Title: Emergent classicality for large channels and states

Speakers: Daniel Ranard

Series: Perimeter Institute Quantum Discussions

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Abstract: In a quantum measurement process, classical information about the measured system spreads through the environment. In contrast, quantum information about the system becomes inaccessible to local observers. In this talk, I will present a result about quantum channels indicating that an aspect of this phenomenon is completely general. We show that for any evolution of the system and environment, for everywhere in the environment excluding an O(1)-sized region we call the "quantum Markov blanket," any locally accessible information about the system must be approximately classical, i.e. obtainable from some fixed measurement. The result strengthens the earlier result of arXiv:1310.8640 in which the excluded region was allowed to grow with total environment size. I will also discuss applications to many-body physics.

Emergent classicality and a bound on the spread of quantum information

Speaker: Daniel Ranard

Joint work with Xiao-Liang Qi

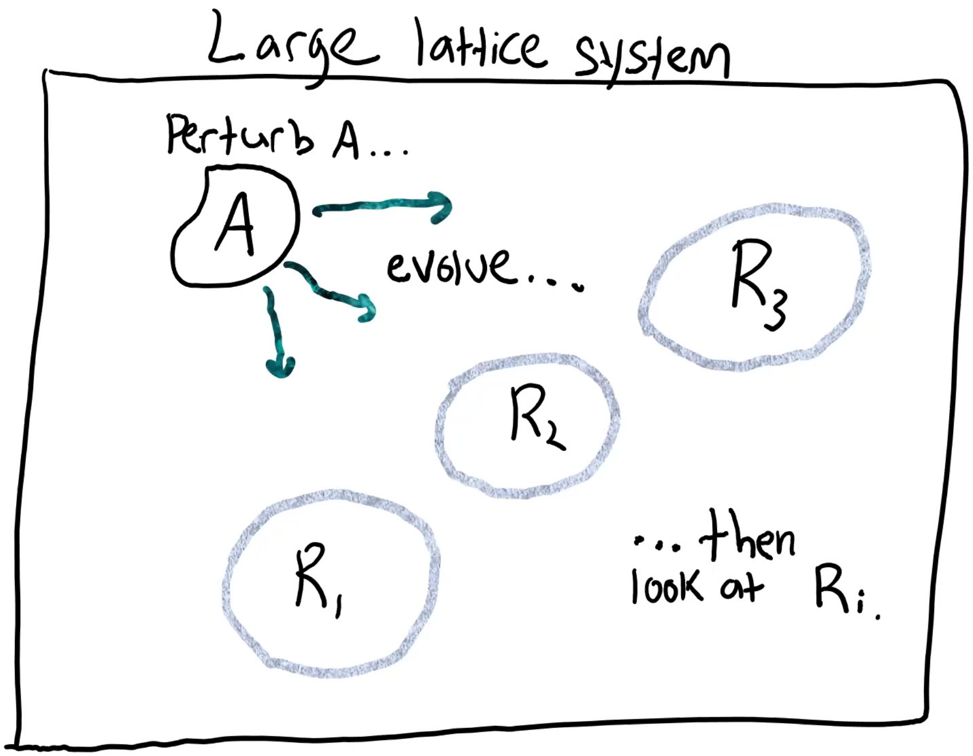
arXiv: 2001.01507

Builds on: Brandão, Piani, Horodecki (2015, *Nat. comm. 6:7908)*

Outline

- Examples of information spreading in different systems
- General constraint on spreading in *all* systems
- Precise theorem statement
- Implications for many-body systems

How can information spread during evolution?



