

Title: Quasinormal modes of black holes in modified and quantum gravity

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Abstract: I will show that quasinormal modes of black holes could be used to investigate quantum gravity or modified gravity in specific situations. Some general comments about isospectrality will also be made. Finally a few other quantum gravity phenomena associated with black holes will be underlined.



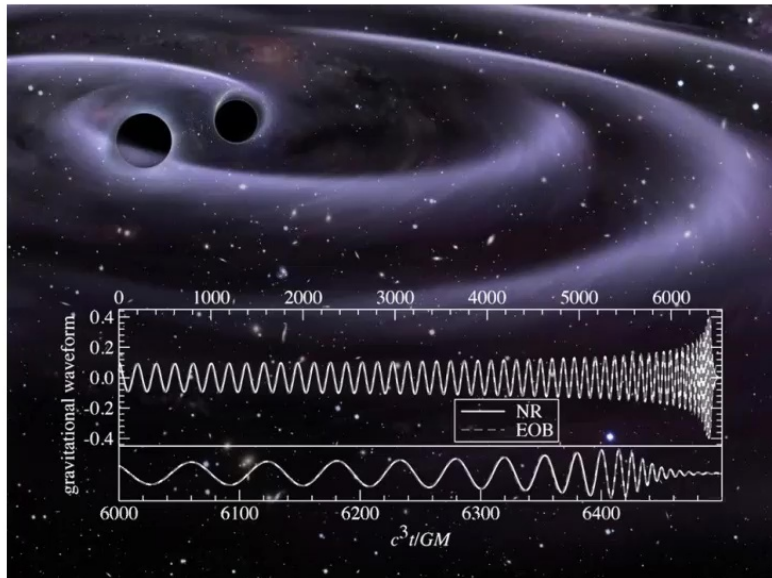
QUASINORMAL MODES OF BLACK HOLES IN MODIFIED AND QUANTUM GRAVITY

Aurélien Barrau

With Jérémy Martinon, Flora Moulin et Killian Martineau

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Quasinormal modes of the Schwarzschild black hole



- BH are simple

QNMs:

- They are free oscillations and do not depend on the initial perturbations
- No matter content, no equation of state, etc.
- « quasi » normal : loss of energy (poles of Green function)



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$$g_{\mu\nu} = g_{\mu\nu}^0 + \delta g_{\mu\nu} \qquad \delta R_{\mu\nu} = 0$$

- purely outgoing at infinity;
- purely ingoing at the event horizon.

$$\psi \propto e^{-i\omega t} = e^{-i(\omega_R + i\omega_I)t} = e^{\omega_I t} \cos(\omega_R t + \phi)$$

$$T = \frac{2\pi}{\omega_R} \qquad \tau = \frac{1}{\omega_I}$$

- $\omega_I < 0$: exponential damping (stable);
- $\omega_I > 0$: exponential growth (unstable).

$$\frac{1}{\sqrt{-g}} \partial_\mu (g^{\mu\nu} \sqrt{-g} \partial_\nu \psi) = 0$$

$$\begin{cases} s = 0 \text{ for scalar perturbations,} \\ s = 1 \text{ for electromagnetic perturbations,} \\ s = 2 \text{ for axial gravitational perturbations,} \end{cases}$$

and it is also worth noticing here that by symmetry we must have $\ell \geq 0$ for scalar perturbations, $\ell \geq 1$ for electromagnetic perturbations and $\ell \geq 2$ for gravitational perturbations.

$$V_\ell(r) = \left(1 - \frac{2M}{r}\right) \left[\frac{\ell(\ell+1)}{r^2} + \frac{2M(1-s)}{r^3} \right]$$

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Review C. Chirenti, 1708.04476v2

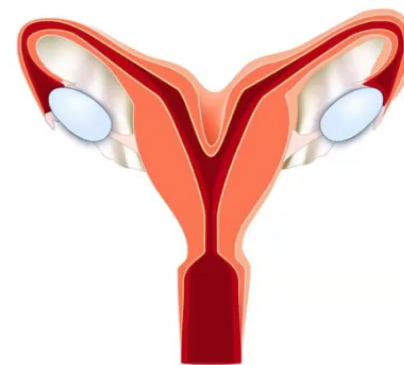
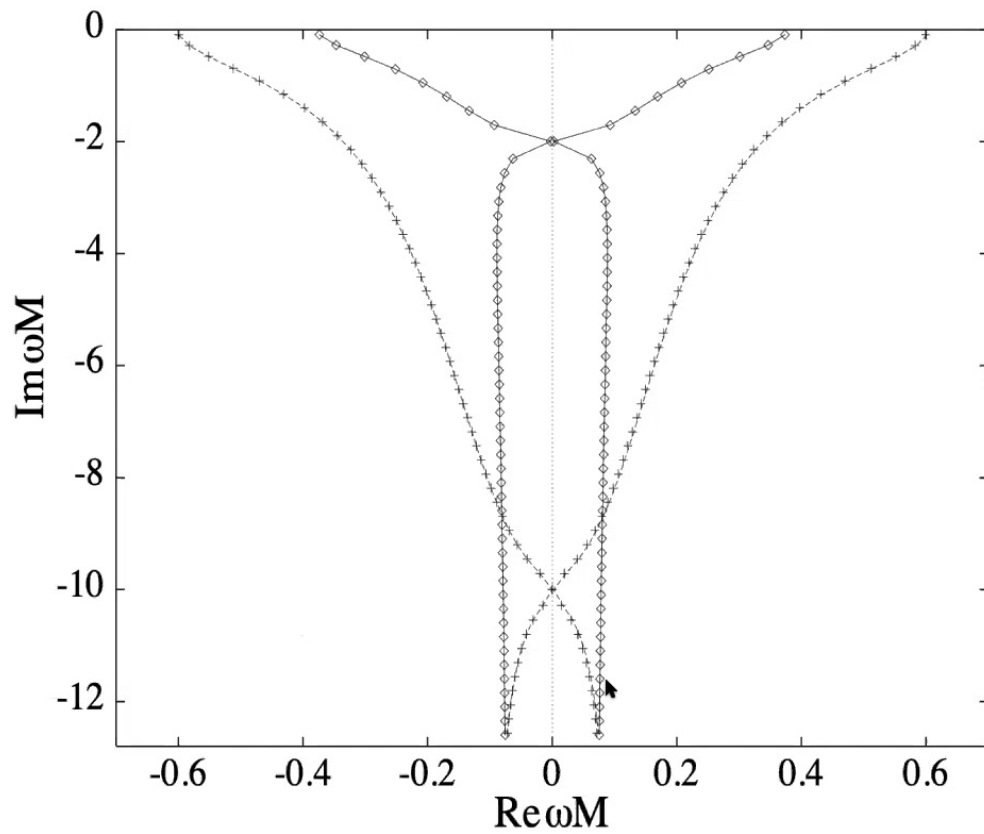
$$ds^2 = -e^{2\nu} dt^2 + e^{2\mu_2} dr^2 + e^{2\mu_3} d\theta^2 + e^{2\psi} (d\phi - \omega dt - q_2 dr - q_3 d\theta)^2,$$

1. the perturbation that gives small values to the metric coefficients that were zero (ω, q_2, q_3): this perturbation induces frame dragging and imparts a rotation to the black hole; this is called the “axial” perturbation.
2. the perturbation that gives small increments to the already non-zero metric coefficients ($e^{2\nu}, e^{2\mu_2}, e^{2\mu_3}, e^{2\psi}$): this is called the “polar” perturbation.

$$V_\ell^{\text{axial}}(r) = \left(1 - \frac{2M}{r}\right) \left[\frac{\ell(\ell+1)}{r^2} - \frac{6M}{r^3} \right]$$

$$V_\ell^{\text{polar}}(r) = \frac{2}{r^3} \left(1 - \frac{2m}{r}\right) \times \frac{9M^3 + 3c^2 M r^2 + c^2 (1+c) r^3 + 9M^2 c r}{(3M + c r)^2}$$

with $c = \ell(\ell+1)/2 - 1$.



