Title: The Fully Constrained Formulation: local uniqueness and numerical accuracy

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Abstract: In this talk I will introduce the Fully Constrained Formulation (FCF) of General Relativity. In this formulation one has a hyperbolic sector and an elliptic one. The constraint equations are solved in each time step and are encoded in the elliptic sector; this set of equations have to be solved to compute initial data even if a free evolution scheme is used for a posterior dynamical evolution. Other formulations (like the XCTS formulation) share a similar elliptic sector. I will comment about the local uniqueness issue of the elliptic sector in the FCF. I will also described briefly the hyperbolic sector. I will finish with some recent reformulation of the equations which keeps the good properties of the local uniqueness, improves the numerical accuracy of the system and gives some additional information.

The Fully Constrained Formulation: local uniqueness and numerical accuracy

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Outline:

· General Relativity: motivation.

· Elliptic equations and hyperbolic equations.

· 3+1 formalism: free evolution schemes vs constrained formulations.

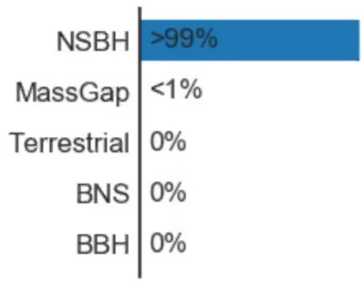
· CFC and the local uniqueness issue.

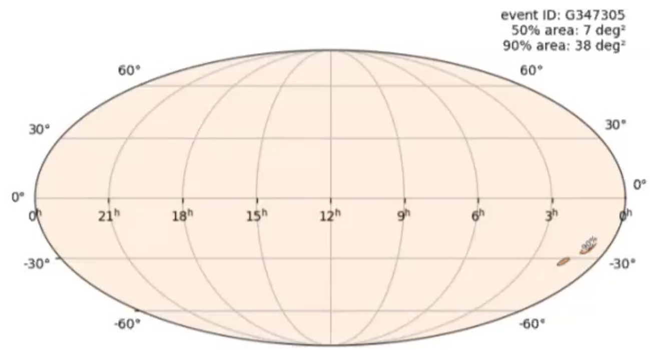
· FCF: structure of the different sectors.

· Conclusions and future steps.

General Relativity: motivation

· Mass / ratio diagram for compact objects do not have a maximum mass in the case of Newtonian gravity.





State-of-the-art topic with recent GWs superevents: S190814bv coming from merger of compact objects with component with low mass (< 3 solar masses).

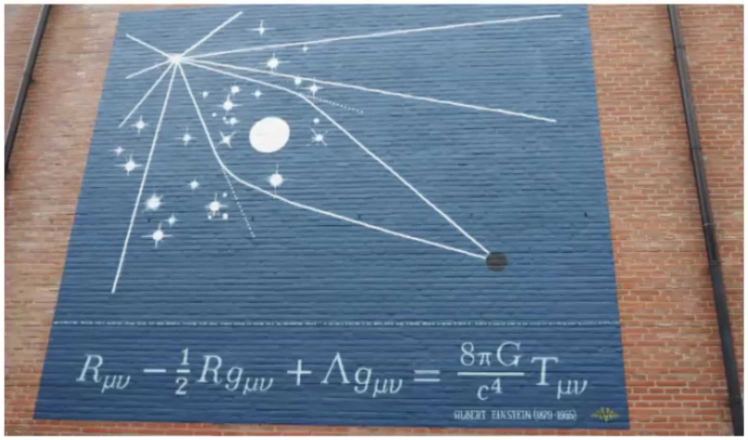
· MHD equations: supernovae core collapse.

· Black holes and relativistic jets.

· Gravitational lensing.

General Relativity: formulations

GR equations have a nice and compact formula...



but actually we are talking about non-linear partial differential equations (PDEs) coupled with the matter evolution content (non-vacuum spacetimes), neutrino radiation...

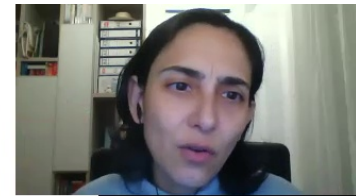
high COMPLEXITY!

· 2+2: null cones, analysis of radiation at spatial / null infinity.

· 3+1: Lichnerowitz 1944, Choquet-Bruhat 1952:

- Foliation of space-time by spacelike hypersurfaces, and evolution of the metric components through hypersurfaces: Cauchy problem (PDE with initial and boundary conditions).

- Most commonly used in numerical relativity.



Brief summary of elliptic and hyperbolic PDEs

· The character of a PDE is determined by the higher order derivatives.

· Elliptic equations:

·· Mathematical definition: if we consider a differential operator of the form

$\sum_{p \leq m} \alpha_p D^p$ and consider a non-null vector $\zeta \neq 0$, then $\sum_{p=m} \alpha_p \zeta^p \neq 0.$

The previous expression has a defined sign.

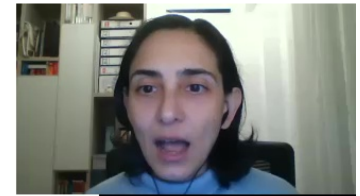
·· Important exemple: Laplace equation in cartesian coordinates:

$$\Delta f = \sum_i \partial^2 f / \partial x_i^2 = 0$$

$$\zeta_1^2 + \zeta_2^2 + \ldots + \zeta_n^2 > 0$$

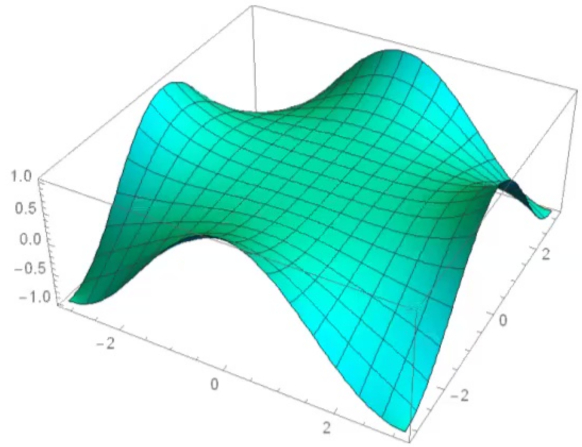
But this is a very theoretical definition... so it would be convenient to know the main properties of this kind of equations.

