Title: The holographic map as a conditional expectation

Speakers: Thomas Faulkner

Series: Quantum Fields and Strings

Date: November 24, 2020 - 2:00 PM

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Abstract: I will study quantum error correcting codes that model aspects of the AdS/CFT correspondence. In an algebraic approach I will demonstrate the existence of a consistent assignment, to each boundary region, of conditional expectations that preserve the code subspace. This allows us to give simple derivations of well known results for these holographic code, and also to derive a few new results.

I will also make a connection to the theory of QFT super-selection sectors.

9:41 AM Tue Jan 9

Conditional Talk

Lec19 Lec20 Lec21 Lec23 Lec24 Lec22 Lec25 Conditional Talk

1

The Holographic Map as a Conditional Expectation

2008.04810

Main take away:

Concrete:

- Quantum Error correcting codes that model AdS/CFT naturally give rise to conditional expectations: well studied mathematical structure in operator algebras

- Strengthens connections to superselection

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- Quantum Error correcting codes that model AdS/CFT naturally give rise to conditional expectations: well studied mathematical structure in operator algebras

- Strengthens connections to superselection sectors elucidated by Casini, Huerta, Magan, Pontello

Speculative:

AdS/CFT naturally give rise to conditional expectations: well studied mathematical structure in operator algebras

- Strengthens connections to superselection sectors elucidated by Casini, Huerta, Magan, Pontello

Speculative:

- Study multiple boundary regions
- Exact recovery (studied here) has short-comings that are (hopefully) not fatal

9:41 AM Tue Jan 9

Conditional Talk

- Exact recovery (studied here) has short-comings that are (hopefully) not fatal
- give a picture of entanglement wedge phase transitions.

(2)

A model of entanglement wedge reconstruction (EWR)

Will call "complementary recovery"

Dong, Harlow, Wall; Harlow; Kang-Kolchmeyer

(∞-dims)

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②

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9:41 AM Tue Jan 9

Conditional Talk

Lec19 Lec20 Lec21 Lec23 Lec24 Lec22 Lec25 Conditional Talk

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K: physical Hilbert space. Boundary CFT Hilbert space

$\mathcal{H}$: code subspace. Bulk gravitational theory / EFT in AdS.

[not clear exactly how to define, but the spirit of QEC approach is that the details

9:41 AM Tue Jan 9

Conditional Talk

Lec19 Lec20 Lec21 Lec23 Lec24 Lec22 Lec25 Conditional Talk

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9:41 AM Tue Jan 9

Conditional Talk

Lec19 Lec20 Lec21 Lec23 Lec24 Lec22 Lec25 Conditional Talk

[not clear exactly how to define, but the spirit of QEC approach is that the details should not matter. More later.]

$V: \mathcal{H} \to K$ is an isometric embedding

$V^\dagger V = \mathbb{1}_{\mathcal{H}}$ $\quad VV^\dagger = e$

e: code subspace projector on K.

(3)

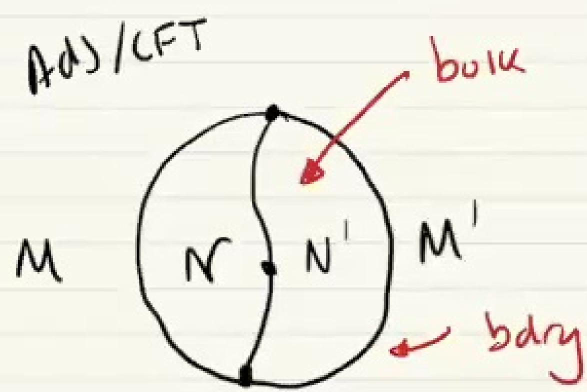
EWR: how does this holographic map restrict

e: code subspace projector on $\mathcal{K}$.

(3)

EWR: how does this holographic map restrict to local algebras on $\mathcal{H}$ & $\mathcal{K}$?

Picture:



t=0 slice

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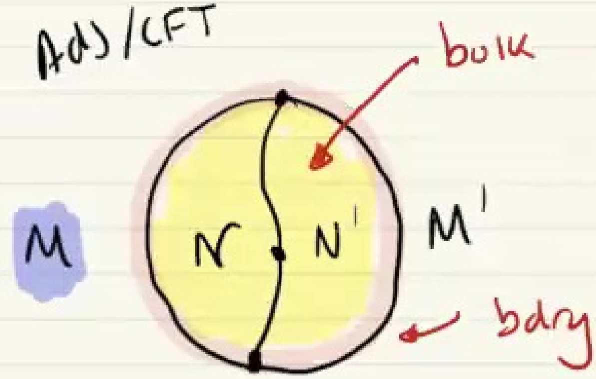

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• M: bdry algebra say  assume

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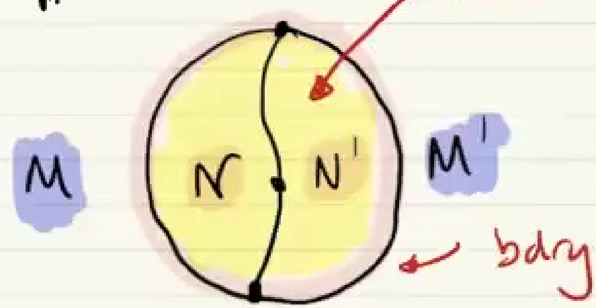

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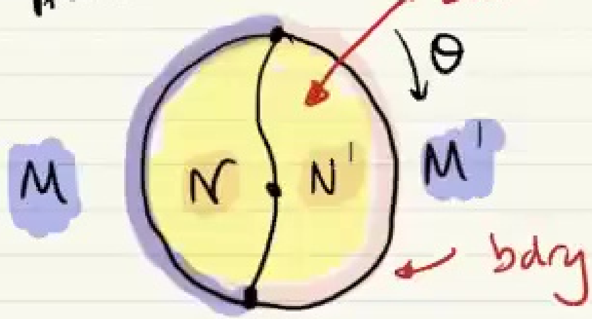

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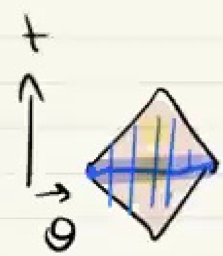

assume it is a vN algebra. $M \subset B(K)$

- N: bulk algebra associated to entanglement wedge. Also assume it is a vN algebra