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PHYSICAL REVIEW LETTERS

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Editors' Suggestion

Testing the No-Hair Theorem with GW150914

Maximiliano Isi, Matthew Giesler, Will M. Farr, Mark A. Scheel, and Saul A. Teukolsky
Phys. Rev. Lett. **123**, 111102 – Published 12 September 2019

Cornell University, Ithaca, New York 14853, USA
(Dated: August 12, 2019)

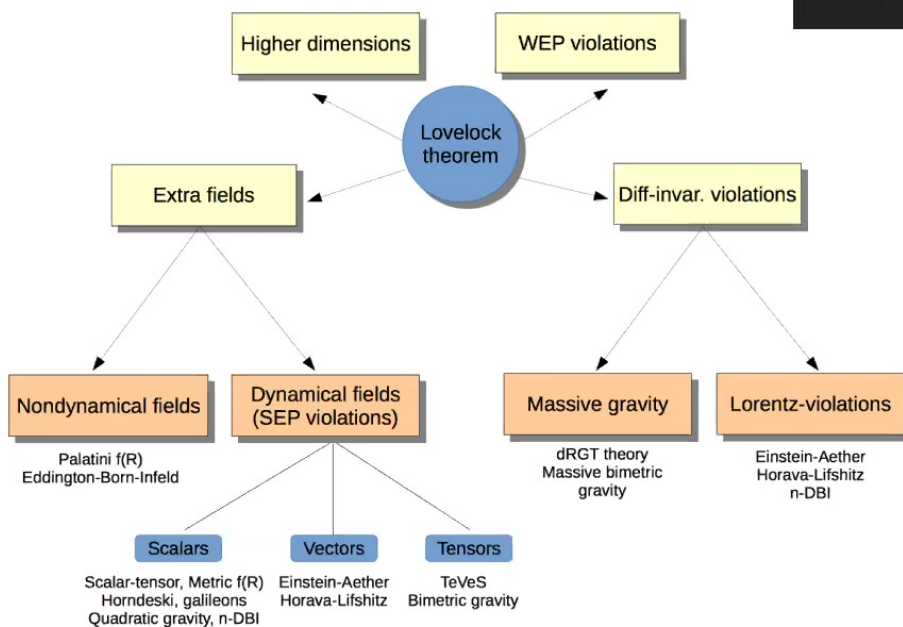
We analyze gravitational-wave data from the first LIGO detection of a binary black-hole merger (GW150914) in search of the ringdown of the remnant black hole. Using observations beginning at the peak of the signal, we find evidence of the fundamental quasinormal mode and at least one overtone, both associated with the dominant angular mode ($\ell = m = 2$), with 3.6σ confidence. A ringdown model including overtones allows us to measure the final mass and spin magnitude of the remnant exclusively from postinspiral data, obtaining an estimate in agreement with the values inferred from the full signal. The mass and spin values we measure from the ringdown agree with those obtained using solely the fundamental mode at a later time, but have smaller uncertainties.

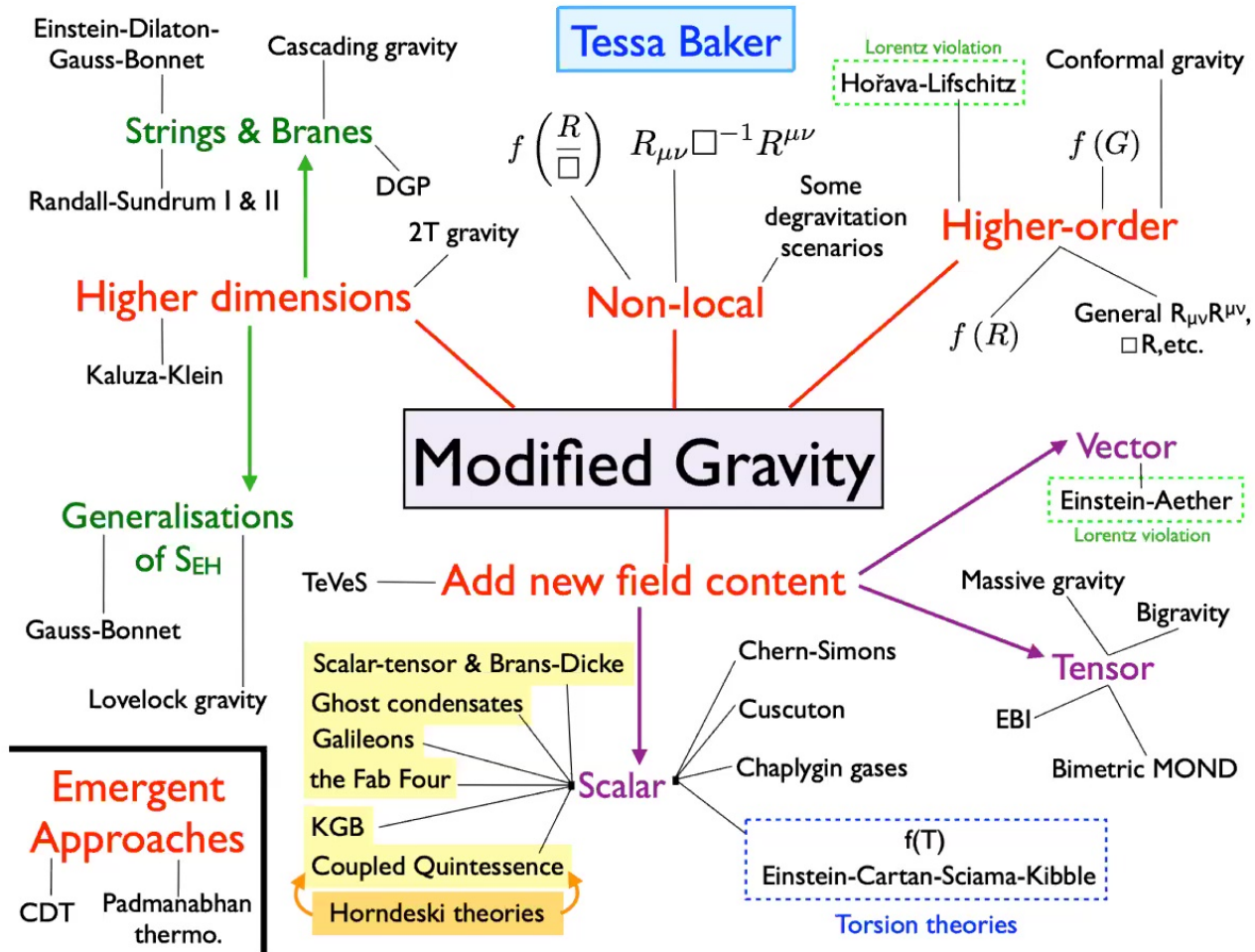


Beyond GR



"The only second-order, local gravitational field equations derivable from an action containing solely the 4D metric tensor (plus related tensors) are the Einstein field equations with a cosmological constant."





From T. Baker

Beyond GR



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$$ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\phi^2$$

$$V(r) = f(r) \left(\frac{\lambda + 2(f(r) - 1)}{r^2} - \frac{f'(r)}{r} \right)$$

$$ds^2 = A(r)dt^2 - B(r)^{-1}dr^2 - H(r)d\theta^2 - H(r)\sin^2\theta d\phi^2$$

$$\frac{d}{dr^*} = \sqrt{AB} \frac{d}{dr}$$

$$\mu^2 = (l-1)(l+2)$$

$$\frac{d^2 Z}{dr^{*2}} + (\omega^2 - V(r))Z = 0$$

$$V(r) = \frac{1}{2H^2} \left(\frac{dH}{dr^*} \right)^2 + \frac{\mu^2 A}{H} - \frac{1}{\sqrt{H}} \frac{d^2}{dr^{*2}} \left(\sqrt{H} \right)$$

We use a 6th order WKB approximation scheme (Konoplya)

$$\omega^2 = V_0 - i\sqrt{-2V_0''} \left(\sum_{j=2}^6 \Lambda_j + n + \frac{1}{2} \right)$$

