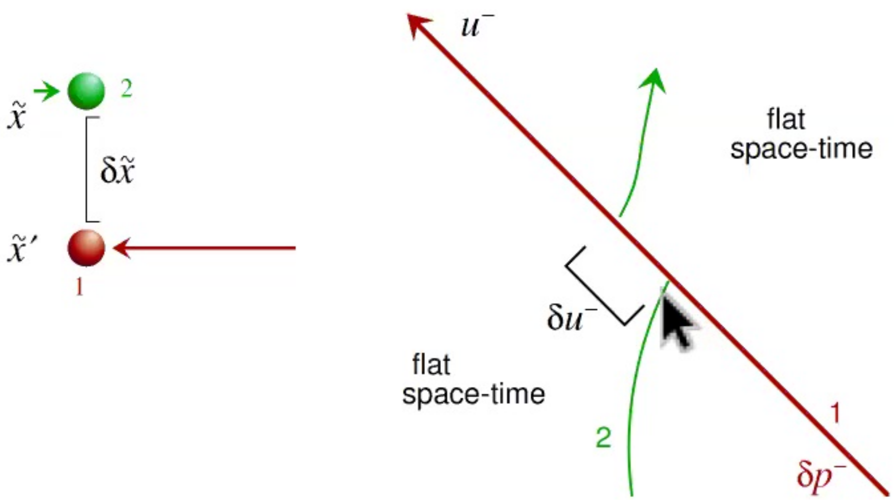

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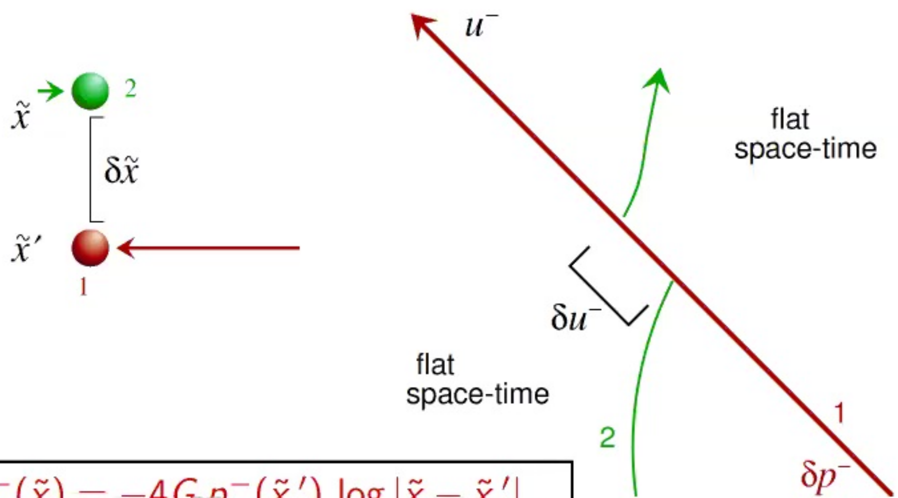

$$\delta u^-(\tilde{x}) = -4G\, p^-(\tilde{x}')\, \log|\tilde{x} - \tilde{x}'|\ .$$

P.C. Aichelburg and R.U. Sexl, J. Gen. Rel. Grav. **2** (1971) 303,
W.B. Bonnor, *Commun. Math. Phys.* **13** (1969) 163,
T. Dray and G. 't Hooft, *Nucl. Phys.* **B253** (1985) 173.

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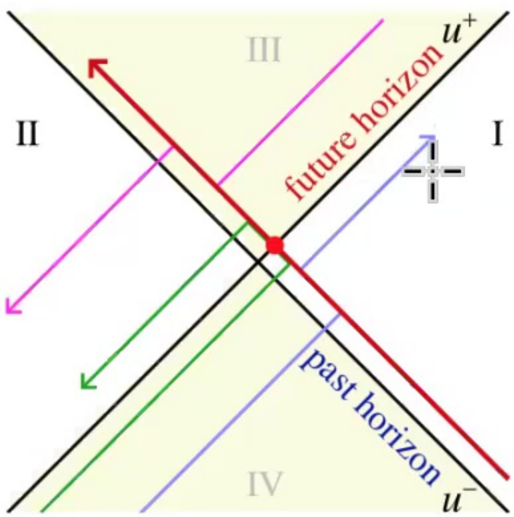
The gravitational back reaction: a given in-going particle (red line) causes all out-going particles (colored lines) to shift by the same amount, δu, which only depends on the angular variables (θ, φ), not on u.



Note sign switches:

the momentum p^- of the particle coming in in region *II*, is *minus* the momentum coming in at *I*.

The variable u^- of the particles going out also switches sign compared to that of the particles going in.

The data are shifted right across the horizon

Same with past event horizon, by time reversal!

