Title: Time's Arrow of a Quantum Superposition of Thermodynamic Evolutions

Speakers: Giulia Rubino

Series: Quantum Foundations

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Abstract: A priori, there exists no preferential temporal direction as microscopic physical laws are time-symmetric. Still, the second law of thermodynamics allows one to associate the 'forward' temporal direction to a positive variation of the total entropy produced in a thermodynamic process, and a negative variation with its 'time-reversal' counterpart.

This definition of a temporal axis is normally considered to apply in both classical and quantum contexts. Yet, quantum physics admits also superpositions between forward and time-reversal processes, thereby seemingly eluding conventional definitions of time's arrow. In this talk, I will demonstrate that a quantum measurement of entropy production can distinguish the two temporal directions, effectively projecting such superpositions of thermodynamic processes onto the forward (time-reversal) time-direction when large positive (negative) values are measured.

Remarkably, for small values (of the order of plus or minus one), the amplitudes of forward and time-reversal processes can interfere, giving rise to entropy-production distributions featuring a more or less reversible process than either of the two components individually, or any classical mixture thereof.

Finally, I will extend these concepts to the case of a thermal machine running in a superposition of the heat engine and the refrigerator mode, illustrating how such interference effects can be employed to reduce undesirable fluctuations.

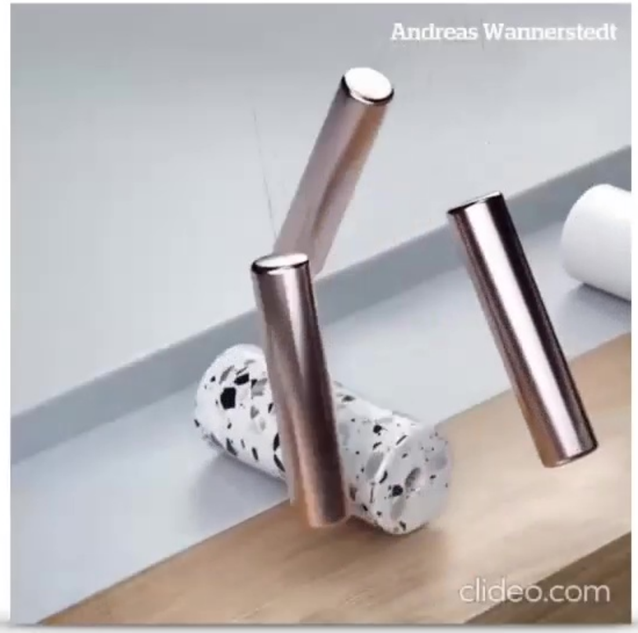
Time's Arrow of a Quantum Superposition of Thermodynamic Evolutions

Quantum Foundations Seminar - Perimeter Institute

Giulia Rubino - November 27th, 2020

Based on: G. Rubino, G. Manzano, C. Brukner, Preprint: arXiv:2008.02818 **[quant-ph]**

- Motivation -



If one wants to establish an arrow of time, one has to look at physical phenomena which are not time-symmetrical

- Motivation -

However, when looking at these phenomena in the macroscopic world, the probability of observing their time-reversal is negligible

- Motivation -

In the microscopic world, the time's arrow can get "blurred" even for physical phenomena able to exhibit a well-defined temporal directionality

- Motivation -

In thermodynamics, the time's arrow is introduced by the second law:

> *The total entropy of the universe can only either increase, or remain constant.*

- Talk Overview -

- Motivation
- Background Information: Fluctuations Theorems
- Effective Projection onto a Definite Time's Arrow
- Interference Effects in the Work Distribution
- Interference of cycles in a SWAP engine
- Conclusion and Outlook

- Fluctuation Theorems -

Suppose that:

- I record the motion of a non-equilibrium thermodynamic process.
- Then I toss a coin. Depending on the outcome I either play the movie as is or its time-reverse.

[1] C. Jarzynski, Annual Review of Condensed Matter Physics **2**, 329 (2011).

- Fluctuation Theorems -

Optimal guessing strategy for a macroscopic system:

If $\langle W \rangle > \Delta F$, the movie proceeds in the right order

If $\langle W \rangle < \Delta F$, the movie proceeds backwards

↑

average work performed on the system by the external driving mechanism

[1] C. Jarzynski, Annual Review of Condensed Matter Physics **2**, 329 (2011).

- Fluctuation Theorems -

Optimal guessing strategy for a macroscopic system:

If $\langle W \rangle > \Delta F$, the movie proceeds in the right order

If $\langle W \rangle < \Delta F$, the movie proceeds backwards

↑ difference in free energies at the beginning and at the end of the movie

[1] C. Jarzynski, Annual Review of Condensed Matter Physics **2**, 329 (2011).

- Fluctuation Theorems -

Optimal guessing strategy for a microscopic system:

Fluctuation Theorems + Bayesian probabilistic reasoning

[1] C. Jarzynski, Annual Review of Condensed Matter Physics **2**, 329 (2011).

- Fluctuation Theorems -

probability that a work W is invested along the forward evolution

In a single shot of the movie

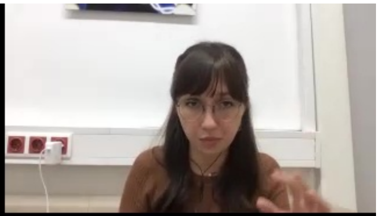
$$\frac{P(+W)}{\tilde{P}(-W)} = e^{\beta W_{\text{diss}}}$$

where $W_{\text{diss}} = W - \Delta F$

probability that a work -W is invested along the backward evolution

[2] G. Bochkov and Y. Kuzovlev, Physica A **106**, 443 (1981).
[3] C. Jarzynski, Phys. Rev. Lett. **78**, 2690 (1997).
[4] G. E. Crooks, Phys. Rev. E **60**, 2721 (1999).

- Fluctuation Theorems -



$\Delta S = \beta W_{\text{diss}}$

Total-entropy-decreasing events ($\beta W_{\text{diss}} < 0$) vanish exponentially with the size of the entropy variation

$$P(\beta W_{\text{diss}} < -\xi) \leq e^{-\xi}$$

in the forward evolution

[5] R. Kawai, *et al.*, Phys. Rev. Lett. **98**, 080602 (2007).
[6] J. M. R. Parrondo, *et al.*, New Journal of Physics **11**, 073008 (2009).

- Fluctuation Theorems -

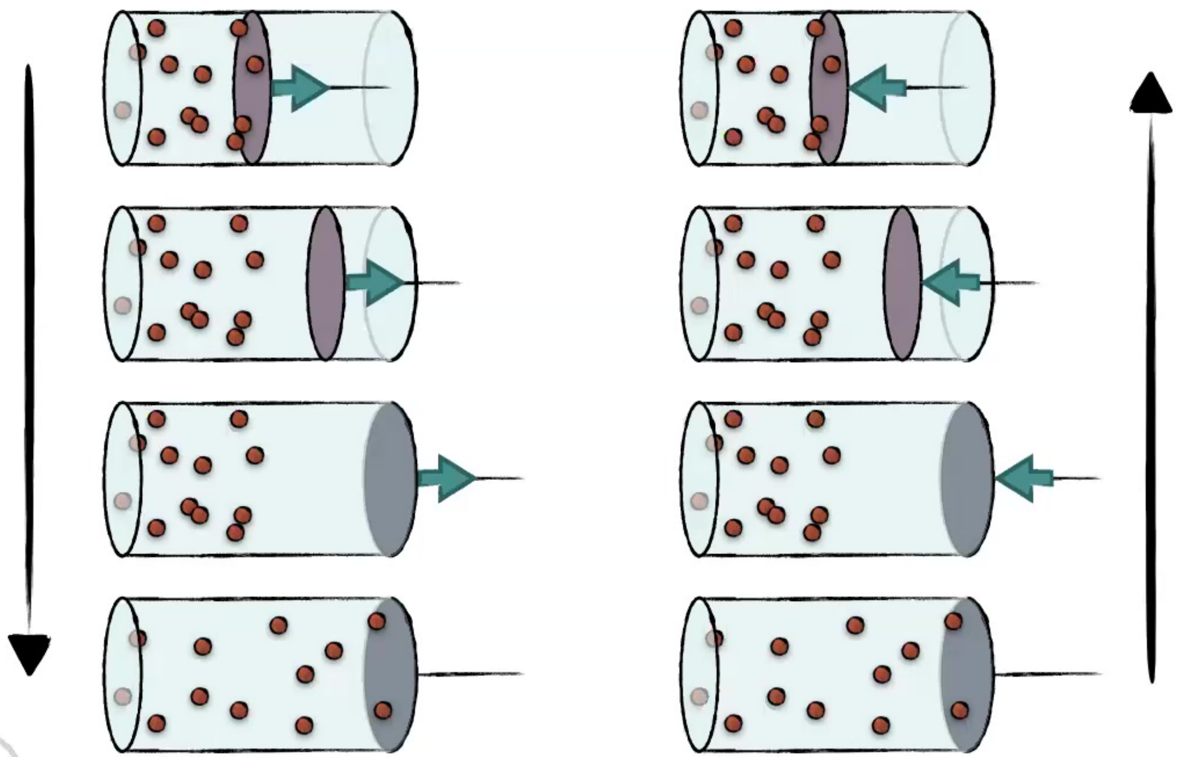
Total-entropy-increasing events ($\beta W_{\text{diss}} > 0$) vanish exponentially with the size of the entropy variation