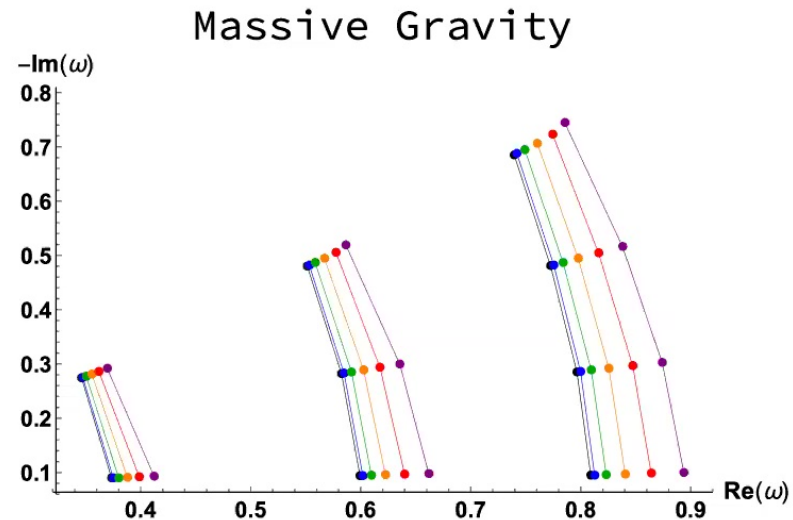


1. Massive gravity

One of the first motivations for modern massive gravity – which can be seen as a generalization of GR – was the hope to account for the accelerated expansion of the Universe by generating a kind of Yukawa-like potential for gravitation. Ghost → dRGT

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} (R + m^2 \mathcal{U}(g, \phi^a))$$

$$f(r) = 1 - \frac{2M}{r} + \frac{\Lambda r^2}{3} + \gamma r + \epsilon$$



2. Modified STV gravity

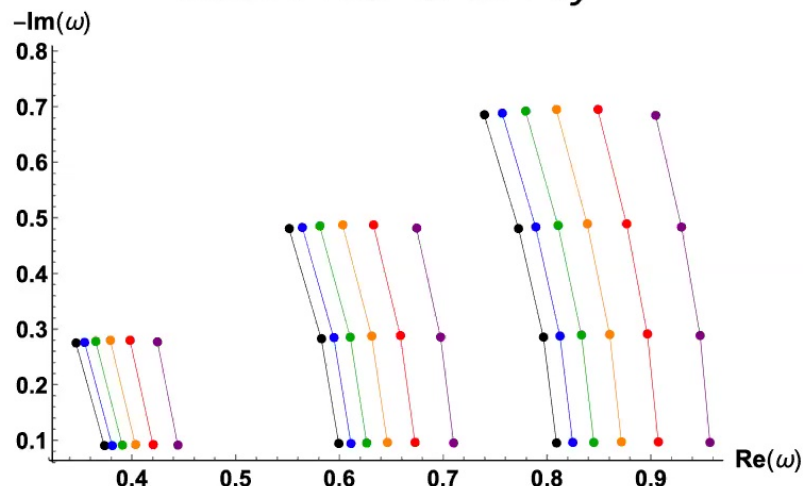
MOG allows the gravitational constant, a vector field coupling, and the vector field mass to vary with space and time. We chose

- the case where the field equations for $B_{\mu\nu}$ are non-linear, as the phenomenology is then richer,
- $\alpha < \alpha_c = 0.67$ where there are two horizons and an appropriate potential behavior for the WKB approximation to hold.

$$G = G_N(1 + \alpha)$$

$$f(r) = 1 - \frac{2M}{r} + \frac{\alpha(1 + \alpha)M^2}{r^2}$$

Modified Gravity



$$B_{\mu\nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu$$

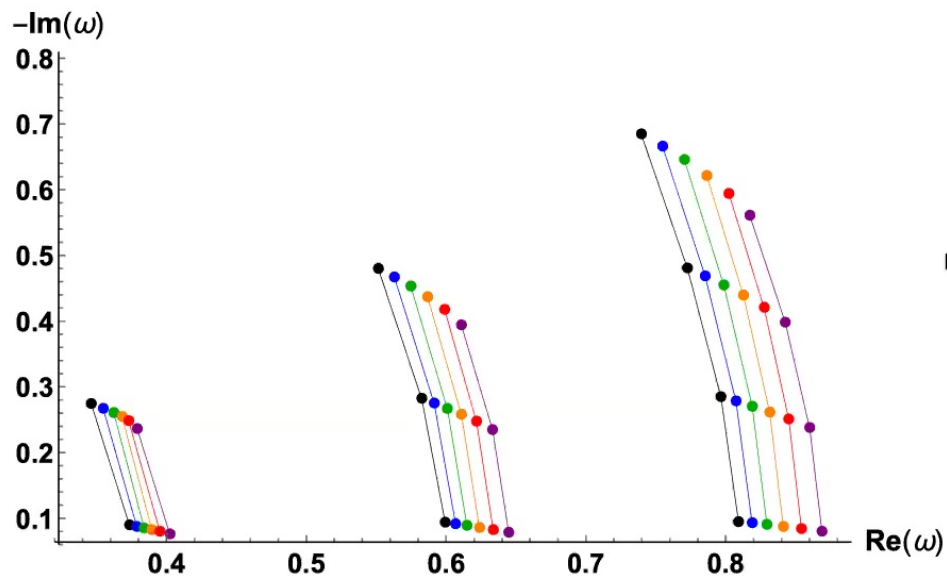


3. Hořava-Lifshitz gravity

Horava-Lifshitz gravity bets on the fundamental nature of the quantum theory instead of relying on GR principles. It is a renormalizable UV-complete gravitational theory which is not Lorentz invariant in 3 + 1 dimensions.



Horava-Lifshitz



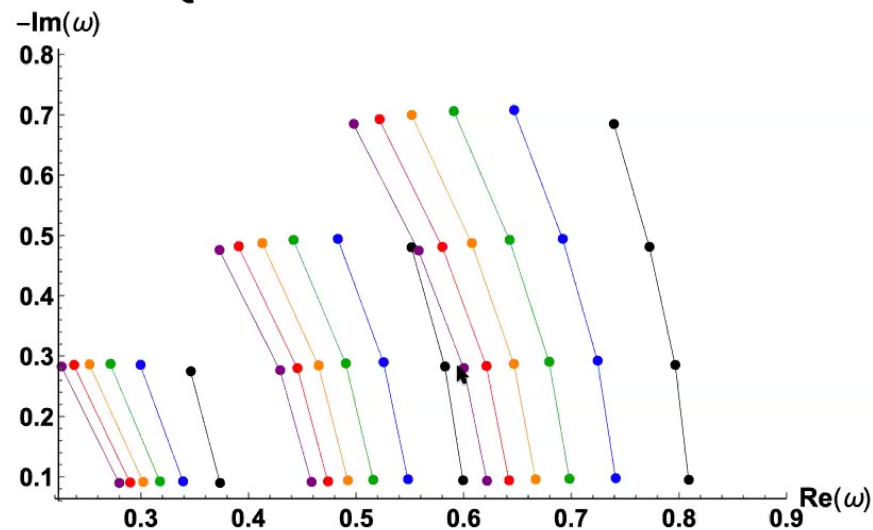
$$f(r) = \frac{2(r^2 - 2Mr + \beta)}{r^2 + 2\beta + \sqrt{r^4 + 8\beta Mr}}$$

4. $\hbar$ correction

Quantum corrections to the Newtonian gravitational potential can be rigorously derived without having a full quantum theory of gravity at disposal.