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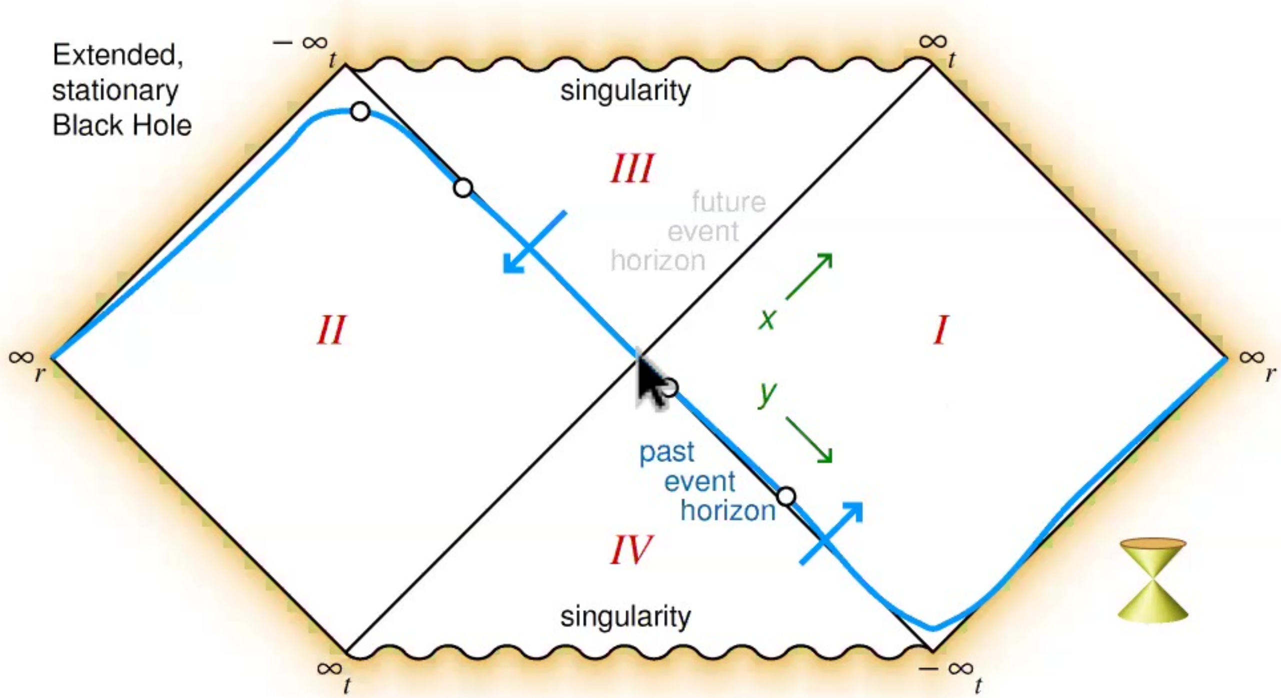
The gravitational back reaction *shifts* the data on the Cauchy surface across the horizon.

Cauchy surfaces must be drawn from ∞_r in region II to ∞_r in region I.



