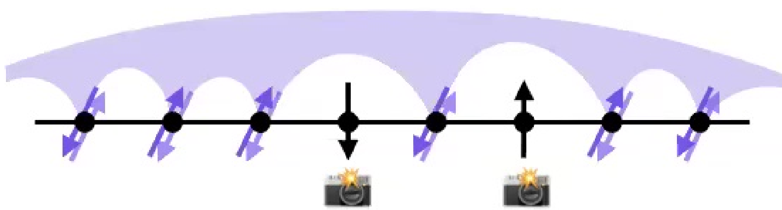
Title: Measurement phase transitions and statistical mechanics

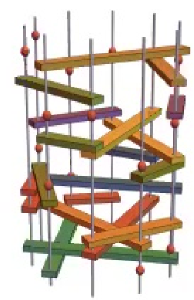
Speakers: Adam Nahum
Date: December 07, 2020 - 12:30 PM

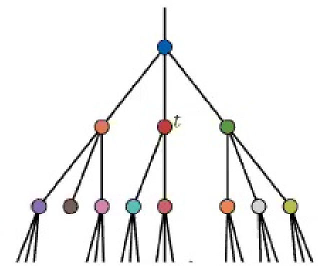
URL: http://pirsa.org/20120000

Abstract: A many-body quantum system that is continually monitored by an external observer may be in distinct dynamical phases, depending on whether or not the observerâ€™s repeated local measurements prevent the buildup of long-range entanglement. The universal properties of the â€œmeasurement phase transitionsâ€• between these phases remain a challenge. In this talk I will describe new theoretical approaches to measurement phase transitions, making connections with problems in statistical mechanics such as disordered magnets and travelling waves. I will show that exact results are possible in some regimes, including for quantum circuits with all-to-all interactions. (Based on arxiv:2009.11311)

Measurement phase transitions and the statistical mechanics of entanglement







Adam Nahum (Oxford)

Quantum Matter Frontier Seminar, Dec 2020

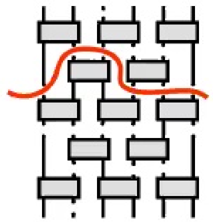
Sthitadhi Roy (Oxford), Brian Skinner (Ohio State), Jonathan Ruhman (Bar Ilan)

arXiv:2009.11311

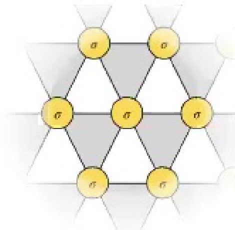
Tianci Zhou (KITP) PRB 19, PRX 20

Plan

1 Measurement phase transition:
A "percolation" transition for quantum info in spacetime

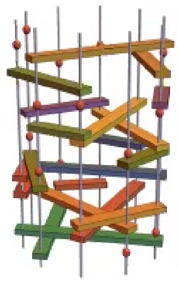


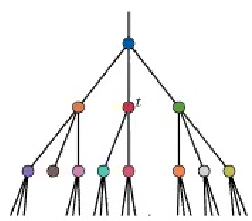
2 Stat mech of entanglement:
Permutations and domain walls



3 Field theories for measurement and entanglement transitions?

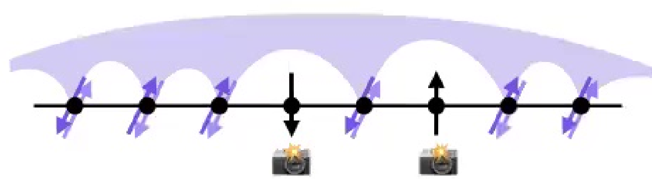
4 All-to-all-coupled measurement dynamics
& entanglement transitions on trees



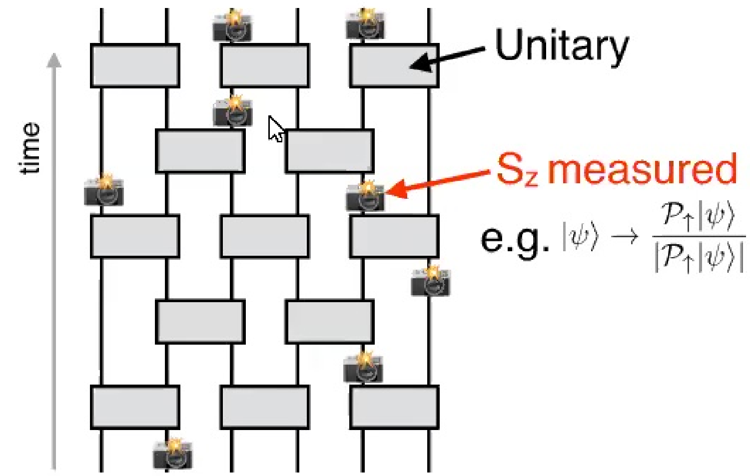


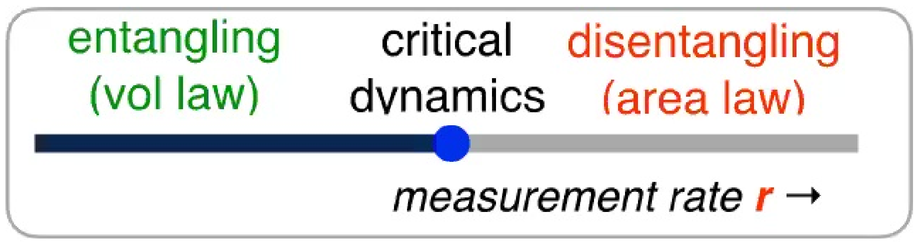
Measurement phase transition

Unitary circuit
+ measurements at rate ***r*** per spin



Stochastic evolution of pure state





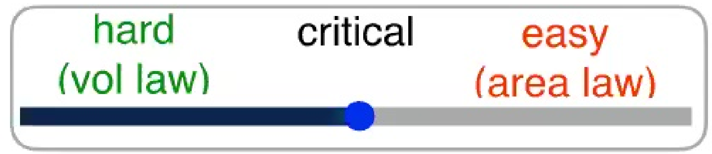
"Percolation" transition for quantum info in spacetime

Skinner, Ruhman, AN '18 Li, Chen, Fisher '18
Chan, Nandkishore, Pretko, Smith '18

Li Chen Fisher 19; Choi Bao Qi Altman 20; Szyniszewski, Romito Schomerus 19; Gullans, Huse 19, 20; Skinner, AN 19; Jian, You, Vasseur, Ludwig 20; Li, Chen, Ludwig, Fisher 20; Zabalo, Gullans, Wilson, Gopalakrishnan, Huse, Pixley 20; Ippoliti et al 20; Tang, Zhu 20; Fan, Vijay, Vishwanath, You, 20; Turkeshi, Fazio, Dalmonte 20; Shtanko et al 20; Li, Fisher 20 (…)

Measurement phase transition

A transition in computational hardness



Can we compute final state from knowledge of Hamiltonian and measurement record?

Can we efficiently simulate open quantum systems with quantum trajectories?

Bonnes, Läuchli 14

Can we efficiently contract shallow-depth circuits? Napp, La Placa, Dalzell, Brandão, Harrow 20

Useful toy model for **quantum** data processing?

Measurement circuits are random quantum error correcting codes

(Ehud's talk 2 wks ago)
Choi, Bao, Qi, Altman 20
Gullans, Huse 19

Universal quantum dynamics of "**monitored**" systems

Unexplored landscape of dynamical phases

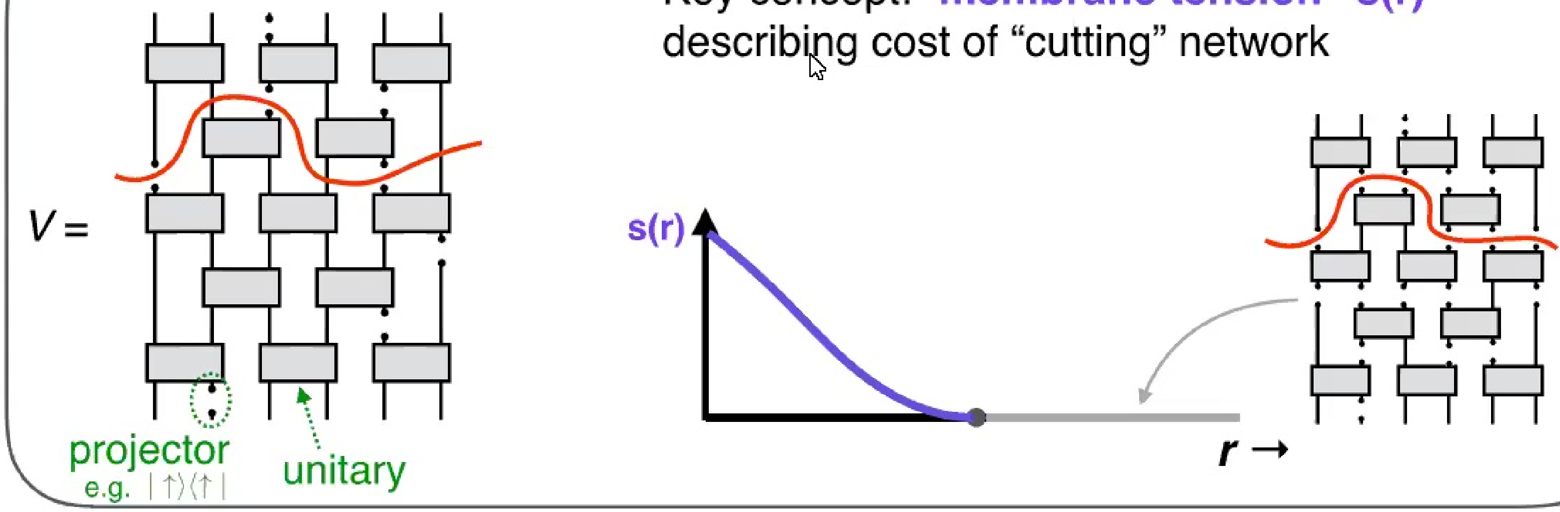
E.g. Role of symmetries/topology
(Sang, Hsieh 20, Lavasani, Alavirad, Barkeshli 20),
Special structure - eg Clifford (many refs), feedback ….