$$f(r) = 1 - \frac{2M}{r} + \gamma \frac{2M}{r^3}$$

Quantum Correction





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5. LQG polymeric BH

$$A(j) = 8\pi\gamma_{BI}\sqrt{j(j+1)}$$

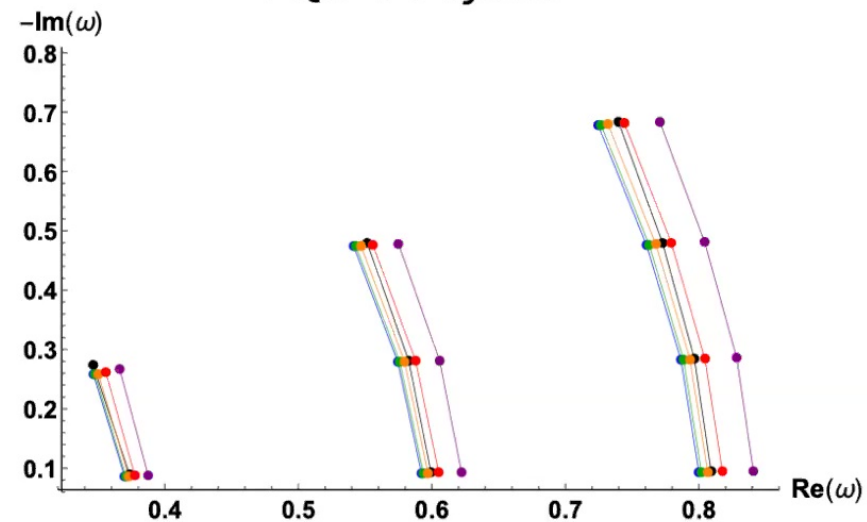
$$ds^2 = -G(r)dt^2 + \frac{dr^2}{F(r)} + H(r)d\Omega^2,$$

$$G(r) = \frac{(r - r_+)(r - r_-)(r + r_*)^2}{r^4 + a_o^2},$$

$$F(r) = \frac{(r - r_+)(r - r_-)r^4}{(r + r_*)^2(r^4 + a_o^2)},$$

$$H(r) = r^2 + \frac{a_o^2}{r^2},$$

LQG Polymer



Distinguishing between those models with observations is challenging :

- there exist degeneracies, for given overtone and multipole numbers, between the models – when taking into account that the values of the parameters controlling the deformation are unknown.
- the intrinsic characteristics of the observed black holes are also unknown, which induces other degeneracies
- the study should be extended to Kerr black hole, which also adds some degeneracies in addition to the complexity

Some interesting trends can however be underlined. For all models, the effect of modifying the gravitational theory are more important for the real part than for the imaginary part of the complex frequency of the QNMs.

Some “trends” are specific to each studied model. In addition, the sign of the frequency shift, and its dependence upon the overtone and multipole numbers is characteristic of a given extension of GR.

If features beyond GR were to be observed, the direction of the frequency shift in the complex plane would already allow to exclude models, as this article shows.

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Possible links between high order overtones and quantum gravity (Hod, Maggiore)

$$\begin{aligned}\Delta M = \hbar \Delta \omega &\Rightarrow \Delta A = \Delta(4\pi R^2) = \Delta(16\pi M^2) = \\ &= 32\pi M \Delta M = 32\pi \hbar M \Delta \omega .\end{aligned}$$

$$\omega = \frac{\ln 3}{8\pi M} - \frac{i}{4M} \left(n + \frac{1}{2} \right)$$



A toy model for cumulative quantum gravity (Rovelli & Haggard)

A.B., Martineau, Moulin, Martinon, Phys.Lett. B795 (2019) 346-350



$$l_R \sim \mathcal{R}^{-1/2} \quad \mathcal{R}^2 := R_{\mu\nu\rho\lambda} R^{\mu\nu\rho\lambda} \quad \text{Disregards cumulative effects}$$

→ Let us assume a « quantumness »

$$\text{Ricci} + l_p^2 \mathcal{R}^2 = 0.$$

$$q = l_P \mathcal{R} \tau$$

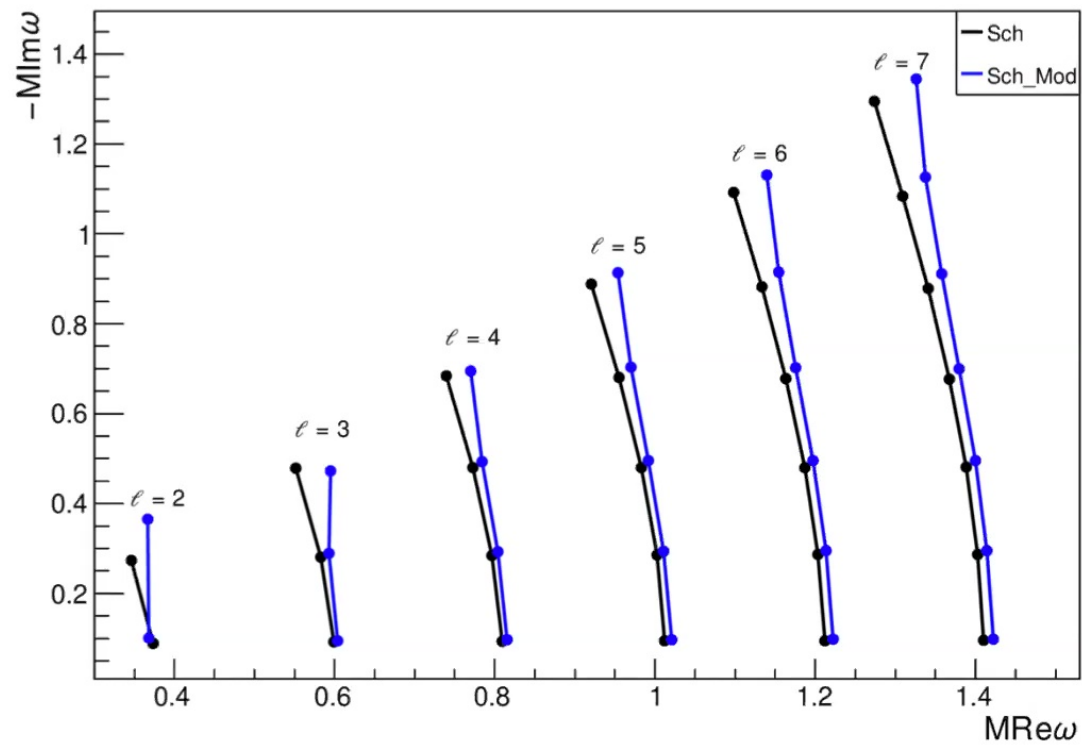
$$\tau = \sqrt{1 - \frac{2M}{r}} t,$$

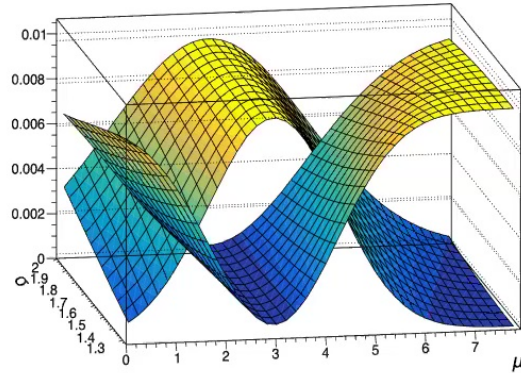
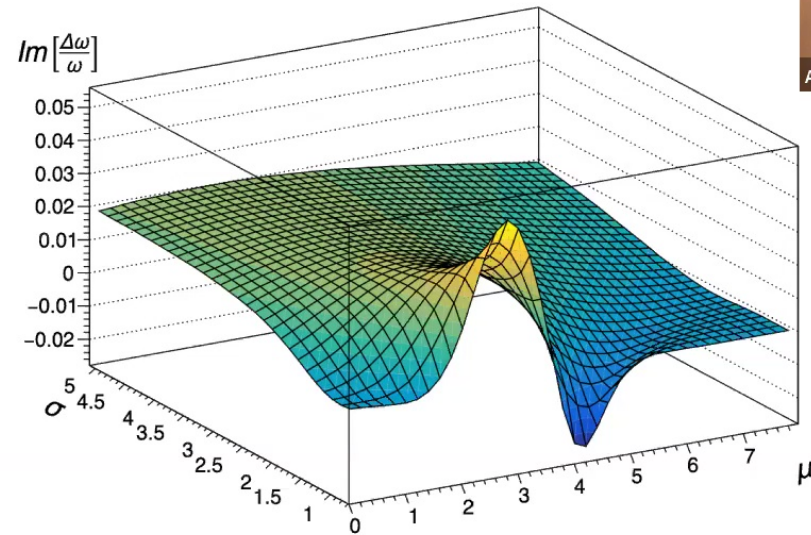
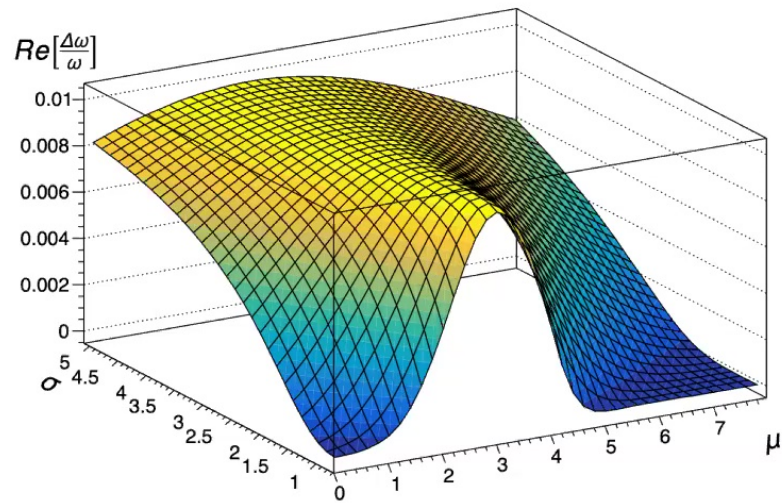
$$q(r) = \frac{M}{r^3} \left(1 - \frac{2M}{r}\right)^{\frac{1}{2}} t. \quad \text{With a maximum at} \quad r = 2M \left(1 + \frac{1}{6}\right)$$

