Title: The geometry of string compactifications

Speakers: Lara Anderson

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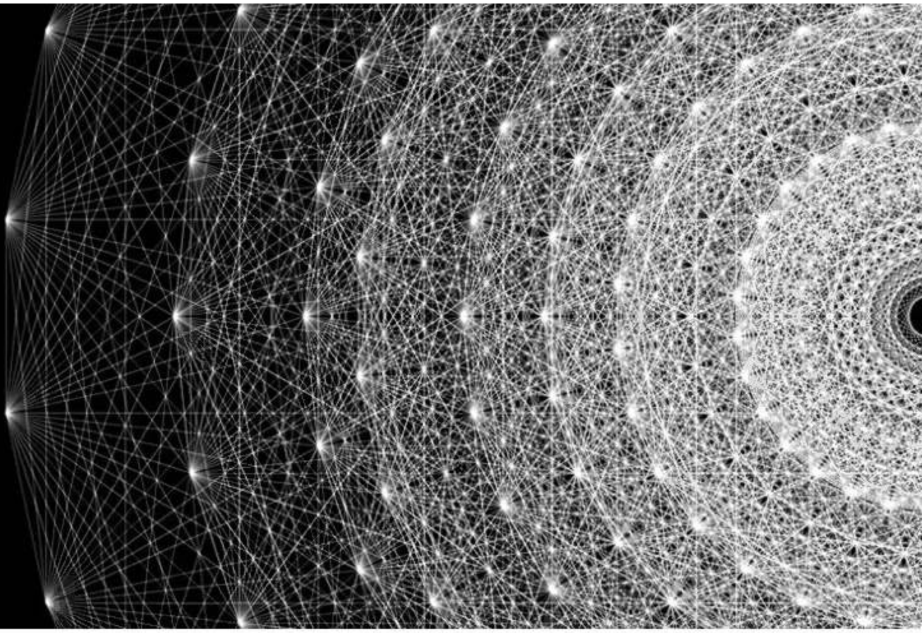
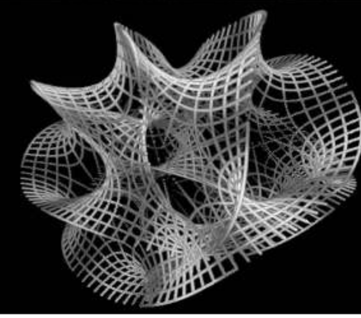
Abstract: In string compactifications the roles of physics and geometry are intrinsically intertwined. While the goals of these 4-dimensional effective theories are physical, the path to those answers frequently leads to cutting-edge challenges in modern mathematics. In this talk, I will describe recent progress in characterizing the geometry of Calabi-Yau manifolds in terms their description as elliptic fibrations. This description has remarkable consequences for the form of the string vacuum space and the properties of string effective theories, including particle masses and couplings.






The Geometry of String Compactifications

Perimeter Institute Colloquium

December 2nd, 2020

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Virginia Tech
Department of Physics

How to find new physics?

The sheer complexity of modern particle experiments makes it very hard to find things you aren't "looking for". What are the "interesting" signals?

Bottom Up

↑

Start with what you "know" at low energies and try to consider the most minimal/interesting modifications to it...

↑

Search for new physics...

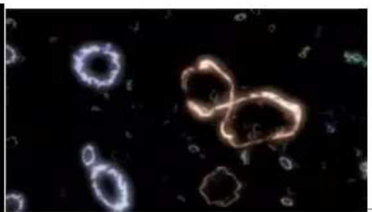
Top Down

↓

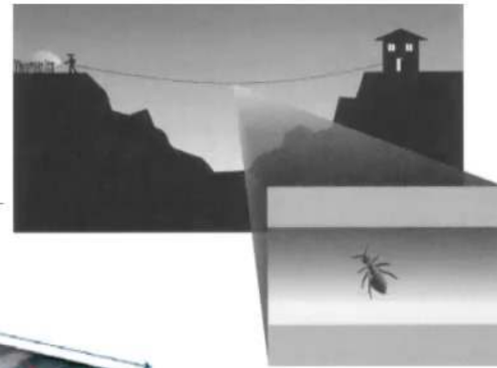
Use a constrained high-energy framework, including Gravity to see what you expect at lower energies...

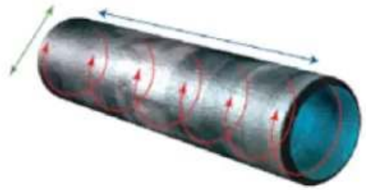
↓

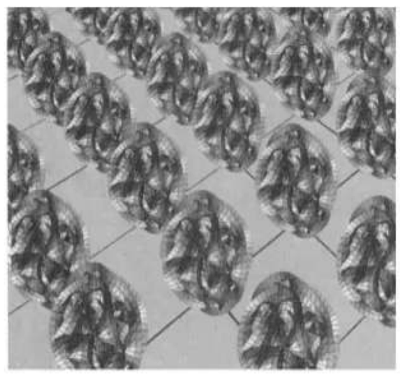
Search for new physics...



Overview







- Known consistent quantum theories of gravity are rare. As one such, string theory can be a powerful extension of quantum field theory

- Many remarkable applications: study of effective field theories, links to geometry/mathematics, or perhaps as a source of physical models…

- But string theory comes with a big catch: inherently formulated in more than 3 spatial dimensions

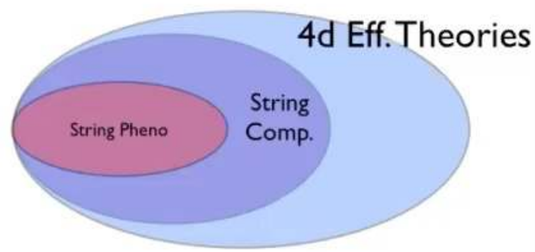
- String compactification (dimensional reduction)=the process of reducing higher-dimensional theories to 3+1 dimensions

Within string theory, the effective physics changes with the extra dimensions...