Unusual critical points for "percolation" of quantum information in spacetime

Spacetime picture

Simplest starting point: large local Hilbert space dimension.
Then, transition determined by circuit geometry (min-cut). "Classical limit"

Key concept: "**membrane tension" s(r)**
describing cost of "cutting" network



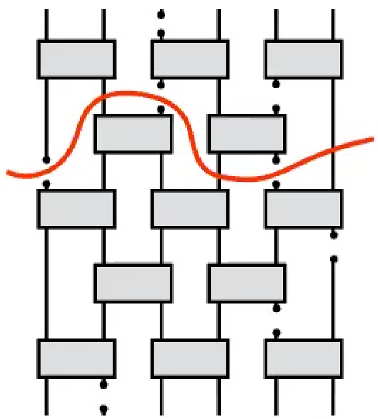
Transition = vanishing of membrane tension.
This idea survives beyond the classical limit

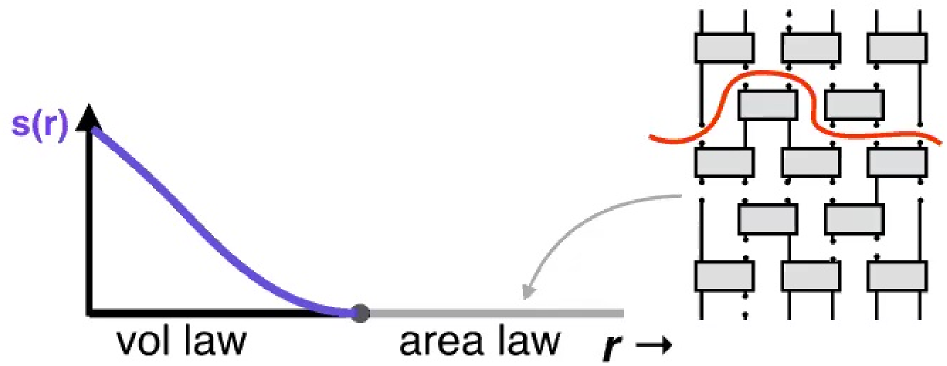
Skinner, Ruhman, AN 19
Bao, Choi, Altman 20
Jian, You, Vasseur, Ludwig 20
Vasseur, Potter, You, Ludwig 19
Zhou, AN 19, AN, Vijay, Haah 18
Hayden et al 16
Aharonov, 00
Li, Fisher, 20
AN, Roy, Skinner, Ruhman 20

Spacetime picture

Key concept: **"membrane tension" s(r)** describing cost of "cutting" network

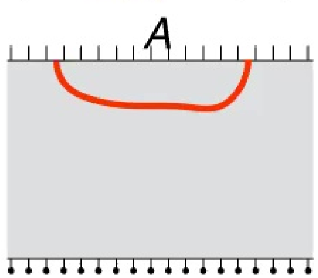
$V =$





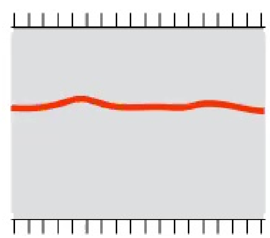
state entanglement

$$S(A) \simeq s(r)\,\mathrm{vol}(A) + \ldots$$



operator entanglement

$$S_{\mathrm{op}} \simeq s(r) N$$



Measurement transition also diagnosed by amount of info "remembered" from initial state

Gullans, Huse 19
Choi, Bao, Qi, Altman 20
Li, Fisher, 20
AN, Roy, Skinner, Ruhman 20

Questions for this talk

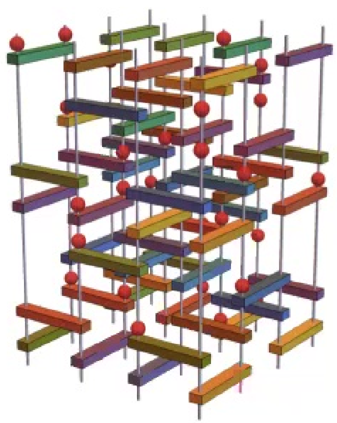
Universal properties of two phases?

Much is understood through refinement of membrane picture

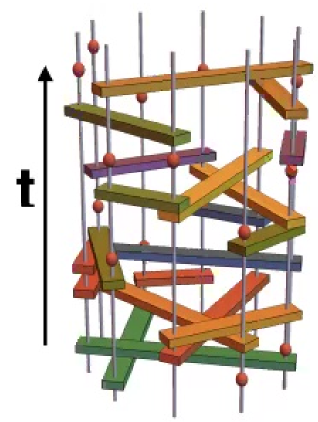
Universal properties of phase transition?

More difficult… but maybe easier if we get away from 1+1D

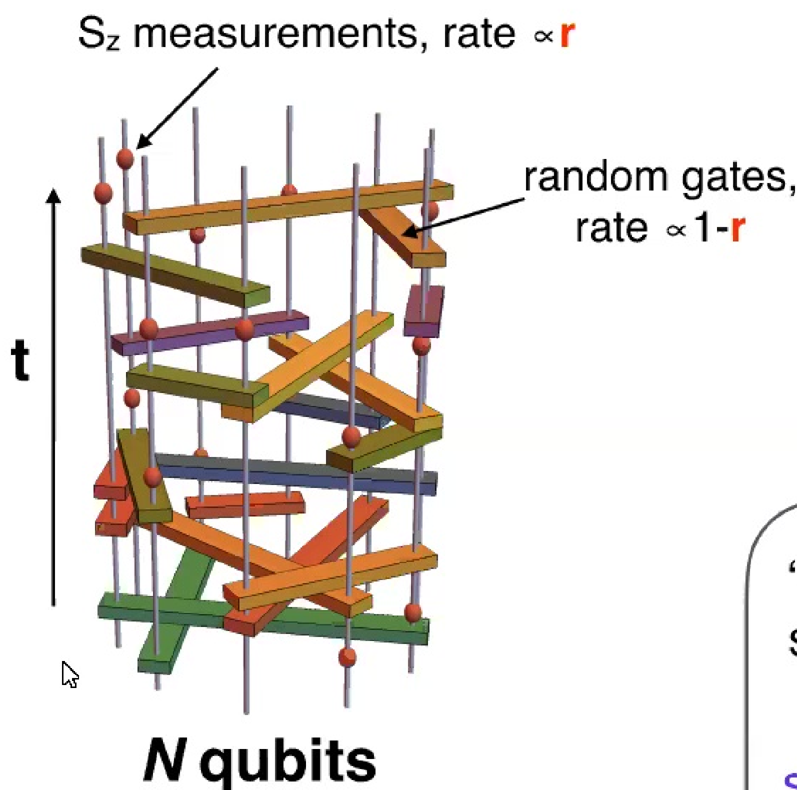
Higher dimensions



Or even all-to-all coupling



Setup for all-to-all circuit



See also:
Vijay '20
Gullans & Huse '20 & unpublished

Distinguish 2 cases:

"True" measurements (MPT):
Born rule → nontrivial, correlated outcome probabilities

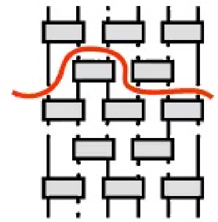
"Forced" measurements (FMPT):
All outcomes spin-up (postselection)

"volumes" and "areas" scale same way (like $d \to \infty$):

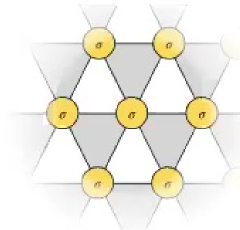
simplest to diagnose transition via propagation of info from initial to final time → operator entang. of V

Plan

1 Measurement phase transition:
A “percolation” transition for quantum info in spacetime

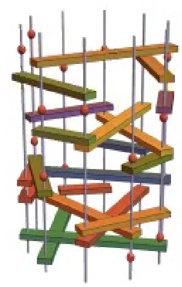


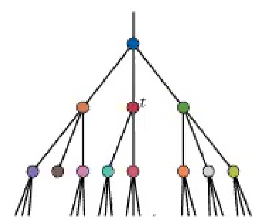
2 Stat mech of entanglement:
Permutations and domain walls



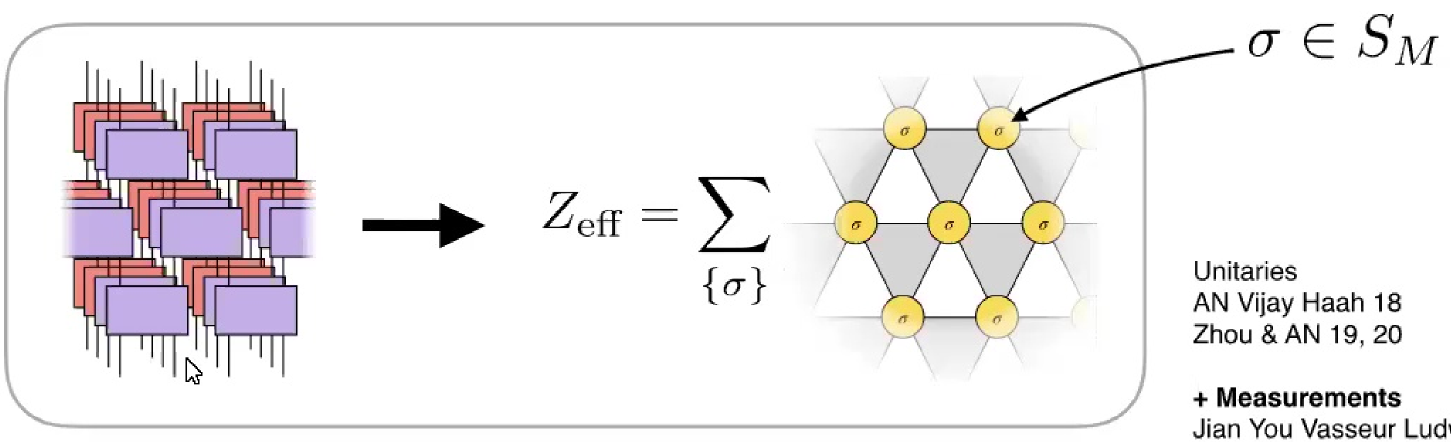
3 Field theories for measurement and entanglement transitions?