We assume in general

$$f(r) = \left(1 - \frac{2M}{r}\right) \left(1 + A e^{-\frac{(r-\mu)^2}{2\sigma^2}}\right)^2$$

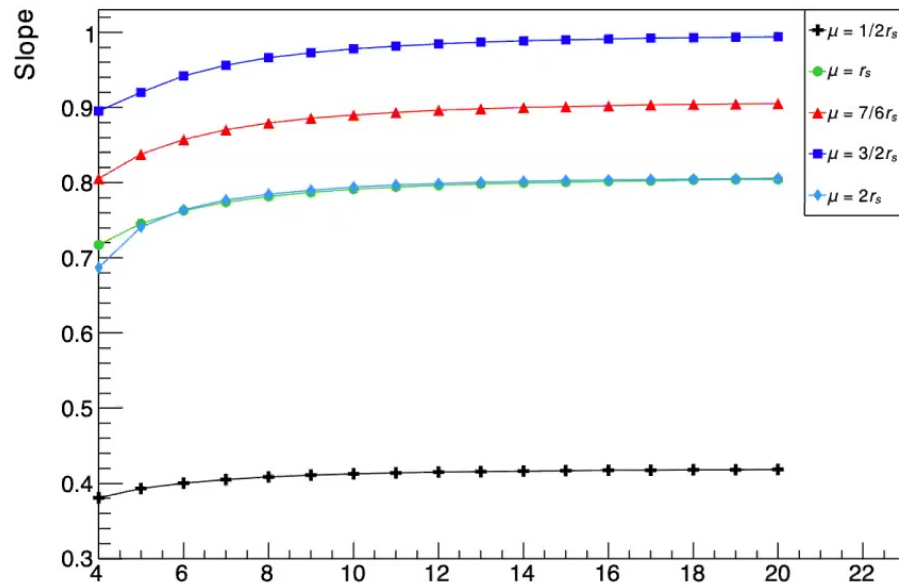




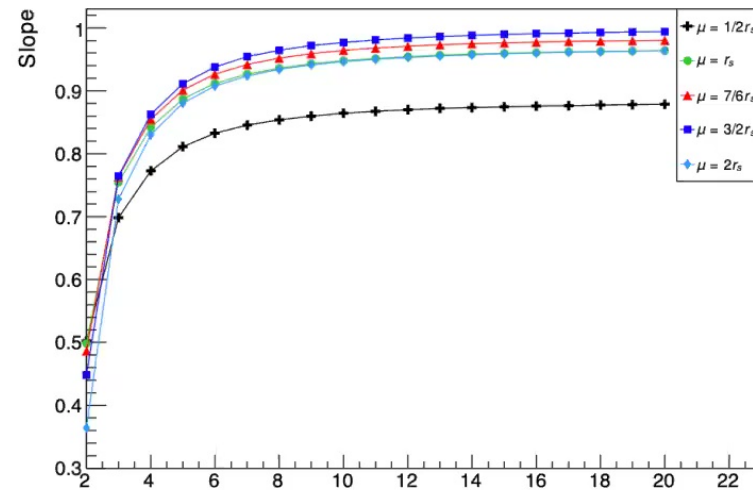


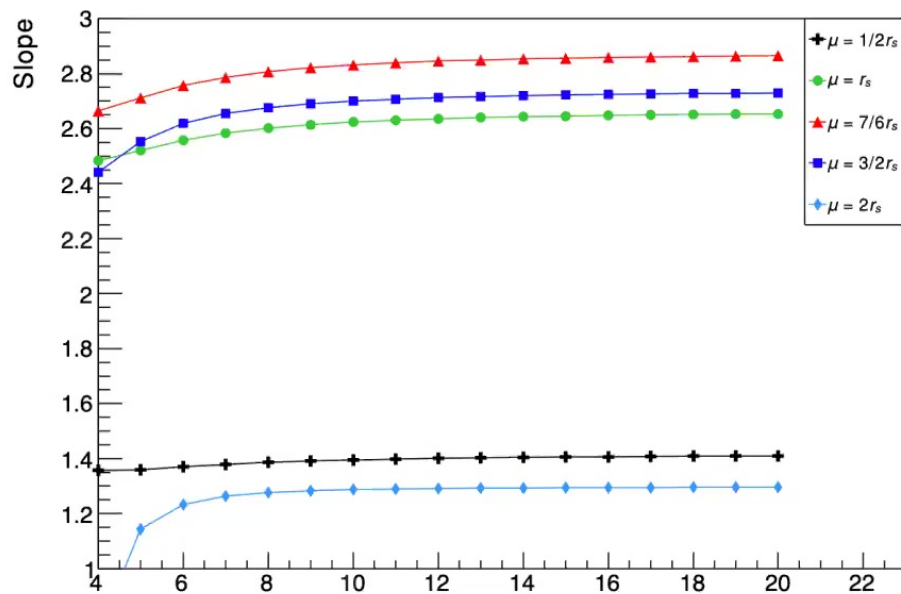


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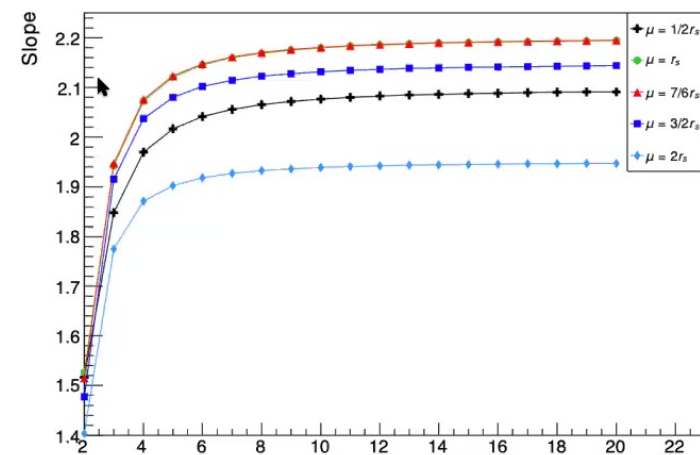
$$\sigma/\dot{M} = 4.$$





$$\text{Im}(\Delta\omega/\omega) = yA$$

$$\sigma/\dot{M} = 1.5$$



$$\sigma/\dot{M} = 4.$$



$$q_{max} = \left(\frac{3}{7}\right)^3 \sqrt{\frac{1}{7}} \frac{t}{M^2}.$$

If one sets t to be the age of the Universe, the mass value required so that the quantum gravity effects can be observed is of the order of $10E-8$ (or less) solar mass. Although far smaller than the mass of stellar black holes, this value is not ridiculously small and way higher than the Planck mass.

In addition, and very speculatively, it could be argued that the maximum possible time to be used to evaluate the mass (the higher the time, the higher the mass) is not necessarily bounded by the age of the Universe: in quantum gravity a “bounce” is possible and black holes could survive during this bounce.

THE HAYWARD METRIC

$$F(r) = 1 - \frac{2m(r)}{r} \qquad m(r) = \frac{M r^3}{r^3 + 2ML^2}$$

$$L/M \sim 0.72, \qquad \left(\frac{\Delta\omega}{\omega}\right) \sim 5\%$$



Many other nice ideas

Variations in EHT images [Giddings](#)



The inconsistency of BH evolution with the principles of quantum mechanics, given the Hawking effect, strongly motivates the need to modify the classical description of a BH at horizon scales. To preserve unitarity, BHs must release their information; many arguments indicate that this should begin midway through a BH's decay, while the BH is comparable to its original size. Such information release requires some modification to the description of a BH that is based local quantum field theory (LQFT) evolution on a classical BH geometry; this modification needs to occur on scales comparable to the size of the event horizon, for an arbitrarily large black hole.