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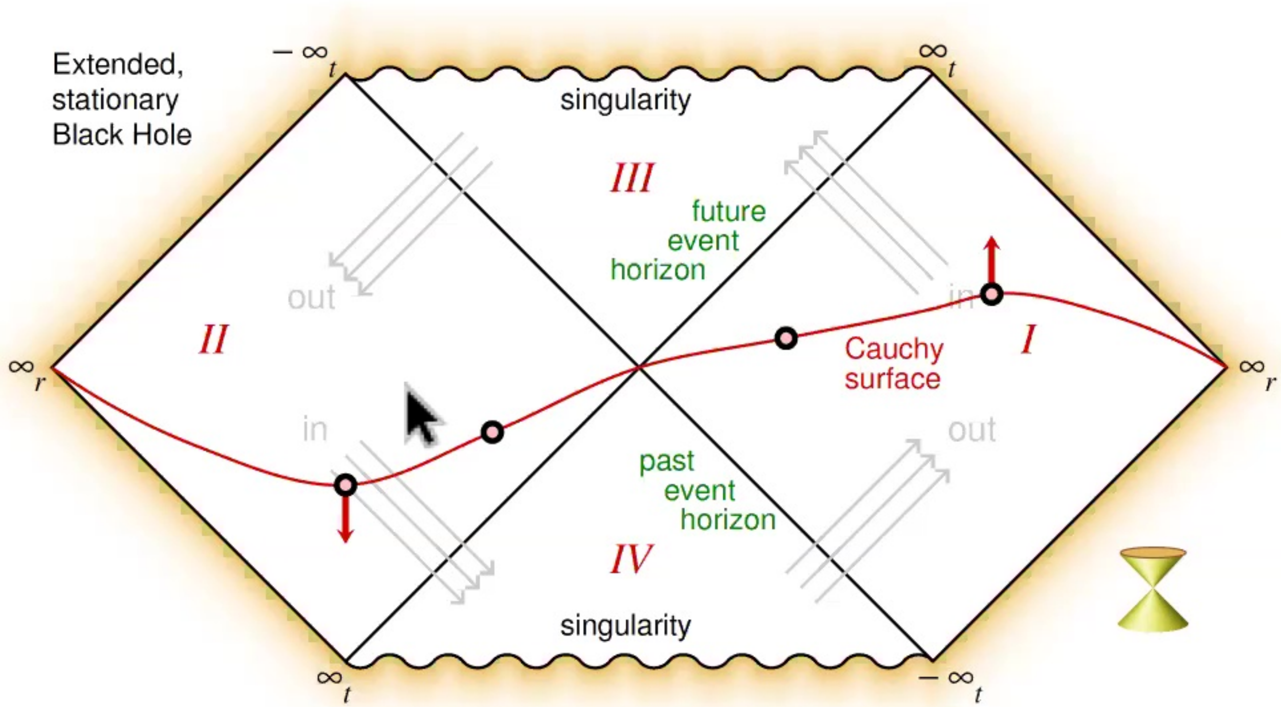
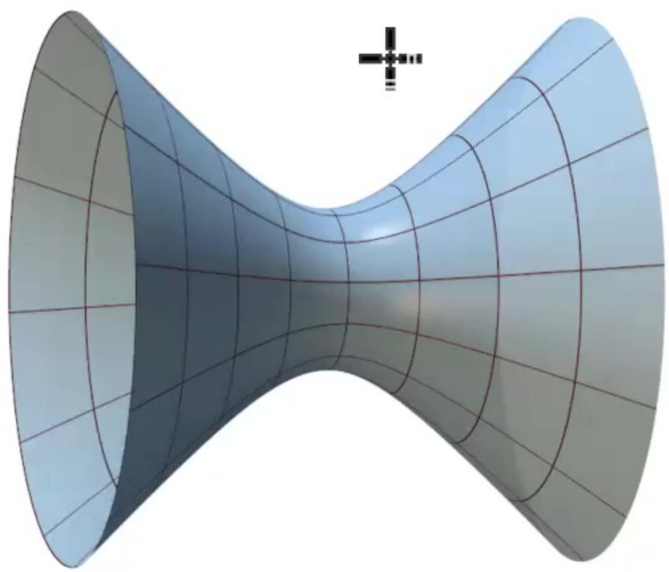
For local observers, the Cauchy surface goes from down to up in both regions.

For distant observers, the direction of time switches in region *II*.



We are forced to include region *II* in our picture. It is an exact copy of region *I*. What does region *II* stand for? The same black hole? Danger: *conic singularities*. How do we divide space-time by $\mathbb{Z}_2$?