4 All-to-all-coupled measurement dynamics
& entanglement transitions on trees





2. Stat mech of entanglement: Permutations and domain walls



Unitaries
AN Vijay Haah 18
Zhou & AN 19, 20

+ Measurements
Jian You Vasseur Ludwig 20
Bao Choi Altman 20

Random tensor networks
Hayden Nezami Qi Walter Yang 16
Vasseur Potter You Ludwig 19

To describe (heuristically):

— Where effective "field" $\sigma_{x,t}$ comes from

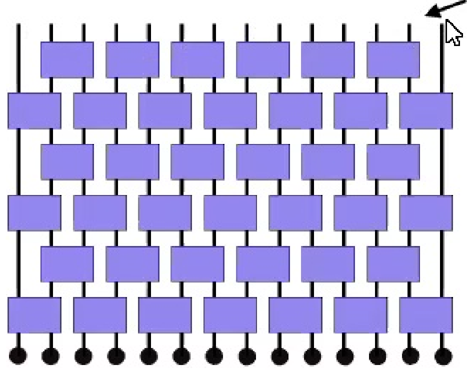
— Symmetry of the models

— What they allow us to address more easily: two phases

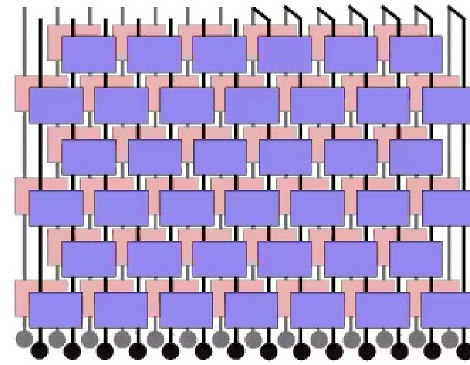
— What is hard: critical point (→ next section)

Multi-sheeted (multi-world) circuit

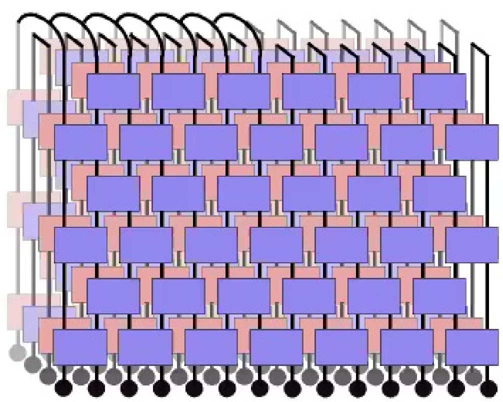
U U^*



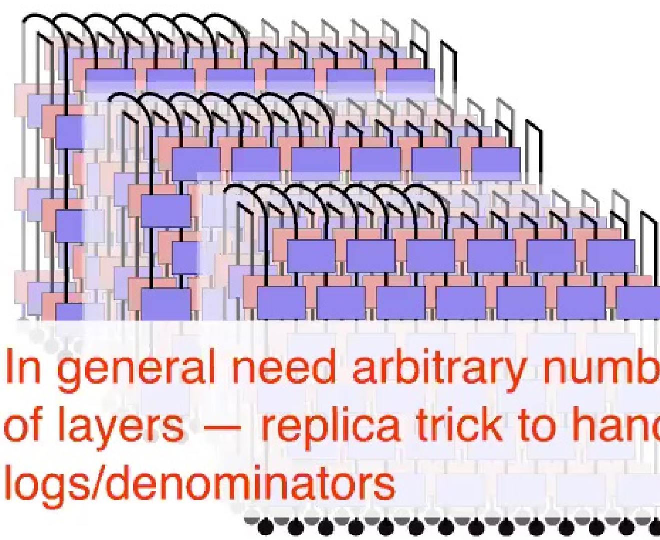
$$|\psi(t)\rangle$$



$$\rho_A(t) = \mathrm{Tr}_{\bar{A}} \, |\psi(t)\rangle \, \langle\psi(t)|$$



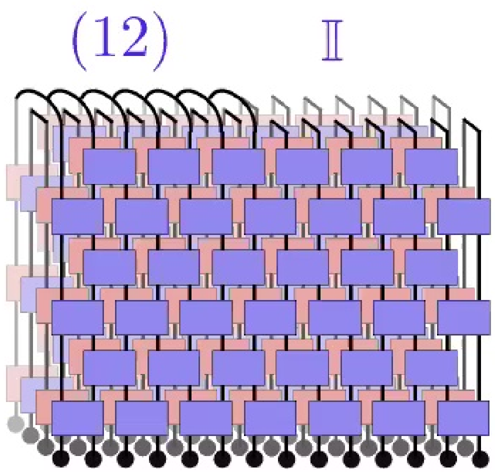
$$e^{-S_2} = \mathrm{Tr}_A \, \rho_A^2$$



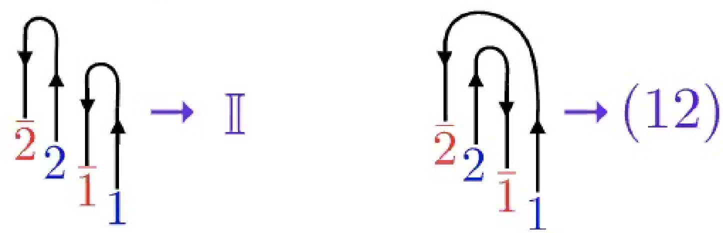
In general need arbitrary number of layers — replica trick to handle logs/denominators

Pairings

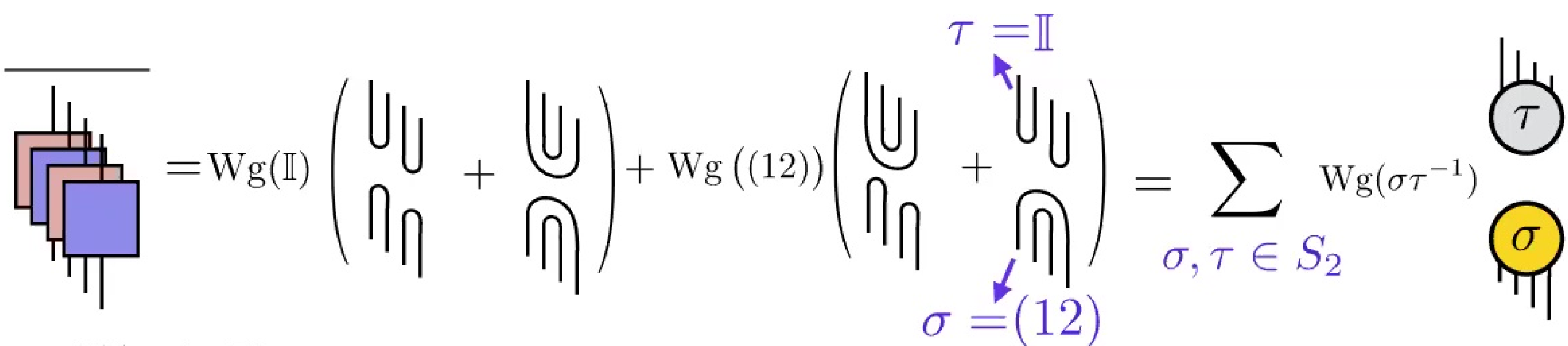
(Example: $\overline{e^{-S_2}}$ in unitary case)



Boundary condns labelled by pairings:

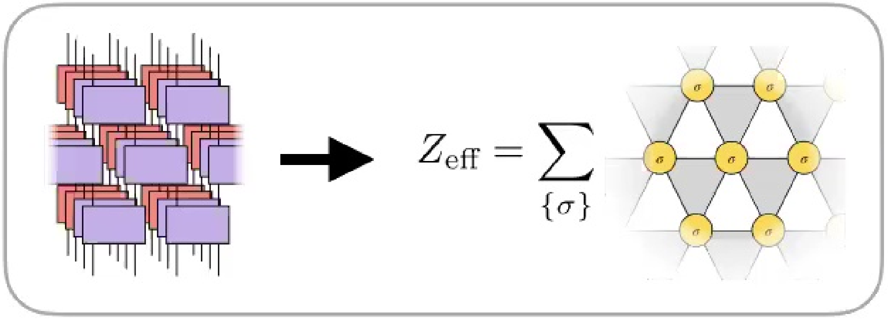


Averaging unitaries gives sums over pairings in bulk:
(a bit like Wick's thm)



Weingarten 78

Effective lattice spin models



Each unitary gives rise to a spin $\sigma_i \in S_M$

(General case: *M* layers of V and *M* layers of V*)

"Ferromagnetic" interactions on triangles dependent on measurement rate

Measurement phase transition = ordering transition (in a replica limit)

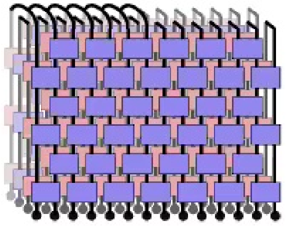
Entanglement entropies = free energies with domain wall bcs

Jian You Vasseur Ludwig 20
Bao Choi Altman 20
Zhou & AN 19
AN Vijay Haah 18

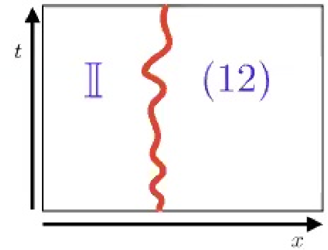
Mappings in random tensor networks
Vasseur Potter You Ludwig 19
Hayden Nezami Qi Walter Yang 16

Domain walls in ordered phase

Toy example: $\overline{e^{-S_2(A)}}$ in unitary case



$e^{-S_2} = \mathrm{Tr}_A\, \rho_A^2$



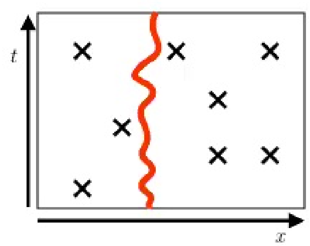
Using replica trick, extend this to $\overline{S_2(A)}$ and to include measurements

Zhou & AN 19
Vasseur Potter You Ludwig 19

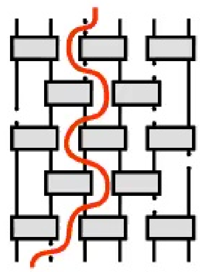
Jian You Vasseur Ludwig 20
Bao Choi Altman 20

Claim: in ordered phase, reduces at large scales to classical domain wall in a quenched random environment.