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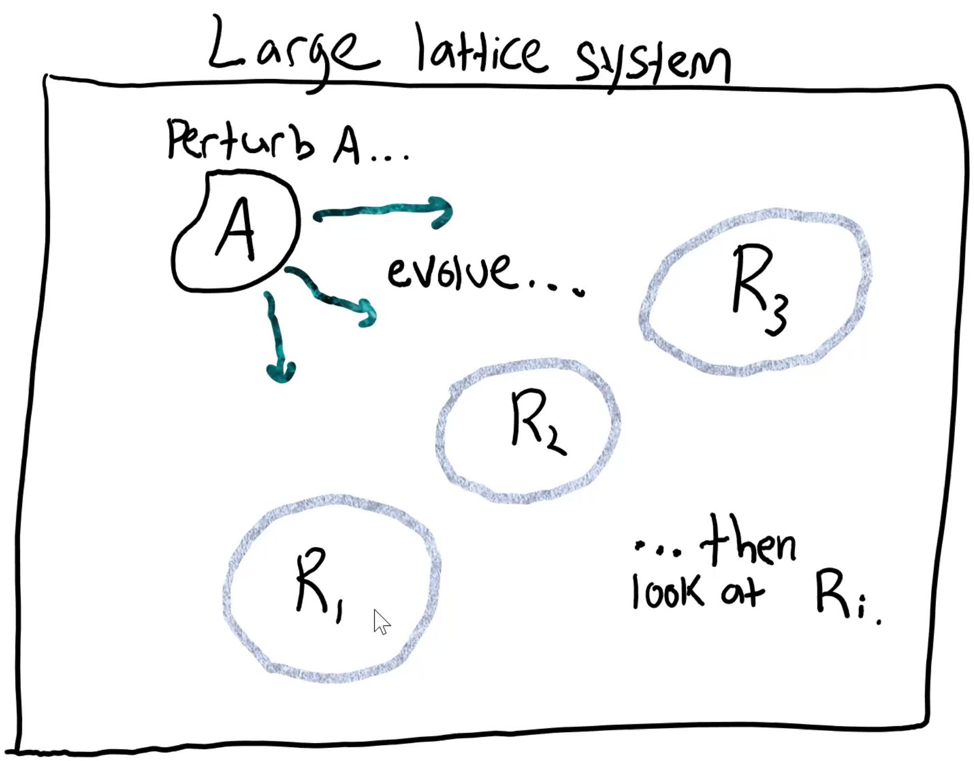

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Example situations:

- Global thermalization

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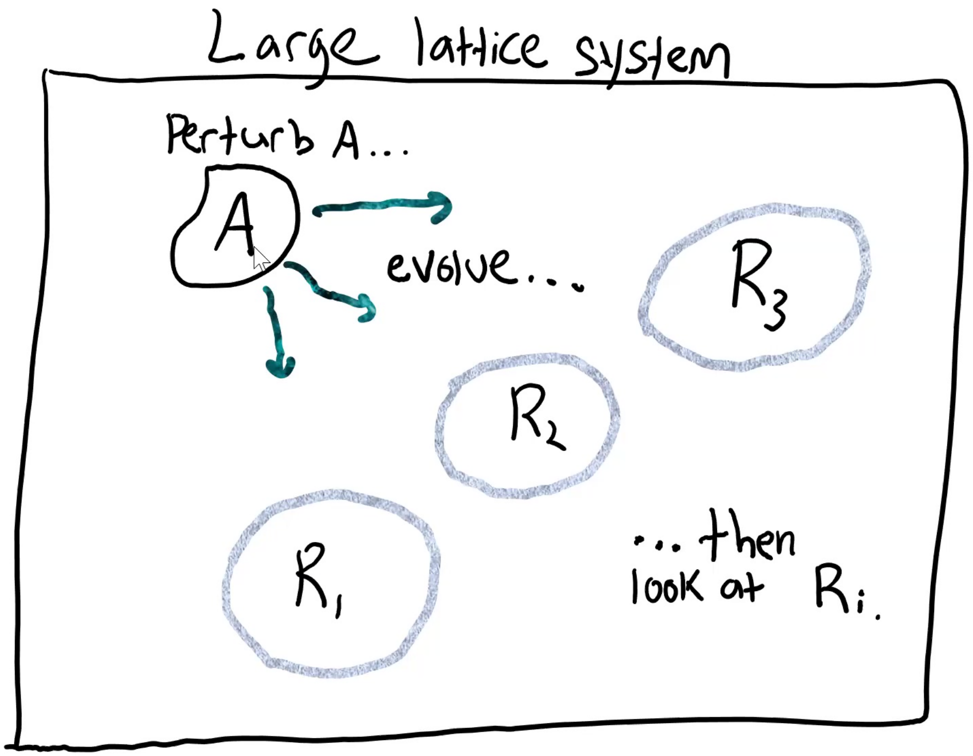