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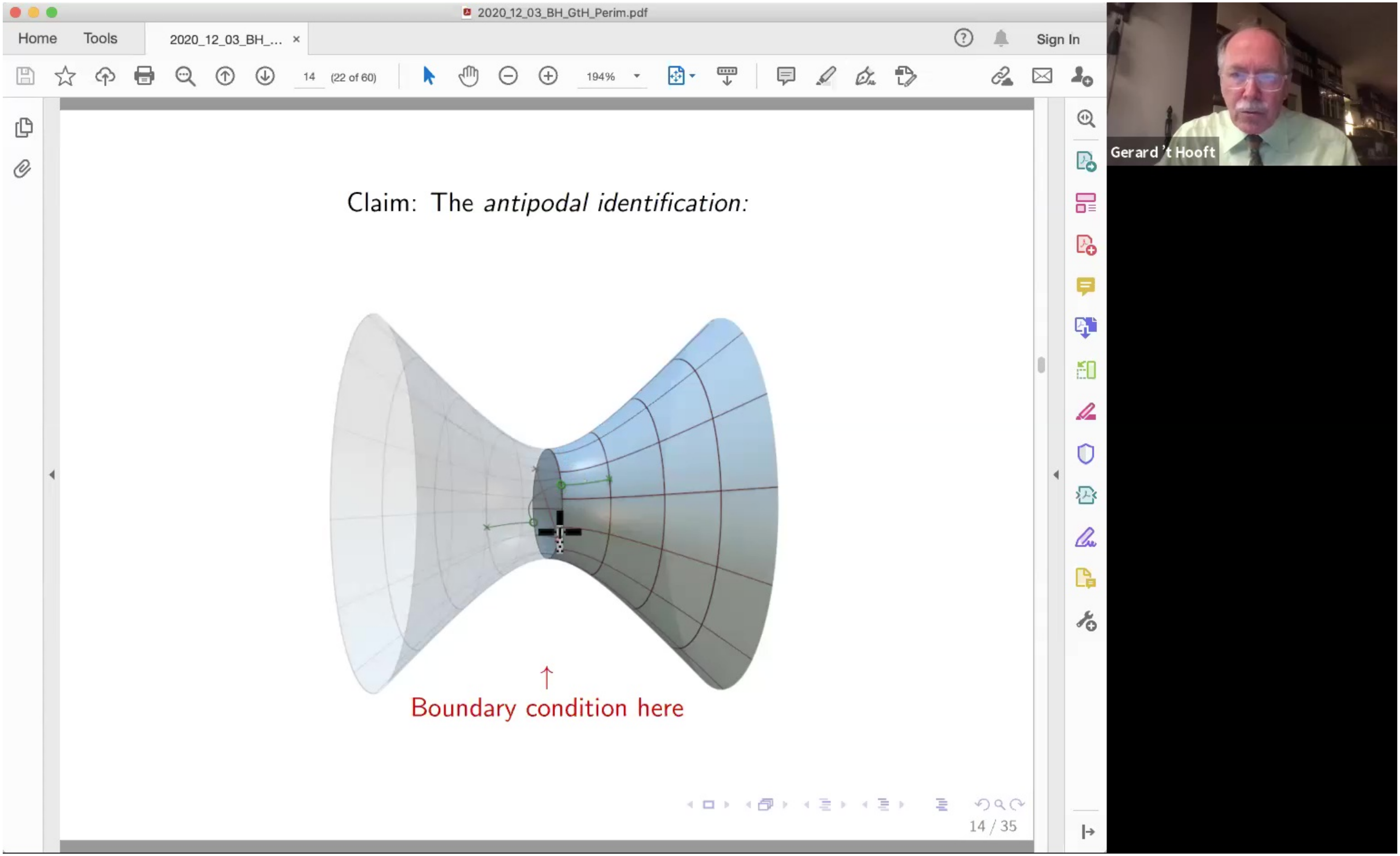
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Claim: The *antipodal identification:*
Boundary condition here
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The black hole space-time manifold is divided by $\mathbb{Z}_2$.
Which mapping can this be? Call it A.

The horizon is an S_2 sphere.
A must be an isometry of this sphere, therefor, $A \in O(3)$.

The mapping squares to one: $A^2 = \mathbb{I}$.
Its eigen values must all be ± 1.
Suppose an eigenvalue $+1$. Then $\exists$ a point that is mapped to itself.
At that point, $p^-(I) = -p^-(II) = 0$.
This would be a singularity that we cannot accept. Conclusion:
all eigen values are -1. Or,

$$A = -\mathbb{I}, \quad \text{the antipodal mapping.}$$

Also time is reversed. In $O(3,1)$:

$$A = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Use CPT invariance.

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Note sign switch problems, see slide # 9

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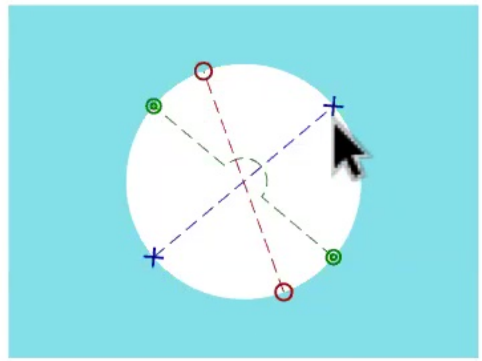
Opening up (collapse) and closing in (final evaporation) of a black hole:



Black emptiness: blue regions are the accessible part of space-time; dotted lines indicate identification.

The white sphere within is *not* part of space-time. Call it a 'vacuole'.

At given time t, the black hole is a 3-dimensional vacuole. The entire life cycle of a black hole is a vacuole in 4-d Minkowski space-time: *an* **instanton**

N.Gaddam, O.Papadoulaki, P.Betzios (Utrecht PhD students)

Space coordinates change sign at the identified points

– *and also time changes sign*

(Note: time stands still at the horizon itself).

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The *gravitational shift.*

An in-particle with momentum p^- at solid angle $\Omega' = (\theta', \varphi')$ causes a shift δu^- at solid angle $\Omega = (\theta, \varphi)$:

$$\delta u^-(\Omega) = 8\pi G\, f(\Omega, \Omega') p^- \ ;$$

$f(\Omega, \Omega')$ obeys an harmonic Laplace equation on the sphere:

$$(1 - \Delta_\Omega) f(\Omega, \Omega') = \delta^2(\Omega, \Omega') \ .$$

If there are many in-particles:

$$\delta p^-(\Omega) = \sum_i p_i^- \delta^2(\Omega, \Omega_i)$$

$$\delta u^-(\Omega) = 8\pi G \int \mathrm{d}^2\Omega' f(\Omega, \Omega') p^-(\Omega')$$

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$$\delta p^-(\Omega) = \sum_i p_i^- \delta^2(\Omega, \Omega_i)$$

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We now replace this by:

$$p^-(\Omega) = \sum_i p_i^- \delta^2(\Omega, \Omega_i)$$

$$u^-(\Omega) = 8\pi G \int \mathrm{d}^2\Omega' f(\Omega, \Omega') p^-(\Omega')$$

Avoid double counting: the in-particles *turn into* out-particles.