Zhou & AN 19, 20
AN, Roy, Skinnner, Ruhman 20



Very much like the min-cut at beginning; now line tension **s(r)** renormalized by fluctuations

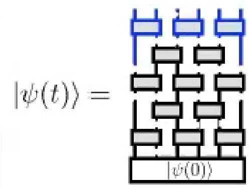


Domain wall phenomenology: survival of information

$S_{op}(t)$: crudely, characterizes deviation from unitarity   Unitary: S_{op} = N log 2   Projector: S_{op} = 0

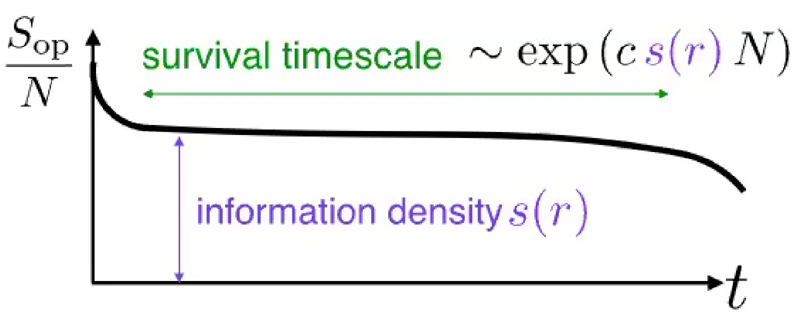
Disentangling phase: S_{op} decays on O(1) timescale

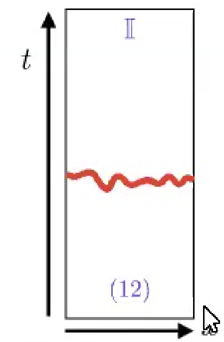
$$\frac{S_{\rm op}(t)}{N} \sim \exp(-t/\tau)$$



Gullans and Huse 19,
Choi, Bao, Qi, Altman 20
Li, Fisher, 20
Fidkowski Haah Hastings 20
AN, Roy, Skinner, Ruhman 20

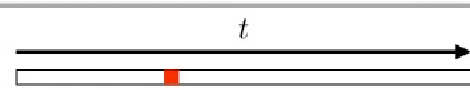
Entangling phase: extensive amount of quantum info, $s(r)\,N$, # spins
survives for **exponentially** long time





cost ~ $s(r)\,N$

Long times — effective **1D** model:



translational entropy of domain wall

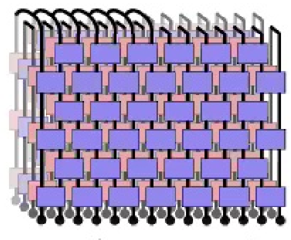
Annealed approx: $S_{\rm op}(t) = s(r)N - \log t + \ldots$ $(1 \ll \log t \ll s(r)N)$

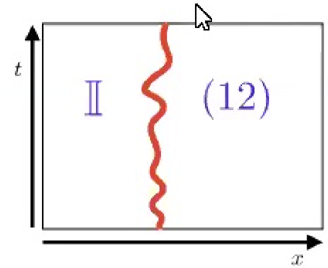
In fact pinning modifies subleading term. Very late times: multiple domain walls.

AN, Roy, Skinner, Ruhman 20
Li, Fisher, 20

Domain walls in ordered phase

Toy example: $\overline{e^{-S_2(A)}}$ in unitary case





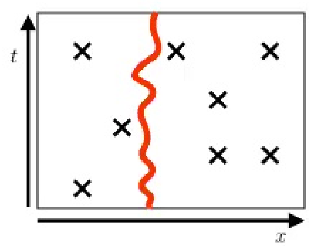
Using replica trick, extend this to $\overline{S_2(A)}$ and to include measurements

Zhou & AN 19
Vasseur Potter You Ludwig 19

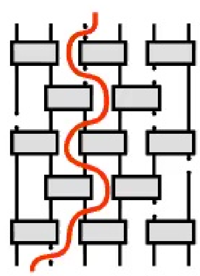
Jian You Vasseur Ludwig 20
Bao Choi Altman 20

Claim: in ordered phase, reduces at large scales to classical domain wall in a quenched random environment.

Zhou & AN 19, 20
AN, Roy, Skinnner, Ruhman 20



Very much like the min-cut at beginning; now line tension **s(r)** renormalized by fluctuations

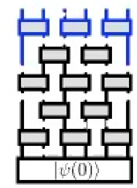


Domain wall phenomenology: survival of information

$S_{op}(t)$: crudely, characterizes deviation from unitarity   Unitary: $S_{op} = N \log 2$ Projector: $S_{op} = 0$

Disentangling phase: S_{op} decays on O(1) timescale

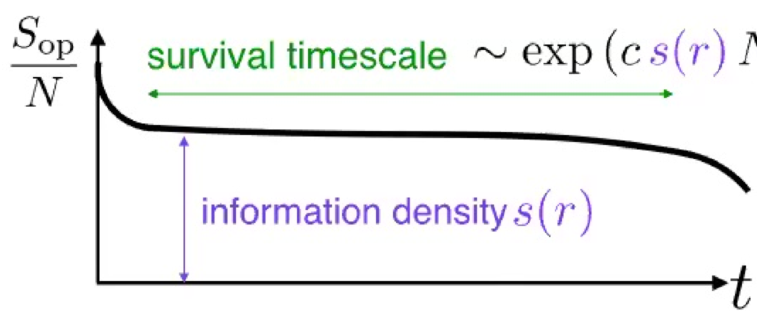
$$\frac{S_{\rm op}(t)}{N} \sim \exp(-t/\tau)$$

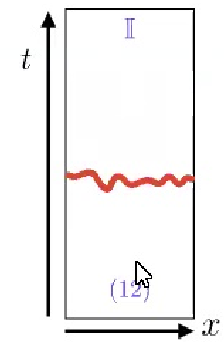
$|\psi(t)\rangle =$ 

Gullans and Huse 19,
Choi, Bao, Qi, Altman 20
Li, Fisher, 20
Fidkowski Haah Hastings 20
AN, Roy, Skinner, Ruhman 20

Entangling phase: extensive amount of quantum info, $s(r)\,N$, survives for **exponentially** long time

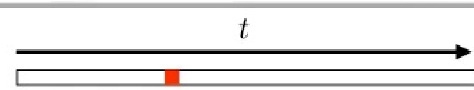
(# spins)





cost ~ $s(r)\,N$

Long times — effective **1D** model:



translational entropy of domain wall

Annealed approx: $S_{\rm op}(t) = s(r)N - \log t + \ldots$ $(1 \ll \log t \ll s(r)N)$

In fact pinning modifies subleading term. Very late times: multiple domain walls.

AN, Roy, Skinner, Ruhman 20
Li, Fisher, 20

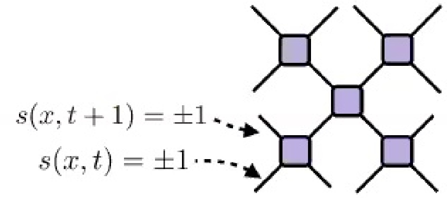
Heuristic picture of pairing field

Zhou, AN, PRX 20
Zhou, AN, PRB 19

Circuit = discrete “path integral”

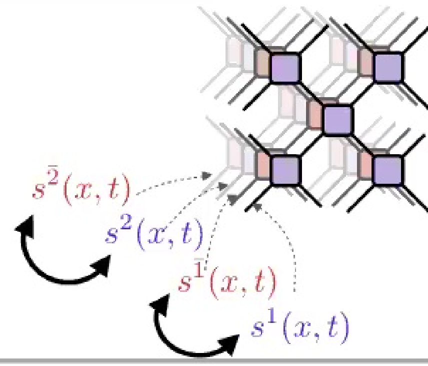
$\sum_{\text{bond indices}}$ = sum over Feynman histories

“ e^{iS} ” = product of matrix elements

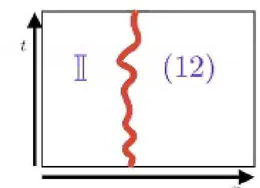


Multi-layer circuit = multi-world path integral
M “forward” and M “backward” histories

Paired configs survive phase cancellation



Effective spin **σ(x,t)** $\in S_M$ is a “pairing field” specifying the dominant pairing pattern in spacetime patch



In fact, paired configs dominate even without any randomness:
pairing field **σ(x,t)** much more general than random models

see
Zhou, AN, PRX 20

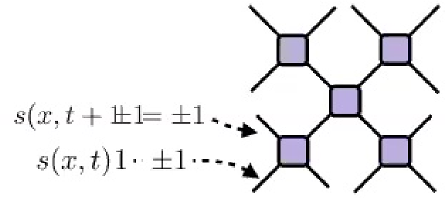
Heuristic picture of pairing field

Zhou, AN, PRX 20
Zhou, AN, PRB 19

Circuit = discrete "path integral"

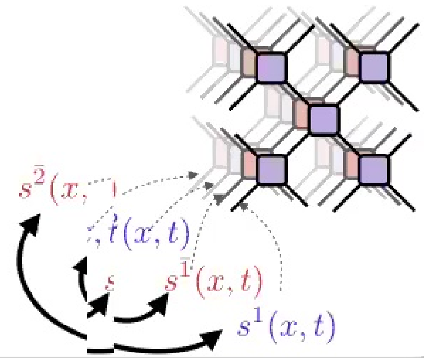
$\sum_{\text{bond indices}}$ = sum over Feynman histories

" e^{iS} " = product of matrix elements

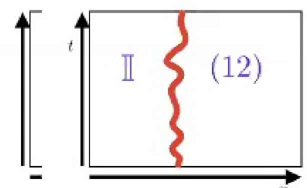


Multi-layer circuit = multi-world path integral
M "forward" and M "backward" histories

Paired configs survive phase cancellation



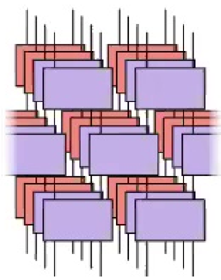
Effective spin **σ(x,t)** ∊ S_M is a "pairing field" specifying the dominant pairing pattern in spacetime patch



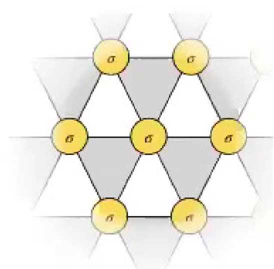
In fact, paired configs dominate even without any randomness:
pairing field **σ(x,t)** much more general than random models

see
Zhou, AN, PRX 20

3. Field theories for measurement and entanglement transitions?



$\rightarrow \quad Z_{\text{eff}} = \sum_{\{\sigma\}}$



$\rightarrow \quad \mathcal{L}(X)?$

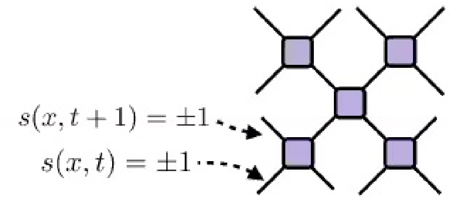
Heuristic picture of pairing field

Zhou, AN, PRX 20
Zhou, AN, PRB 19

Circuit = discrete "path integral"