$$\tilde{P}(\beta W_{\text{diss}} > +\xi) \leq e^{-\xi}$$

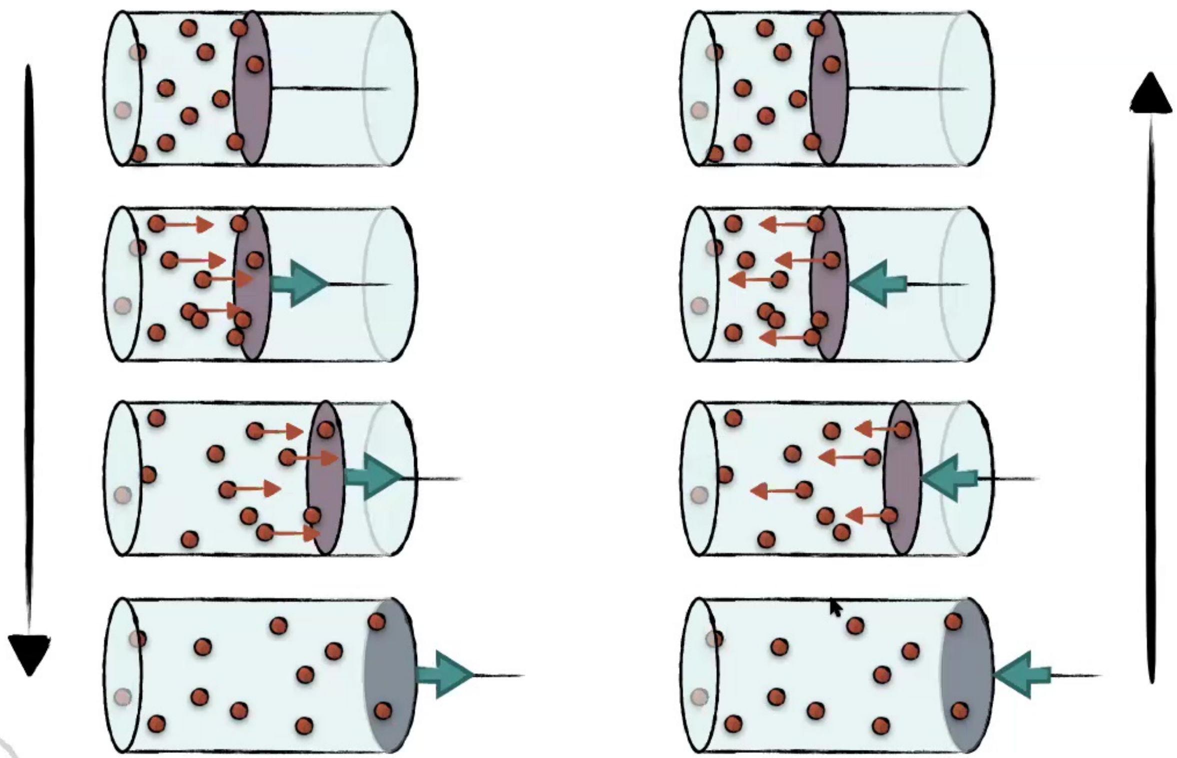
in the backward evolution

[5] R. Kawai, *et al.*, Phys. Rev. Lett. **98**, 080602 (2007).
[6] J. M. R. Parrondo, *et al.*, New Journal of Physics **11**, 073008 (2009).

- Fluctuation Theorems -



- Fluctuation Theorems -



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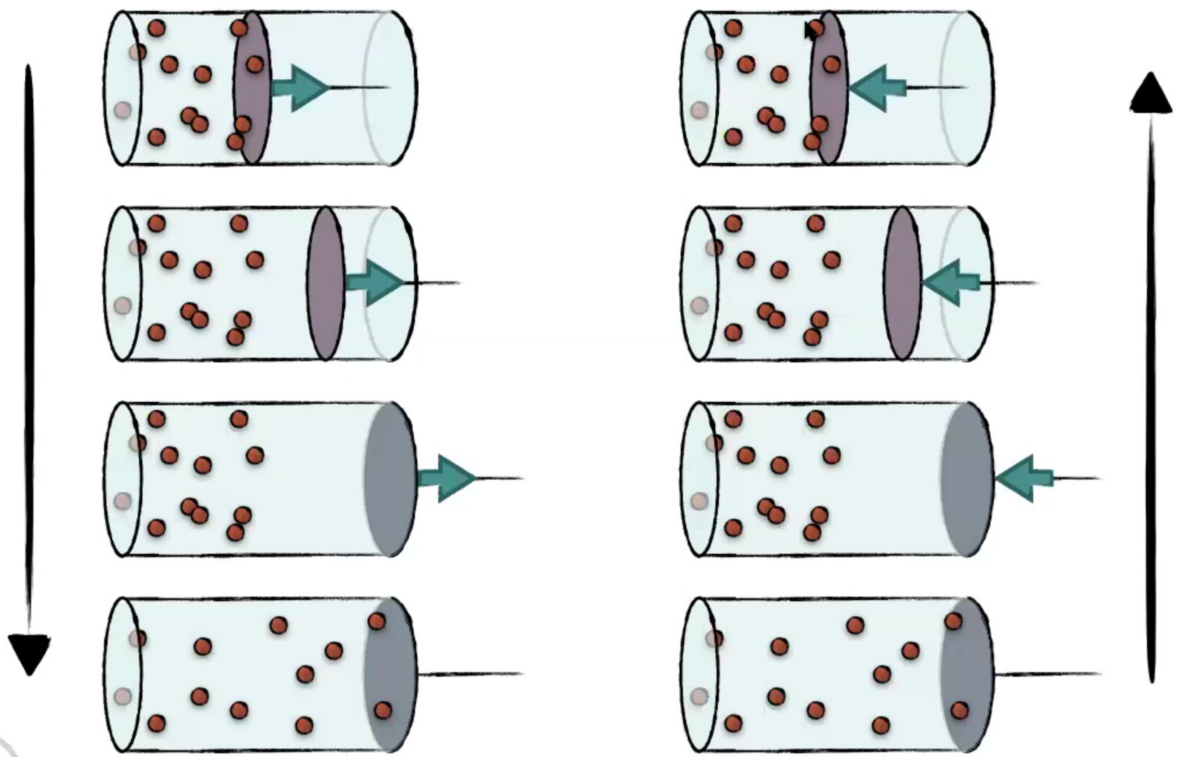
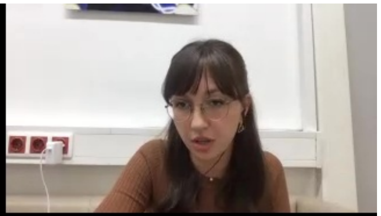
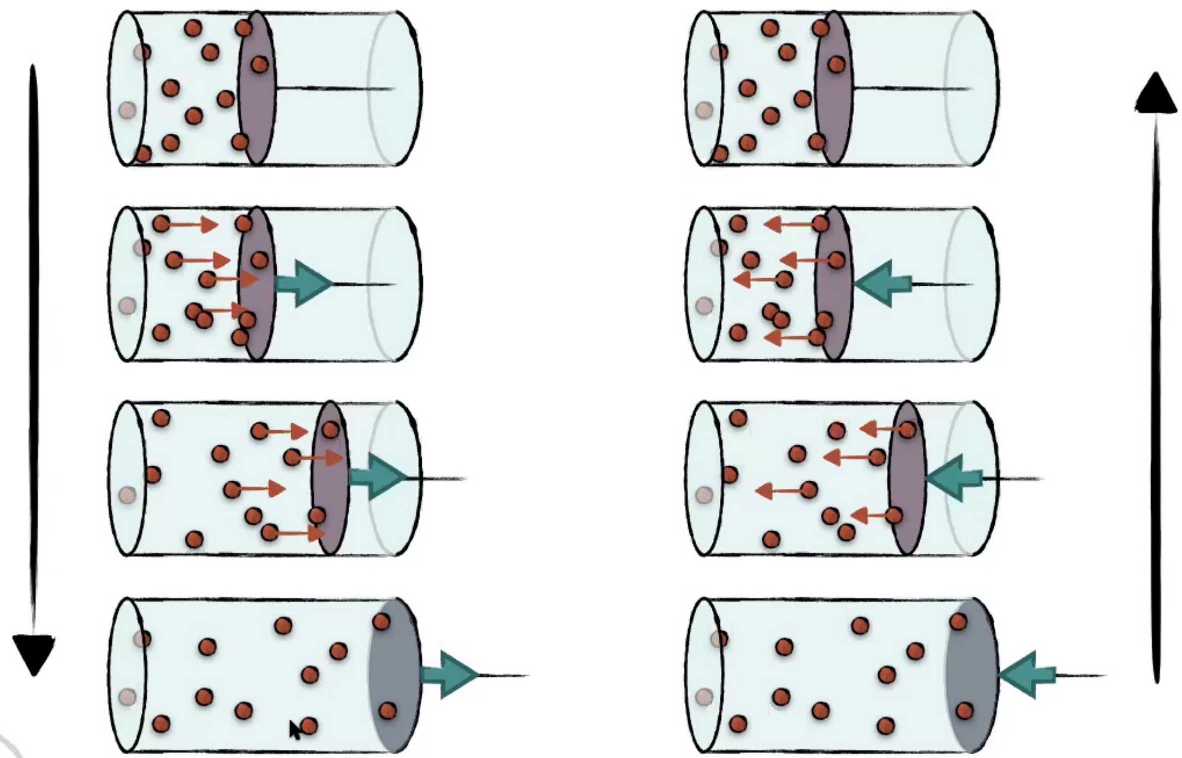