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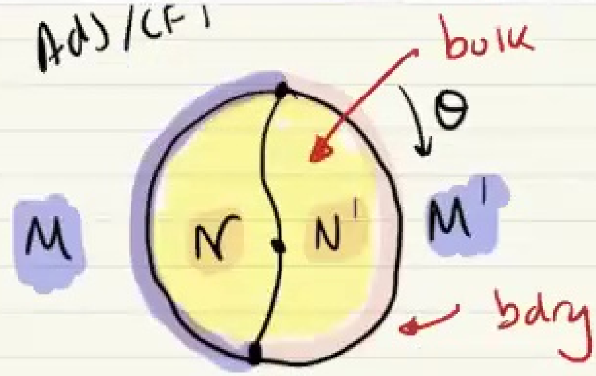

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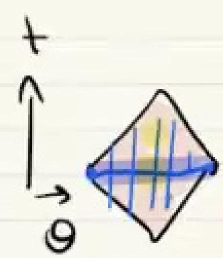

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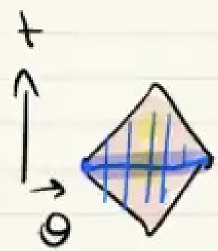

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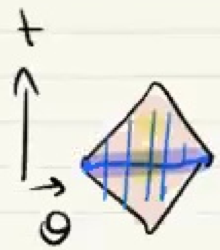
• M: bdry algebra say 

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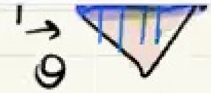
$$M' = \{ m' \in B(\mathcal{H}) : [M, M] = 0 \}$$



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4

- New developments in QI approach to AdS/CFT it is a vN algebra. $M \subset B(\mathcal{K})$

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④

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(4)

- Now developments in QI approach to AdS/CFT

RT → FLM → JLMS allowed for a

precise characterization of how N & M

are related

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(4)

- Now developments in QI approach to AdS/CFT
  RT $\rightarrow$ FLM $\rightarrow$ JLMS allowed for a precise characterization of how N & M are related

Def: • $\underline{\text{DHW}}$ condition: $\underline{\forall}$ $\psi_1, \psi_2 \in \mathcal{H}$ then:

$$\underline{\omega_{\psi_1}|_{N'} = \omega_{\psi_2}|_{N'}} \Rightarrow \underline{\omega_{V\psi_1}|_{M'} = \omega_{V\psi_2}|_{M'}}$$

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Will say "N is reconstructable from M"

Comments: (i) $\omega_\psi(\cdot) \equiv \langle\psi|\cdot|\psi\rangle$ is a positive linear functional ~ "State"

algebraic version of density matrix

ω_ψ is global defined on $B(\mathcal{H}) \rightarrow$ all correlation functions for $|\psi\rangle$

9:41 AM Tue Jan 9

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$\omega_\psi|_{N'}$ means restrict to $N' \subset B(\mathcal{H})$

Algebraic version of a reduced dens. matrix

9:41 AM Tue Jan 9

Conditional Talk

Lec19 Lec20 Lec21 Lec23 Lec24 Lec22 Lec25 Conditional Talk

are related

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(5)

(ii) physically: information about $\mathcal{N}$ (via differences in the state $\omega_{t_1}|_{\mathcal{N}} \neq \omega_{t_2}|_{\mathcal{N}}$) cannot be detected on $\underline{\mathcal{M}'}$. A version of privacy

(iii) often stated as:

$$S_{rel}(\psi_1 | \psi_2 ; \mathcal{N}') = 0 \implies S_{rel}(V\psi_1 | V\psi_2 ; \mathcal{M}') = 0$$

derived in AdS/CFT from JLMS

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9:41 AM Tue Jan 9

Conditional Talk

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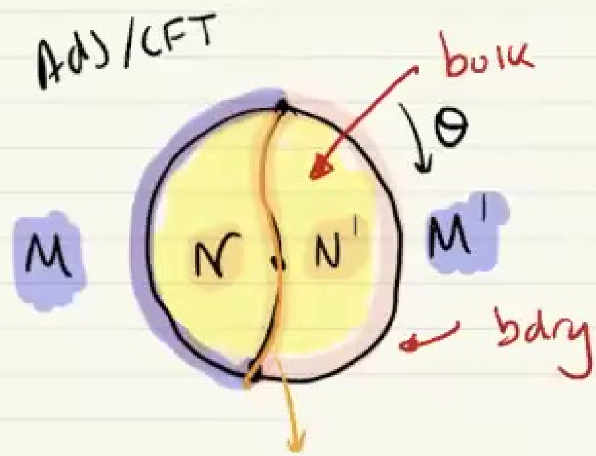

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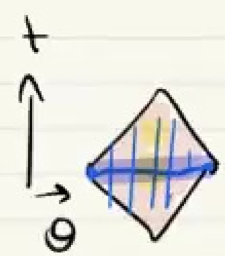

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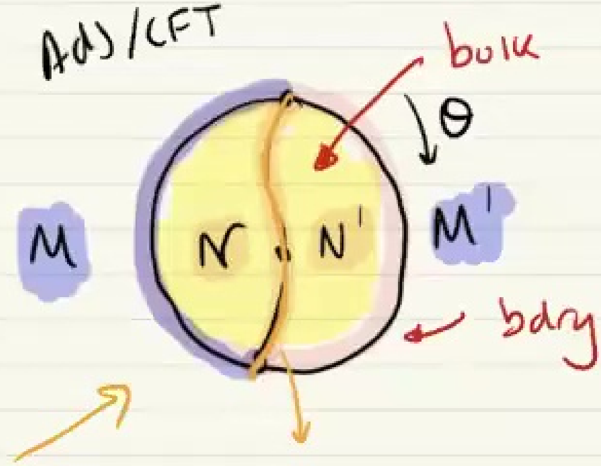

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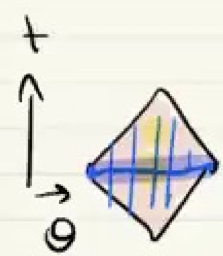

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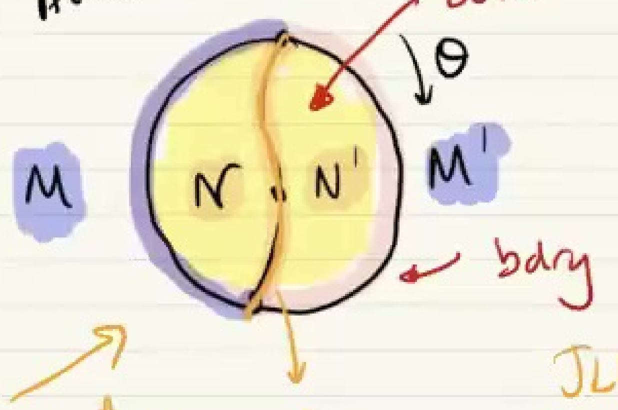