Brief summary of elliptic and hyperbolic PDEs



·· Main properties of elliptic equations:

The solution in the whole domain is determined by the values at the boundary:



$$\partial^2 f/\partial x_1^2 + \partial^2 f/\partial x_2^2 = 0$$

$$f(x_1, -\pi) = \cos(x_1) = f(x_1, \pi)$$

$$f(-\pi, x_2) = \cos(x_2) = f(\pi, x_2)$$

This can be interpreted as a infinite speed propagation.

If I change slightly the value of the function somewhere at the boundary, the whole solution changes.

Physically, these equations used to represent constraints or restrictions.

Brief summary of elliptic and hyperbolic PDEs

· Hyperbolic equations:

·· Mathematical definition:
we have an evolution equation of the form

$$\partial_t U + A^i \partial_i U = G$$

We also consider a general vector ζ_i .

Then, the eigenvalues and eigenvectors system associated to previous equation,

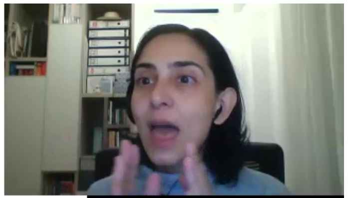
$$(I - \lambda\, \zeta_i A^i) r = 0$$

forms a set of real eigenvalues and a basis of real eigenvectors.

·· Important exemple: spherical wave $\partial^2 f / \partial t^2 = \partial^2 (ru) / \partial r^2$

with general solution $f(t,r) = G_1(r-t)/r + G_2(r+t)/r$

·· Any kind of waves are modeled by hyperbolic equations.



Brief summary of elliptic and hyperbolic PDEs

·· Main properties of hyperbolic equations:

Finite propagation (eigenvalues) in known directions (eigenvectors); in GR eigenvalues are bounded by speed of light in vacuum.

Given some initial data, the values of the function at the boundaries depend on the directions determined by the eigenvalues: consistent propagation of the information.

If I change slightly the value of the function somewhere, the whole solution will only change locally according to the eigenvalues and eigenvectors system.

Physically, these equations used to represent evolution of a system: fluids, traffic...

It is common that shocks and discontinuities appear in the evolution of these equations even if we consider smooth initial data.

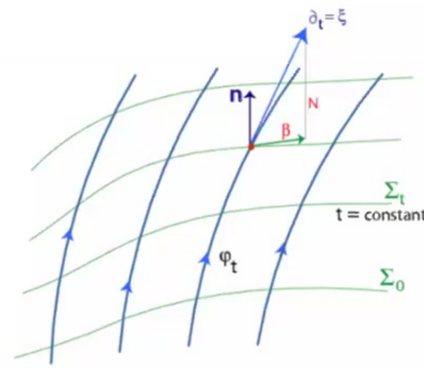
3+1 formulations

Gauge freedom: free choice of the more desired / convenient foliation.

· Physical motivations: e.g., maximal slicing in order to avoid singularities...

· Mathematical motivations: e.g., well-posedness of the resulting PDE system...

· Numerical motivations: e.g., stability of long-term numerical simulations, efficient evolution...



Decomposition of Einstein equations: $\xi = N\boldsymbol{n} + \boldsymbol{\beta}$. $\quad \boldsymbol{\gamma} = \gamma_{ij} dx^i dx^j$

Metric line element: $g_{\alpha\beta} dx^\alpha dx^\beta = -N^2 dt^2 + \gamma_{ij} \left(dx^i + \beta^i dt\right) \left(dx^j + \beta^j dt\right)$

Extrinsic curvature: $\boldsymbol{K} := -\frac{1}{2}\mathcal{L}_n \boldsymbol{\gamma}$

3+1 formulations

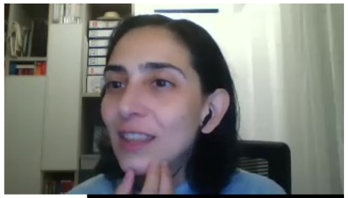
Splitting: · ij components: evolution equations (6+6) for the 3-metrix and extrinsic curvature.
· 00, 0i components: constraint equations (4), Hamiltonian and momentum constraints.

If constraints equations are fulfiled initially, then they are also fulfiled ANALYTICALLY through the evolution... but this does not happen always numerically:

- Free evolution schemes: evolve the evolution equations and MONITOR the error in the constraint equations: BSSN (strongly hyperbolic and well-possed), ccZ4...

- Constrained evolution schemes: SOLVE the evolution and constraint equations on each spatial hypersurface: CFC, FCF...

Different formulations:
- New variables defined.
- Gauge conditions.



CFC

CFC (Conformally Flat Condition): Isenberg 1979/2008, Wilson and Mathews 1989:

· Spatial 3-metric is assumed to be conformally flat. Gravitational radiation encoded in the neglected terms:

$$\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij}, \gamma_{ij} = f_{ij} + h_{ij}, h_{ij} \approx 0.$$

· Exact in spherical symmetry (CC 2011). Very accurate for axisymmetric rotating NSs.

· Set of elliptic equations for the metric variables (including the constraint equations): lapse, shift, conformal factor.

· Applications: collapse of rotating cores of massive stars and GWs catalogue (H. Dimmelmeier), evolution of binary systems NS-NS/BH (A. Bauswein)...

· Shares similar structure with XCTS, used in generation of initial data.

Local uniqueness issue

CFC (Conformally Flat Condition): Isenberg 1979/2008, Wilson and Mathews 1989:

$$\Delta\psi = -2\pi\psi^{-1}\left[E^* + \frac{\psi^6 K_{ij}K^{ij}}{16\pi}\right]$$

$$\Delta(N\psi) = 2\pi N\psi^{-1}\left[E^* + 2S^* + \frac{7\psi^6 K^{ij}K_{ij}}{16\pi}\right]$$

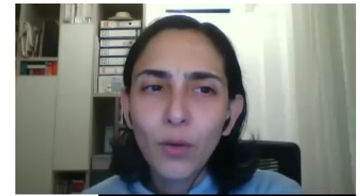
$$\Delta\beta^i + \frac{1}{3}\mathcal{D}^i\mathcal{D}_j\beta^j = 16\pi N\psi^{-2}(S^*)^i + 2\psi^{10}K^{ij}\mathcal{D}_j\frac{N}{\psi^6}$$

The original set of elliptic equations suffer from a non-local uniqueness pathology at extreme curvature or very high density regimes [CC et al. 2009].