The corresponding periods are

$$P = \frac{2\pi}{\omega_T} \simeq 0.93 \left(\frac{M}{4.3 \times 10^6 M_\odot} \right) \left(\frac{1}{2} + \frac{1}{2\sqrt{1-a^2}} \right) \text{ hr} \\ \simeq 59 \left(\frac{M}{6.5 \times 10^9 M_\odot} \right) \left(\frac{1}{2} + \frac{1}{2\sqrt{1-a^2}} \right) \text{ d} ,$$

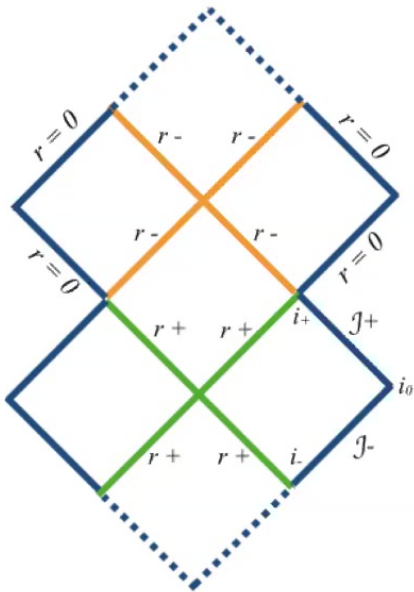
for Sgr A* and M87, with respective masses $M \simeq 4.3 \times 10^6 M_\odot$ and $6.5 \times 10^9 M_\odot$.

Quantum fields in the background spacetime of a polymeric BH

Moulin, Martineau, Grain, A.B., Class.Quant.Grav. 36 (2019) no.12, 125003



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In practice, the procedure consists in first defining the Hamiltonian constraint by the use of holonomies along the considered fixed graph. Both the diffeomorphism and Gauss constraints are identically vanishing: the first one is zero because of homogeneity and the second one is zero because the spacetime is of the Kantowski-Sachs form. The Hamiltonian constraint is solved after replacing the connection by the holonomy. Finally, the solution is expanded to the full spacetime, leading to the effective LQG-corrected geodesically complete Schwarzschild metric.

$$\frac{dN}{dt} = \frac{1}{e^{\frac{\omega}{T_H}} \pm 1} \sigma(M, s, \omega) \frac{d^3 k}{(2\pi)^3}$$

$$\sigma(M, s, \omega) = \sum_{l=0}^{\infty} \frac{(2j+1)\pi}{\omega^2} |A_{l,s}|^2$$

$$ds^2 = G(r)dt^2 - \frac{dr^2}{F(r)} - H(r)d\Omega^2 ,$$

$$G(r) = \frac{(r - r_+)(r - r_-)(r + r_x)^2}{r^4 + a_o^2} ,$$

$$F(r) = \frac{(r - r_+)(r - r_-)r^4}{(r + r_x)^2(r^4 + a_o^2)} ,$$

$$H(r) = r^2 + \frac{a_o^2}{r^2} ,$$

$$\frac{\sqrt{GF}}{H} \frac{\partial}{\partial r} \left(H \sqrt{GF} \frac{\partial R(r)}{\partial r} \right) + \left(\omega^2 - \frac{G}{H} l(l+1) \right) R(r) = 0,$$

$$\left(\frac{\partial^2}{\partial r^{*2}} + \omega^2 - V(r^*) \right) \Psi(r) = 0,$$

$$V(r) = \frac{G}{H} l(l+1) + \frac{1}{2} \sqrt{\frac{GF}{H}} \frac{\partial}{\partial r} \left(\sqrt{\frac{GF}{H}} \frac{\partial H}{\partial r} \right) .$$

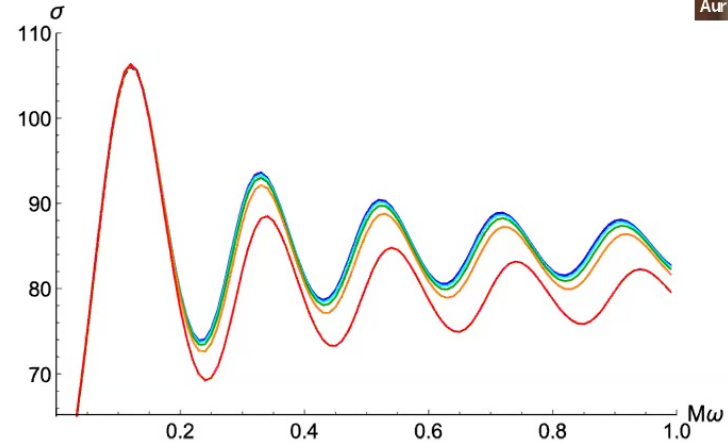


FIG. 2. Emission cross section for a scalar field with energy ω in the background spacetime of a LBH of mass M for different values of ϵ ($\epsilon = \gamma\delta$ measures the “quantumness” of spacetime). From bottom to top: $\epsilon = 10^{\{-0.3, -0.6, -0.8, -1, -3\}}$. The blue line, corresponding to $\epsilon = 10^{-3}$ is superposed with the cross section for a Schwarzschild BH.





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$$\begin{aligned}
 (D + \epsilon - \rho)P^0 + (\delta^* + \pi - \alpha)P^1 &= i\mu_* \bar{Q}^{1'}, \\
 (\Delta + \mu - \gamma)P^1 + (\delta + \beta - \tau)P^0 &= -i\mu_* \bar{Q}^{0'}, \\
 (D + \epsilon^* - \rho^*)\bar{Q}^{0'} + (\delta + \pi^* - \alpha^*)\bar{Q}^{1'} &= -i\mu_* P^1, \\
 (\Delta + \mu_* - \gamma^*)\bar{Q}^{1'} + (\delta^* + \beta^* - \tau^*)\bar{Q}^{0'} &= i\mu_* P^0,
 \end{aligned}$$

with

$$D = l^i \partial_i; \quad \Delta = n^i \partial_i; \quad \delta = m^i \partial_i; \quad \delta^* = \bar{m}^i \partial_i.$$

$$\frac{1}{C_1} \frac{\partial^2 R_+}{\partial r^{*2}} - \frac{1}{2\sqrt{C_1}} \frac{\partial R_+}{\partial r^{*2}} + \left(\frac{\omega^2}{C_1} + i \frac{\omega}{\sqrt{C_1}} \right) R_+ = 0.$$

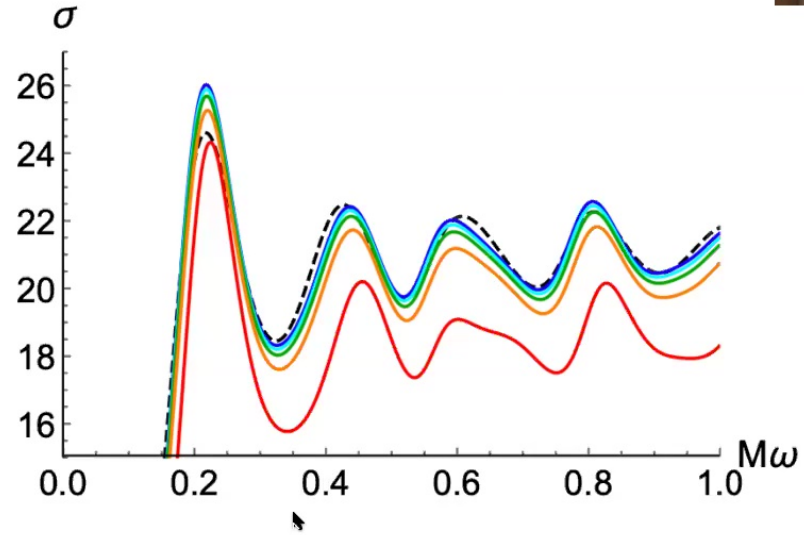


FIG. 3. Emission cross section for a fermionic field, with energy ω , in the background spacetime of a LBH of mass M . From bottom to top: $\epsilon = 10^{\{-0.3, -0.6, -0.8, -1, -3\}}$. The dashed dark curve corresponds to the Schwarzschild cross section.