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JLMS

$$S_M = \frac{\text{Area}}{4G_N} + S_N \qquad S_{rel}(\psi_1$$

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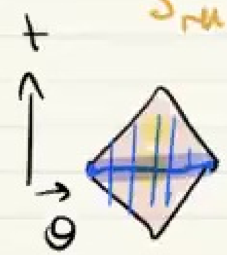

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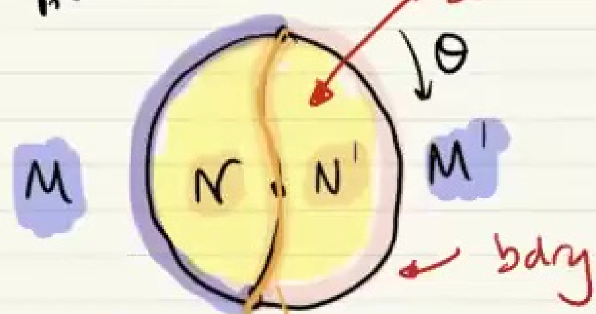

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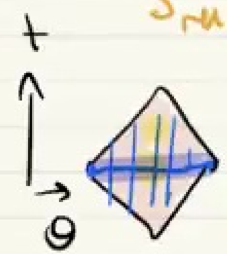

assume

9:41 AM Tue Jan 9

Conditional Talk

Lec19 Lec20 Lec21 Lec23 Lec24 Lec22 Lec25 Conditional Talk

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$\omega_\psi|_{N'}$ means restrict to $N' \subset B(\mathcal{H})$

(6)

Complementary Recovery:

Def: "N is c-reconstructable from M if"

a) N is reconstructable from M

b) N' is reconstructable from M'

To understand this structure we give a well known theorem (Accardi, Cecchini; DHW & Harlow; Kang-Kolchmeyer)

Main theorem

The following are equivalent:

9:41 AM Tue Jan 9

Conditional Talk

Lec18 Lec19 Lec20 Lec21 Lec23 Lec24 Lec22 Lec25 Conditional Talk

$$S_{rel}(\psi_1|\psi_2;\mathcal{N}') = 0 \Rightarrow S_{rel}(V\psi_1|V\psi_2;\mathcal{M}') = 0$$

derived in AdS/CFT from JLMS

(iv) correct up to non-perturbative errors in G_N, although only for "reconstructable wedge" & not "entanglement wedge": these are only different for very large code subspaces or near EW transitions

We will assume EXACT

9:41 AM Tue Jan 9

Conditional Talk

Lec19 Lec20 Lec21 Lec23 Lec24 Lec22 Lec25 Conditional Talk

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$\frac{1}{G_N}$

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9:41 AM Tue Jan 9

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b) N' is reconstructable from M'

To understand this structure we give a well known theorem (Accardi, Cecchini; DHW & Harlow; Kang-Kolchmeyer)

Main Theorem

The following are equivalent:

(i) DHW $(\omega_{\psi_1} - \omega_{\psi_2})|_{N'} = 0 \Rightarrow (\omega_{V\psi_1} - \omega_{V\psi_2})|_{M'} = 0$

$\forall \; \psi_1, \psi_2 \in \mathcal{K}$

(ii) $\left(\alpha_t : \; B(\mathcal{K}) \to B(\mathcal{H}) \quad \alpha_t = V^\dagger (\cdot) V \right)$

(bdry) (bulk)

restrict to:

9:41 AM Tue Jan 9

Conditional Talk

Lec19 Lec20 Lec21 Lec23 Lec24 Lec22 Lec25 Conditional Talk

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restricts to:

$\alpha_t|_{M'} = \alpha' \qquad \alpha': M' \to N'$

9:41 AM Tue Jan 9

Conditional Talk

Lec19 Lec20 Lec21 Lec23 Lec24 Lec22 Lec25 Conditional Talk

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(ii) $\left(\alpha_t : B(\mathcal{K}) \to B(\mathcal{H}) \quad \alpha_t \equiv V^{\dagger}(\cdot)V \right)$

(bdry) (bulk)

restricts to:

$\alpha_t|_{M'} \equiv \alpha' \qquad \alpha' : M' \to N'$

$(\alpha'$ is faithful$)$

9:41 AM Tue Jan 9

Conditional Talk

Lec18 Lec19 Lec20 Lec21 Lec23 Lec24 Lec22 Lec25 Conditional Talk

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(ii) $\Big(\alpha_b : \underset{(\text{bdry})}{B(\mathcal{K})} \to \underset{(\text{bulk})}{B(\mathcal{H})} \qquad \alpha_b = V^\dagger (M') V \Big)$

restricts to:

$\alpha_b|_{M'} = \alpha' \qquad \boxed{\alpha' : M' \to N'}$

(α' is faithful) (injective)

(iii) $\exists \; \beta : N \to M$ a 1 $*$-homomorphism

7

9:41 AM Tue Jan 9

Conditional Talk

Lec18 Lec19 Lec20 Lec21 Lec23 Lec24 Lec22 Lec25 Conditional Talk

Heisenberg Picture

(ii) $\left(\alpha_t : B(K) \to B(H) \quad \alpha_t = V^\dagger (m') V \right)$

(bdry) (bulk)

restricts to:

$\alpha_t|_{M'} = \alpha'$ $\quad \alpha' : M' \to N'$

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(injective) ⑦

(iii) $\exists$ β $N \to M$ a (*-homomorphism

$\beta(n_1 n_2) = \beta(n_1)\beta(n_2)$

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$$\boxed{\beta(n) V = V n}$$

Comments:

- $\alpha' \leftrightarrow \beta$ called the dual map. it is a version of the Petz map (without J's)
- α' & β are quantum channels in the Heisenberg picture (completely positive normal)

(unital $\beta(1) = 1$) & such that

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Comments:

- $\alpha' \leftrightarrow \beta$ called the dual map. it is a version of the Petz map (without J's)
- α' & β are quantum channels in the Heisenberg picture (completely positive, normal)
- Explains the name "reconstructable"