A maximum principle can be used to prove local uniqueness of the solutions of elliptic equations of the form

$$\Delta u + hu^p = g$$

as long as $sign(p) \neq sign(h)$.

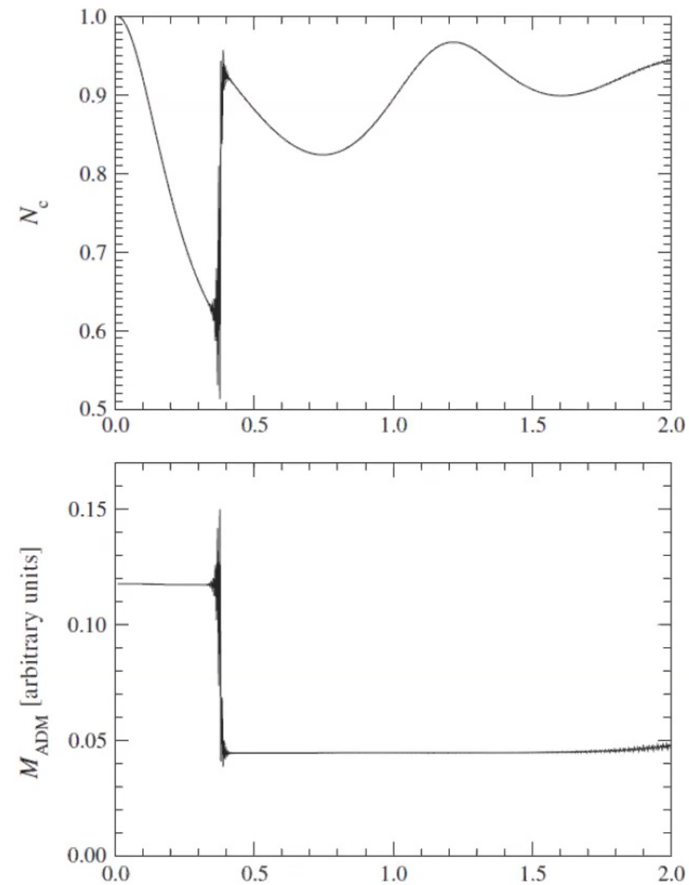


Local uniqueness issue

Full relativistic case (problem does not come from CFC simplification) with a similar elliptic sector:

· Evolution of a vacuum spacetime, with initial data formed by a Gaussian wave packet with small amplitude: dispersion of wave packet to infinity and no numerical problems.

· When a high amplitude is used instead: wave packet collapses to a BH and wrong behaviour of central lapse and ADM mass (also for conformal factor). Convergence to another (unphysical) solution.



Local uniqueness issue

Conformal decomposition of extrinsic curvature and introduction of auxiliar vector:

$$K^{ij} = \psi^{10}\hat{A}^{ij}, \hat{A}^{ij} = (LX)^{ij} + \hat{A}^{ij}_{TT} \approx (LX)^{ij} = \mathcal{D}^i X^j + \mathcal{D}^j X^i - \frac{2}{3}\mathcal{D}_k X^k f^{ij}$$

From the momentum constraint we can deduce an elliptic equation for the new auxiliar vector and rewrite the whole elliptic system:

$$\Delta X^i + \tfrac{1}{3}\mathcal{D}^i\mathcal{D}_j X^j = 8\pi f^{ij} S^*_j$$

$$\Delta\psi = -2\pi\psi^{-1}E^* - \psi^{-7}\frac{f_{il}f_{jm}\hat{A}^{lm}\hat{A}^{ij}}{8}$$

$$\Delta(\psi N) = 2\pi N\psi^{-1}(E^* + 2S^*) + N\psi^{-7}\frac{7f_{il}f_{jm}\hat{A}^{lm}\hat{A}^{ij}}{8}$$

$$\Delta\beta^i + \tfrac{1}{3}\mathcal{D}^i(\mathcal{D}_j\beta^j) = \mathcal{D}_j(2N\psi^{-6}\hat{A}^{ij})$$

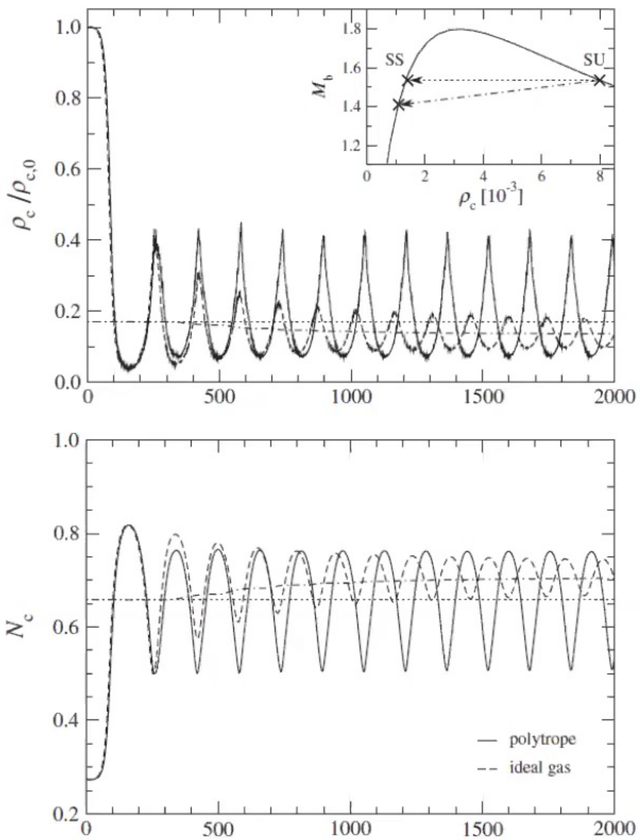
Local uniqueness is guaranteed and hierarchical structure is found: xCFC scheme. Implemented in several codes: CoCoNuT, x-Echo (Bucciantini and Del Zanna, 2011).