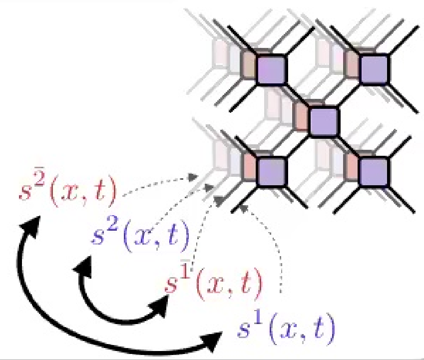
$\sum_{\text{bond indices}}$ = sum over Feynman histories

" e^{iS} " = product of matrix elements

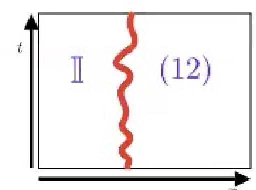


Multi-layer circuit = multi-world path integral
M "forward" and M "backward" histories

Paired configs survive phase cancellation



Effective spin **σ(x,t)** ∊ S_M is a "pairing field" specifying the dominant pairing pattern in spacetime patch



In fact, paired configs dominate even without any randomness:
pairing field **σ(x,t)** much more general than random models

see
Zhou, AN, PRX 20

Replica symmetry

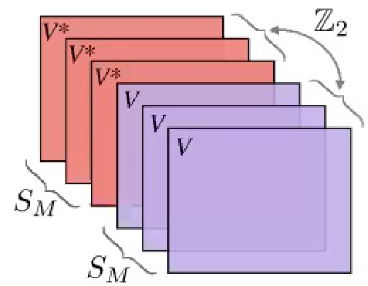
$\boldsymbol{M}$ layers each of V and V^*

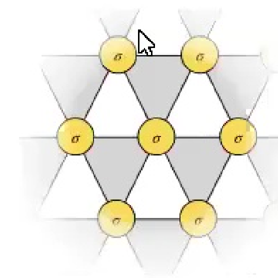
A "ferromagnet": ordering transition = measurement transition

Global symmetry

$$G_M = (S_M \times S_M) \rtimes \mathbb{Z}_2$$

$$\sigma \to g_L \sigma g_R^{-1}, \; \sigma \to \sigma^{-1}$$





Vasseur Potter You Ludwig 19
Zhou, AN 19 & 20
AN, Roy, Ruhman, Skinner 20

Two different replica limits!

$M \to 0$ forced measurements/ random tensor network

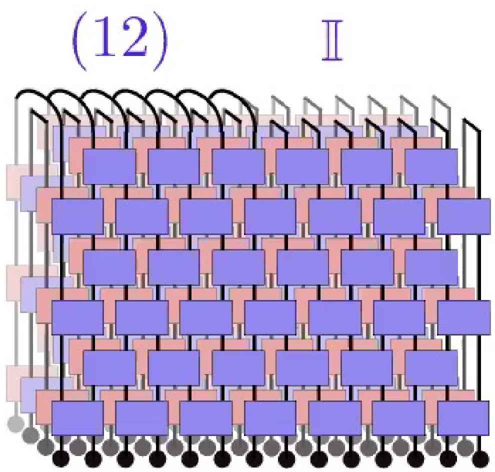
$M \to 1$ true measurements

Jian You Vasseur Ludwig 20
Bao Choi Altman 20

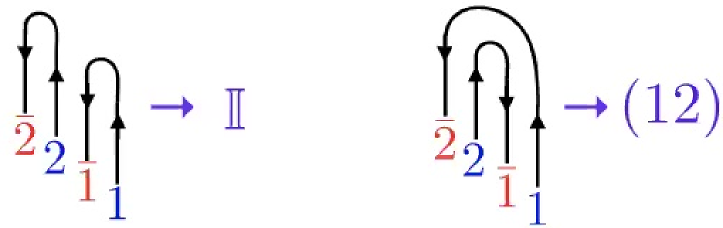
Additional factor of $P(\text{outcomes}) = \langle \psi(0)|V^\dagger V|\psi(0)\rangle$ in averages

Pairings

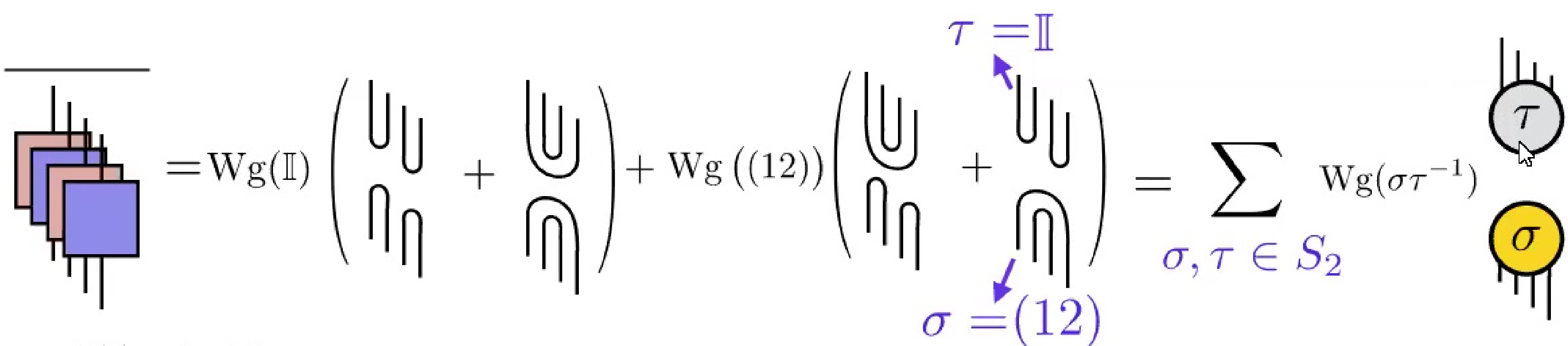
(Example: $\overline{e^{-S_2}}$ in unitary case)



Boundary condns labelled by pairings:



Averaging unitaries gives sums over pairings in bulk:
(a bit like Wick's thm)



Weingarten 78

Replica symmetry

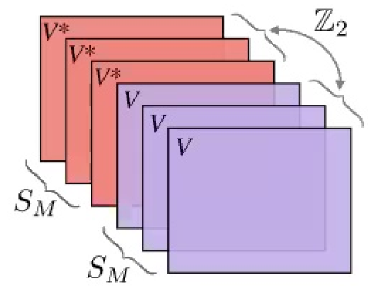
M layers each of *V* and *V**

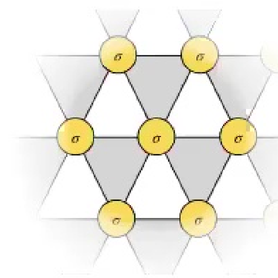
A "ferromagnet": ordering transition = measurement transition

Global symmetry

$$G_M = (S_M \times S_M) \rtimes \mathbb{Z}_2$$

$$\sigma \to g_L \sigma g_R^{-1}, \ \ \sigma \to \sigma^{-1}$$





Vasseur Potter You Ludwig 19
Zhou, AN 19 & 20
AN, Roy, Ruhman, Skinner 20

Two different replica limits!

$M \to 0$ forced measurements/ random tensor network

$M \to 1$ true measurements

Jian You Vasseur Ludwig 20
Bao Choi Altman 20

Additional factor of $P(\text{outcomes}) = \langle \psi(0) | V^\dagger V | \psi(0) \rangle$ in averages

Replica symmetry

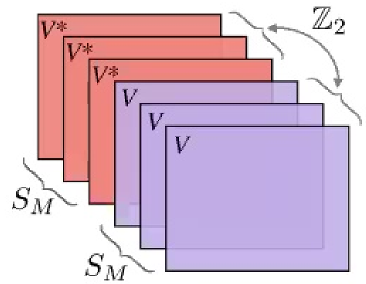
M layers each of V and V^*

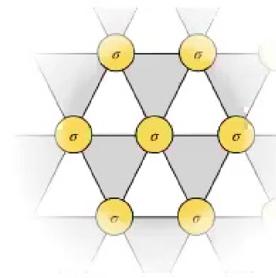
A "ferromagnet": ordering transition = measurement transition

Global symmetry

$$G_M = (S_M \times S_M) \rtimes \mathbb{Z}_2$$

$$\sigma \to g_L \sigma g_R^{-1}, \;\; \sigma \to \sigma^{-1}$$





asse
ou,
, R

asseur Potter You Ludwig 19
hou, AN 19 & 20
N, Roy, Ruhman, Skinner 20

Va
i Alt

ir

Jian ..u Vasseur Ludwig 20
Bao Choi Altman 20

Two different replica limits!

$M \to 0$ forced measurements/ random tensor network

$M \to 1$ true measurements

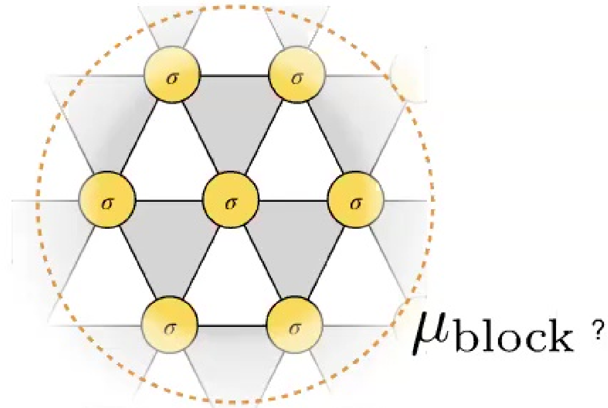
Additional factor of $P(\text{outcomes}) = \langle \psi(0) | V^\dagger V | \psi(0) \rangle$ in averages

Coarse-graining

$\sigma \in S_M$ $M \to 1$ or $M \to 0$

Can we form a "block spin" ?

$$\mu_{\text{block}} = \sum_{i \,\in\, \text{block}} \sigma_i \quad ?$$



This is equivalent to treating order parameter as a vector with $M!$ components, one for each possible value of σ as in Vasseur Potter You Ludwig 19

Problem: μ_{block} splits into arbitrarily many representations of G_M.
→ would yield Lagrangian with infinitely many fields!