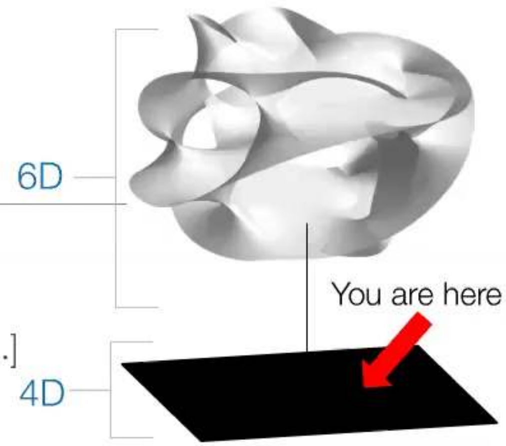
String theory theory predicts extra dimensions, we need to ask:

- What kinds/shapes of extra dimensions are allowed in string theory?
- How do these different geometries change the physical theories (including symmetries, particles and interactions) appearing in 4D?
- Which geometries agree with the physics we already know? And what other particles/physics could they predict to exist?

This area of investigation is called "String Phenomenology"



String Compactification (Heterotic)



- E.g. Start in 10D:

$$S_{het} = \frac{1}{2\kappa^2}\int d^{10}x(-G)^{1/2}[R - \frac{\kappa^2}{4g^2\phi}Tr(|\ F\ |^2) + \ldots]$$

- Look solutions of the form: $\mathbb{R}^{1,3} \times X_6 \longrightarrow$ (X_6 compact manifold)
- "Integrate out" The dependence on X $\longrightarrow$ 4D Effective Field theory!
- E.O.M. (for the simplest class of solutions):

X_6 is a complex manifold w/
SU(3) Holonomy

↕

Tr(R)=0 (Ricci Flat, Kahler)

↕

A Calabi-Yau Manifold

Introducing complex coordinates (a,b=1,2,3) on X...

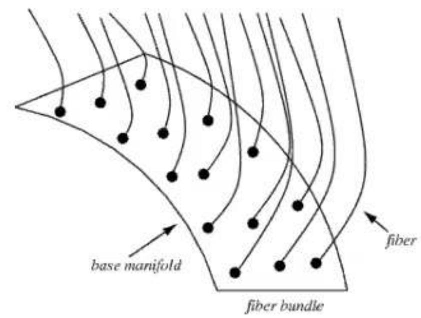
Hermitian Yang-Mills Eqs:

$$g^{a\bar{b}}F_{a\bar{b}} = 0$$

$$F_{ab} = F_{\bar{a}\bar{b}} = 0$$

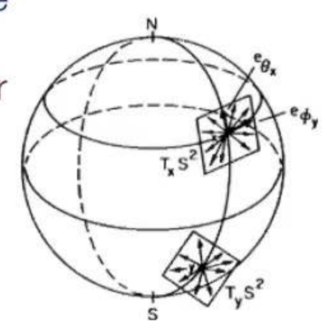
Spacetime Geometry, physics, and symmetries

- What physical theories can exist and what spaces they live in are correlated questions: Symmetries of spaces impact physical laws (e.g. Newton's laws of motion, Maxwell's equations, special relativity, general relativity, etc)



- Geometrically, the symmetries of physical laws can be encoded through vector bundles

Bundle ≈
Base
+
Fiber



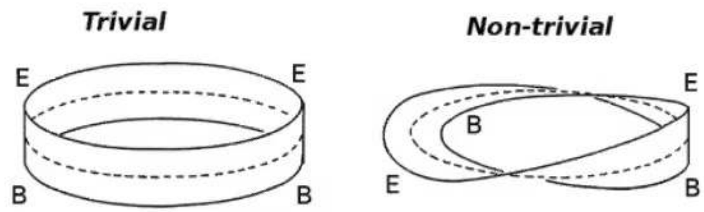
- A surjective morphism, $\pi : V \to X$ with $\pi^{-1}(x) = \pi^{-1}(x')\ \forall x, x' \in X$
- Fiber= $\pi^{-1}(x)$ (vector space), Base=X

- Example: U(1) gauge freedom → Abelian vector bundle

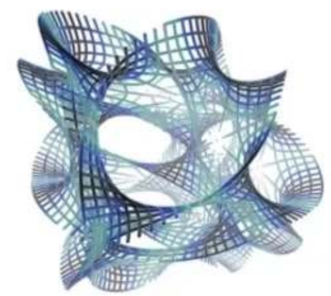
$d * F = 0 \qquad dF = 0$

Base space and fiber geometry constrain one another...

Neat fact:
The "shape" of the underlying space and the shape of the fibers constrain one another



Standard Model Symmetry:
SU(3) x SU(2) x U(1)?



Can use this to figure out...

a) What gauge theories can arise when we have compact directions?

b) How can the vector bundles constrain the size/shape of those new directions?

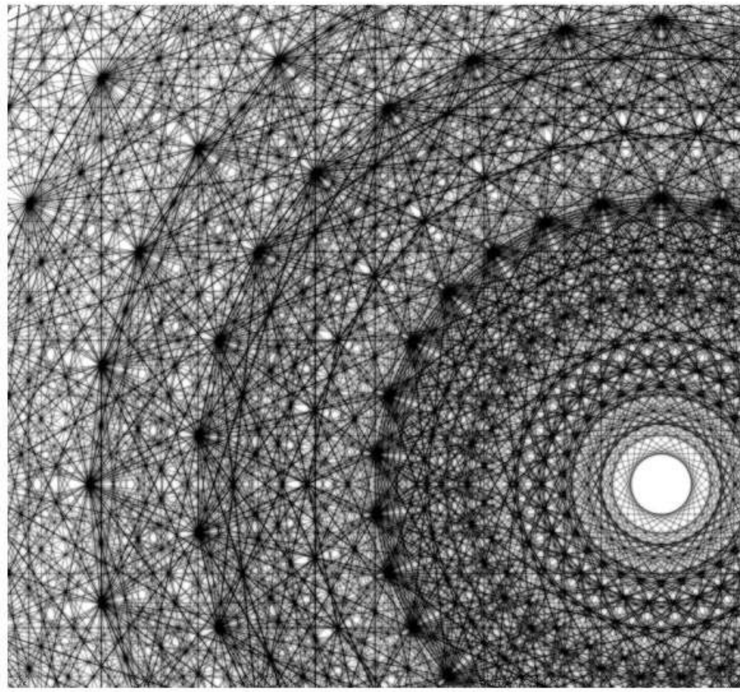
what bundles?