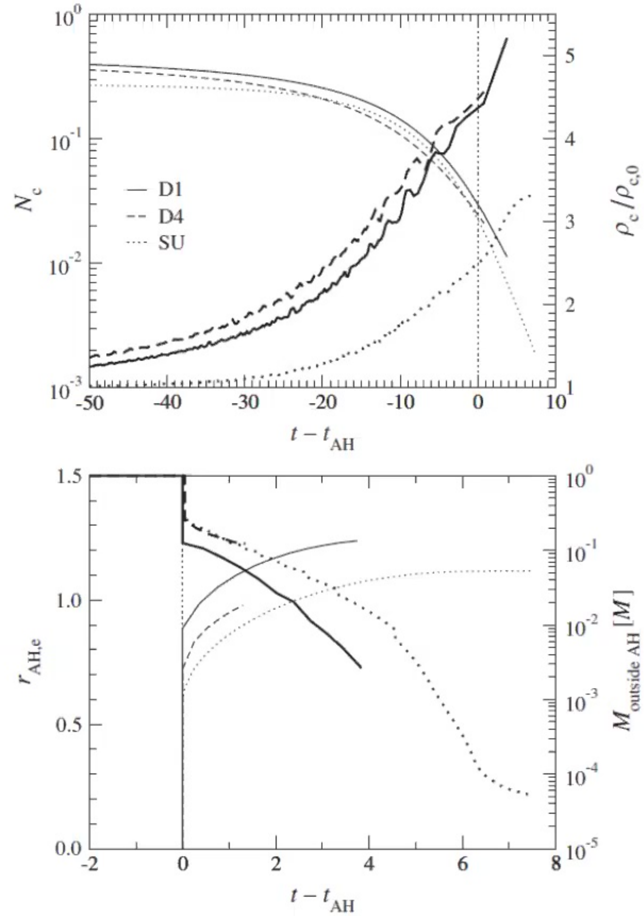
Local uniqueness issue

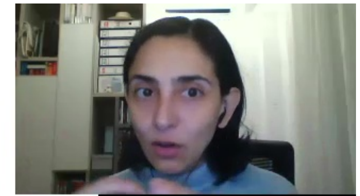
Succesful simulations in xCFC:

· Migration of unstable NS to the stable branch.

· Collapse of unstable NS to a BH.







Fully relativistic scheme: FCF

FCF (Fully Constrained Formulation): Bonazzola 2004:

· Maximal slicing and generalized Dirac gauge: $K = 0, \quad D_i h^{ij} = 0$

· Conformal 3-metric and conformal decomposition of extrinsic curvature.

· Extension of CFC: similar elliptic system with extra source terms, and additional hyperbolic sector with the evolution of $h^{ij}, \hat{A}^{ij}_{TT}$ encoding the GW radiation.

· Elliptic equations are more stable but difficult to solve and parallelize: Chevishev-Jacobi methods (CJM) (Adsuara et al. 2017) could solve the parallelization problem...

· Hyperbolic equations: Partially Implicit Runge-Kutta (PIRK) methods (CC and Cerdá-Durán 2012) developed for these equations and afterwards applied to other formulations (BSSN).

Fully relativistic scheme: FCF

FCF (Fully Constrained Formulation): Bonazzola 2004: elliptic sector

$$\Delta X^i + \frac{1}{3}\mathcal{D}^i\mathcal{D}_j X^j = -\tilde{\gamma}^{im}\left(\mathcal{D}_k\tilde{\gamma}_{ml} - \frac{1}{2}\mathcal{D}_m\tilde{\gamma}_{kl}\right)\hat{A}^{kl} + 8\pi\tilde{\gamma}^{ij}(S^*)_j,$$

$$\tilde{\gamma}^{kl}\mathcal{D}_k\mathcal{D}_l\psi = -2\pi\psi^{-1}E^* - \frac{1}{8}\psi^{-7}\tilde{\gamma}_{il}\tilde{\gamma}_{jm}\hat{A}^{lm}\hat{A}^{ij} + \frac{1}{8}\psi\tilde{R},$$

$$\tilde{\gamma}^{kl}\mathcal{D}_k\mathcal{D}_l(N\psi) = N\psi\left(2\pi\psi^{-2}(E^* + 2S^*) + \frac{7}{8}\psi^{-8}\tilde{\gamma}_{il}\tilde{\gamma}_{jm}\hat{A}^{lm}\hat{A}^{ij} + \frac{1}{8}\tilde{R}\right),$$

$$\Delta\beta^i + \frac{1}{3}\mathcal{D}^i\mathcal{D}_j\beta^j = 16\pi N\psi^{-6}\tilde{\gamma}^{ij}(S^*)_j - h^{kl}\mathcal{D}_k\mathcal{D}_l\beta^i - \frac{1}{3}h^{ik}\mathcal{D}_k\mathcal{D}_j\beta^j + \hat{A}^{ij}\mathcal{D}_j(2N\psi^{-6})$$
$$-N\psi^{-6}\tilde{\gamma}^{im}(\mathcal{D}_k\tilde{\gamma}_{ml} + \mathcal{D}_l\tilde{\gamma}_{km} - \mathcal{D}_m\tilde{\gamma}_{kl})\left((LX)^{kl} + \hat{A}^{kl}_{TT}\right).$$