This also removes firewalls

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The *positions* u^- of the out particles are generated by the *momenta* p^- of the in-particles. in partial waves:

$$u^-_{\ell m} = \frac{8\pi G}{\ell^2+\ell+1}\, p^-_{\ell m}\ , \qquad u^+_{\ell m} = -\frac{8\pi G}{\ell^2+\ell+1}\, p^+_{\ell m}\ ,$$

$$[u^\pm_{\ell m},\, p^\mp_{\ell' m'}] = i\hbar\, \delta_{\ell\ell'}\, \delta_{mm'}\ .$$

In terms of the coordinates u^+ and u^-, the wave functions in u^+ and u^- are each other's Fourier transform.

For distant observers, $u^\pm$ and $p^\pm$ depend exponentially on time $\tau = t/4GM$:

$$u^\pm,\ p^\pm \ \rightarrow\ e^{\mp\tau}$$

Introduce tortoise coordinates (close to horizon)

$$u^+_{\rm in} = \sigma_{\rm in}\, e^{\varrho_{\rm in}}\ , \quad u^-_{\rm out} = \sigma_{\rm out}\, e^{\varrho_{\rm out}}\ ; \qquad \sigma_{\rm in} = (\pm)\, , \quad \sigma_{\rm out} = [\pm]$$

Rephrase the Fourier transformations in terms of the ϱ coordinates.

The *signs* of $\sigma_{\rm in}$ and $\sigma_{\rm out}$ *do not commute*.

The relation is now invariant under time shifts:

$$\varrho_{\rm in} \rightarrow \varrho_{\rm in} - \tau\ ; \quad \varrho_{\rm out} \rightarrow \varrho_{\rm out} + \tau .$$

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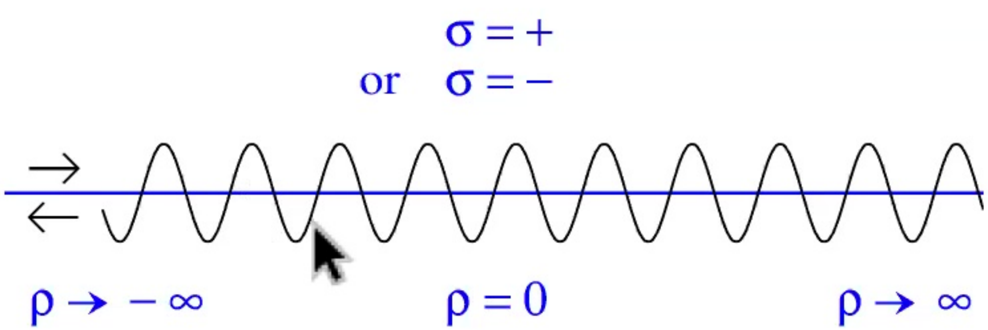
Tortoise coordinates

$$x = \sigma\, e^{\varrho} \quad \sim \quad r - 2GM$$

Plane waves $e^{-i\kappa(\tau \pm \varrho)}$ in ϱ coordinates



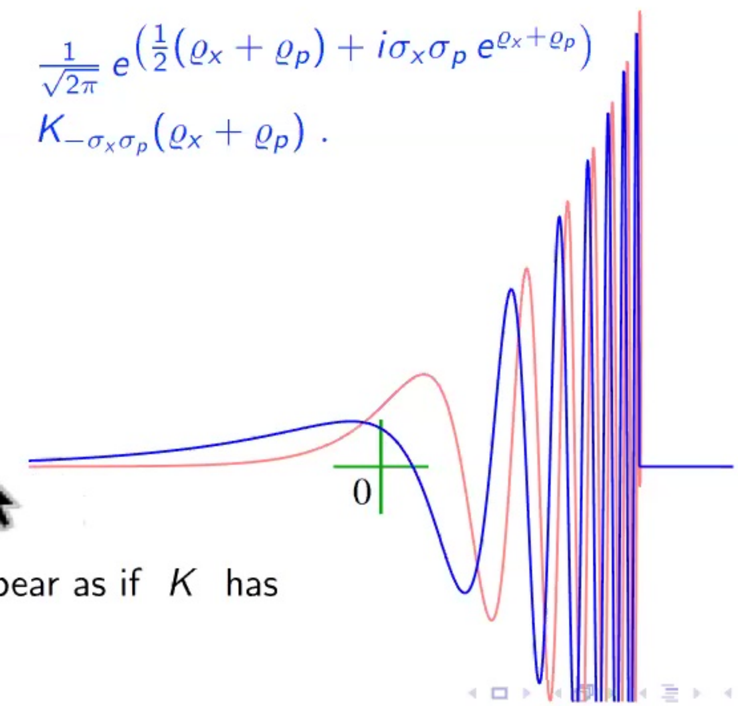
The Fourier transform in x, p space is entirely non-local:

$$\langle x|p\rangle = \frac{1}{\sqrt{2\pi}}\, e^{ipx}$$

But if we write $x = \sigma_x\, e^{\varrho_x}$ and $p = \sigma_p\, e^{\varrho_p}$, where σ_x and σ_p are signs $\pm$, then the relation becomes:

$$\langle \varrho_x, \sigma_x | \varrho_p, \sigma_p \rangle = \frac{1}{\sqrt{2\pi}}\, e^{\left(\frac{1}{2}(\varrho_x+\varrho_p) + i\sigma_x\sigma_p\, e^{\varrho_x+\varrho_p}\right)} = K_{-\sigma_x\sigma_p}(\varrho_x+\varrho_p)\ .$$

$K_+(x)$:



In practice it will appear as if K has a finite support.

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Tortoise coordinates

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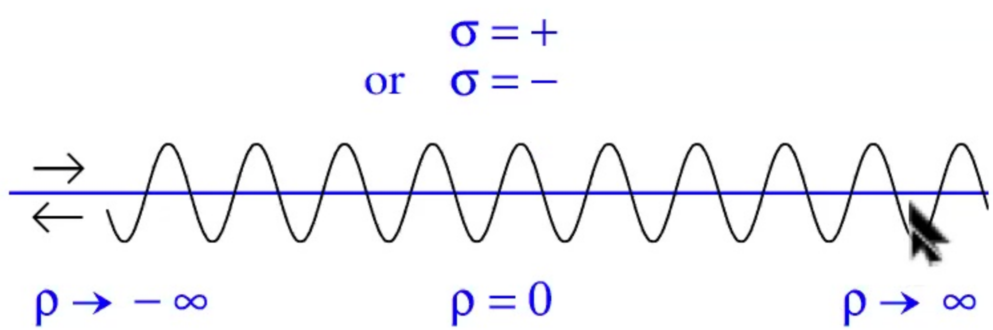
Plane waves $e^{-i\kappa(\tau \pm \varrho)}$ in ϱ coordinates



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$$\sigma = \pm 1 \,, \qquad -\infty < \varrho < \infty :$$

$$\psi_{\rm in}(\varrho,\sigma) = \psi_\sigma^{\rm in}\, e^{-i\kappa(\varrho+\tau)} \;; \qquad \psi_{\rm out}(\varrho,\sigma) = \psi_\sigma^{\rm out}\, e^{i\kappa(\varrho-\tau)}$$

The Fourier transformation gives at fixed ℓ, m

$$\begin{pmatrix}\psi_+^{\rm out}\\ \psi_-^{\rm out}\end{pmatrix} = \frac{e^{-\frac{\pi i}{4}}}{\sqrt{2\pi}}\Gamma(\tfrac12 - i\kappa)\begin{pmatrix} e^{-\frac12\pi\kappa} & ie^{+\frac12\pi\kappa}\\ ie^{+\frac12\pi\kappa} & e^{-\frac12\pi\kappa}\end{pmatrix}\begin{pmatrix}\psi_+^{\rm in}\\ \psi_-^{\rm in}\end{pmatrix}$$

Of course, the Fourier transform is unitary. Unitarity follows from:

$$|\Gamma(\tfrac12 - i\kappa)|^2 = \frac{\pi}{\cosh \pi\kappa}$$

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Add the scale factor $\frac{8\pi G}{\ell^2+\ell+1}$ (describing the extent of the gravitational shift effect), to get, if $u^\pm = \sigma_\pm e^{\varrho^\pm}$,

$$\psi^{\rm in}_{\sigma_+}\, e^{-i\kappa\varrho^+} \;\to\; \psi^{\rm out}_{\sigma_-} e^{i\kappa\varrho^-} \ ,$$

$$\psi^{\rm out}_{\sigma_-} \;=\; \sum_{\sigma_+} F_{\sigma_+\sigma_-}(\kappa)\, e^{-i\kappa \log\left(8\pi G/(\ell^2+\ell+1)\right)}\, \psi^{\rm in}_{\sigma_+}$$

Here:

$$F_\sigma(\kappa) = \frac{1}{\sqrt{2\pi}} \int_0^\infty \frac{{\rm d}y}{y}\, y^{\frac12 - i\kappa}\, e^{-i\sigma y} \;=\; \frac{1}{\sqrt{2\pi}}\, \Gamma(\tfrac12 - i\kappa)\, e^{-\frac{i\sigma\pi}{4} - \frac{\pi}{2}\kappa\sigma} \ .$$

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$$\sigma = \pm 1\ , \qquad -\infty < \varrho < \infty :$$

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What do these equations describe?