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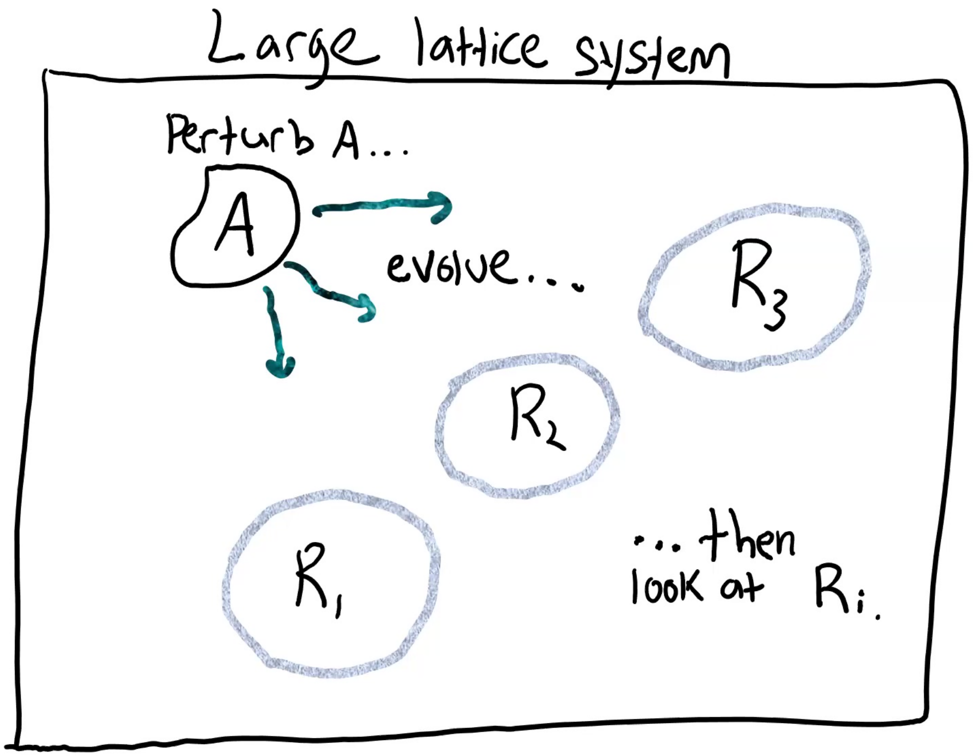