- Fluctuation Theorems -



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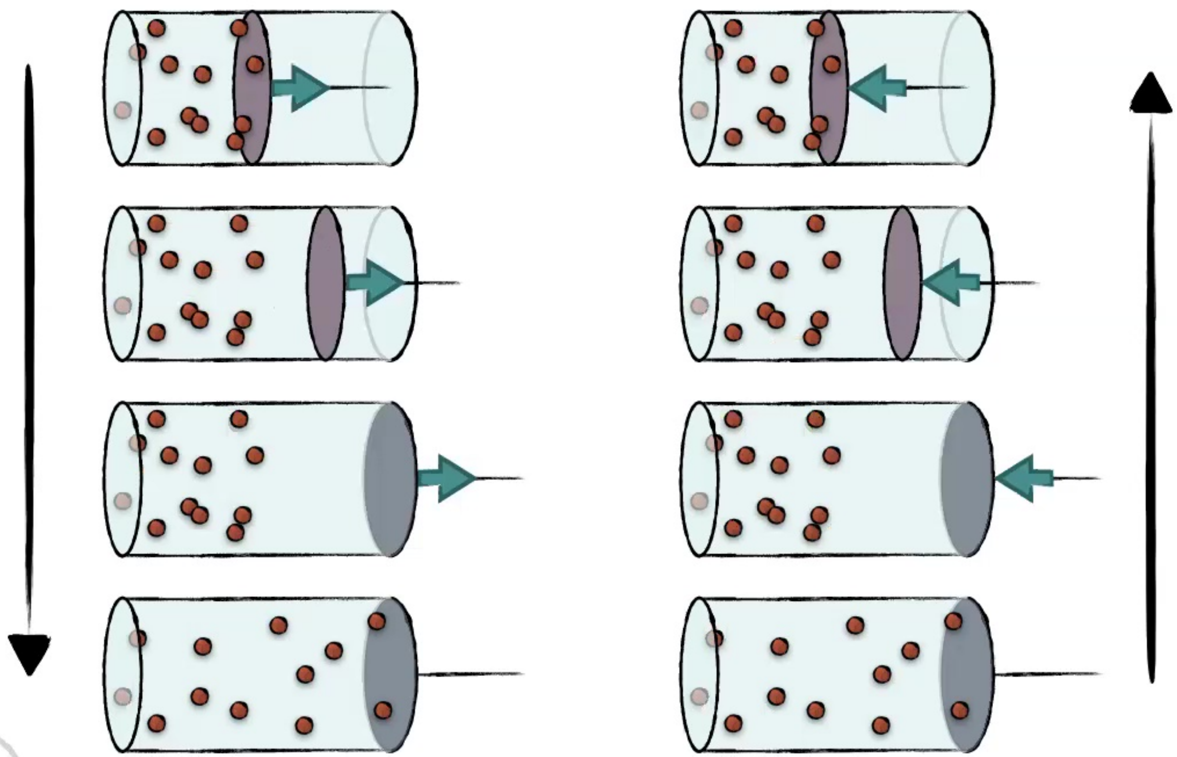