What symmetries/bundles arise in string theory? An example: $E_8 \times E_8$ Heterotic String Theory...

- Comes equipped with vector bundles in 10 dimensions. The higher dimensional gauge symmetry is the exceptional Lie group E_8. It has 248 dimensions and a rich and complicated structure!

- This bundle/symmetry can be broken into parts that live over the the compact space, X, and parts that are ``left over" in our 4-dimensions

- The Hard Part: Choose the bundle on the compact directions to "leave behind" a piece of E_8 that gives the symmetry and particles of the Standard Model in our world



Dimensional Reduction

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The origin of matter...

All charged matter in the 4D theory must come from the 10D gauge field (Adjoint-valued=**248**). Let A,B=0,..9

4D Gauge fields $$A^B_{10D} = (A^\mu_{4D}, A^a_{6D})$$

4D Scalar fields $$A_{\bar{a}} = A^0_{\bar{a}} + \delta C_x T^{xi} \omega_{i\bar{a}}$$

where T^{xi} are structure constants of $G \times H \subset E_8$

For 4D (massless modes): $$\bar{D}\omega = \bar{D} * \omega = 0$$

(harmonic H-valued 1-forms on X)

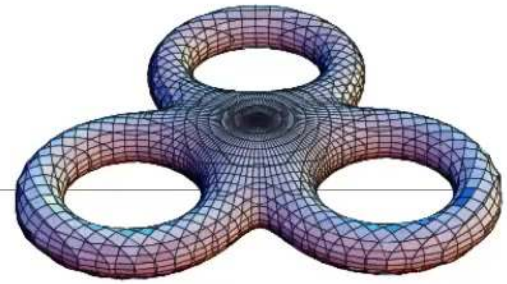
(We're solving a V-twisted Dirac Eq on X):

$$\nabla\!\!\!/_X \Psi = 0$$

$F = F^0 + 2\bar{D}\delta A$ (So a harmonic perturbation doesn't change the E.O.M --> Flat directions in the potential)

Gauge field vevs valued in H, break the 4D symmetry to G

Matter + Moduli



E.g. $E_8 \to SU(5) \times SU(5)$

$$\mathbf{248}_{E_8} \to [(\mathbf{1},\mathbf{24}) \oplus (\mathbf{5},\bar{\mathbf{10}}) \oplus (\bar{\mathbf{5}},\mathbf{10}) \oplus (\mathbf{10},\mathbf{5}) \oplus (\bar{\mathbf{10}},\bar{\mathbf{5}}) \oplus (\mathbf{24},\mathbf{1})]_{\mathbf{SU(5)\times SU(5)}}$$

This tells the type of matter that could exist in 4D, but not how much of it there is?...(i.e. what multiplicity? ⟷ How many harmonic forms in the basis? $(\delta C_\rho \omega^\rho)$

The space of all such "closed but not exact" 1-forms is called a Cohomology Group, $H^1(X,\Lambda^k V)$

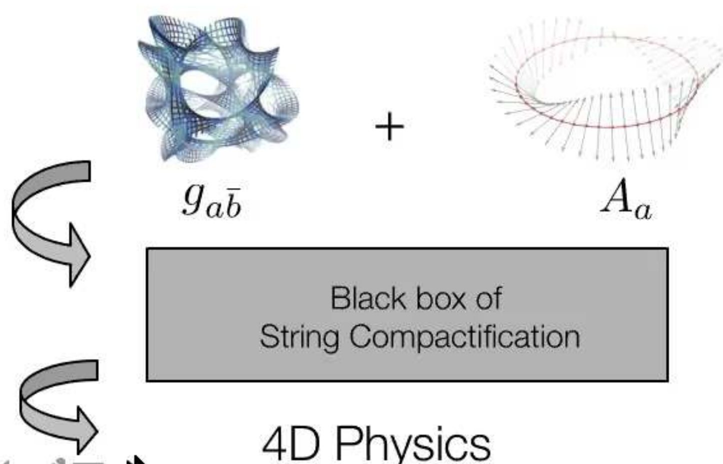
The particles and symmetries of the 4D theory are fixed by the compact geometry!

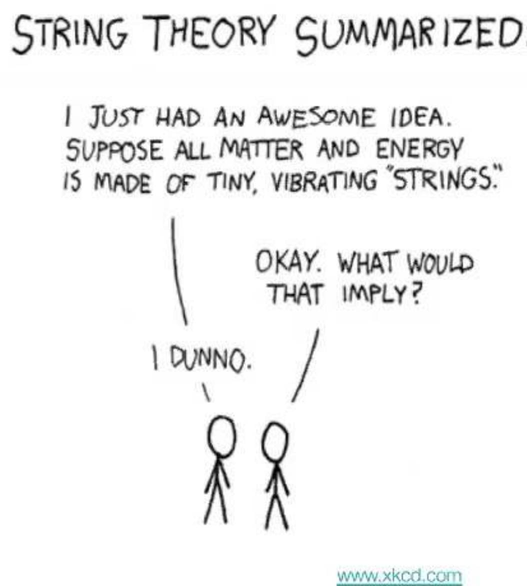
Can we get the Standard Model?
3 families of quarks + leptons?

$$SU(3) \times SU(2) \times U(1)$$

A physical theory is simply a tool to make predictions...

- The ideas of string pheno have been attractive for over 20 years… in principle every tune-able parameter in the low energy physics (for example all 25 free parameters in the SM) could be fixed by the choice of geometry.
- So, what's taken so long?
- What does string theory predict?





We want a LOT...