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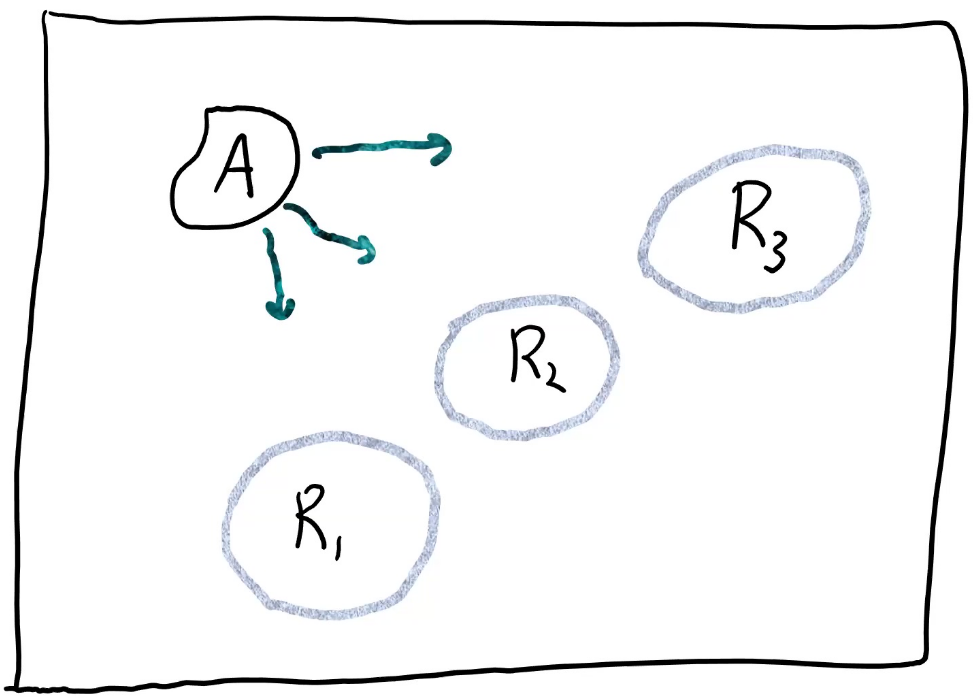
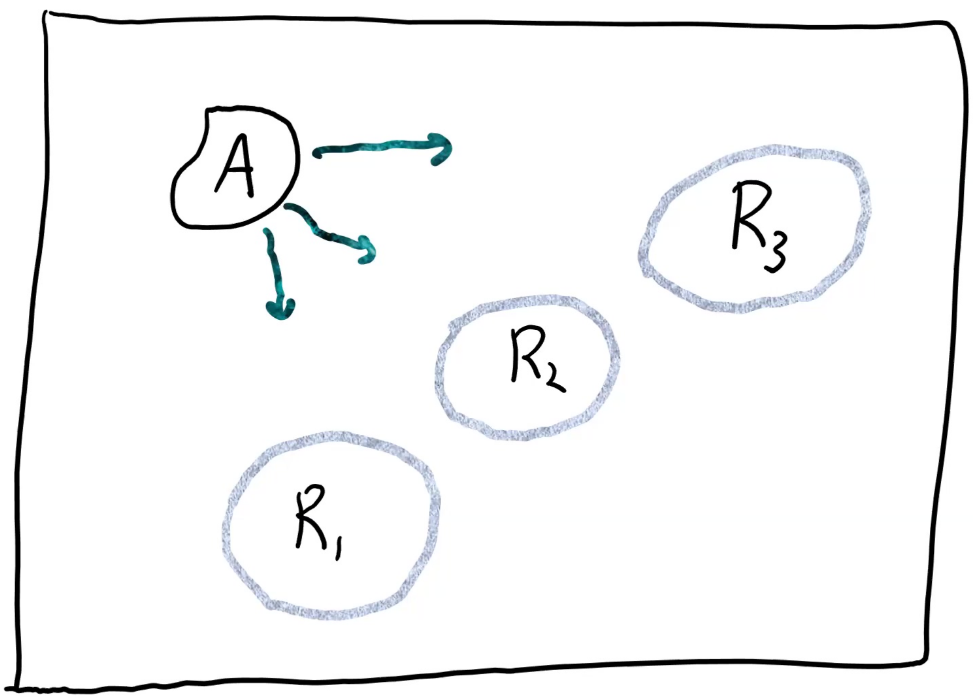
Decoherence example



A in superposition

$$|\psi_0\rangle = \underbrace{(c_0|0\rangle_A + c_1|1\rangle_A)}|0\rangle_{R_1}|0\rangle_{R_2}|0\rangle_{R_3}$$