9:41 AM Tue Jan 9

Conditional Talk

s.t $\beta(n)V|\psi\rangle = Vn|\psi\rangle$ so acts

8

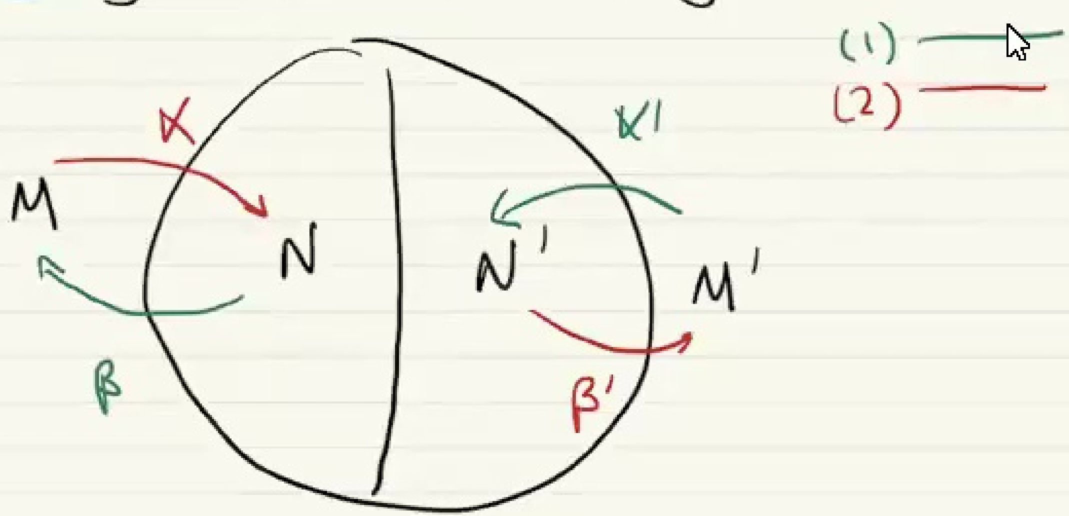
correctly on the code subspace.

- we have stated a version which requires a single cyclic & sep. vector $V|\psi\rangle$ for $\mathcal{M}$
- $\beta(N) \cong N^{\beta}$ is a vN subalgebra of $\mathcal{M}$ b/c of the homomorphism property.

a single cyclic & sep. vector $|\Psi\rangle$ for $\mathcal{M}$

- $\beta(\mathcal{N}) \equiv \mathcal{N}^{\beta}$ is a vN subalgebra of $\mathcal{M}$ b/c of the homomorphism property.

Complementary recovery: apply theorem twice:



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Conditional Talk

Lec18 Lec19 Lec20 Lec21 Lec23 Lec24 Lec22 Lec25 Conditional Talk

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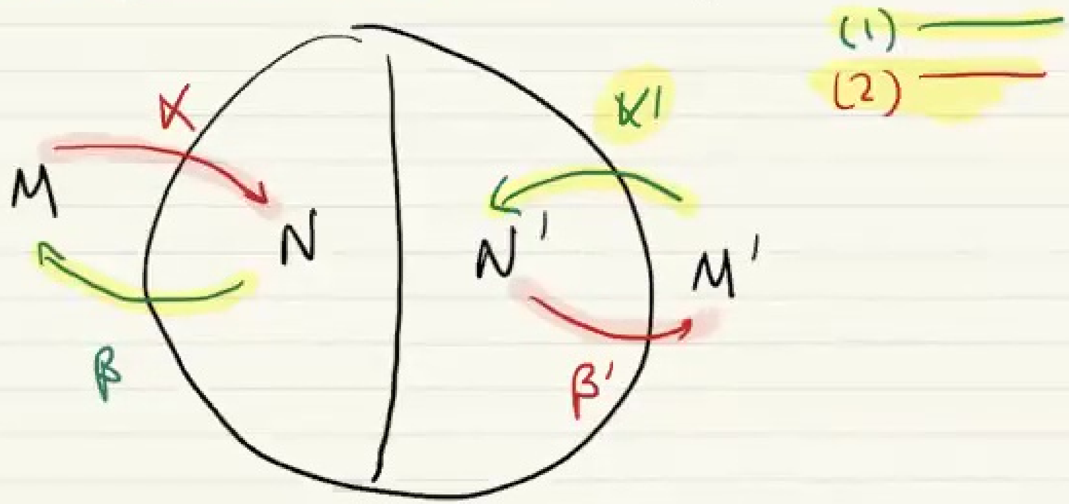

"N is C-reconstructable from M"

* You should think of N above as defining Entanglement Wedge

as defining Entanglement Wedge

* C-reconstructability is a strong condition.

* reconstructable is weaker

- $\tilde{N} \subset N$ any subalgebra is reconstructable

[will show iff statement later]

9:41 AM Tue Jan 9

Conditional Talk

– $\tilde{N}_a, \tilde{N}_b$ reconstructable $\tilde{N}_a \vee \tilde{N}_b$ is reconstructable (from M)

However N is unique [if it exists]

Claim: N c-reconstructable from M then:

$$E = (\beta \circ \alpha) : M \to N^\beta \subset M$$

is a faithful conditional expectation

that preserves the code subspace:

$$\omega_{V\psi} \circ E = \omega_{V\psi} \quad \forall \psi \in \mathcal{H}$$

Conditional Expectation:

- $\underset{B \subset A}{E : A \to B}$ $\quad B \subset A$ Subalgebra $\quad E(b) = b$

s.t $\quad E(b_2 a b_1) = b_2 E(a) b_1 \quad (E(b) = b)$

$b_1, b_2 \in B \quad a \in A \quad \uparrow$ "bimodule property"