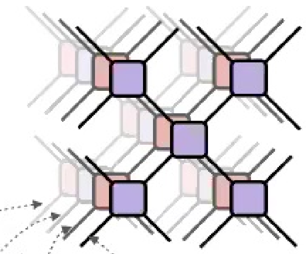
AN, Roy, Skinner, Ruhman 20

Hope: maybe only one/a few of these fields are massless at transition

Today: explore the simplest possibility

Reformulate in a different language

Alternative point of view: **overlap** between spin trajectories in forward/backward layers



Define pairing field as $X_{a,\bar{b}} \sim s^a s^{\bar{b}}$

Just an *M*x*M* matrix, rather than an abstract group element in S_M.
Analogy: Edwards-Anderson order parameter (different symmetry)

Landau theory for X? $\mathcal{L}(X)?$

– G_M symmetry $X \to M_L X M_R^T, \quad X \to X^T$

– Replica limit M→1 (measurements) or M→0 (forced measurements/RTN)

Interestingly, simplest candidates are different for these two limits

Replica group theory

$X_{a,\bar{b}} \sim S^a S^{\bar{b}}$ *M x M* matrix

***M→1* limit:** candidate Lagrangian for **measurement transition**

$$\mathcal{L} = \sum_{ab} \left[(\partial_t X_{ab})^2 + (\nabla X_{ab})^2 + (r - r_c) X_{ab}^2 + X_{ab}^3 \right]$$

$$\sum_a X_{ab} = 0 \qquad \sum_b X_{ab} = 0$$

- A single rep of ***G***$_M$;
- Upper critical dimension 6 $\quad s(r) \sim (r_c - r)^{5/2}$ above UCD

***M→0* limit:** candidate Lagrangian for random tensor networks and for **forced measurement transition**

$$\mathcal{L} = \sum_{ab} \left[(\partial_t X_{ab})^2 + (\nabla X_{ab})^2 + (r - r_c) X_{ab} + X_{ab}^3 \right] + \sum_{abcd} X_{ab} F_{ab,cd} X_{cd}$$

$$F_{ab,cd} = \delta_{ac} + \delta_{bd}$$

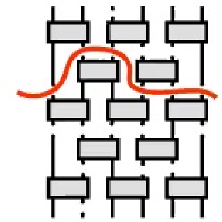
• 3 reps of $\boldsymbol{G_M}$ become simultaneously massless; • UCD = 10

these Lagrangians are speculative (work in progress…)

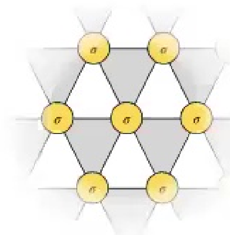
AN, Roy, Skinner, Ruhman 20

Plan

1 Measurement phase transition:
A “percolation” transition for quantum info in spacetime

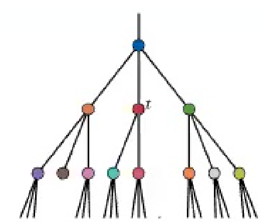


2 Stat mech of entanglement:
Permutations and domain walls

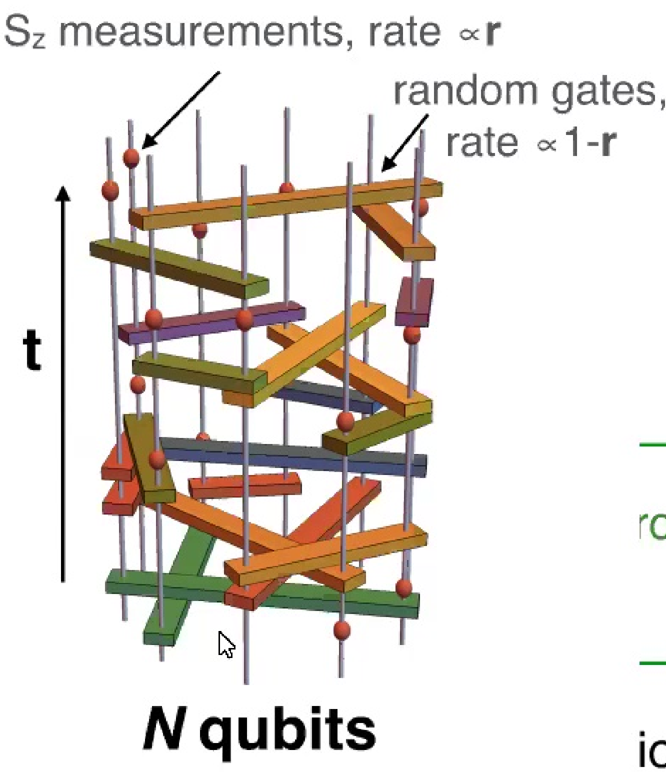


3 Field theories for measurement and entanglement transitions?

4 All-to-all-coupled measurement dynamics
& entanglement transitions on trees

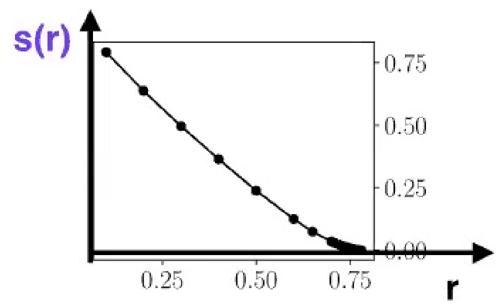


4. All-to-all measurement circuits & trees



Classical limit (large bond dim.):
a solvable percolation model:

$$s(r) \sim (r_c - r)^{5/2} \qquad r_c = 4/5$$



rom

ion:

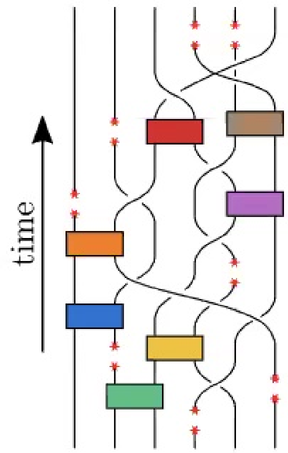
Can we obtain exact results away from the classical limit, e.g. for spin-1/2?

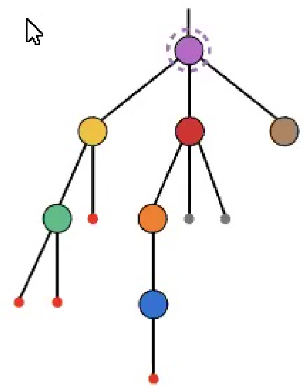
Classical problem is solved by field theory mapping
AN, Roy, Skinner, Ruhman 20
Different approach: Gullans, Huse, unpublished

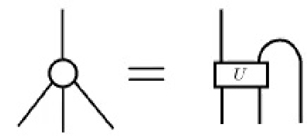
Exploit lesson from classical solution:
use local tree structure

Tree structure

Exploit tree structure of N→∞ limit (loops have size log N → ∞)







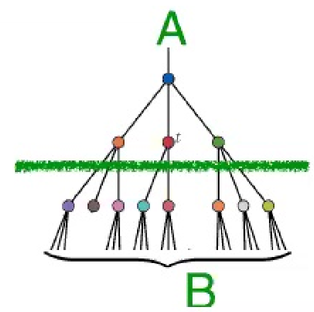
Classical percolation transition separates finite from infinite trees.
Compute average branching number → critical frac. of missing bonds = 2/3

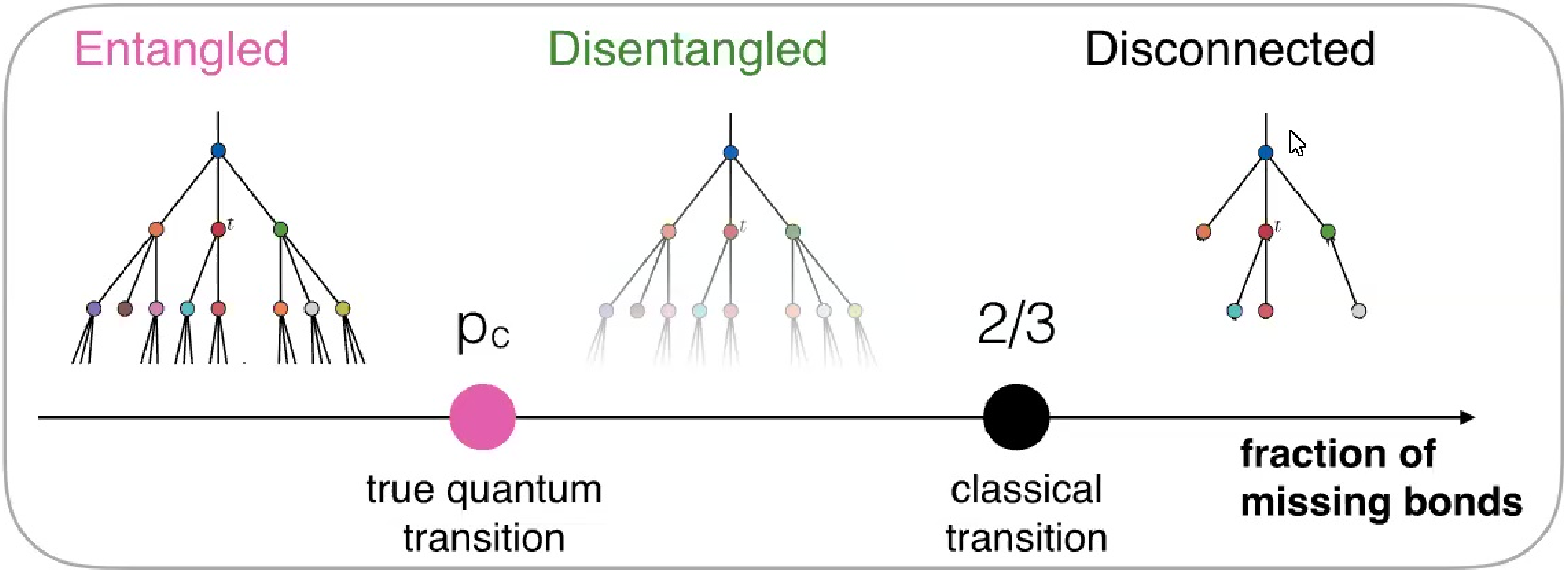
Quantum transition for spin-1/2 defined by “quantum” connectivity of tree (conjecture)

Study case with forced (postselected) measurements

Quantum tree

Is root of large tree entangled with leaves?