Decoherence example



A in superposition

$$|\psi_0\rangle = \underbrace{(c_0|0\rangle_A + c_1|1\rangle_A)}|0\rangle_{R_1}|0\rangle_{R_2}|0\rangle_{R_3}$$

evolve

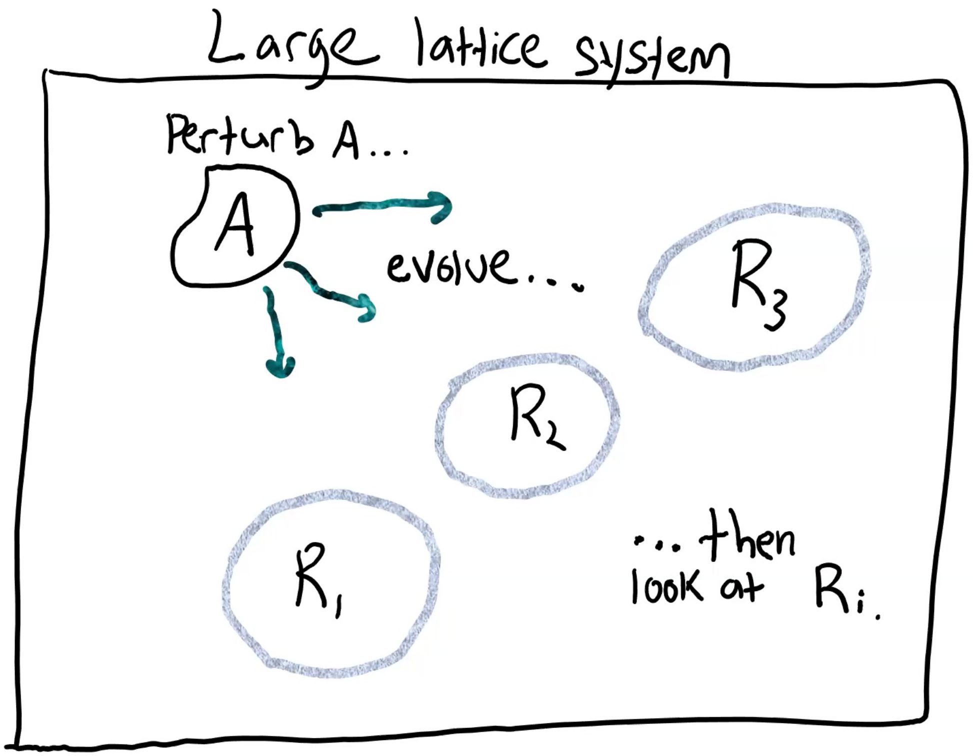
$$|\psi_t\rangle = c_0|0\rangle_A|0\rangle_{R_1}|0\rangle_{R_2}|0\rangle_{R_3} + c_1|1\rangle_A|1\rangle_{R_1}|1\rangle_{R_2}|1\rangle_{R_3}$$

Each R_i records state of A in 0,1 basis

$$\rho_{R_i} = |c_0|^2|0\rangle\langle 0| + |c_1|^2|1\rangle\langle 1|$$

Phase info. about original state on A is lost when just looking at A or R_i.

How can information spread during evolution?