Fully relativistic scheme: FCF

FCF (Fully Constrained Formulation): Bonazzola 2004: hyperbolic sector

$$\partial_t h^{ij} = \beta^k \mathcal{D}_k h^{ij} + 2N\psi^{-6}\hat{A}^{ij}_{TT} + 2N\psi^{-6}(LX)^{ij} - \tilde{\gamma}^{ik}\mathcal{D}_k\beta^j - \tilde{\gamma}^{kj}\mathcal{D}_k\beta^i + \frac{2}{3}\tilde{\gamma}^{ij}\mathcal{D}_k\beta^k,$$

$$\begin{aligned}
\partial_t \hat{A}^{ij}_{TT} = {} & \mathcal{D}_k\left(\beta^k \hat{A}^{ij}\right) - \hat{A}^{kj}\mathcal{D}_k\beta^i - \hat{A}^{ik}\mathcal{D}_k\beta^j + \frac{2}{3}\hat{A}^{ij}\mathcal{D}_k\beta^k + 2N\psi^{-6}\tilde{\gamma}_{kl}\hat{A}^{ik}\hat{A}^{jl} \\
& + N\psi^2\tilde{R}^{ij}_{*} - \frac{1}{3}N\psi^2\tilde{R}\tilde{\gamma}^{ij} - \frac{1}{2}(\tilde{\gamma}^{ik}\mathcal{D}_k h^{lj} + \tilde{\gamma}^{kj}\mathcal{D}_k h^{il})\mathcal{D}_l(N\psi^2) - (L\dot{X})^{ij} \\
& + \mathcal{D}_k\left(\frac{N\psi^2}{2}\right)\tilde{\gamma}^{kl}\mathcal{D}_l h^{ij} + \frac{N\psi^2}{2}\tilde{\gamma}^{kl}\mathcal{D}_k\left(\mathcal{D}_l h^{ij}\right) + 4\psi(\tilde{\gamma}^{ik}\tilde{\gamma}^{jl}\mathcal{D}_k\psi\mathcal{D}_l N + \tilde{\gamma}^{ik}\tilde{\gamma}^{jl}\mathcal{D}_k N\mathcal{D}_l\psi) \\
& + 8N\tilde{\gamma}^{ik}\tilde{\gamma}^{jl}\mathcal{D}_k\psi\mathcal{D}_l\psi - \frac{8}{3}\tilde{\gamma}^{ij}\tilde{\gamma}^{kl}\mathcal{D}_k\psi\mathcal{D}_l(N\psi) - \tilde{\gamma}^{ik}\tilde{\gamma}^{jl}\mathcal{D}_k\mathcal{D}_l(N\psi^2) + \frac{1}{3}\tilde{\gamma}^{ij}\tilde{\gamma}^{kl}\mathcal{D}_k\mathcal{D}_l(N\psi^2) \\
& - 8\pi N\psi^6\left(\psi^4 S^{ij} - \frac{S\tilde{\gamma}^{ij}}{3}\right).
\end{aligned}$$

Fully relativistic scheme: FCF

FCF (Fully Constrained Formulation): Bonazzola 2004: elliptic sector

$$\Delta X^i + \frac{1}{3}\mathcal{D}^i\mathcal{D}_j X^j = -\tilde{\gamma}^{im}\left(\mathcal{D}_k\tilde{\gamma}_{ml} - \frac{1}{2}\mathcal{D}_m\tilde{\gamma}_{kl}\right)\hat{A}^{kl} + 8\pi\tilde{\gamma}^{ij}(S^*)_j,$$

$$\tilde{\gamma}^{kl}\mathcal{D}_k\mathcal{D}_l\psi = -2\pi\psi^{-1}E^* - \frac{1}{8}\psi^{-7}\tilde{\gamma}_{il}\tilde{\gamma}_{jm}\hat{A}^{lm}\hat{A}^{ij} + \frac{1}{8}\psi\tilde{R},$$

$$\tilde{\gamma}^{kl}\mathcal{D}_k\mathcal{D}_l(N\psi) = N\psi\left(2\pi\psi^{-2}(E^* + 2S^*) + \frac{7}{8}\psi^{-8}\tilde{\gamma}_{il}\tilde{\gamma}_{jm}\hat{A}^{lm}\hat{A}^{ij} + \frac{1}{8}\tilde{R}\right),$$

$$\Delta\beta^i + \frac{1}{3}\mathcal{D}^i\mathcal{D}_j\beta^j = 16\pi N\psi^{-6}\tilde{\gamma}^{ij}(S^*)_j - h^{kl}\mathcal{D}_k\mathcal{D}_l\beta^i - \frac{1}{3}h^{ik}\mathcal{D}_k\mathcal{D}_j\beta^j + \hat{A}^{ij}\mathcal{D}_j(2N\psi^{-6})$$
$$-N\psi^{-6}\tilde{\gamma}^{im}(\mathcal{D}_k\tilde{\gamma}_{ml} + \mathcal{D}_l\tilde{\gamma}_{km} - \mathcal{D}_m\tilde{\gamma}_{kl})\left((LX)^{kl} + \hat{A}^{kl}_{TT}\right).$$

Fully relativistic scheme: FCF

FCF (Fully Constrained Formulation): Bonazzola 2004:

The elliptic sector has a similar structure to the CFC scheme and a new hyperbolic sector has to be considered.

Numerical accuracy?

Expansion of metric variables in terms of 1/c powers [Blanchet, Damour, Schäfer 1990]:

$$\beta_i = \mathcal{O}\left(\frac{1}{c^3}\right)$$

$$N = 1 - \frac{U}{c^2} + \mathcal{O}\left(\frac{1}{c^4}\right)$$

$$h^{ij} = \mathcal{O}\left(\frac{1}{c^4}\right)$$

$$\psi = 1 + \frac{U}{2c^2} + \mathcal{O}\left(\frac{1}{c^4}\right)$$

$$N\psi^2 = 1 + \mathcal{O}\left(\frac{1}{c^4}\right)$$

$$\hat{A}^{ij} = \hat{A}^{ij}_{TT} + (LX)^{ij} = \mathcal{O}\left(\frac{1}{c^3}\right)$$

$$X^i = \mathcal{O}\left(\frac{1}{c^3}\right), \quad \hat{A}^{ij}_{TT} = \mathcal{O}\left(\frac{1}{c^5}\right)$$