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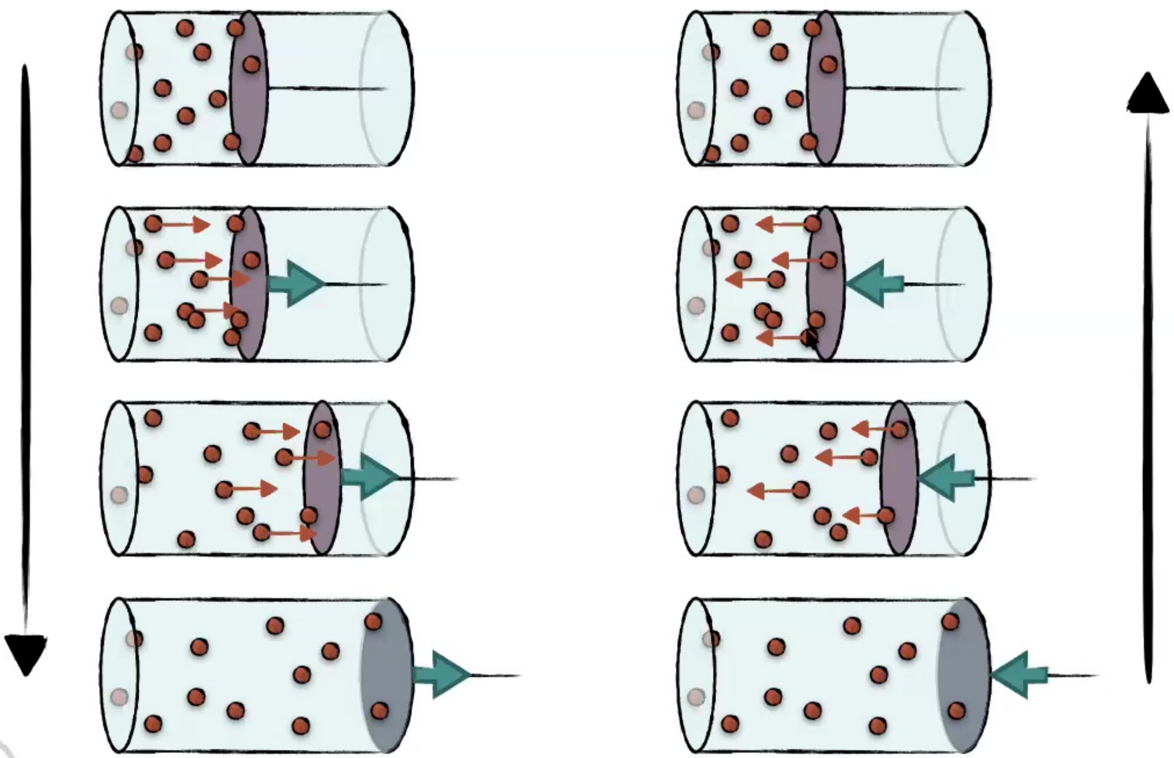
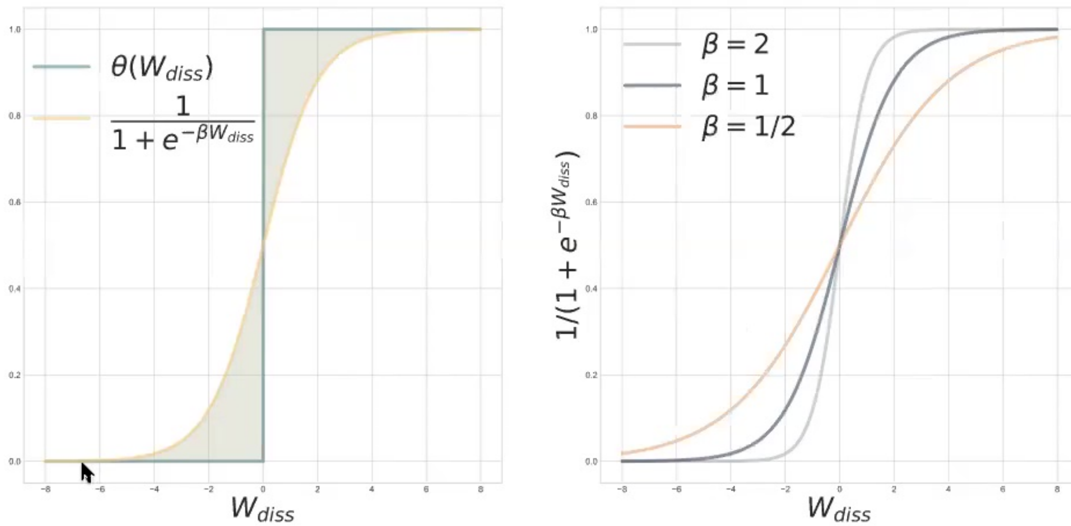


- Fluctuation Theorems -



- Fluctuation Theorems -

Remarks:

1. Here, ‘forward’ and ‘backward’ are interchangeable labels, since each process represents the time-inverted version of the other

- Fluctuation Theorems -

$$\Delta S > 0$$

$$W = E_2 - E_1$$

$$W_{\text{diss}} = W - \Delta F$$

$$\frac{P(+W)}{\tilde{P}(-W)} = e^{\beta W_{\text{diss}}}$$

- Fluctuation Theorems -

$$\Delta S < 0$$

$$\tilde{W} = E_1 - E_2 = -W$$

$$\tilde{W}_{\text{diss}} = \tilde{W} - \Delta\tilde{F}$$

$$\Delta\tilde{F} = -\Delta F$$

$$\frac{\tilde{P}(+\tilde{W})}{P(-\tilde{W})} = e^{\beta \tilde{W}_{\text{diss}}}$$

$$(\Delta\tilde{S} > 0)$$

- Fluctuation Theorems -

Remarks:

2. Considerations on time-inversion assume relevance in the absence of complete time-symmetry

- Fluctuation Theorems -

Remarks:

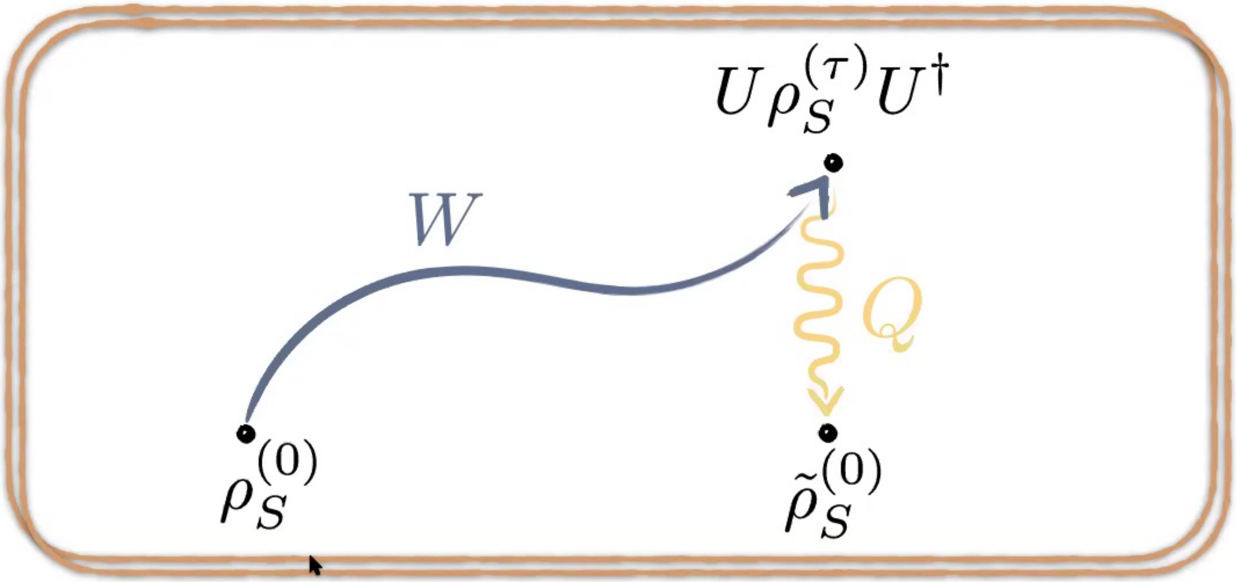
(Complete time-symmetry leads to $\Delta S_{\mathrm{tot}} = 0$ in every single realisation)

- Fluctuation Theorems -

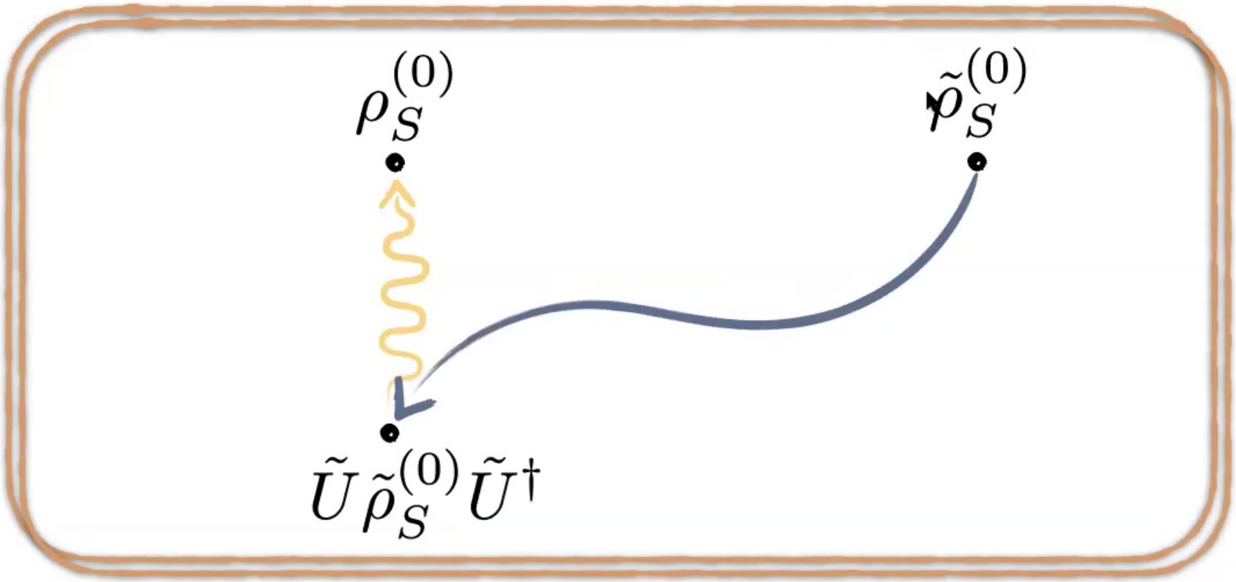
Remarks:

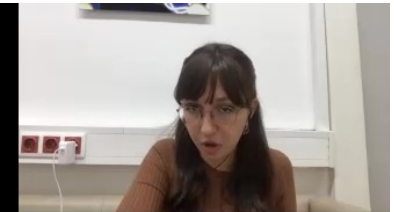
So, in order to exhibit time-asymmetry, the two conjugated processes are assumed to start from equilibrium states

- Fluctuation Theorems -



- Fluctuation Theorems -





- Fluctuation Theorems -

Suppose that:

- I record the motion of a non-equilibrium thermodynamic process.

- Then I toss a coin. Depending on the outcome I either play the movie as is or its time-reverse.

[1] C. Jarzynski, Annual Review of Condensed Matter Physics **2**, 329 (2011).

- Fluctuation Theorems -

To sum up:

- When $\beta W_{\mathrm{diss}} \sim 1$, the directionality of time flow cannot be inferred

- When $\beta |W_{\mathrm{diss}}| \gg 1$, a clear temporal directionality is reestablished

- Effective Projection onto a Definite Time's Arrow -

Definition of the framework

Consider:

- a thermodynamic system S
- an environment E (including the thermal reservoir and, eventually, other degrees of freedom getting entangled with the system)
- an auxiliary system A

- Effective Projection onto a Definite Time's Arrow -

Definition of the framework

The overall initial state is

$$|\Psi_0\rangle_{S,E,A} = \alpha_0 \, |\psi_0\rangle_{S,E} \otimes |0\rangle_A + \alpha_1 \, |\tilde{\psi}_0\rangle_{S,E} \otimes |1\rangle_A$$

where $\alpha_0, \alpha_1 \in \mathbb{C}$, $|\alpha_0|^2 + |\alpha_1|^2 = 1.$

- Effective Projection onto a Definite Time's Arrow -

Definition of the framework

Initial state of the forward process:

$$|\psi_0\rangle_{S,E} = \sum_k \sqrt{\frac{e^{-\beta E_k^{(0)}}}{Z_0}}\, |E_k^{(0)}\rangle_S \, |\varepsilon_k^{(0)}\rangle_E$$

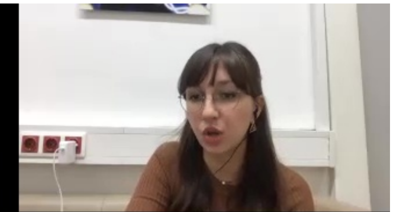
Initial state of the backward process:

$$|\tilde{\psi}_0\rangle_{S,E} = \sum_k \sqrt{\frac{e^{-\beta E_k^{(\tau)}}}{Z_\tau}}\, \Theta |E_k^{(\tau)}\rangle_S \, |\varepsilon_k^{(\tau)}\rangle_E$$

$$Z_{0,\tau} = \mathrm{Tr}\left(e^{-\beta H[\lambda(0,\tau)]}\right)$$

- Effective Projection onto a Definite Time's Arrow -

Definition of the framework



Initial state of the forward process:

$$|\psi_0\rangle_{S,E} = \sum_k \sqrt{\frac{e^{-\beta E_k^{(0)}}}{Z_0}}\, |E_k^{(0)}\rangle_S \, |\varepsilon_k^{(0)}\rangle_E$$

Initial state of the backward process:

$$|\tilde{\psi}_0\rangle_{S,E} = \sum_k \sqrt{\frac{e^{-\beta E_k^{(\tau)}}}{Z_\tau}}\, \Theta |E_k^{(\tau)}\rangle_S \, |\varepsilon_k^{(\tau)}\rangle_E$$

anti-unitary time-reversal operator
(flips the sign of observables with odd parity)

[7] M. Campisi, P. Hänggi, and P. Talkner, Rev. Mod. Phys. **83**, 771 (2011).

- Effective Projection onto a Definite Time's Arrow -

Definition of the framework

The overall initial state is

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where $\alpha_0, \alpha_1 \in \mathbb{C}$, $|\alpha_0|^2 + |\alpha_1|^2 = 1$.

- Effective Projection onto a Definite Time's Arrow -

Definition of the framework

The evolved state at some arbitrary instant of time $t \in [0, \tau]$ is given by

$$|\Psi(t)\rangle_{S,E,A} = \alpha_0 \left[U(t,0) \otimes \mathcal{I}_{E,A}\right] |\psi_0\rangle_{S,E} \otimes |0\rangle_A$$
$$+\alpha_1 | \left[\tilde{U}(t,0) \otimes \mathcal{I}_{E,A}\right] |\tilde{\psi}_0\rangle_{S,E} \otimes |1\rangle_A$$

So, during the the quenches the system does not interact with the environment

After the quench, the system thermalises through the interaction with the thermal reservoir

- Effective Projection onto a Definite Time's Arrow -

The extended two-point measurement (TPM) scheme

In the TPM scheme, work is defined as the energy difference between the initial and final states of the system before and after the thermodynamic process

[7] M. Campisi, P. Hänggi, and P. Talkner, Rev. Mod. Phys. **83**, 771 (2011).

Implementations:

[8] R. Dorner, *et al.*, Phys. Rev. Lett. **110**, 230601 (2013).
[9] L. Mazzola, *et al.*, Phys. Rev. Lett. **110**, 230602 (2013).
[10] T. B. Batalhao, *et al.*, Phys. Rev. Lett. **113**, 140601 (2014).
[11] A. J. Roncaglia, *et al.*, Phys. Rev. Lett. **113**, 250601 (2014).
[12] G. D. Chiara, *et al.*, New Journal of Physics **17**, 035004 (2015).

- Effective Projection onto a Definite Time's Arrow -

The extended two-point measurement (TPM) scheme

This allows one to compute the stochastic work invested in a single realisation of the protocol:

$$P(W) = \sum_{n,m} p_{n,m} \cdot \delta(W - W_{n,m})$$

(for the forward process)

$p_{n,m} = p_{m|n}\, p_n^{(0)}$, where $p_n^{(0)} = \dfrac{e^{-\beta E_n^{(0)}}}{Z_0}$ and

$$p_{m|n} = \left| \langle E_m^{(\tau)} | \, U(\tau, 0) | E_n^{(0)} \rangle \right|^2$$

- Effective Projection onto a Definite Time's Arrow -

The extended two-point measurement (TPM) scheme

This allows one to compute the stochastic work invested in a single realisation of the protocol:

$$P(W) = \sum_{n,m} p_{n,m} \cdot \delta(W - W_{n,m})$$

(for the forward process)

$$\ln\left(\frac{p_{n,m}}{\tilde{p}_{n,m}}\right) = \Delta S_{n,m} = \beta(W_{n,m} - \Delta F)$$

- Effective Projection onto a Definite Time's Arrow -

The extended two-point measurement (TPM) scheme

This allows one to compute the stochastic work invested in a single realisation of the protocol:

$$\tilde{P}(W) = \sum_{n,m} \tilde{p}_{n,m} \cdot \delta(W - \tilde{W}_{n,m})$$

(for the backward process)