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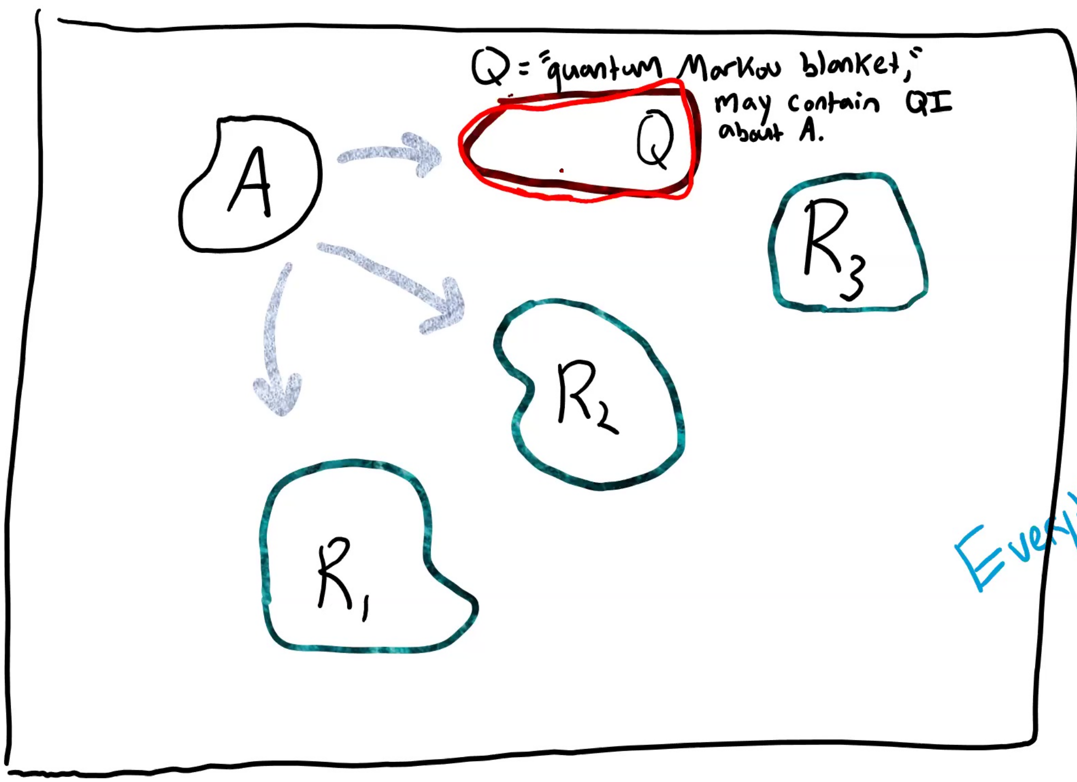

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Everything!

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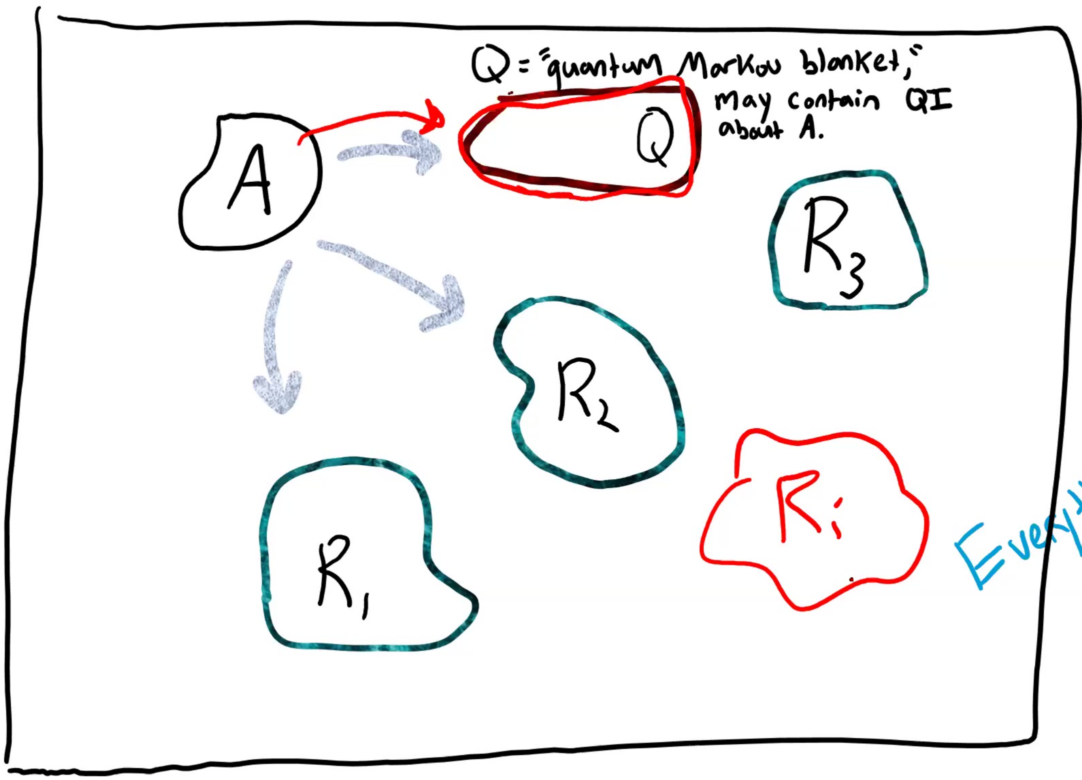

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Everything!