FCF: numerical accuracy

Work in progress in collaboration with S. Santos (PhD student): Evolution equations can have numerical accuracy problems due to cancellations:

$$\overbrace{\partial_t h^{ij}}^{1/c^5} = \overbrace{\beta^k \mathcal{D}_k h^{ij}}^{1/c^7} + \overbrace{2N\psi^{-6}\hat{A}^{ij}_{TT}}^{1/c^5} + \overbrace{2N\psi^{-6}(LX)^{ij} - \tilde{\gamma}^{ik}\mathcal{D}_k\beta^j - \tilde{\gamma}^{kj}\mathcal{D}_k\beta^i + \frac{2}{3}\tilde{\gamma}^{ij}\mathcal{D}_k\beta^k}^{1/c^3}$$

$$\overbrace{\partial_t \hat{A}^{ij}_{TT}}^{1/c^6} = S^{ij}_{\hat{A}} - (L\dot{X})^{ij} =$$

$$\mathcal{D}_k\left(\beta^k \hat{A}^{ij}\right) - \hat{A}^{kj}\mathcal{D}_k\beta^i - \hat{A}^{ik}\mathcal{D}_k\beta^j + \frac{2}{3}\hat{A}^{ij}\mathcal{D}_k\beta^k + 2N\psi^{-6}\tilde{\gamma}_{kl}\hat{A}^{ik}\hat{A}^{jl} + \frac{3}{4}N\psi^{-6}\tilde{\gamma}^{ij}\tilde{\gamma}_{lk}\tilde{\gamma}_{nm}\hat{A}^{km}\hat{A}^{ln}$$

$$+N\psi^2\tilde{R}^{ij}_* + \mathcal{D}_k\left(\frac{N\psi^2}{2}\right)\tilde{\gamma}^{kl}\mathcal{D}_l h^{ij} - \frac{1}{4}N\psi^2\tilde{R}\tilde{\gamma}^{ij} - \frac{1}{2}(\tilde{\gamma}^{ik}\mathcal{D}_k h^{lj} + \tilde{\gamma}^{kj}\mathcal{D}_k h^{il})\mathcal{D}_l(N\psi^2)$$

$$+4\psi^{-1}\tilde{\gamma}^{ik}\tilde{\gamma}^{jl}\mathcal{D}_k\psi\mathcal{D}_l(N\psi^2) + 4\psi^{-1}\tilde{\gamma}^{ik}\tilde{\gamma}^{jl}\mathcal{D}_l\psi\mathcal{D}_k(N\psi^2) - 2\psi^{-1}\tilde{\gamma}^{ij}\tilde{\gamma}^{kl}\mathcal{D}_k\psi\mathcal{D}_l(N\psi^2)$$

$$\overbrace{+\frac{N\psi^2}{2}\tilde{\gamma}^{kl}\mathcal{D}_k\left(\mathcal{D}_l h^{ij}\right) - 8N\tilde{\gamma}^{ik}\tilde{\gamma}^{jl}\mathcal{D}_l\psi\mathcal{D}_k\psi + 2N\tilde{\gamma}^{ij}\tilde{\gamma}^{kl}\mathcal{D}_k\psi\mathcal{D}_l\psi - \tilde{\gamma}^{ik}\tilde{\gamma}^{jl}\mathcal{D}_k\mathcal{D}_l(N\psi^2) - (L\dot{X})^{ij}}^{1/c^4}$$

$$\overbrace{-8\pi N\psi^{10}S^{ij} + 4\pi N S^*\tilde{\gamma}^{ij}}^{1/c^4}.$$

FCF: numerical accuracy

Definition of new auxiliary variable: $V^i = 2N\psi^{-6}X^i - \beta^i$

Elliptic sector:

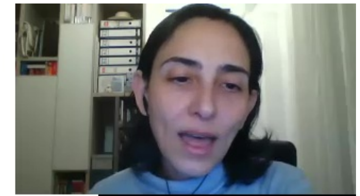
- Keep elliptic equations for: X^i, ψ

- Replace elliptic equation for $N\psi$ by elliptic equation for: $N\psi^2 \sim 1 + \mathcal{O}(c^{-4})$

- Replace elliptic equation for β^i by elliptic equation for: $V^i \sim \mathcal{O}(c^{-5})$

- Add elliptic equation for: $\dot{X}^i \sim \mathcal{O}(c^{-3})$

Still guarantee of local-uniqueness and hierarchical structure.



FCF: numerical accuracy

Hyperbolic sector:

$$\partial_t h^{ij} = \overbrace{\beta^k \mathcal{D}_k h^{ij} - h^{ik}\mathcal{D}_k \beta^j - h^{kj}\mathcal{D}_k \beta^i + \frac{2}{3} h^{ij}\mathcal{D}_k \beta^k}^{1/c^7}$$

$$\overbrace{+2N\psi^{-6}\hat{A}^{ij}_{TT} + (LV)^{ij} - X^j \mathcal{D}^i(2N\psi^{-6}) - X^i \mathcal{D}^j(2N\psi^{-6}) + \frac{2}{3} f^{ij} X^k \mathcal{D}_k(2N\psi^{-6})}^{1/c^5}$$

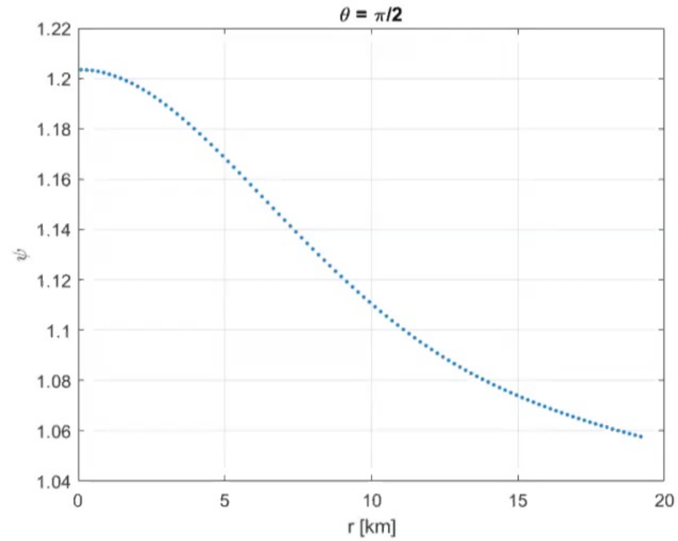
$$\begin{aligned}\partial_t \hat{A}^{ij}_{TT} = {} & \frac{N\psi^2}{2}\mathcal{D}^l \mathcal{D}_l h^{ij} - 8N\mathcal{D}^i\psi\mathcal{D}^j\psi + 2Nf^{ij}\mathcal{D}^l\psi\mathcal{D}_l\psi - \mathcal{D}^i\mathcal{D}^j(N\psi^2) \\ & -(L\dot{X})^{ij} - 8\pi N\psi^{10} S^{ij} + 4\pi N S^* \tilde{\gamma}^{ij} + \mathcal{O}\left(\frac{1}{c^6}\right),\end{aligned}$$