- Existence is non-trivial for general vN algebras

algebras

- Non-commutative version of conditioning on some random variable B

$$B(\mathcal{H}_A) \supset B(\mathcal{H}_B) \otimes \mathbb{1}_C$$

- E.g: $\mathcal{H}_A = \overbrace{\mathcal{H}_B} \otimes \mathcal{H}_C$

$$E(a) = \frac{\mathbb{1}_C}{\dim \mathcal{H}_C} \otimes \mathrm{tr}_C(a)$$

10

OR $\int dg \; V(g)^\dagger a V(g)$ Haar measure

$A \to B$ = fixed pt algebra of $V(g)$

9:41 AM Tue Jan 9

Conditional Talk

Lec18 Lec19 Lec20 Lec21 Lec23 Lec24 Lec22 Lec25 Conditional Talk

10

OR $\int dg\, V(g)^{\dagger} a V(g)$ Haar measure

$A \to B$ = fixed pt algebra of $V(g)$

Back to claim: Check it fixes sub-algebra

- $E(\underline{\beta(n)}) = \beta \circ \alpha \circ \beta(n) = \beta(n)$

$V^{\dagger}\beta(n)V = V^{\dagger}Vn = n$

reconstruction

QInfo

9:41 AM Tue Jan 9

Conditional Talk

Lec18 Lec19 Lec20 Lec21 Lec23 Lec24 Lec22 Lec25 Conditional Talk

- $E(\beta(n)) = \beta \circ \cancel{\alpha} \circ \beta(n) = \beta(n)$

$\alpha \circ \beta = \mathrm{Id}$

$V^\dagger \beta(n) V = V^\dagger V n = n$

reconstruction

[QInfo

α = Noise

β = Decoding

$\alpha \circ \beta = \mathrm{Id}$

$\Rightarrow D \circ N = \mathrm{Id}$

- $\omega_{V\psi} \circ E(m) = \langle \psi | V^\dagger E(m) V | \psi \rangle$

$= \langle \psi | V^\dagger \beta(\alpha(m)) V | \psi \rangle$

$= \langle \psi | \alpha(m) | \psi \rangle = \omega_{V\psi}(m)$

Consequences: • Equality of $S_{rel.}$ through the code (JLMS) [Petz]

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9:41 AM Tue Jan 9

Conditional Talk

Lec18 Lec19 Lec20 Lec21 Lec23 Lec24 Lec22 Lec25 Conditional Talk

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Consequences: • Equality of $S_{rel.}$ through the code (JLMS) [Petz]

• Equality of bulk & bdry mod

flow [well known Takesaki] (11)

Also see: Gesteau & Karg for the converse

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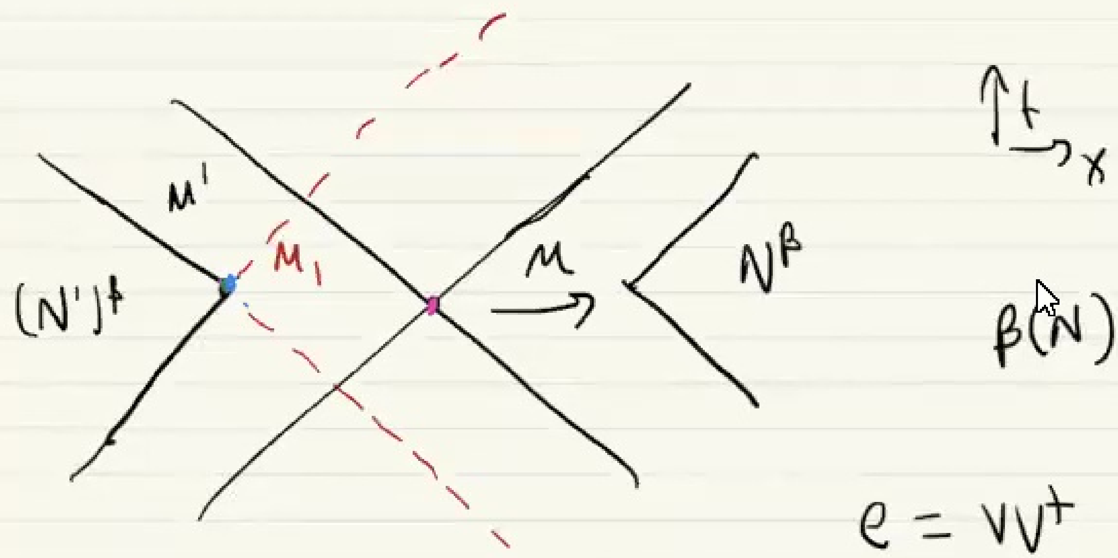
A schematic (somewhat misleading) picture:



$(N')^\beta \equiv \beta'(N')$

• One can show: $\left|(N')^\beta\right|' = (M \vee e)$

Also see: ... for the converse

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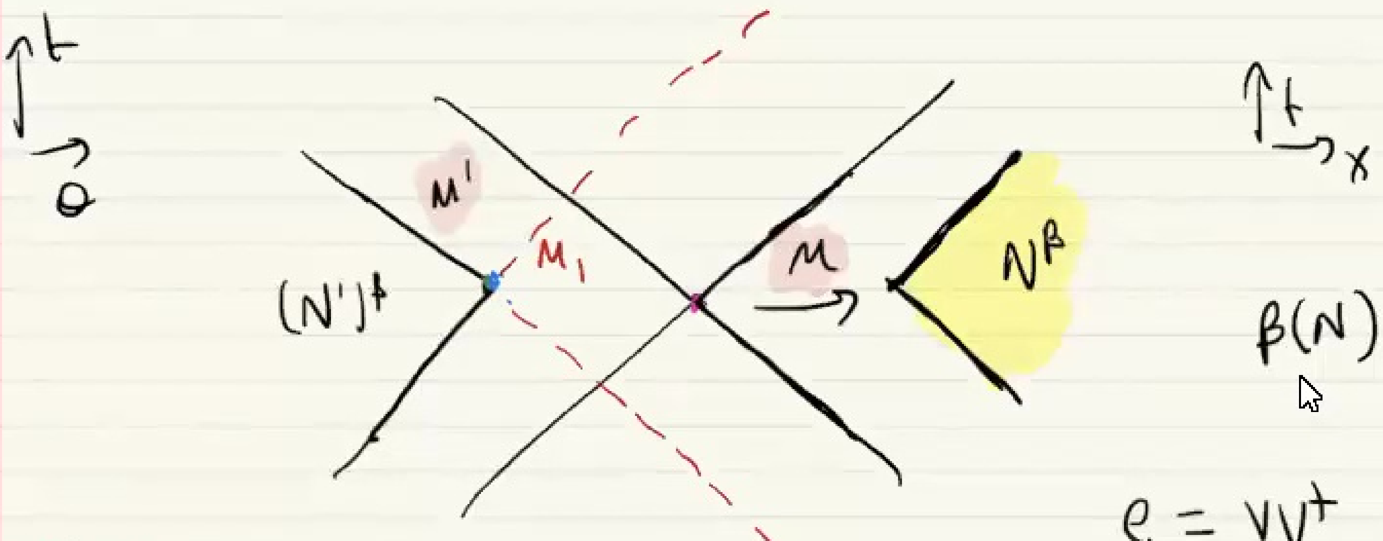
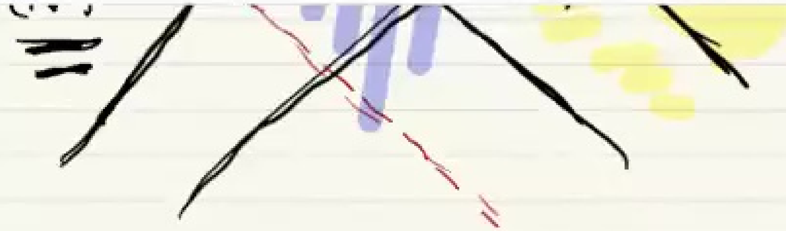


$\beta(N)$

$e = VV^{\dagger}$

$(N')^{\beta} \equiv \beta'(N')$

- One can show: $\left[(N')^{\beta}\right]' = (M \vee e)$



$\beta(N')$

$e = vv^\dagger$

$(N')^\beta \equiv \beta'(N')$

- One can show: $\left[(N')^\beta\right]' = (M \vee e)$

gen by these

where $e = vv^\dagger$

- $M \vee e \equiv M_1$ is called the Jones extension, plays important role in Index theory for subfactors.