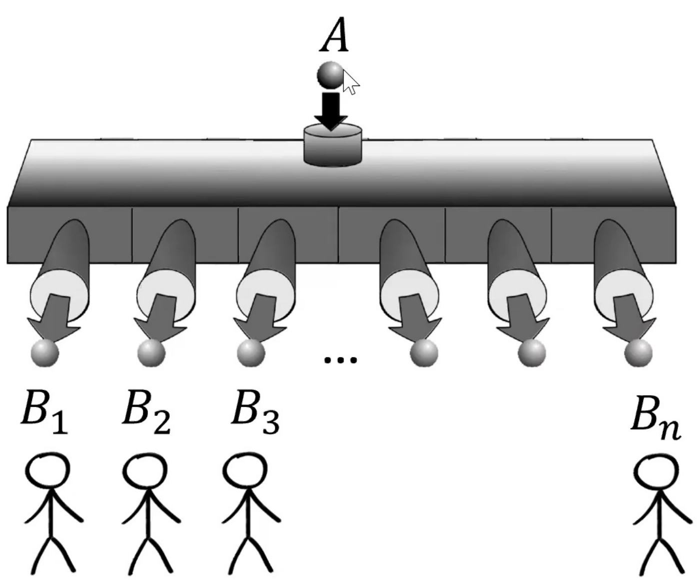
Earlier work

Inspired by very similar ideas in the excellent paper:

Generic emergence of classical features in quantum Darwinism, Brandão, Piani, Horodecki.

The present work proves a stronger statement, with a simple + constructive argument.

Studying local sub-systems of the environment



Input system

Imagine input A couples to environment B, then study local sub-systems of B.

$B = B_1 \otimes \cdots \otimes B_n$ Output, or "environment"

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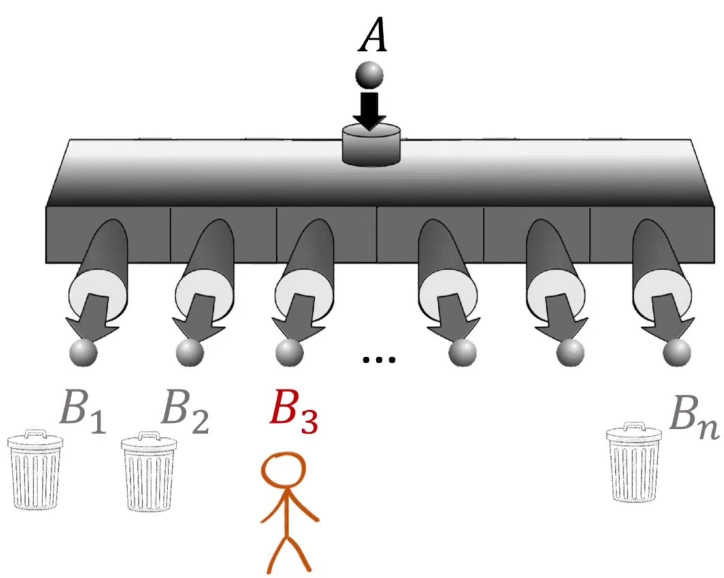

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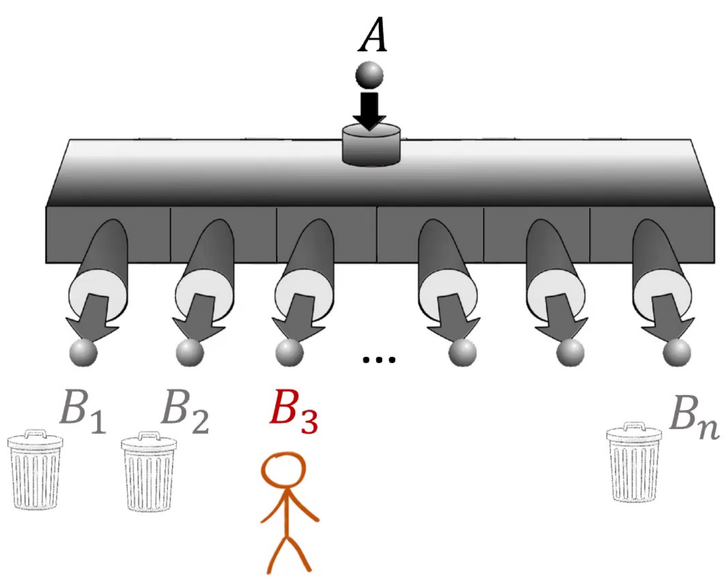