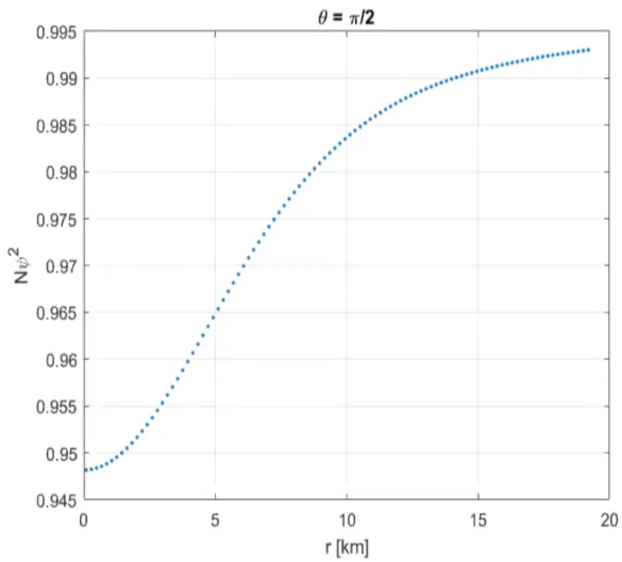
We have fixed the right accuracy for the first evolution equation.

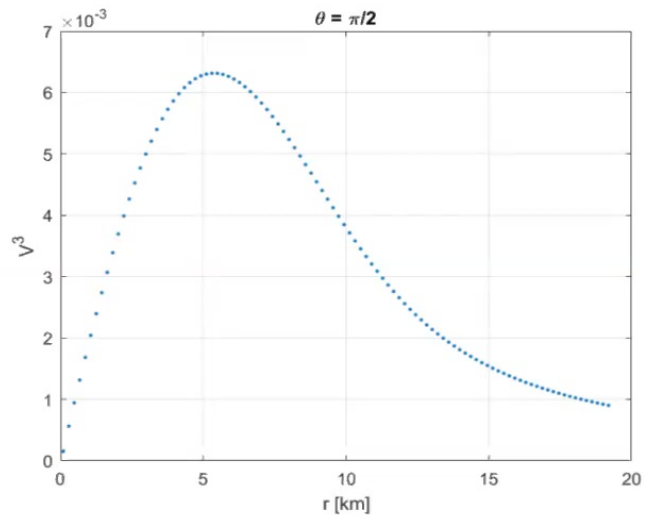
For the second one, we can derive an estimate for the h tensor.

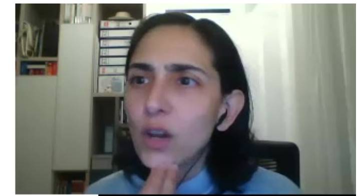
Numerical computation of initial data

· Rotating neutron star (LORENE code): 12.859 km star radius, 7.91g/cm^3 central density, 606 Hz (1/s), politropic fluid with Gamma=2.
· Spherical orthonormal coordinates.