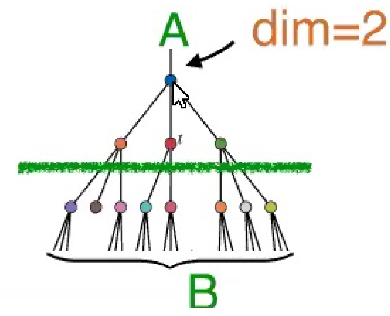


Exact results for genuine quantum transition
Direct method (no replicas)
Generalises to other random tree tensor network states

AN, Roy, Skinner, Ruhman 20

Study of a different random tree tensor network (numerics & replica approach):
Lopez-Piqueres, Ware and Vasseur, '20

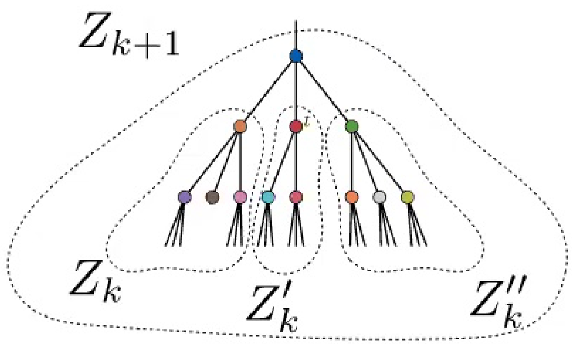
Recursive construction



Characterize tree by singular values $\left\{\lambda_{\min},\ \sqrt{1-\lambda_{\min}^2}\right\}$

Define $Z = \lambda_{\min}^2$ Close to transition: $Z \sim S_2^{\text{tree}} \ll 1$

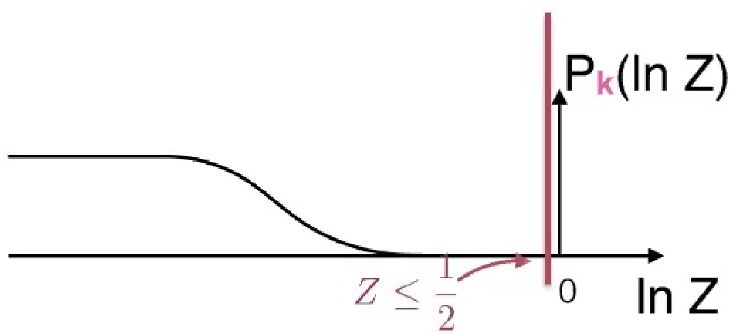
Z obeys random recursion relation:



$$Z_{k+1} = A_1 Z_k + A_2 Z'_k + A_3 Z''_k - \mathcal{O}(Z^2)$$

A_i are rational functions of **random** node tensor elements,
Or **zero** if the branch is missing.

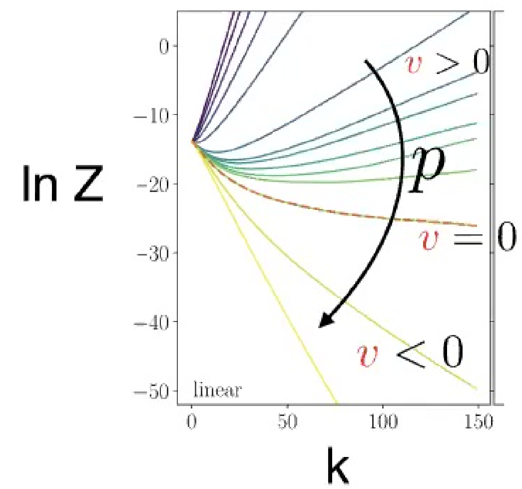
Recursively defines
cumulative probability $P_k(\ln Z)$:



Linear vs nonlinear problem

Linearized problem: $Z_{k+1} = A_1 Z_k + A_2 Z_k' + A_3 Z_k''$

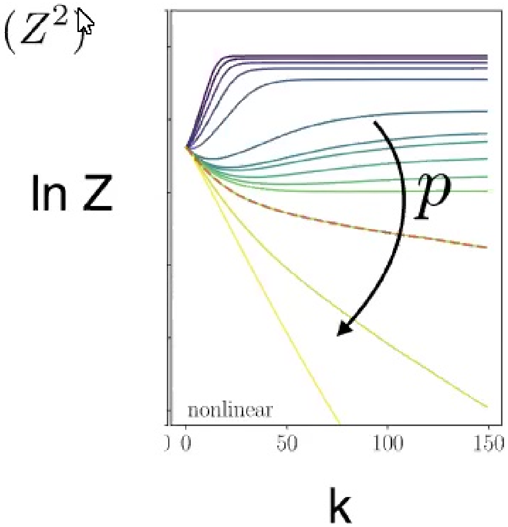
Exponential growth/decay depending on p: $Z_k^{\text{typ}} \sim e^{vk}$



Nonlinear problem: $Z_{k+1} = A_1 Z_k + A_2 Z_k' + A_3 Z_k'' - \mathcal{O}(Z^2)$

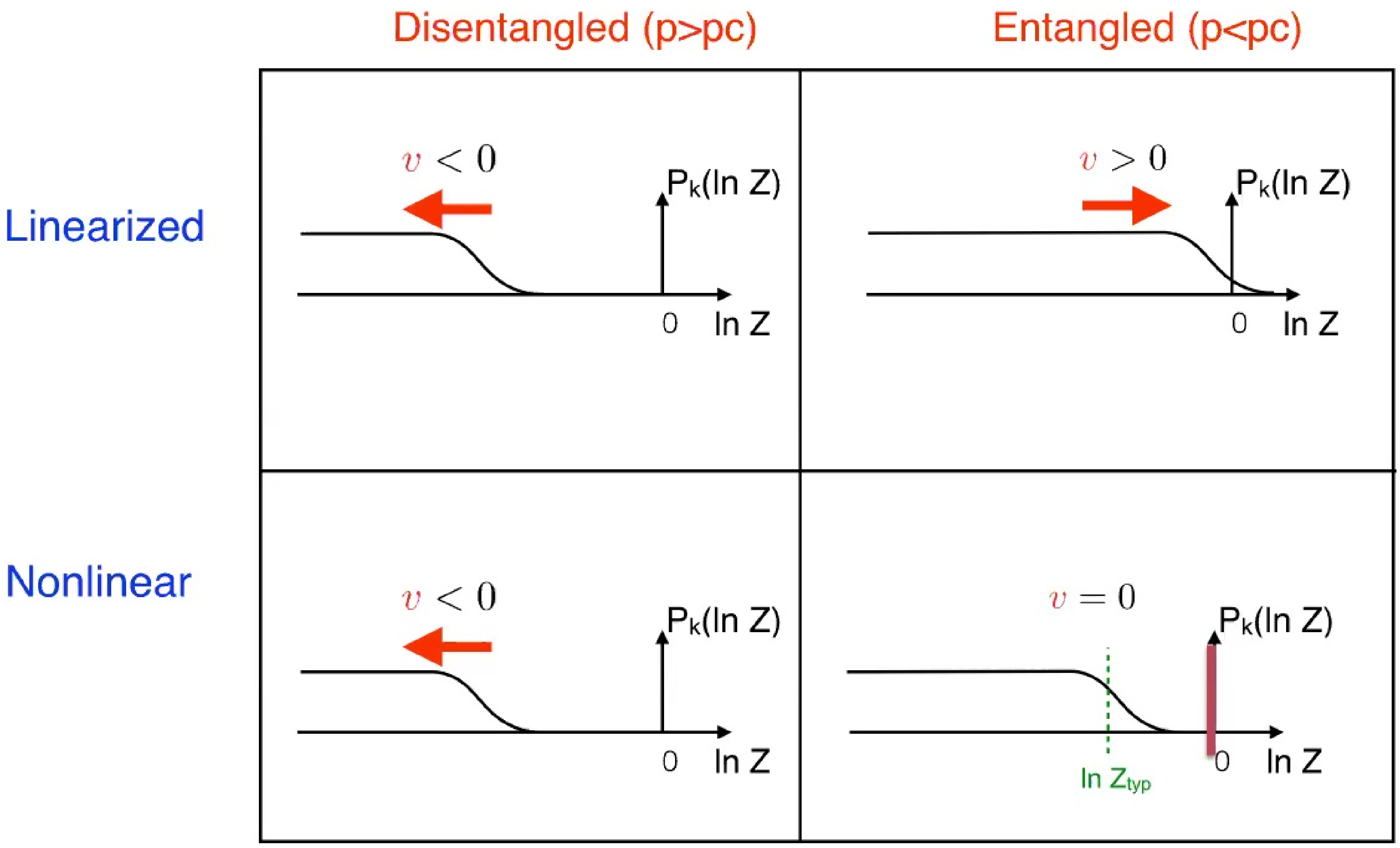
Regime with exponentially decay unaffected
→ disentangled phase $S_{\text{tree}} \sim Z^{\text{typ}} \to 0$

Regime w/ growth replaced w/ saturation
→ entangled phase



Linearized problem captures p_c. Need nonlinearity for scaling when $p<p_c$.

Linear vs nonlinear problem



Linearized problem captures p_c. Need nonlinearity for scaling when $p<p_c$.

Traveling wave

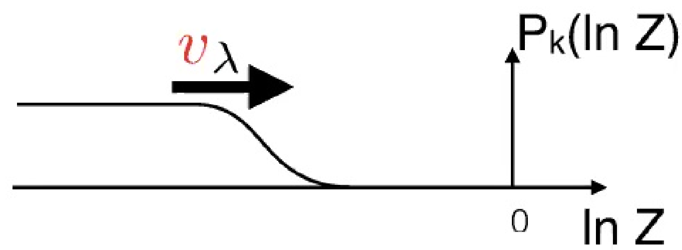
Beautiful connection with mathematics of travelling waves e.g. Fisher-KPP

Derrida, Spohn 88

Linearized problem $Z_{k+1} = A_1 Z_k + A_2 Z_k' + A_3 Z_k''$

$$P_k(\ln Z) = F_\lambda(\ln Z - v_\lambda k)$$

$\ln Z \sim$ space, $k \sim$ time



Many traveling wave solns with different wavefronts!

$$F_\lambda(x) \sim e^{-\lambda x}$$

(have simplified - should consider generating fn)