The information going in and out was expanded in partial waves. At each value of the pair (ℓ, m), there is exactly one number indicating either the average position of the in going material, or its Fourier transform, the average momentum, both with the angular position (θ, φ) replaced by the harmonic numbers (ℓ, m).

The nice thing about the partial waves is that the equations for the grav. shift *decouple*.

So it all becomes undergraduate quantum mechanics. Everything can now be followed in detail.

A consequence of the antipodal identification is that the even values of ℓ do not participate. But at odd values of ℓ we over-count states of Hilbert space by a factor 2,

since our two regions *I* and *II* both describe the outside space-time.

This is why, far from the black hole, we recover all conventional space time points exactly once.

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Important question: *How did antipodal identification 'suddenly' emerge when the black hole was formed?*

An other improvement compared to other approaches: our background space-time is an eternal black hole, with a time-like Killing vector. So we can set up our Hilbert space by listing all energy eigen states.

We wish to ignore all states containing super energetic particles. But then it seems that, since time evolution now corresponds to local Lorentz boosts, we can follow what happens only in limited stretches of time.

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But now our S-matrix turns in-going particles into out-going ones. The momentum of the in-particles is transformed into position of out particles. This is a replacement: particles are replaced by their Shapiro footprints and vice versa. We can always do this such that high momentum in, is replaced by large position operator out.

Thus, if an in-particle gains too much momentum, it is replaced by the out-particles whose position is increasing – it moves out of the black hole and that is that.

Vice versa, we can go back to the past, replacing Hawking particles with still very large momentum by in-particles on the way in, but still far away.

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Vice versa, we can go back to the past, replacing Hawking particles with still very large momentum by in-particles on the way in, but still far away.

If this possibility weren't there, the particles accumulationg at horizons wouyld have generated impenetrable firewalls. these are absent now.

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Replacement of an in particle by an out particle or *vice versa* has an important effect on the horizon: it readjusts the Shapiro shift. By removing all early in-particles, we went to a different basis of Hilbert space, where you don't see the particles that imploded to make the black hole, having them replaced by the Hilbert space of out going Hawking particles. Thus we 'changed the distant past', without changing the present, actually only by changing the past horizon by one without the growing Shapiro shifts. We went to a totally different basis of Hilbert space.

> This means that we changed the metric that was there before the black hole was formed. It's all a formality since the past is over, but now we have a new, artificial space-time in our past.

This is the one with the antipodal identification.

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To recuperate:

The *early in-going* particles have exponentially enhanced momenta, but their position operators are quite small.

If we replace them by their footprints, we see *early out-going* particles, whose momenta are now quite low (Hawking radiation), while their position operators are large (they left the black hole and are now far away).

The *late out-going* particles have not yet left, their momenta are high but positions small: they are still very close to the horizon. We replace them the same way by late in-going particles: these are still far way, on their way to fall in.

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If we do all this wisely, we only need to talk of *low momentum* particles, whose gravitational imprints are still weak. This way we can be sure that, at all times, only low momentum particles fill up all of Hilbert space. This ensures that our space-time will never be strongly deformed by the gravity fields of matter, and justifies the use of the eternal Penrose diagram that contains no matter sources.

Yet this is not the "usual" eternal Penrose; unitarity requires antipodal identification. So now we see how our philosophy needs to be applied: either you show the full load of grav. back reaction of matter (the Vaidya metric, and classical gravity effects of Hawking particles), or the antipodal description of space-time.

Finally, *by checking unitarity*, we justify our procedure a posteriori.

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The antipodal identification comes with an inversion of time. Consequently, also the Hamiltonian is mapped to *minus the Hamiltonian*. Now in field theory, the Hamiltonian is always positive (at large scales). So it is not exactly *minus* the Hamiltonian that we see in region II, but it must be $E^{\max} - H$ that we see there, where $E^{\max}$ is the maximal possible energy (density).[1]

Consequently, the antipodal identification maps the vacuum state onto the anti-vacuum state, which is the state to which no more energy can be added.

In theories of deterministic quantum mechanics, the antivacuum and the vacuum are identical, mathematically.

[1] No need to worry about the gravitational effects of $E^{\max}$, as we had just decided that, in the antipodal picture, gravity effects of matter had to be ignored.