Particle Physics:

- Gauge and matter structure of the Standard Model
- Hierarchy of scales + masses (including Neutrinos)
- Flavor CKM, PMNS mixing, CP, no FCNC
- Hierarchy of gauge couplings (unification?)
- ‘Stable’ proton + baryogenesis
- Concrete, consistent predictions for “new” particles

Cosmology:

- Inflation or alternative for CMB fluctuations
- Dark matter (+avoid over-closing)
- Dark Radiation $N_{eff}>3$
- Dark Energy

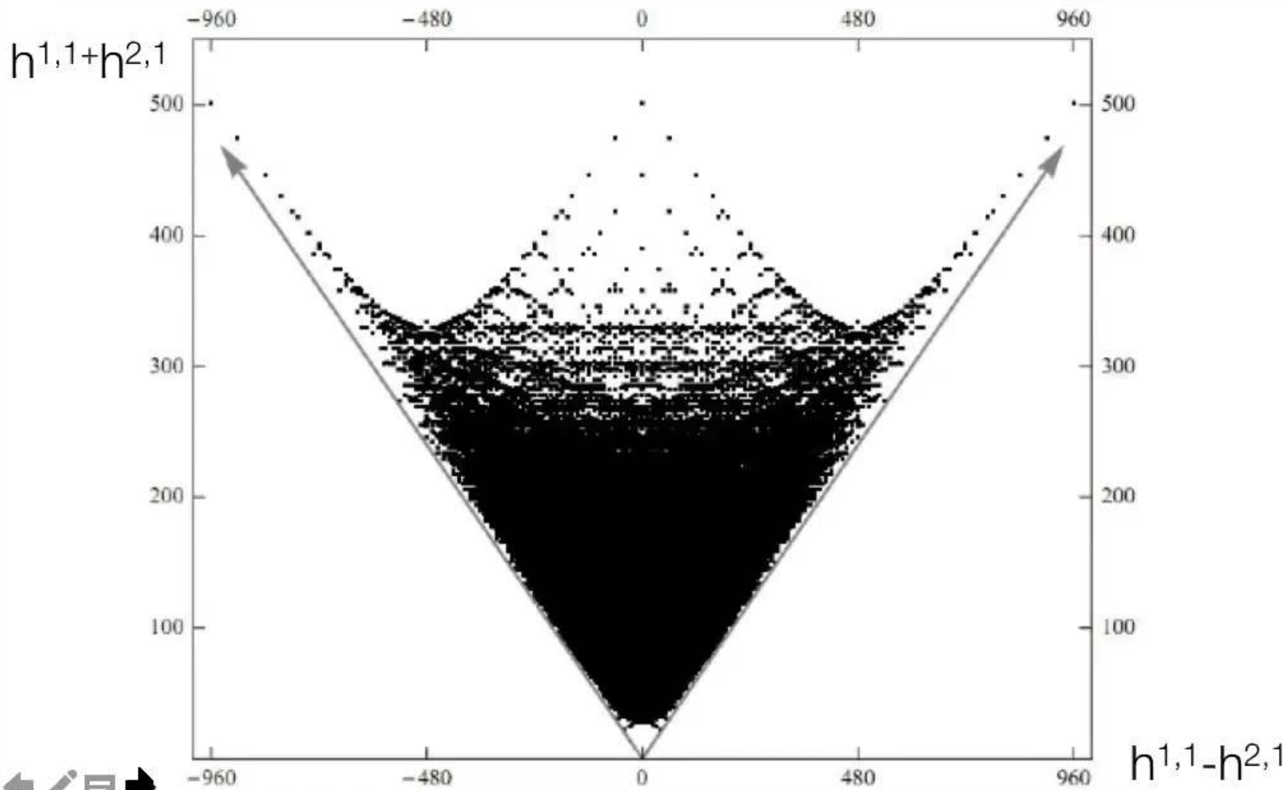
Important: If any ONE of these doesn’t work, can rule out the model!

Substantial challenges...

1) There are many consistent (though not realistic) geometries known.
(Not even known if the list of all such things is finite).

Half a billion Calabi-Yau manifolds and counting...

(Kreuzer + Skarke 2000)



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3) Some generic features of the effective theory are problematic $\rightarrow$ e.g. "Moduli"=massless scalars.

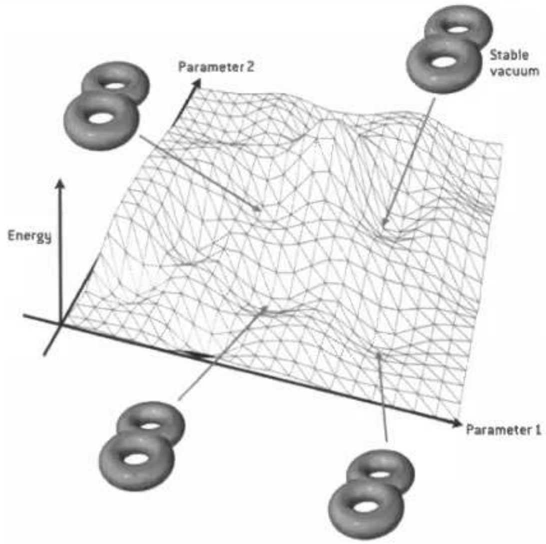
Moduli Problem

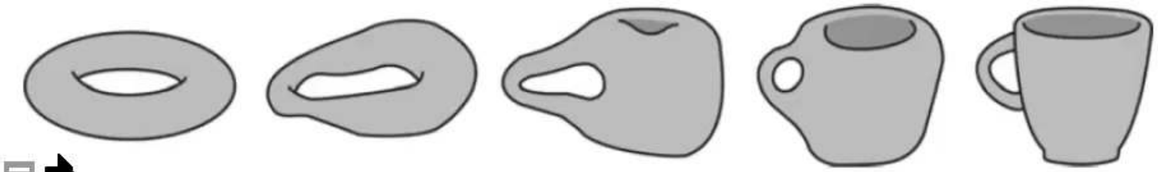
The 6-dimensional Calabi-Yau manifold has parameters (called "Moduli") associated to

1) Its size
2) Its shape

Order ~100 of such parameters and string theory does not tell us their values.

Unless we add something to the theory to fix the size/shape these lead to unphysical massless particles in 4D...