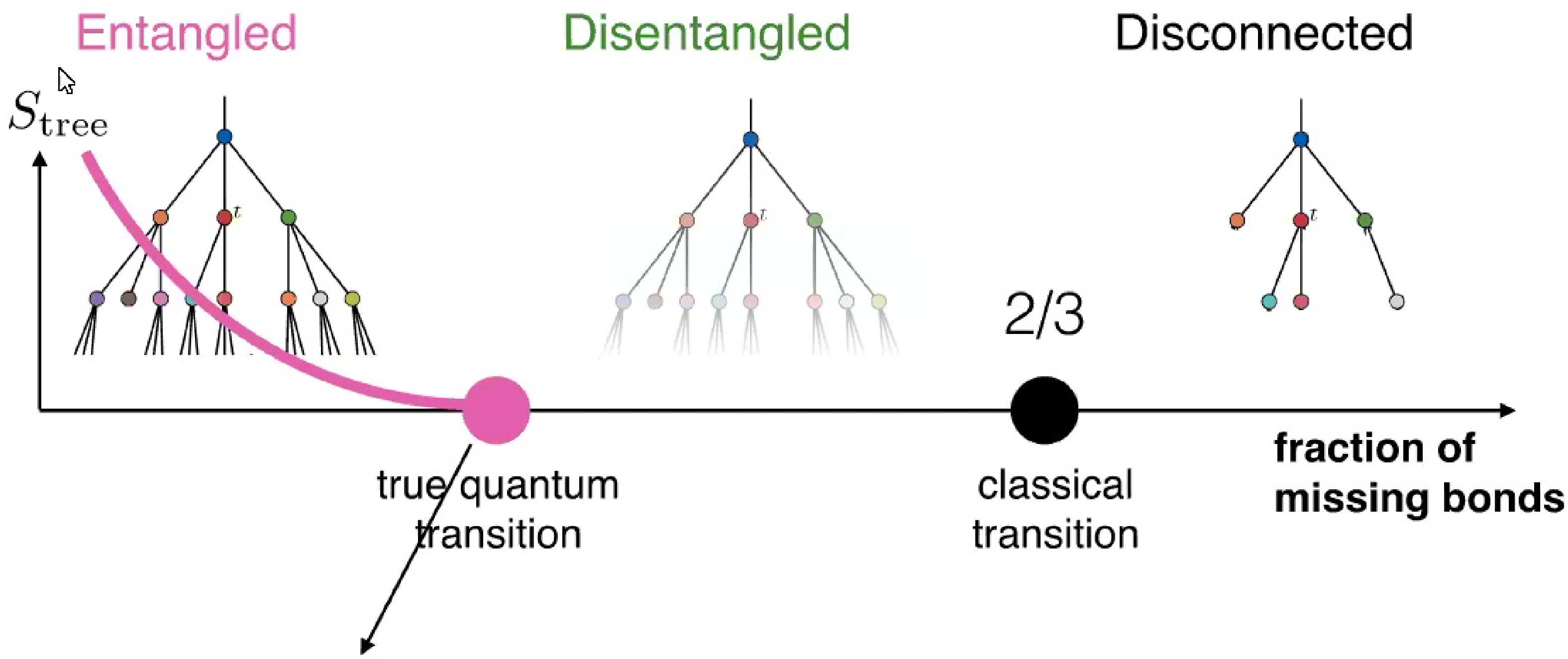
The stable solution is the slowest:

$$Z \sim e^{v_* k}$$

disentangled phase

entangled phase

Quantum tree

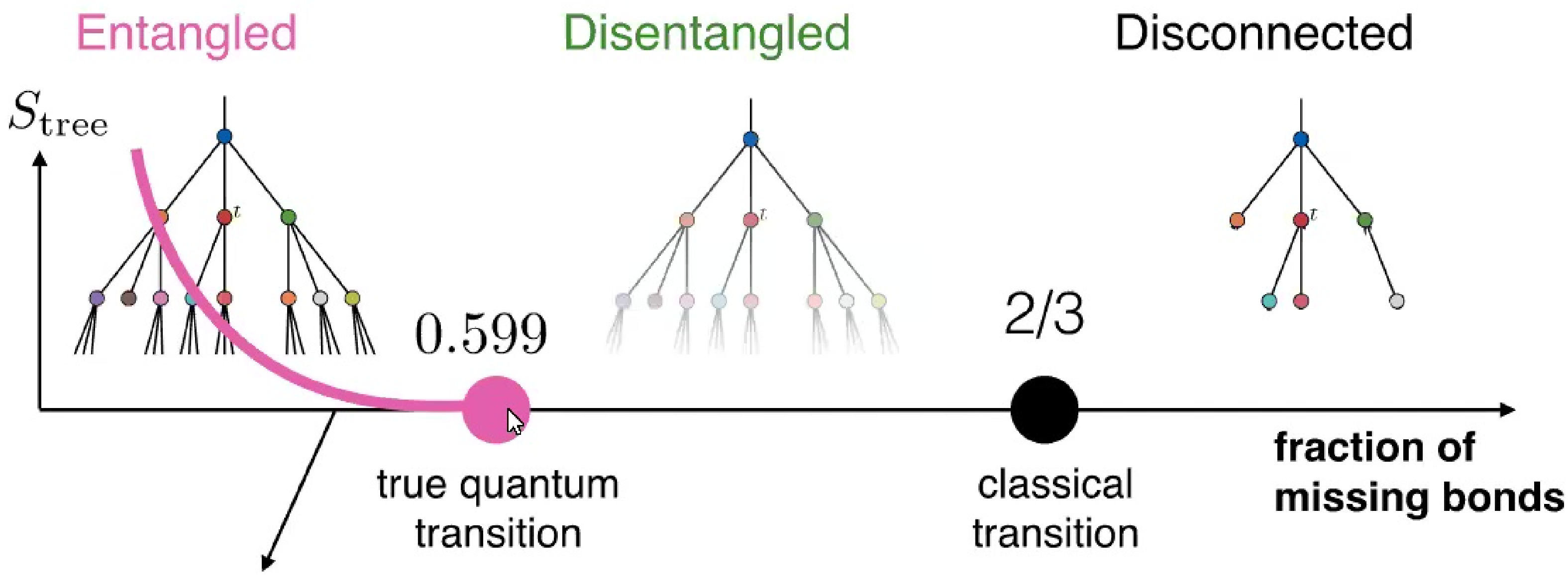


$$p_c = \frac{212 + 75\pi}{512 + 75\pi} \quad (\simeq 0.599)$$

Next: critical scaling of S_{tree} ($\sim Z$) in entangled phase?

Quantum tree

AN, Roy, Skinner, Ruhman 20
for details



$$S_{\text{tree}} \sim \exp\left(-\frac{C}{\sqrt{r_c - r}}\right)$$

Surprisingly rapid critical scaling
Contrast “mean-field-like” classical tree

- Exact results for tree and for all-to-all circuit
- Scaling qualitatively different from classical limit
- Suggests that, in high dimensions, field theories from earlier part of talk are not the whole story

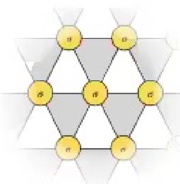
Summary

arXiv:2009.11311

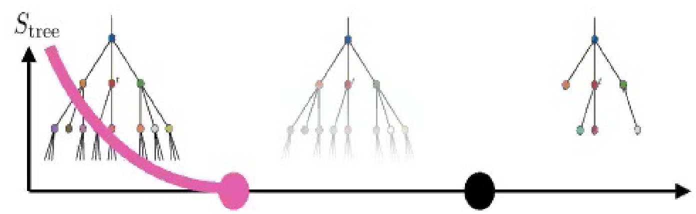
Measurement transition:
new universality class(es) for “percolation” of quantum information

Replica approach — candidate field theories

$$Z_{\text{eff}} = \sum_{\{\sigma\}}$$



Circuits/tensor networks with local tree structure → random recursion relations

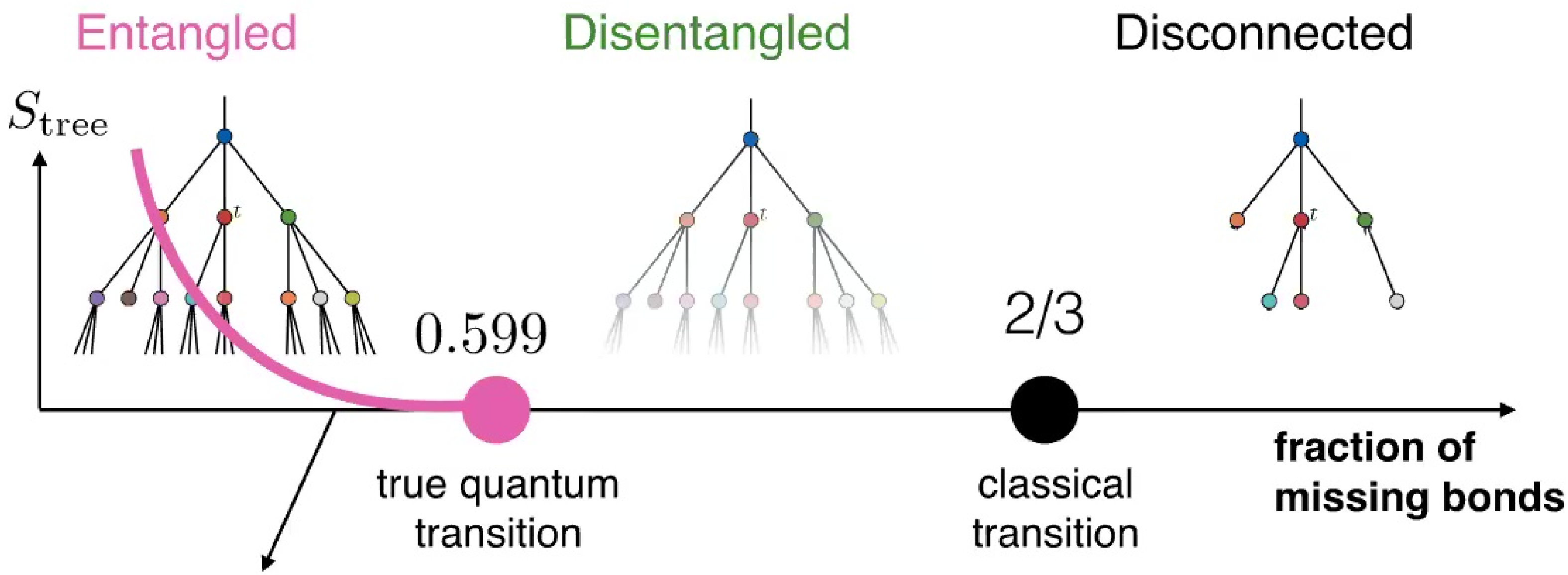


Many exciting directions for future

Different monitoring protocols
Role of symmetries
Special gate sets
Free fermions
…

Quantum tree

AN, Roy, Skinner, Ruhman 20
for details



$$S_{\text{tree}} \sim \exp\left(-\frac{C}{\sqrt{r_c - r}}\right)$$

Surprisingly rapid critical scaling
Contrast "mean-field-like" classical tree

- Exact results for tree and for all-to-all circuit
- Scaling qualitatively different from classical limit
- Suggests that, in high dimensions, field theories from earlier part of talk are not the whole story