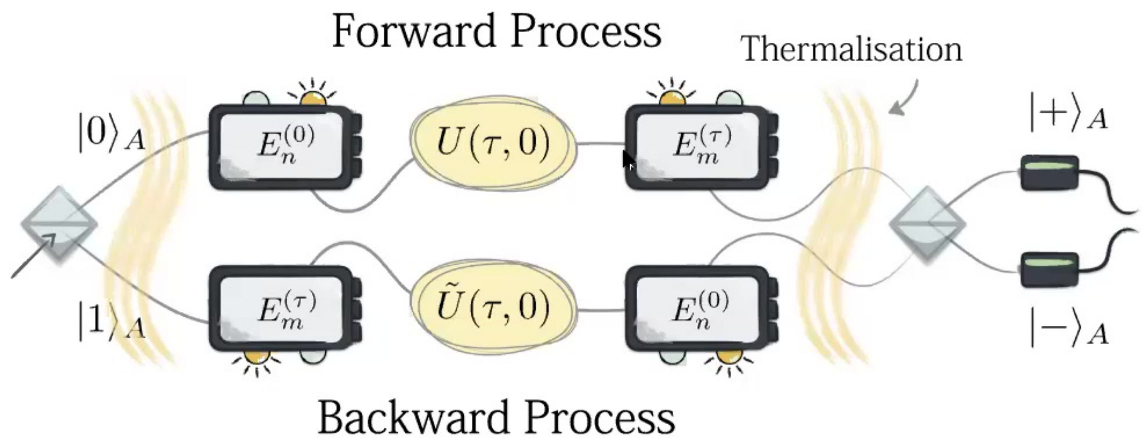
$$\tilde{W}_{n,m} = -W_{n,m}$$

[7] M. Campisi, P. Hänggi, and P. Talkner, Rev. Mod. Phys. **83**, 771 (2011).

- Effective Projection onto a Definite Time's Arrow -

The extended two-point measurement (TPM) scheme

Extension of the TPM scheme (we include energy measurements at $t = 0$ and $t = \tau$ in both branches of the superposition)



- Effective Projection onto a Definite Time's Arrow -

The extended two-point measurement (TPM) scheme

Extension of the TPM scheme (we include energy measurements at $t = 0$ and $t = \tau$ in both branches of the superposition)

$$M_W = \sum_{n,m} \Big[|E_m^{(\tau)}\rangle\langle E_m^{(\tau)}| U(\tau,0) |E_n^{(0)}\rangle\langle E_n^{(0)}| \otimes \mathcal{I}_E \otimes |0\rangle\langle 0|_A$$

$$+ \Theta |E_n^{(0)}\rangle\langle E_n^{(0)}| \Theta^\dagger \tilde{U}(\tau,0) \Theta |E_m^{(\tau)}\rangle\langle E_m^{(\tau)}| \Theta^\dagger \otimes \mathcal{I}_E \otimes |1\rangle\langle 1|_A \Big]$$

$$\cdot \, \delta\big(W - W_{n,m}\big)$$

(the measurements must preserve the coherence between the forward and backward processes)

[13] G. Rubino, *et al.*, Science Advances 3 (2017).

- Effective Projection onto a Definite Time's Arrow -

The extended two-point measurement (TPM) scheme

After M_W, we project the auxiliary qubit onto an arbitrary state $|\xi\rangle_A$

$$|\Psi_W^{\xi}\rangle_{S,E,A} \equiv (\mathcal{I}_{S,E} \otimes |\xi\rangle\langle\xi|_A) \circ M_W |\Psi_0\rangle_{S,E,A}$$

The joint probability of measuring the work W and projecting the auxiliary state onto $|\xi\rangle_A$ is

$$\mathcal{P}(\xi, W) = \left|\left| |\Psi_W^{\xi}\rangle_{S,E,A} \right|\right|^2$$

- Effective Projection onto a Definite Time's Arrow -

The extended two-point measurement (TPM) scheme

Let me call:

- $\mathcal{P}_\xi(W) := \mathcal{P}(W|\xi) = \mathcal{P}(\xi, W)/\mathcal{P}(\xi)$
  with $\mathcal{P}(\xi) = \int dW\, \mathcal{P}(\xi, W)$
- $q_0^\xi = |\alpha_0|^2 \big|\langle \xi | 0 \rangle\big|^2 / \mathcal{P}(\xi)$
- $q_1^\xi = |\alpha_1|^2 \big|\langle \xi | 1 \rangle\big|^2 / \mathcal{P}(\xi)$

- Effective Projection onto a Definite Time's Arrow -

The extended two-point measurement (TPM) scheme

We obtain

$$\mathcal{P}_\xi(W) = q_0^\xi \; P(W) + q_1^\xi \; \tilde{P}(-W) + 2\,\mathbb{R}\big(I_\xi(W)\big)$$

where

$$I_\xi(W) = \frac{\alpha_0^* \alpha_1 \langle 0|\xi\rangle\langle\xi|1\rangle}{\mathcal{P}(\xi)} \sum_{n,m} \sum_{n',m'} \sqrt{p_{n,m}\, p_{n',m'}}\, e^{-\frac{\beta}{2}(W_{n',m'} - \Delta F)}$$
$$e^{-i(\Phi_{n,m} + \Phi_{n',m'})} \langle E_m^{(\tau)}|\Theta|E_{n'}^{(0)}\rangle \langle \varepsilon_n^{(0)}|\varepsilon_{m'}^{(\tau)}\rangle$$
$$\cdot\delta(W - W_{n,m})\, \delta(W - W_{n',m'})$$

- Effective Projection onto a Definite Time's Arrow -

The extended two-point measurement (TPM) scheme

We obtain

$$\mathcal{P}_\xi(W) = q_0^\xi \; P(W) + q_1^\xi \; \tilde{P}(-W) + 2\,\mathbb{R}\big(I_\xi(W)\big)$$

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- Effective Projection onto a Definite Time's Arrow -

The extended two-point measurement (TPM) scheme

We obtain

$$\mathcal{P}_\xi(W) = \underbrace{q_0^\xi \; P(W) + q_1^\xi \; \tilde{P}(-W)} + 2\,\mathbb{R}\big(I_\xi(W)\big)$$

"incoherent" part: compatible with running the process in one or the other time's direction with a given probability

- Effective Projection onto a Definite Time's Arrow -

The extended two-point measurement (TPM) scheme

We obtain

$$\mathcal{P}_\xi(W) = q_0^\xi \; P(W) + q_1^\xi \; \tilde{P}(-W) + \underbrace{2\,\mathbb{R}\big(I_\xi(W)\big)}$$

"coherent" part: quantum feature arising from the superposition of the two temporal directions

- Effective Projection onto a Definite Time's Arrow -

For $\Delta S = \beta W_{\rm diss} \gg 1$

$$\mathcal{P}_\xi(W) = P(W)\big(q_0^\xi + q_1^\xi e^{-\beta W_{\rm diss}}\big) + 2\,\mathbb{R}\big(I_\xi(W)\big)$$
$$\approx q_0^\xi \; P(W)$$

i.e., the state of the system is projected onto the forward component of the quantum superposition
(without having measured the auxiliary qubit)

- Effective Projection onto a Definite Time's Arrow -

For $\Delta S = \beta W_{\text{diss}} \ll -1$

$$\mathcal{P}_\xi(W) = \tilde{P}(-W)\big(q_0^\xi e^{\beta W_{\text{diss}}} + q_1^\xi\big) + 2\,\mathbb{R}\big(I_\xi(W)\big)$$
$$\approx q_1^\xi\ \tilde{P}(-W)$$

i.e., the state of the system is projected onto the backward component of the quantum superposition

(where we used the fact that $\tilde{p}_{n,m} = p_{n,m}\, e^{-\beta(W_{n,m} - \Delta F)}$)