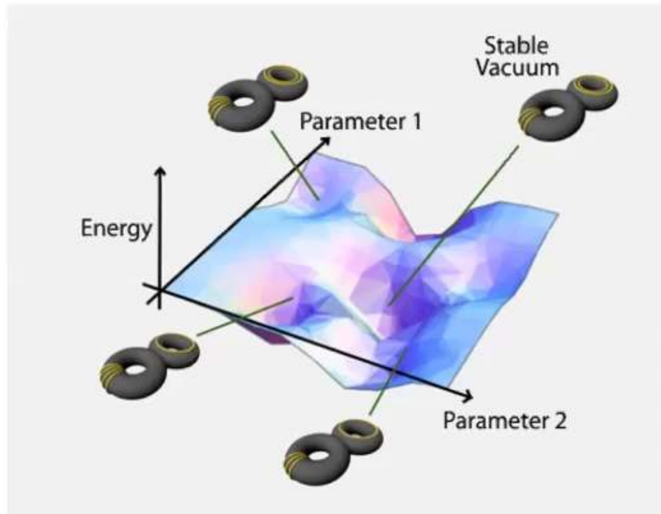


We're not here for the "Landscape"...

- Once we add flux potentials to lift moduli there is the possibility of getting many vacua in the theory...

- 10^{500} vacua??

- But this counting was done for a physically uninteresting set of geometries (none of those vacua have an electron!)



For 4D physics, we've replaced the question:

"Which field theory?" with "Which Geometry?"

Need to characterize "physically relevant geometries"

Substantial challenges...

1) There are many consistent (though not realistic) geometries known. (Not even known if the list of all such things is finite).

2) Incredible mathematical complexity involved in working out the 4D physics. Many parts of the calculation of the string compactification (e.g.) unknown...$g^{a\bar{b}}, A_{\bar{a}}$

3) Some generic features of the effective theory are problematic $\rightarrow$ e.g. "Moduli"=massless scalars.

The road to string phenomenology leads to the forefront of modern geometry….

String Geometry

- A serious and systematic approach to string phenomenology must include a systematic study of geometry. Need:

1. A study of string vacua that is algorithmic, computational and large scale (huge numbers of possible geometries/vacua).

2. New mathematical tools must be developed to fully extract and characterize the low energy theories resulting from compactification (as a function of the background geometry).

3. Novel mechanisms must be developed to address obstacles (i.e. stabilize moduli, etc).

These goals motivate my research. Here is some recent progress…

An algorithmic search for Standard Model vacua...

(w/ Constantin, Gray, Lukas, and Palti, e.g arXiv:1307.4787)

My collaborators and I attempted to generate a large dataset of geometries with the exact charged matter spectrum (i.e. quarks and leptons) of the SM

1) We chose a simple, Abelian "probe" construction of vector bundles over Calabi-Yau manifolds which allowed us to systematically produce large numbers of consistent string geometries.

2) Scanned over 10^{40} string geometries to search for those that produced exactly the particle spectrum of the Standard Model

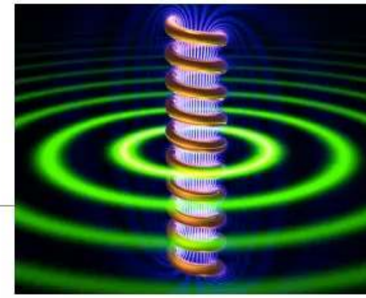
3) This is the largest and most systematic dataset of its kind in string theory

4) Scanned for coarse properties of the 4D physics consistent with experiment. Even "roughly" realistic string geometry is rare! We are working now to identify patterns...e.g.

SM Spectra	1 Higgs Pair	2 Higgs Pair	3 Higgs Pair	$rk(Y^{(u)}) > 0$	No proton decay	1 Higgs, No PD, $rk(Y^{(u)})>0$
407	262	77	63	45	198	13

Patterns/Predictions

(w/ Constantin, Lee and Lukas, e.g. arXiv:1507.03235)



$$< A > \neq 0$$
$$< F > = 0$$

- Simply having a large number of SM-like theories isn't good enough. Must be able to determine what they have in common, or where they all fail.
- Hope is to spot general structure…One example: Symmetry breaking down to SU(3) x SU(2) x U(1)

- Different ways to break symmetry: Higgsing, hypercharge flux, or through "Wilson Lines" (the Hosotani Mechanism) similar to an Aharanov-Bohm type of gauge field configuration in the extra dimensions.

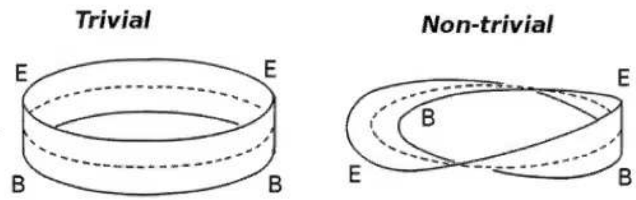
> We proved that only Wilson line breaking is compatible with SM spectra in a heterotic theory… independent of manifold/bundle/construction!

- Leads to important 4D signatures (here no doublet-triplet splitting problem, no relations between couplings, possible residual discrete symmetries).

- More structure like this that we are in the process of finding....

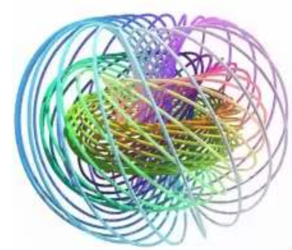
Fibrations + Calabi-Yau manifolds

(w/ Gao, Gray, and Lee, e.g. arXiv:1608.07555)