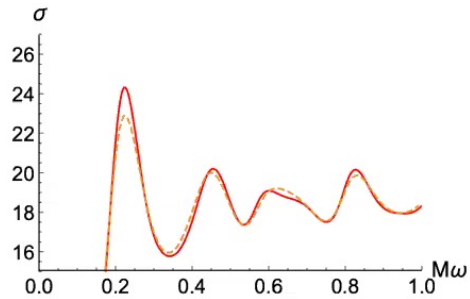
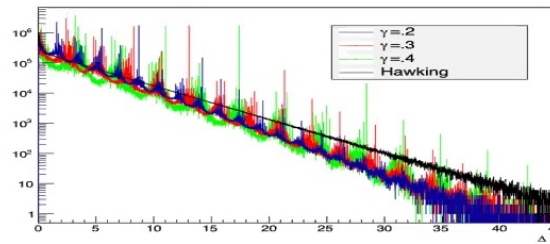
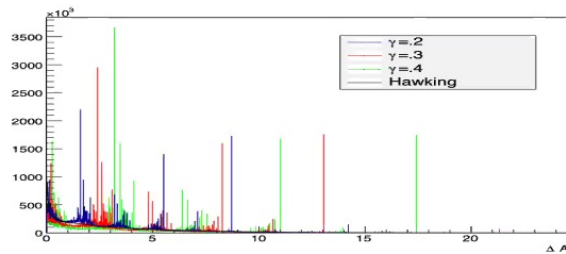
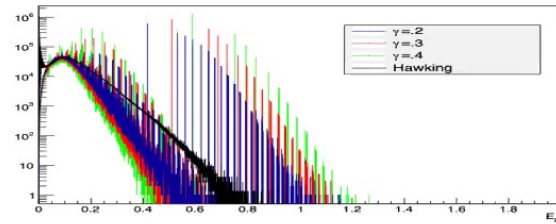
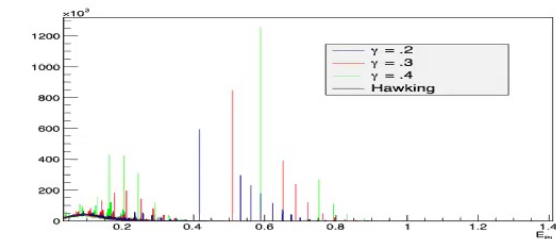


FIG. 4. Emission cross section for a fermionic field, with energy ω , in the background spacetime of a LBH of mass M , for $\epsilon = 10^{-0.3}$. The dashed curved corresponds to $a_0 = 0$ and the plain curve to the usual LQG value, $a_0 = A_{\min}/8\pi = \sqrt{3}\gamma/2$.

The effects are generically small but the trend is quite clear. Phenomenologically, large values of the polymerization parameter could be probed by a decreased cross section, together with a slight frequency shift for fermions. In addition, the non-vanishing minimum area leaves a specific footprint on the first peak.

A.B., Cao, Noui, Perez:



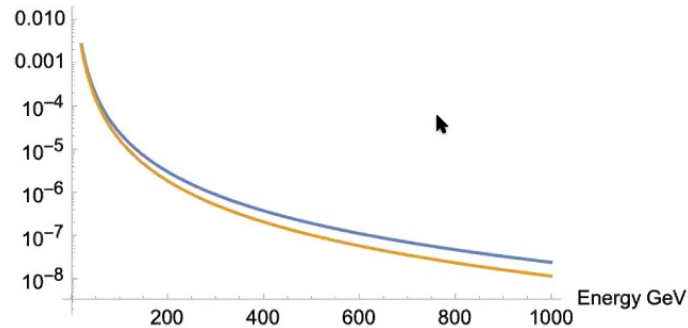
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Evaporation from a local quantum gravity perspective

A.B., Phys. Rev. Lett. 117 (2016) 271301



Spectrum (arbitrary units)



$$A_j = 8\pi\gamma l_{Pl}^2 \sum_{n=1}^N \sqrt{j_n(j_n + 1)},$$

Hawking spectrum characteristics recovered

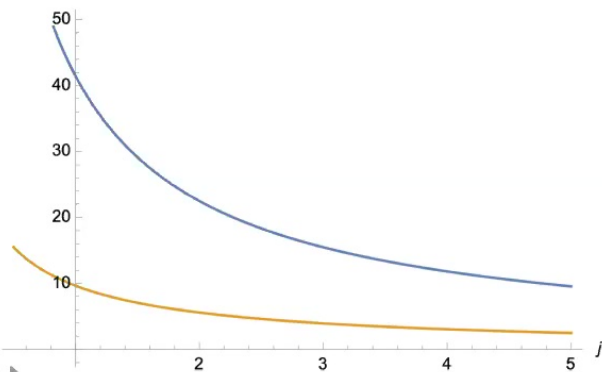
Effect in principle detectable even at arbitrary high masses

The continuous gamma signal does not wash out the effect

The dynamics does not wash out the effect

Specific LQG features could be measured

relative error %



Isopectrality

Moulin, A.B., Gen.Rel.Grav. 52 (2020) no.8, 82



$$ds^2 = B(r)dt^2 - B(r)^{-1}dr^2 - r^2d\Omega^2.$$

$$\frac{1}{r\sqrt{2}}\left(\mathcal{L}_0 + 2\cot\theta\right)\Psi_0 - \left(\mathcal{D}_0 + \frac{4}{r}\right)\Psi_1 = 3\kappa\Psi_2 + [R_1], \quad (52)$$

$$-\frac{B}{2}\left(\mathcal{D}_0^\dagger + \frac{2B'}{B} + \frac{1}{r}\right)\Psi_0 - \frac{1}{r\sqrt{2}}\left(\mathcal{L}_0^\dagger - \cot\theta\right)\Psi_1 = 3\sigma\Psi_2 + [R_2], \quad (53)$$

$$\left(\mathcal{D}_0 + \frac{2}{r}\right)\sigma - \frac{1}{r\sqrt{2}}\left(\mathcal{L}_0^\dagger - \cot\theta\right)\kappa = \Psi_0, \quad (54)$$

$$\left(\mathcal{D}_0 + \frac{1}{r}\right)\Psi_4 - \frac{1}{r\sqrt{2}}\left(\mathcal{L}_0 - \cot\theta\right)\Psi_3 = -3\Psi_2\lambda + [R_3], \quad (55)$$

$$\frac{1}{r\sqrt{2}}\left(\mathcal{L}_0^\dagger + 2\cot\theta\right)\Psi_4 + \frac{B}{2}\left(\mathcal{D}_0^\dagger - \frac{B'}{B} + \frac{4}{r}\right)\Psi_3 = -3\Psi_2\nu + [R_4], \quad (56)$$

$$-\frac{B}{2}\left(\mathcal{D}_0^\dagger - \frac{B'}{B} + \frac{2}{r}\right)\lambda - \frac{1}{r\sqrt{2}}\left(\mathcal{L}_0 - \cot\theta\right)\nu = \Psi_4. \quad (57)$$