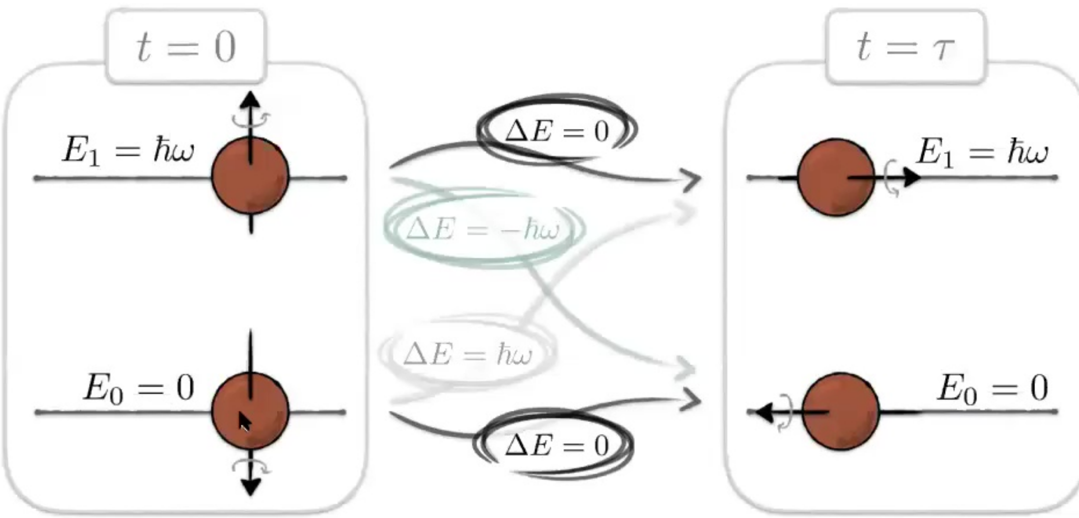
- Interference Effects in the Work Distribution -

Consider a spin-1/2 system subjected to a magnetic field whose direction is rotating within the *x* - *z* plane at constant angular velocity Ω around the *y*-axis

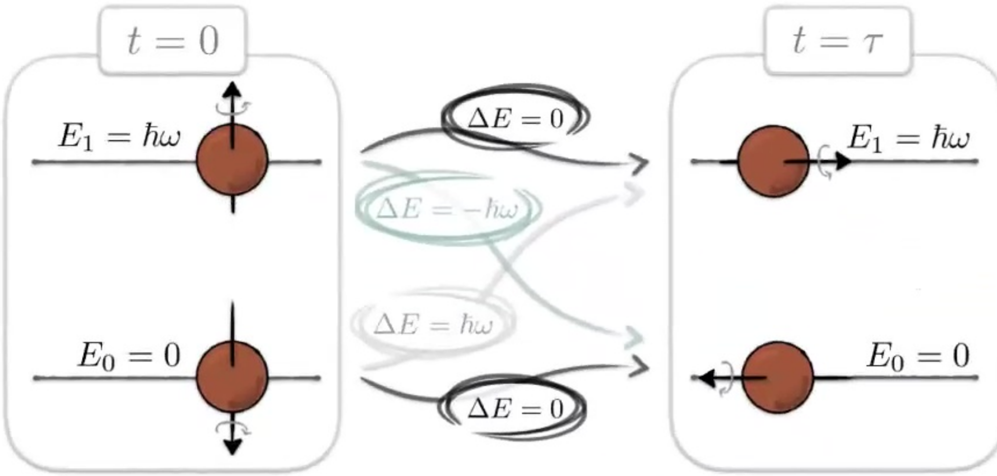
$$H(\Omega t) = \frac{\hbar\omega}{2}\left[\mathcal{I} + \cos(\Omega t)\,\sigma_z + \sin(\Omega t)\,\sigma_x\right]$$

- Interference Effects in the Work Distribution -

Consider a spin-1/2 system subjected to a magnetic field whose direction is rotating within the *x* - *z* plane at constant angular velocity Ω around the *y*-axis



- Interference Effects in the Work Distribution -

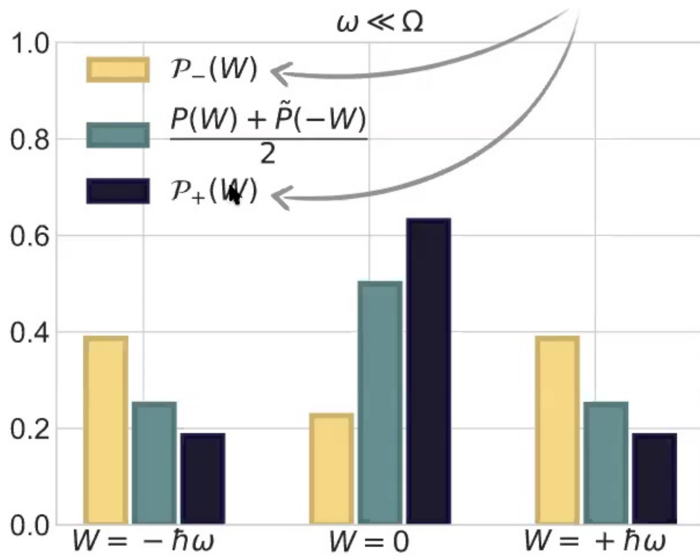


We superpose the forward and backward quenches, and we project the auxiliary system onto the diagonal basis $\left\{|\pm\rangle_A = (|0\rangle_A \pm |1\rangle_A)/\sqrt{2}\right\}$

- Interference Effects in the Work Distribution -

Limit of a rapid quench (and hence of a large degree of irreversibility)

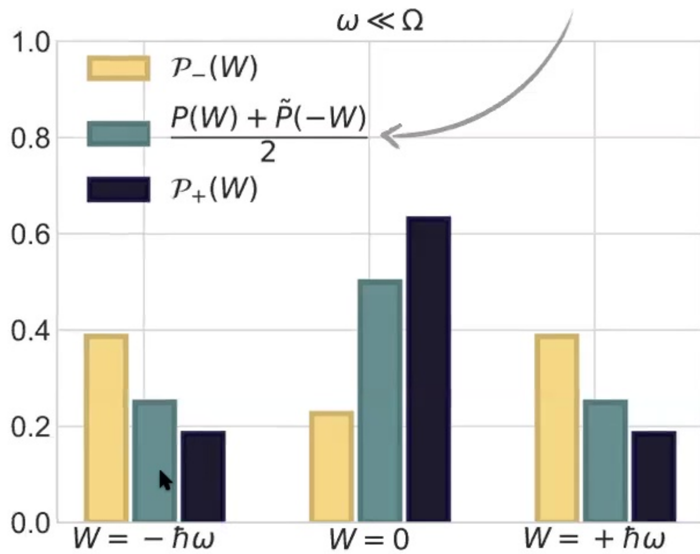
Work probability distributions from the extended TPM scheme



- Interference Effects in the Work Distribution -

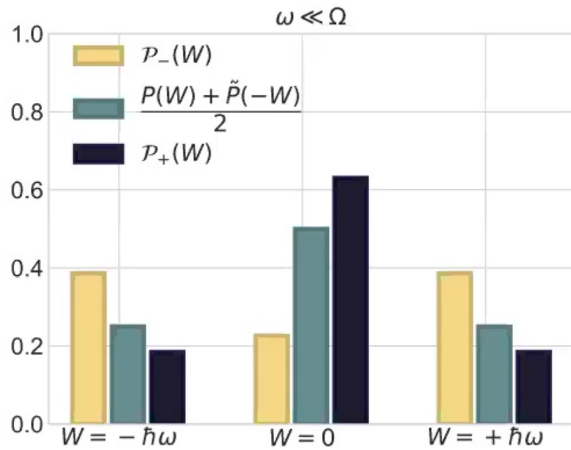
Limit of a rapid quench (and hence of a large degree of irreversibility)

classical mixture of the forward and backward processes



- Interference Effects in the Work Distribution -

Limit of a rapid quench



Here reversibility = adiabaticity, so in the post-selected case, we see the probability distribution of a slower quench