9:41 AM Tue Jan 9

Conditional Talk

Lec18 | Lec19 | Lec20 | Lec21 | Lec23 | Lec24 | Lec22 | Lec25 | Conditional Talk

extension, plays important role in Index theory for subfactors.

(12)

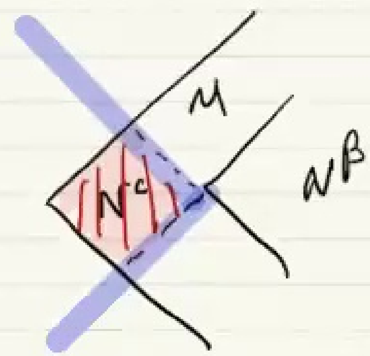
- Relative commutant plays an important role:

$$N^c \equiv M \cap (N^\beta)'$$

- Ryu Takayanagi Area operator: [Harlow]

• Relative commutant plays an important role.

$$N^c \equiv M \cap (N^\beta)'$$



• Ryu Takayanagi Area operator: [Harlow]

Naturally associated to E: [Takasaki]

Factorization $M \simeq \overbrace{(M \cap (N^\beta)')}^{N^c} \otimes N^\beta$

(almost: requires factors & normalcy)

Naturally associated to E: [Takasaki]

Factorization $M \cong \overbrace{(M \wedge (N^\beta)')}^{N^c} \otimes N^\beta$

(almost: requires factors & normalcy)

States fixed by E: $\rho \circ E = \rho$ ρ on M

$$\rho = \rho|_{N^c} \otimes \rho|_{N^\beta}$$

$\rho = \omega_{v\psi}$

$$S_M(\rho) = S_{N^c}(\rho) + \overline{S_{N^\beta}(\rho)} = S_N(\omega_\psi)$$

Bulk Entropy.

Bulk Entropy.

(13)

State on $\underline{N^c}$: $\rho(n^c) = \rho_0 E(n^c)$

but $E(n^c) \in N^\beta$ and commutes:

$$E(n^c)\, n^\beta = E(n^c n^\beta) = E(n^\beta n^c) = n^\beta E(n^c)$$

thus $E(n^c) \in Z(N^\beta)$ the center!

central projections.

$$\Rightarrow E(n^c) = \sum \pi_a \, \gamma_a(n^c)$$

9:41 AM Tue Jan 9

Conditional Talk

Lec19 Lec20 Lec21 Lec23 Lec24 Lec22 Lec25 Conditional Talk

$$E(N^c)\,N^\beta = E(N^c N^\beta) = E(N^\beta N^c) = N^\beta E(N^c)$$

$$N^\beta \wedge (N^\beta)'$$

thus $E(N^c) \in Z(N^\beta)$ the center!

minimal central projections.

$$\Rightarrow \quad E(N^c) = \sum_a \Pi_a \left(\gamma_a(N^c)\right)$$

$$S_M(\rho) = \sum_a \rho(\Pi_a)\, S_{N^c}(\gamma_a) + S_{N^\beta}(\rho)$$

$$\downarrow \quad \rho = \omega_{V\psi}!$$

$$\boxed{S_M(\omega_{V\psi}) = \omega_\psi(\hat{L}_N) + S_N(\omega_\psi)}$$

9:41 AM Tue Jan 9

Conditional Talk

Lec18 Lec19 Lec20 Lec21 Lec23 Lec24 Lec22 Lec25 Conditional Talk

$$\Rightarrow \quad E(n^c) = \sum_a \underline{\underline{\pi_a}} \, \underline{\underline{\gamma_a(n^c)}}$$

linear functional on N^c

$$\underline{\underline{S_M(\rho)}} = \sum_a \underline{\underline{\rho(\pi_a)}} \, \underline{S_{N^c}(\gamma_a)} + S_{N'}(\rho)$$

$\downarrow \quad \rho = \omega_{V\psi}$:

$$\boxed{S_M(\omega_{V\psi}) = \omega_\psi(\hat{\ell}_N) + \underline{S_N(\omega_\psi)}}$$

$$\hat{\ell}_N = \sum_a \pi_a \left(S_{N^c}(\gamma_a)\right)$$

central projections for N

$P_a = \omega_+(\Pi_a)$ only depends on E

This is exactly the "RT formula" with associated area operator $\hat{\mathcal{L}}_N$ as discussed in Harlow

Note: really type-I since we have assumed S_M etc. are finite. In type-III setting [QFT] we can still define $\hat{\mathcal{L}}_N$

A minimization formula: Plays role of area

9:41 AM Tue Jan 9

Conditional Talk

Note: really type-I since we have assumed

S_M etc. are finite. In type-III

setting [QFT] we can still define $\hat{\mathcal{L}}_N$

A minimization formula:

Plays role of area term.

Thm: $$S_M(\omega_{\psi}) \leq \inf_{\{N:\ \text{reconstructable from } M\}} \left(\overbrace{S_{N^c}(\omega_{\psi})} + S_N(\omega_{\psi}) \right)$$

$\sim S_{gen}$?

$$N^c = M \wedge (\beta(N))'$$

(15)

& equality is achieved iff $\exists$ an N

s.t N is C-reconstructable from M

Very suggestive, but some issues with analogy

to the RT formula minimization:

(1) not all subalgebras of EW have a

larger S_{gen} (S_{gen} decreases along

"EW horizon" by QFC)

15

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Very suggestive, but some issues with analogy to the RT formula minimization:

(1) not all subalgebras of EW have a larger S_{gen} (S_{gen} decreases along "EW horizon" by QFC)

(2) area operator is no longer an operator

(3) If we achieve minimum for one state we achive it for all (assuming faithfulness

9:41 AM Tue Jan 9

Conditional Talk

Lec18 Lec19 Lec20 Lec21 Lec23 Lec24 Lec22 Lec25 Conditional Talk

(1) not all subalgebras of EW have a

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"EW horizon" by QFC)

(2) area operator is no longer an operator

(3) If we achieve minimum for one state

we achive it for all (assuming faithfulness

of the state) → Exact Recovery.