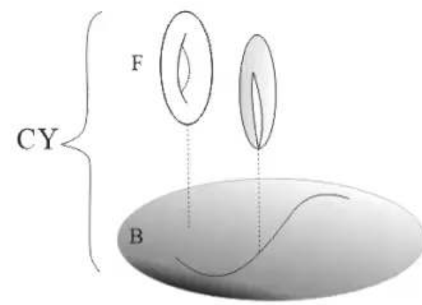


Although we do not know if there are finite # of CY manifolds, the number that can be written as a fibration

$$\pi : X \to B$$



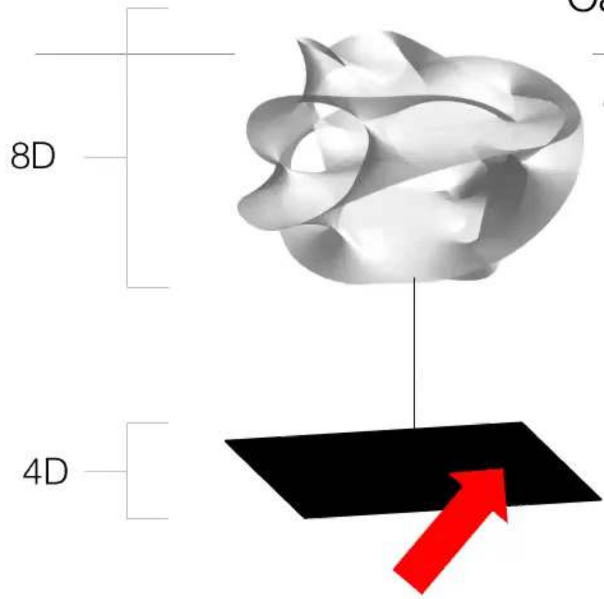
have been classified (by M. Gross) and are known to be finite.



Fibrations also play a remarkable role in "string dualities" where different branches of string theory (or different geometric backgrounds) give rise to the same effective theories.

F-theory: A 12 dimensional theory…

Calabi-Yau 4-manifold

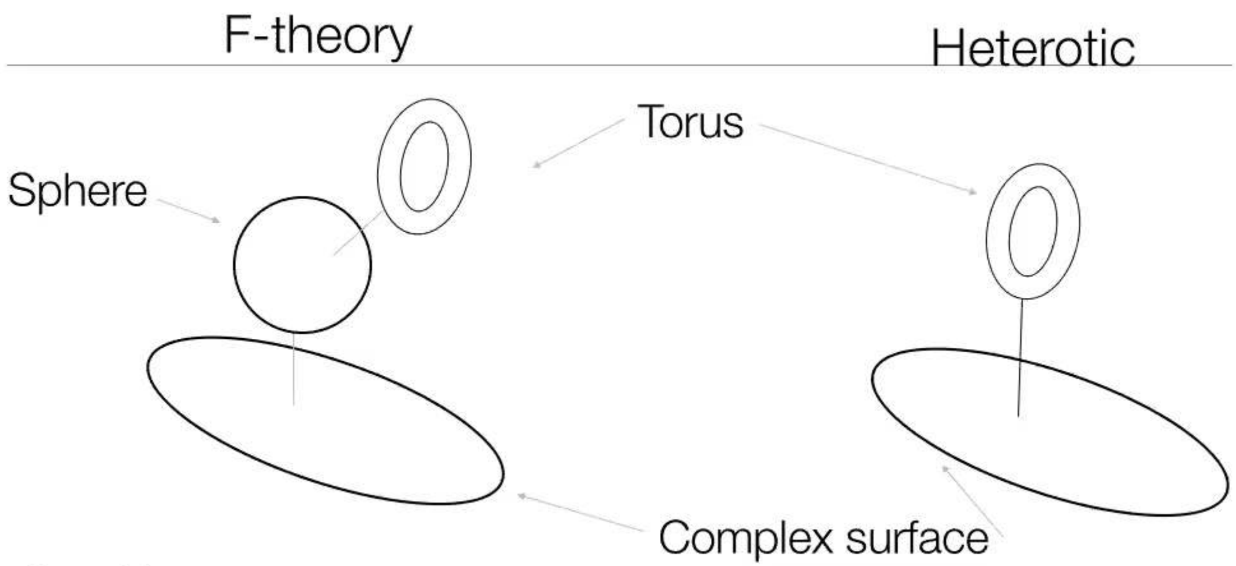


F-theory geometrizes the axio-dilaton of IIB string theory via an elliptic fibration

You are still here

This construction can also give rise to four dimensional physics that is at least semi-realistic…

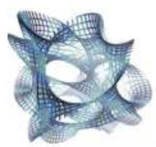
Heterotic/F-theory Duality:



- Consider:
- A heterotic theory on an elliptically fibered CY 3-fold: $\pi : X \rightarrow B$
- F-theory compactified on a CY 4-fold built by stacking K3 surface (i.e. a sphere and torus) over the same base B
- These two solutions to string theory give rise to the same 4-dimensional physics.

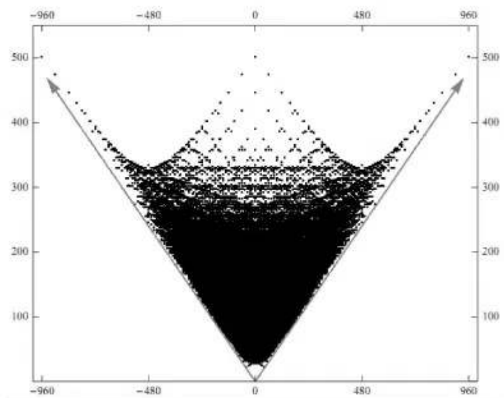
Fibrations

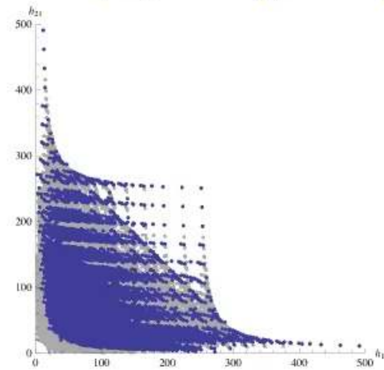
- By breaking a manifold into fiber + base, much easier to describe/classify. This decomposition makes many features of the effective theory calculable -> particle spectrum, form of metric, etc.



What fibrations?

- We have begun to systematically characterize this. Surprising observation: >99% of known CY manifolds admit fibrations! (LA, J. Gray, W. Taylor…)



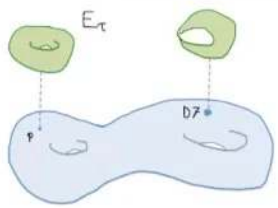


(Taylor, arXiv:1205.0952)

Finiteness of CY manifolds

- Studying and classifying fibrations has allowed us to "pull apart" complicated CY manifolds into simpler pieces: CY3 → ("fiber" + "base").