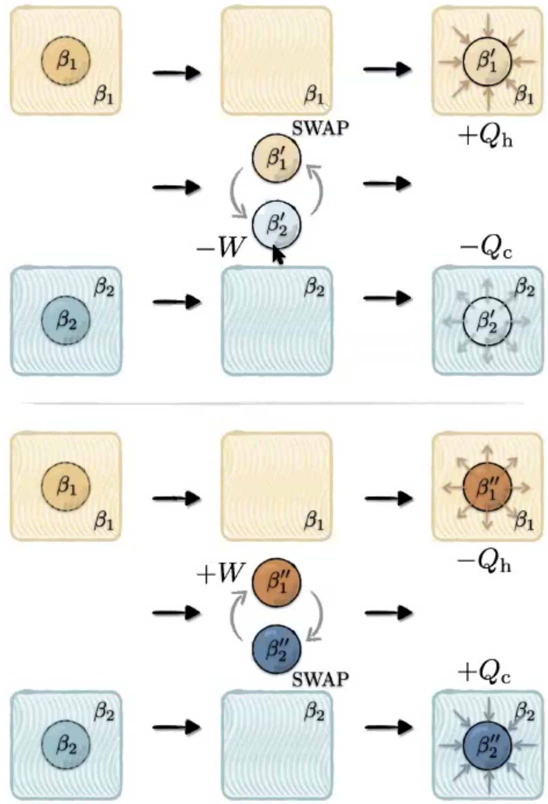
- Interference of cycles in a SWAP engine -

We can also apply the extended TPM scheme to a pair of generic processes, not necessarily temporally related

Let us consider, for instance, two different modes of operation of a thermal machine: a heat engine and a refrigerator

- Interference of cycles in a SWAP engine -

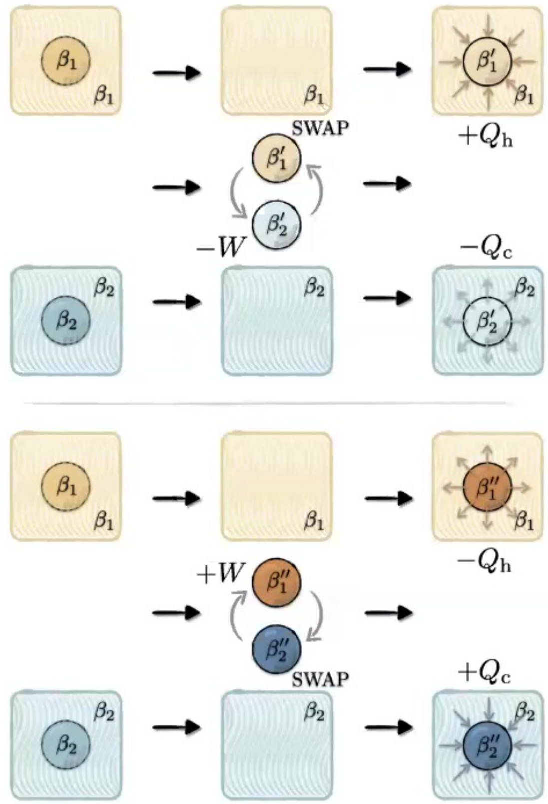
Cyclic SWAP engine with two qubits and two thermal reservoirs at different temperatures

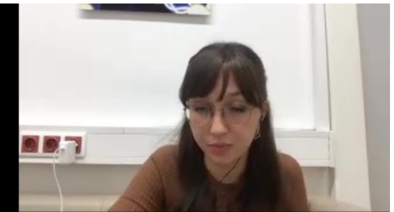


- Interference of cycles in a SWAP engine -

Heat engine: extracts work $W < 0$ out of a heat current from the hot to the cold reservoir $Q_c < 0$

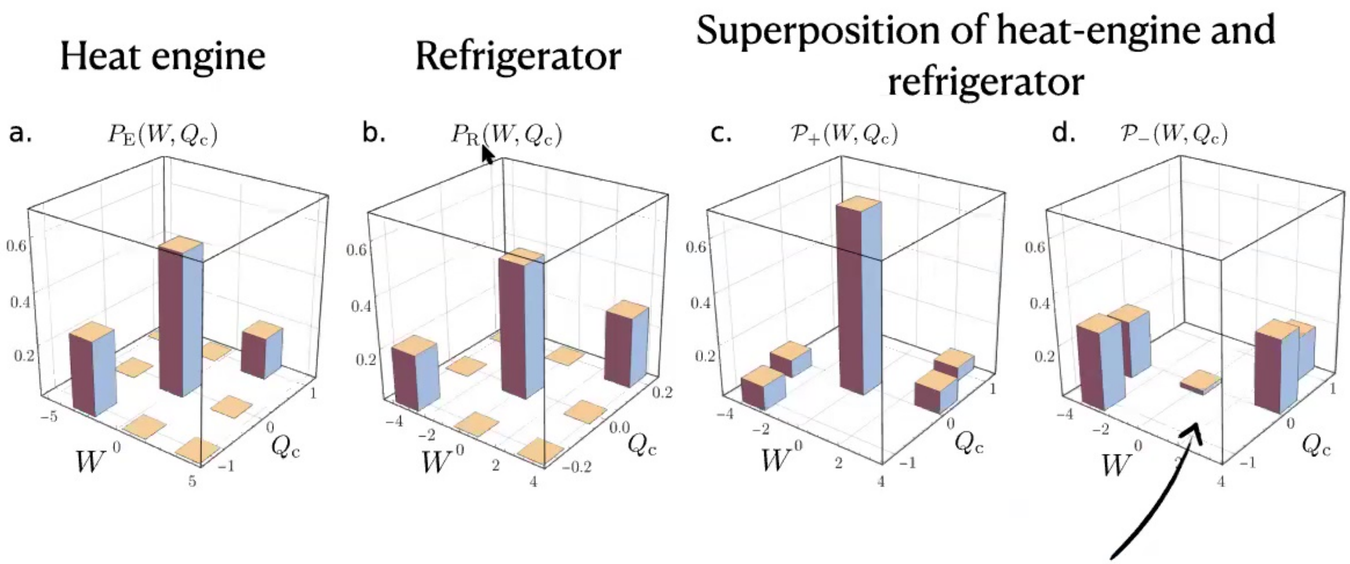
Power-driven refrigerator: extracts heat from the cold reservoir $Q_c > 0$ at the price of an input work $W > 0$





- Interference of cycles in a SWAP engine -

Joint probability distributions of performing work W and absorbing heat Q_c



The interference effect can diminish the probability that the machine fails to perform either of the two tasks

- Conclusion and Outlook -

Questions I have addressed within this talk:

1) How a definite (thermodynamic) arrow of time can emerge from arbitrary superpositions of forward and backward processes?

2) Can forward and backward processes interfere and what is the signature of this?

Future directions

- Conclusion and Outlook -

Questions I have addressed within this talk:

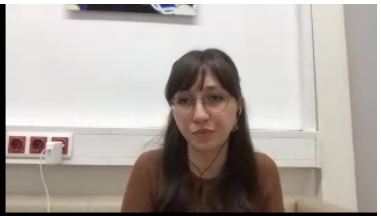
1) How a definite (thermodynamic) arrow of time can emerge from arbitrary superpositions of forward and backward processes?

Observing a large increase (decrease) of dissipative work effectively projects the system in the forward (backward) temporal direction

2) Can forward and backward processes interfere and what is the signature of this?

Future directions

- Conclusion and Outlook -



Questions I have addressed within this talk:

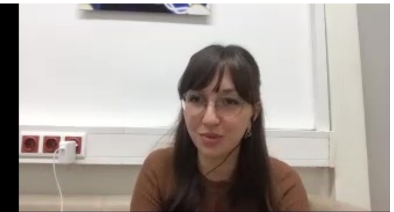
1) How a definite (thermodynamic) arrow of time can emerge from arbitrary superpositions of forward and backward processes?

2) Can forward and backward processes interfere and what is the signature of this?

- They can interfere for small values of the dissipative work
- The quantum superposition between the two irreversible processes can result in a dynamics which is no longer irreversible
- Similar techniques can be applied to reduce undesirable fluctuations in the performance of a thermal machine

Future directions

- Conclusion and Outlook -



Questions I have addressed within this talk:

1) How a definite (thermodynamic) arrow of time can emerge from arbitrary superpositions of forward and backward processes?

2) Can forward and backward processes interfere and what is the signature of this?

Future directions

- Investigate superpositions of more complex quantum thermal devices
- Applying these findings to other sort of entropies (*e.g.*, coarse-grained entropy)

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