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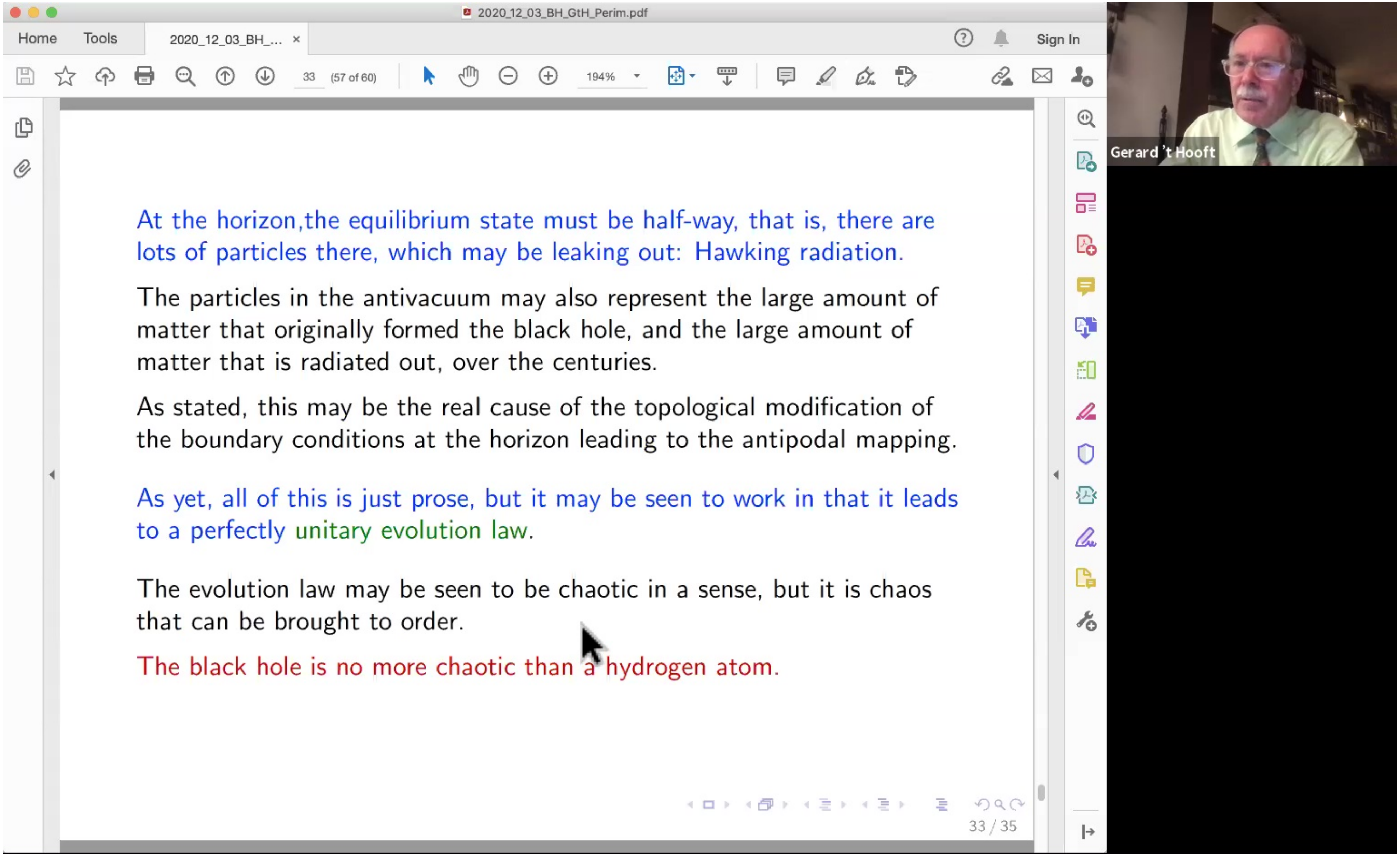
2020_12_03_BH_GtH_Perim.pdf
Gerard 't Hooft
At the horizon,the equilibrium state must be half-way, that is, there are lots of particles there, which may be leaking out: Hawking radiation.
The particles in the antivacuum may also represent the large amount of matter that originally formed the black hole, and the large amount of matter that is radiated out, over the centuries.
As stated, this may be the real cause of the topological modification of the boundary conditions at the horizon leading to the antipodal mapping.
As yet, all of this is just prose, but it may be seen to work in that it leads to a perfectly unitary evolution law.
The evolution law may be seen to be chaotic in a sense, but it is chaos that can be brought to order.
The black hole is no more chaotic than a hydrogen atom.
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Many questions remain:

1. State counting: still qualitative
2. Rewrite in-out momentum and position amplitudes in terms of the SM particles in and out
3. Horizon is very similar to, but not the same as, the string world sheet. Particles are vertex insertions, Find Kac-Moody algebras etc.
4. Further understanding of the vacuole - instanton (virtual emerging and disappearing black hole)
5. Further ideas about connection with SM and with deterministic quanyum schemes (ther "anti-vacuum")

See further explanations on web site G. 't Hooft home page

More work done by N.K. Gaddam, O. Papadoulaki and P. Betzios.

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THANK YOU

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