- All "Fibers" in CY manifolds must be other CY manifolds! Base manifolds can be characterized/classified using "birational geometry".



Unique CY 1-fold: T^2 (elliptic curve)
Unique CY 2-fold: K3 (complex surface)

- Systematic efforts underway to characterize what base manifolds can be combined with T^2 or K3 fibers (Gross, Grassi, di Cerbo, Svaldi, etc)

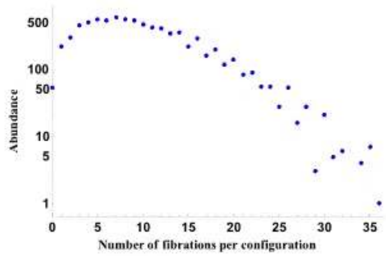
- We have developed algorithmic tools to find all fibrations for known datasets of CY manifolds

Even more remarkably…

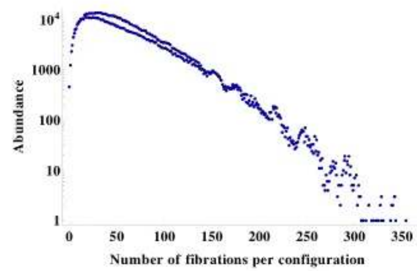
- We find that a generic CY admits more than one fibration….A LOT more!

$$
\begin{array}{ccccc}
 & & X_n & & \\
 & \swarrow^{\mathbb{E}_{(1)}} & \downarrow^{\mathbb{E}_{(2)}} & \searrow^{\mathbb{E}_{(i)}} & \\
B^{(1)}_{n-1} & & B^{(2)}_{n-1} \cdots & & B^{(i)}_{n-1}
\end{array}
$$

- CY 3-folds



4-folds



- Hope: may be able to establish that every CY manifold with large enough topology (i.e. $h^{1,1}$>18) admits an elliptic fibration. Now trying to use this to establish finiteness of the set of all CY manifolds!
- Fibrations a powerful tool to describe the physics. Working to build a duality cartography characterizing more fully redundancies in the set of string vacua.

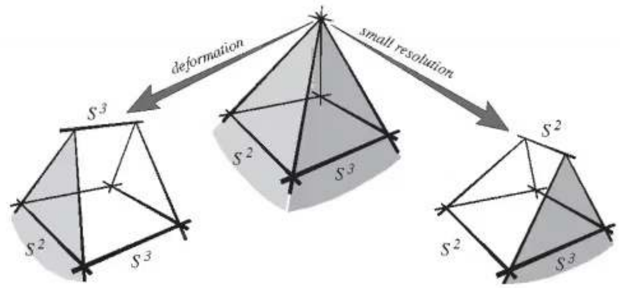
Geometric Transitions and Fibrations

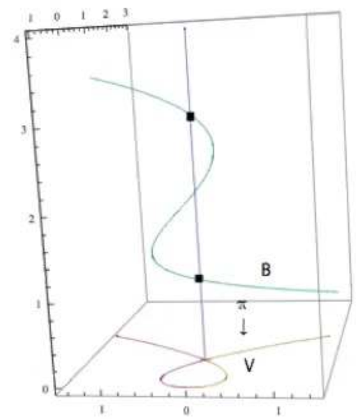
- Steps towards finiteness: We have proven that for known datasets of CY manifolds, any manifold can be connected to an elliptically fibered manifold via a finite number of geometric transitions (i.e. flops/conifolds)

- Working now on extending this to general CY 3-fold…

$$X_{def} \Leftrightarrow X_{sing} \Leftrightarrow X_{res}$$

Not fibered　　　　Fibered





Fibered manifolds and Particle Couplings

- Another application of fibered manifolds arises in heterotic compactification where the existence of a fibration in X gives rise to so called ``topological vanishings” of couplings.

- Basic Idea: Frequently we are interested in particle textures/hierarchies in couplings. That is, we would like allowed gauge-invariant couplings to vanish. Eg.s

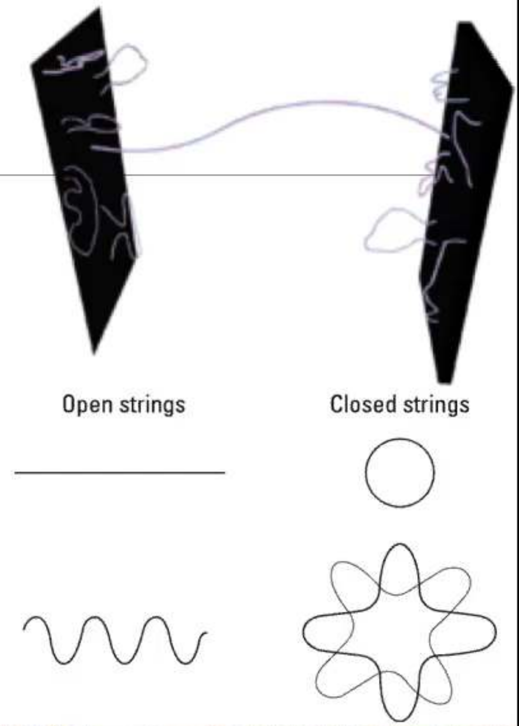
  - Examples: Standard Model (heavy top quark), The “mu problem” in supersymmetric theories ($\mu H_u H_d$), Forbid operators facilitating rapid proton decay, etc.

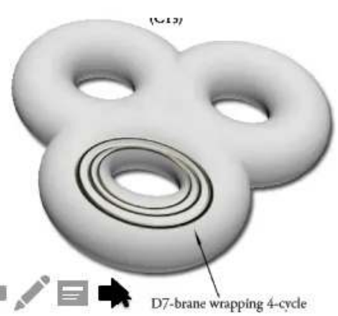
- For heterotic theories on elliptically fibered CY manifolds, topological vanishings are generic (can be either useful or dangerous).

Non-perturbative physics

- Strings are not the only dynamical objects in string theory… Also can have dynamical boundary conditions for open strings → D-branes .