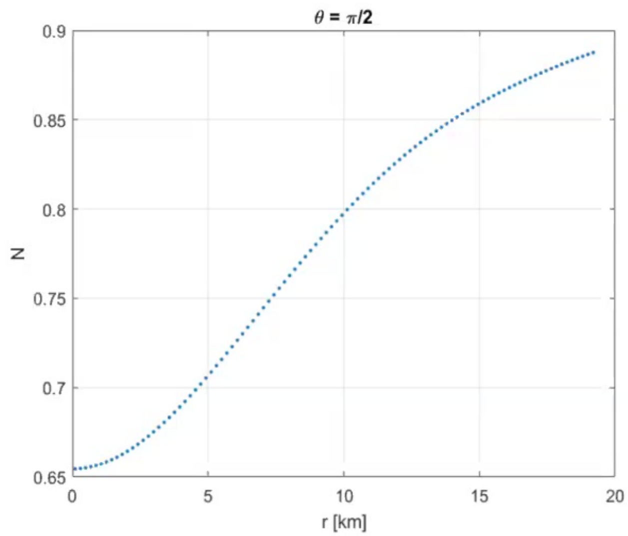


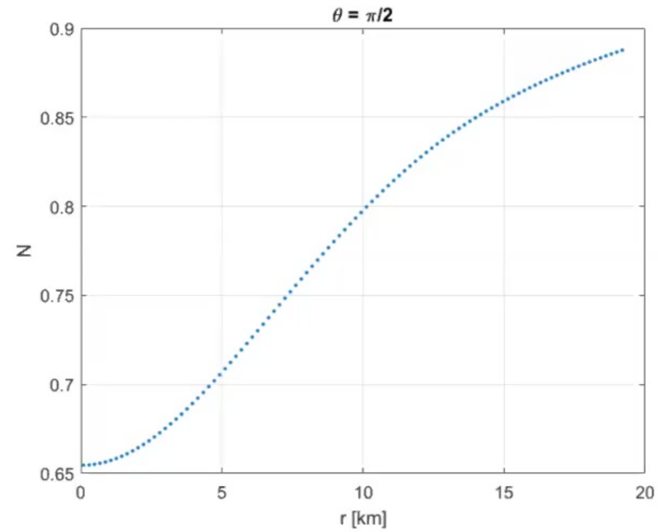


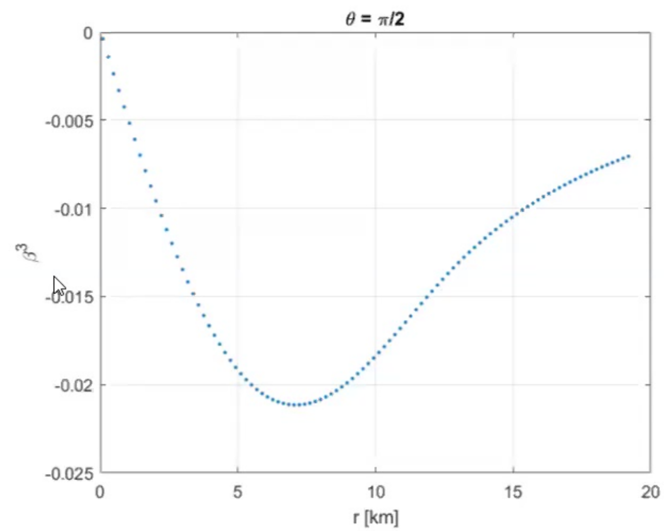


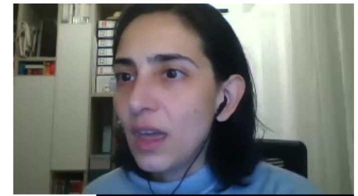
Numerical computation of initial data

· Rotating neutron star (LORENE code): 12.859 km star radius, 7.91g/cm^3 central density, 606 Hz (1/s), politropic fluid with Gamma=2.
· Spherical orthonormal coordinates.









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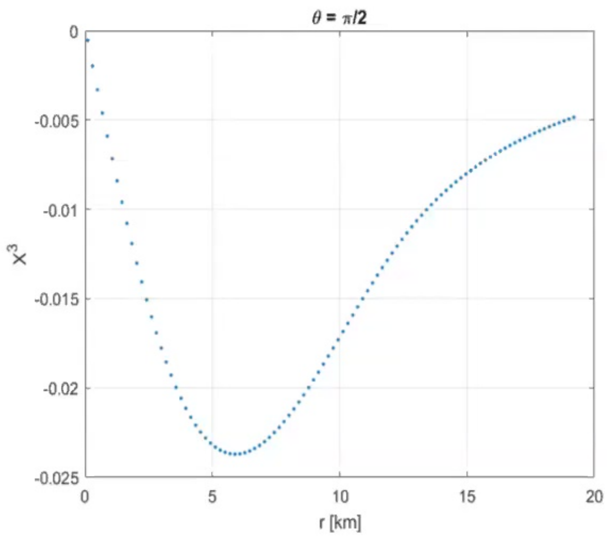
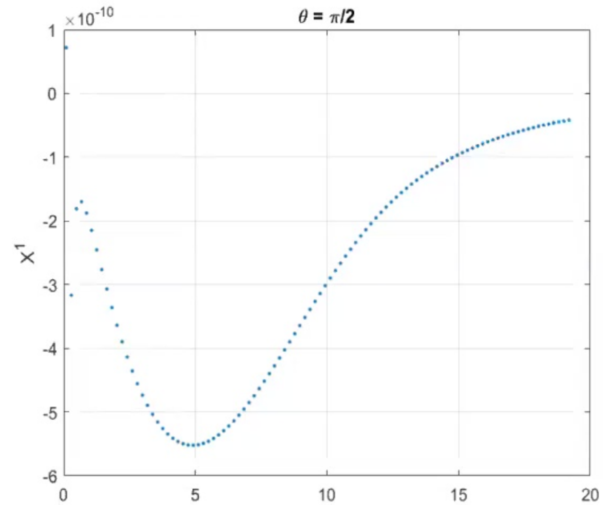
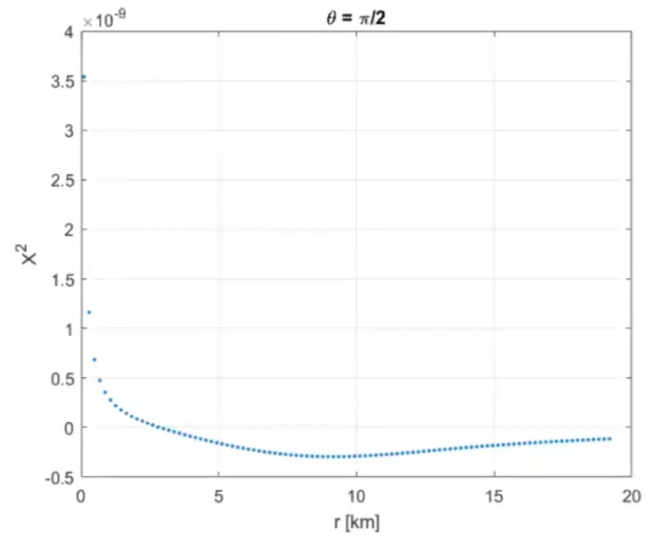
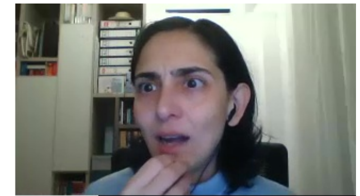


X^3 on the left; $\dot{X}^1, \dot{X}^2$ on the right.







Conclusions:

- 3+1: Free evolution schemes vs constrained schemes.

- Need of solving the constraint equations for initial data in both cases.

- CFC and the local uniqueness problem: derivation of xCFC.

- Fully relativistic scheme: FCF.

· Same elliptic sector of xCFC (local uniqueness and hierarchical structure) and extra hyperbolic sector.

· Numerical accuracy potential problem in the hyperbolic sector.

· Definition of new variables.
·· Keep good properties in the elliptic sector and improves accuracy with respect to xCFC equations.
·· Work in progress in the hyperbolic sector: numerical accuracy problem partially solved, estimate of tensor encoding the gravitational radiation.

Future steps:

· Check accuracy in more complex numerical simulations
→ importance in calibration of GWs templates.

· Check remaining constraints (determinant of $h^{ij}\ldots$) in different numerical simulations.

· Possible use of leading terms in simplified simulations: cosmological applications.

Thank you for your attention and we are looking forward to visit the Perimeter Institute!!!