16

Generalities: How far can we push this?

- Multiple boundary alg. M_1, M_2

① Entanglement Wedge Nesting: $\mathcal{E}_1, \mathcal{E}_2 \sim$ Entanglement Wedges.

$\mathcal{E}_1$ is C-reconstructable from M_1

$\mathcal{E}_2$ is C-reconstructable from M_2

Then:

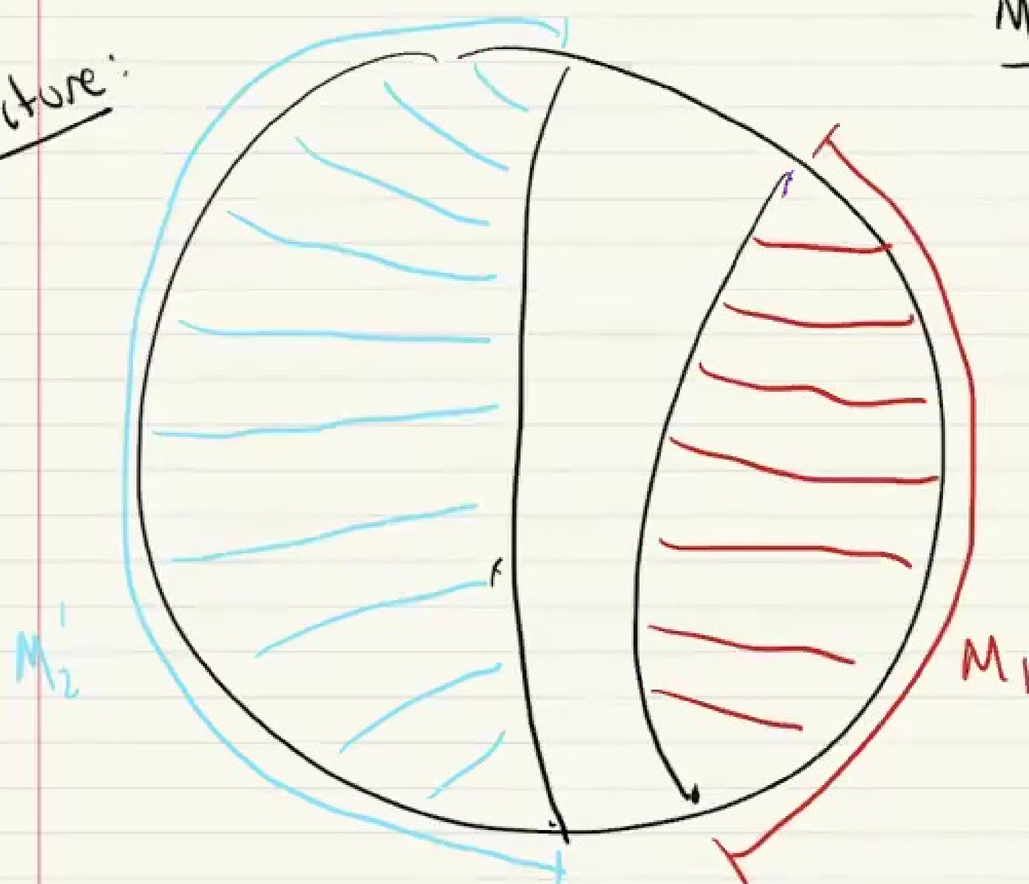
$$M_1 \subset M_2 \Rightarrow \mathcal{E}_1 \subset \mathcal{E}_2$$

$M_1 \subset M_2 \rightarrow$

Then:

$$M_1 \subset M_2 \Rightarrow \mathcal{E}_1 \subset \mathcal{E}_2$$

Picture:



$M_1 \subset M_2 \to$

$$[M_1, M_2'] = 0$$

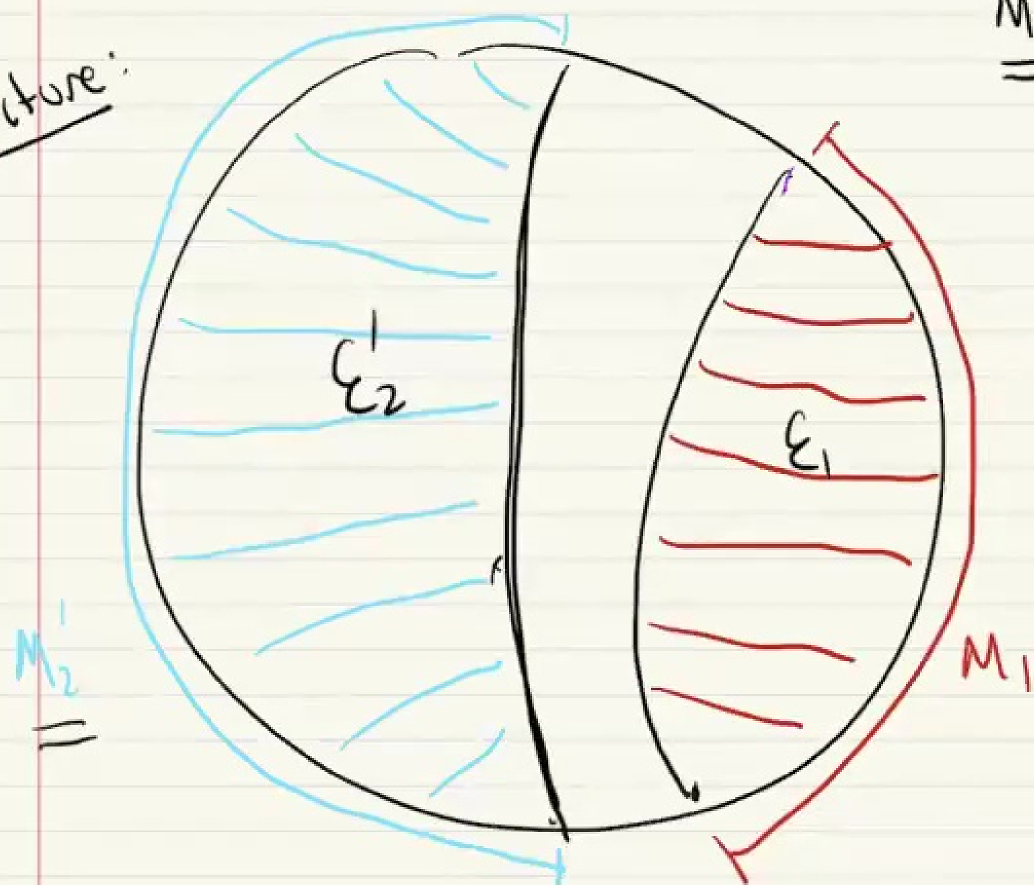
$$\Rightarrow$$

$$[\mathcal{E}_1, \mathcal{E}_2'] = 0$$

Then:

$$M_1 \subset M_2 \Rightarrow \mathcal{E}_1 \subset \mathcal{E}_2$$

Picture:



$M_1 \subset M_2 \to$

$$[M_1, M_2'] = 0$$

$\Rightarrow$

$$[\mathcal{E}_1, \mathcal{E}_2'] = 0$$

true $M_2 = M_1$, $N_2 = \mathcal{E}_1 \Rightarrow N_1 \subset \mathcal{E}_1$

$\Rightarrow$ Reconstructable iff $N_1 \subset \mathcal{E}_1$

(2) Consistency of $\underline{\underline{E}}$: $\underline{\underline{M_1 \subset M_2}}$

with c-reconstructable algebras $N_1 \subset N_2$ then

$$E_2 : M_2 \to (N_2)^\beta \quad \text{restricts:}$$

$$E_2|_{M_1} = E_1 : M_1 \to (N_1)^\beta$$

This consistency proves that exact complementary recovery is the same structure as a

Consistency of E: $M_1 \subset M_2$

with C-reconstructable algebras $N_1 \subset N_2$ then

$$E_2 : M_2 \to (N_2)' \quad \text{restricts:}$$

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This consistency proves that exact complementary recovery is the same structure as a net of inclusions $(N_i') \subset M_i$

Longo Rehren

Consistency of E: $M_1 \subset M_2$

with c-reconstructable algebras $N_1 \subset N_2$ then

$$E_2 : M_2 \to (N_2)^c \quad \text{restricts:}$$

$$E_2|_{M_1} = E_1 : M_1 \to (N_1)^c$$

This consistency proves that exact complementary recovery is the same structure as a net of inclusions $(N_i^c) \subset M_i$

Longo Rehren

Casini, Huerta, Magán, Pontello

Here $N^{\beta} \subset M$ is the sub-algebra of neutral operators under some global charge [orbifolded theory] of the local operators in M

E: projects to this neutral sector

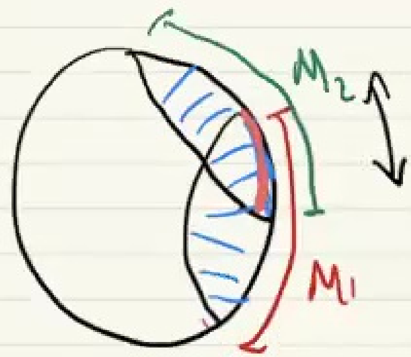
$$\frac{1}{|G|}\sum_{g\in G} V(g)^{\dagger} m\, V(g)$$

In AdS/CFT: E projects to the low-energy gravity degrees of freedom.

I have no specific proposal for E

→ Q. Info arguments tell us one should exist

③ There is an important issue: [W. Kelly]



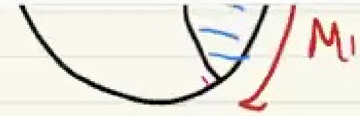
Overlapping regions

Thm: N_i is c-reconstructable from M_i

if $\exists\ \psi$ s.t $V\psi$ is cyclic & sep for

$M_1 \wedge M_2$ then:

$N_1 \vee N_2$ is c-reconstructable from $M_1 \vee M_2$



Thm: N_i is c-reconstructable from M_i

if $\exists\ \psi$ s.t $V\psi$ is cyclic & sep for

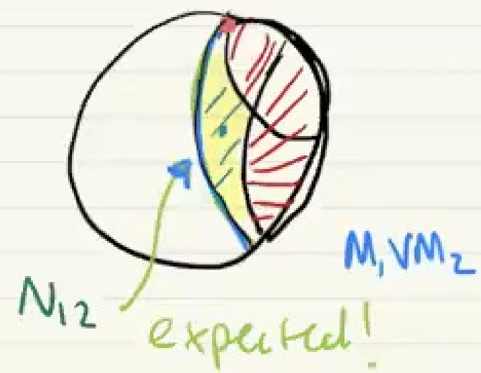
$M_1 \wedge M_2$ then:

$N_1 \vee N_2$ is c-reconstructable from $M_1 \vee M_2$

(19)

i.e. the bulk algebras are "additive" for

But: not true in AdS/CFT



Uses in an essential way the cyclicity for $M_1 \wedge M_2$

i.e. $\{ x\, V|\psi\rangle \,;\, x \in M_1 \wedge M_2 \}$ is a dense subset of K.

• **Fix:** (Kelly) is to have approximate recovery i.e. $\{ x\, V|t\rangle \; ; \; x \in M_1 \cap M_2 \}$ is a dense subset of K.

- Fix: (Kelly) is to have approximate recovery

then cyclicity loses its power!

Cyclicity still holds but:

$$x\, V|t\rangle \to |\xi\rangle \qquad \|x\| \to \infty$$

&

$$N_{12} \overset{c.rec}{\sim} M_1 \vee M_2$$

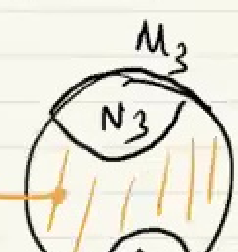
$$\Rightarrow \left(N_{12}' \overset{c.rec}{\sim} (M_1 \vee M_2)' \right)$$

But what do we pick for N_{12} ?

Holography suggests two options:

(21)

$$N_{12} = \underline{N_1 \vee N_2} \quad \text{or} \quad N_{12} = (\underline{N_3 \vee N_4})'$$



- We call such a code

"Dual Additive"

Future work:

* Beyond Exact Recovery
  - α-bit Story (Hayden Penington)
  - breakdown of complementary recovery (Akers, Leichenauer, Levine & Akers, Penington)
* Lofty goal: axiomatic

9:41 AM Tue Jan 9

Conditional Talk

Lec18 Lec19 Lec20 Lec21 Lec23 Lec24 Lec22 Lec25 Conditional Talk

- breakdown of complementary recovery (Akers, Leichenauer, Levine & Akers, Penington)

* Lofty goal: axiomatic characterization of quantum gravity (in low energy / EFT limit) in AdS