- Within F-theory, "back-reaction" of the branes on the geometry leads to singular CY spaces. My collaborators and I have developed a new way to compute the "imprint" of D-branes within a CY space.



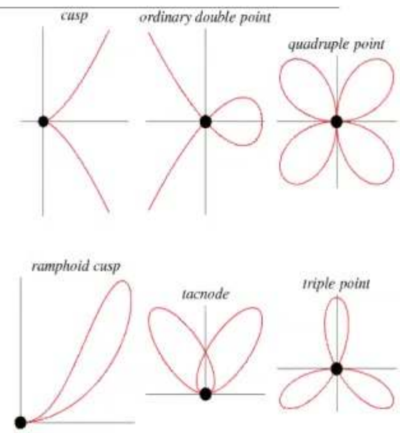


Elliptic fibrations also provide a remarkable window into new mathematics and physics…

(w/ Heckman, Katz, and Schaposnik, e.g. ArXiv:1702.06137)

- Thus far have described perturbative aspects of string theory, but non-perturbative contributions (i.e. branes) are also crucial.
- F-theory encodes information about 7-branes within the geometry of an elliptically fibered CY manifold. But open string (local models) and closed string (global models and CY moduli) descriptions can be hard to relate.
- One example: T-brane solutions involve brane worldvolume degrees of freedom that can provide novel fluxes -> leading to symmetry breaking, new particle spectra, Yukawa couplings, etc.
- Hitchin System: $F - \frac{i}{2}[\Phi, \Phi^\dagger] = 0 \quad , \quad \bar{\partial}_A \Phi = 0 \qquad (\Phi \in H^1(End(V) \times K))$
- i.e. "Higgs Bundle" = vector bundle and a scalar field defined over a complex curve (or higher dimensional variety)

The elephant in the medicine cabinet…



- Building on work by Diaconescu, Donagi and Pantev, my collaborators and I began to study CY elliptic fibrations in the limit of singular geometry and how to define their moduli spaces.
- Using the theory of limiting mixed Hodge structures we studied the intermediate Jacobian of CY 3-folds and find "emergent" Hitchin systems!
- That is, the CY moduli contain the open string d.o.f. in singular limit! (i.e. found a "transition function" relating open/closed string d.o.f.)
- Theory of 3-dimensional complex manifolds completely different to a theory of Higgs bundles on curves. Remarkable correspondence $\longrightarrow$ new physics and new mathematics!

$$\begin{array}{ccc} & & M \\ & \nearrow & \downarrow \\ \pi^* H & \rightarrow & \tilde{M}_{\rm cx} \\ \downarrow & & \downarrow \pi \\ H & \rightarrow & M_{\rm loc} \end{array}$$

Summary and Conclusions

The work described here is only a part of a broader program...

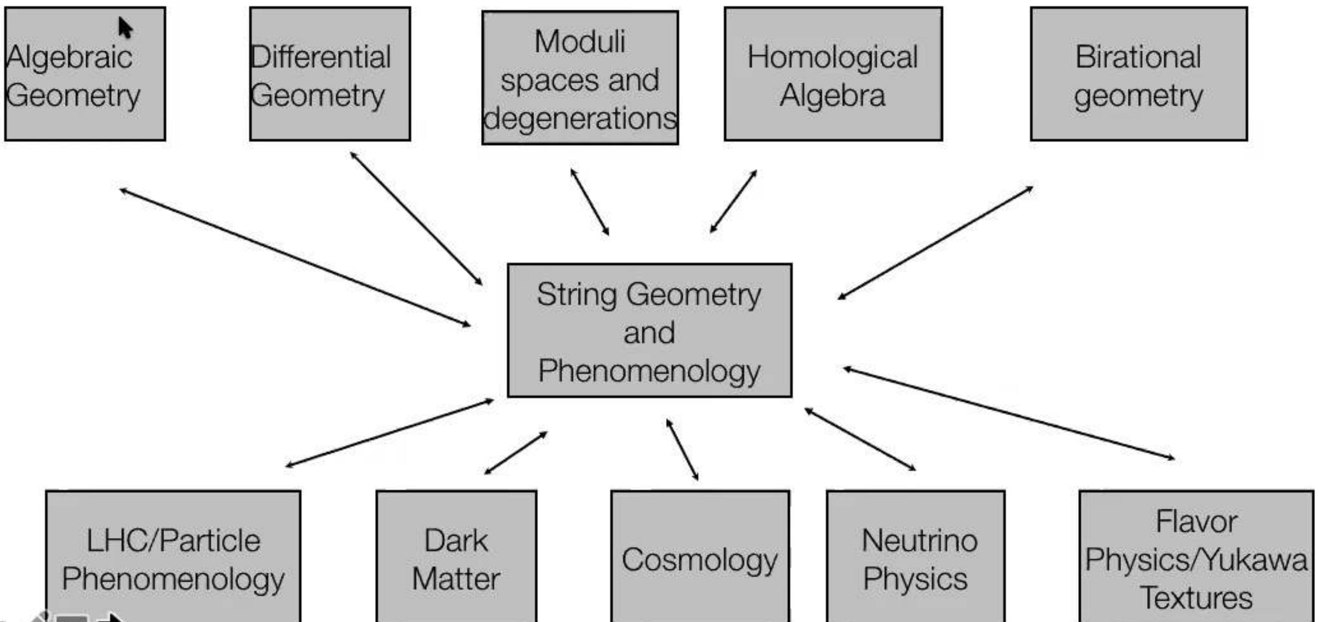
Today: String compactifications = “Manifolds/Bundles for fun and profit”

Recent and substantial progress:

- Engineering quantum field theories in string theory to match observable particle physics
- Bounding and constraining string geometry → bounding/classifying field theories that can arise in string theory
- Novel mathematical tools opening up new arenas in physics/mathematics (i.e. CY Manifold/Hitchin System correspondences)

More to come...

The interdisciplinary boundary of string theory and geometry is just beginning to be explored in earnest and there is much exciting work to be done... so stay tuned!



Thank you!

And thanks to my collaborators:

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