$$B(r) = 1 + \frac{D_1}{r} + D_2r^2,$$

$$Y = V^\pm Z^\pm + (W^\pm + 2i\omega)\Lambda_+ Z^\pm,$$

$$\Lambda_- Y = \mp\gamma\frac{B^2}{r^4}Z^\pm + Q\Lambda_+ Z^\pm,$$

$$K^\pm = \kappa \pm 2i\omega\gamma,$$

$$K^\pm Z^\pm = \frac{r^4}{B^2}[QY - (W^\pm + 2i\omega)\Lambda_- Y],$$

$$K^\pm \Lambda_+ Z^\pm = \frac{r^4}{B^2}V^\pm \Lambda_- Y \pm \gamma Y.$$

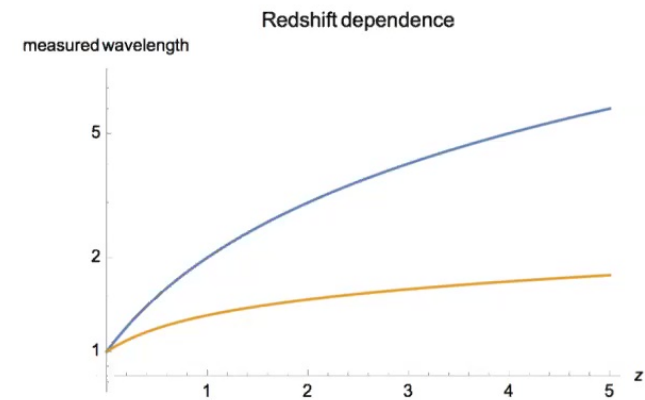
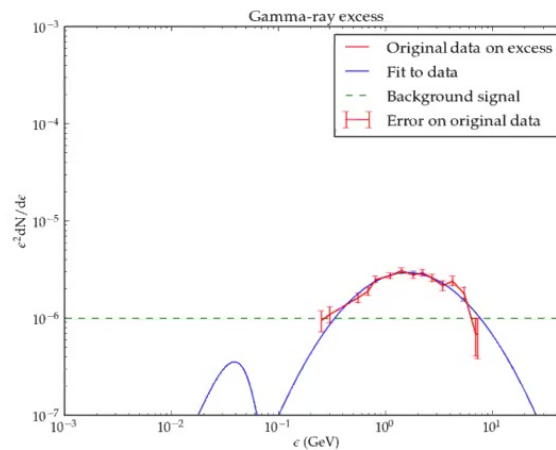
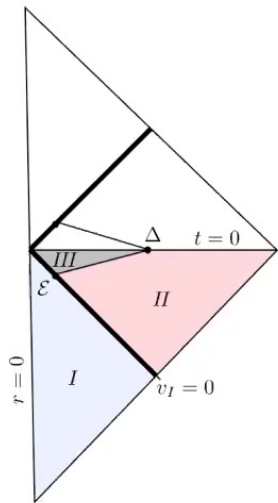


Other approaches : bouncing black holes (Rovelli, Vidotto, Haggard...)

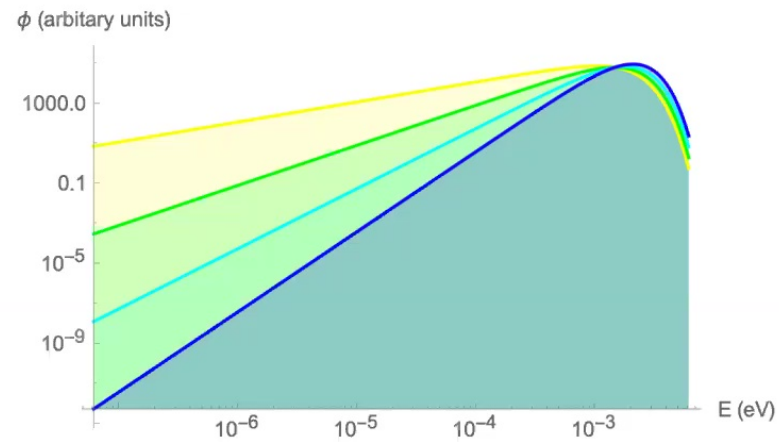
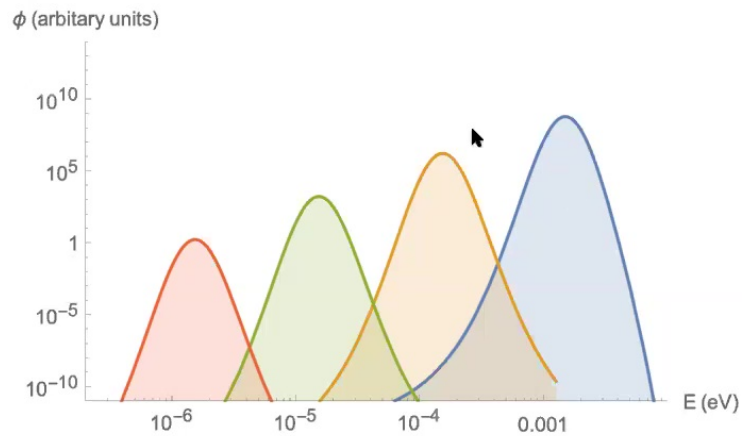
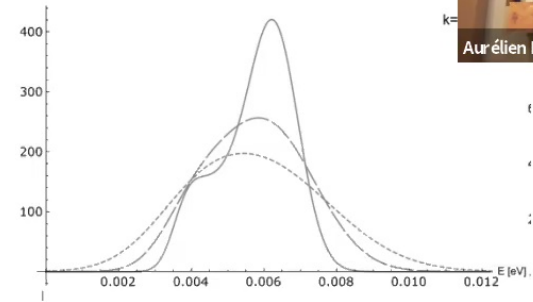
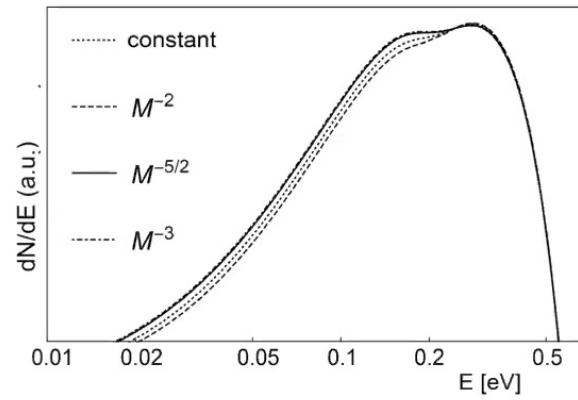
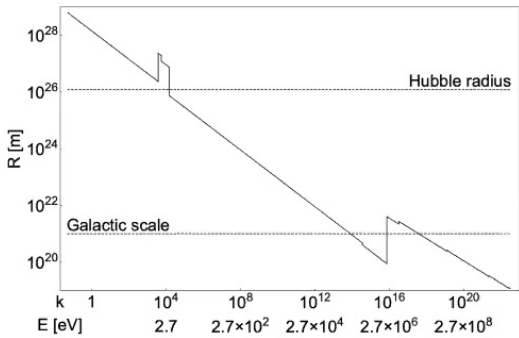
A.B., Moulin, Martineau, Phys.Rev. D97 (2018) no.6, 066019



$$\tau = -8m \ln v_o > \tau_q = 4k \frac{m^2}{l_p}$$



- Two possible origins for the signal
- Redshift dependence remains



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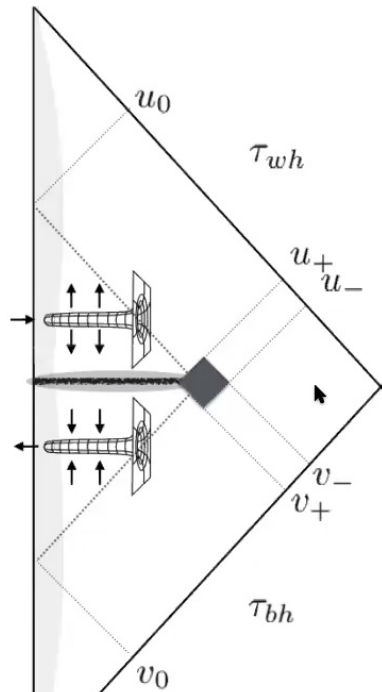
A.B., Moulin, Martineau, Phys.Rev. D97 (2018) no.6, 066019

$$\tau \sim M_i^4.$$



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$$p \sim e^{-S_E/\hbar}$$



White hole dark matter (Rovelli & Vidotto)

- Not possible at the reheating (BBN)

$$\beta' = M_i e^{-N_R} \frac{\Omega_{WH}}{\Omega_r}$$

- Not possible before inflation
- Constrained in a bounce model without inflation, studied in too extreme cases.
- Coalescence

$$dP = \frac{3}{58} \left[- \left(\frac{t}{\frac{3}{170} \alpha^4} \right)^{3/8} + \left(\frac{t}{T \frac{3}{170} \alpha^4} \right)^{3/37} \right] \frac{dt}{t},$$

with

$$\alpha = \left(\frac{1}{\rho_{rel}(z_{eq})} \right)^{1/3},$$

Bianchi, Christodoulou, D'Ambrosio, Rovelli, Haggard