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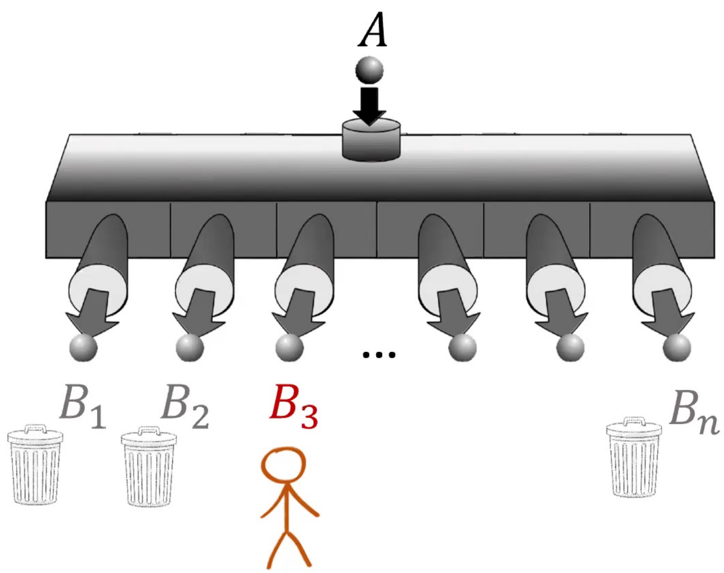
Almost everywhere in the environment B, the locally accessible information about A looks *classical*, i.e. can be obtained from a measurement of A in some fixed basis.

Studying local sub-systems of the environment



ρ_A

Model evolution as coupling to environment, evolving, then tracing out all except B_i

$$\rho_{B_i} = Tr_{A\,\overline{B_i}}\left(U_{AB}(\rho_A \otimes \tau_B)U_{AB}^{+}\right)$$

Environment starts in state τ_B
Then both systems evolve by unitary U_{AB}

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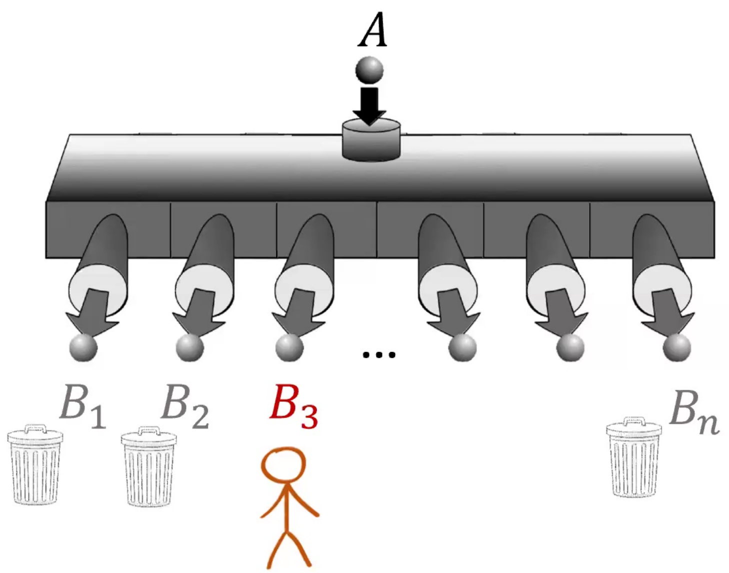

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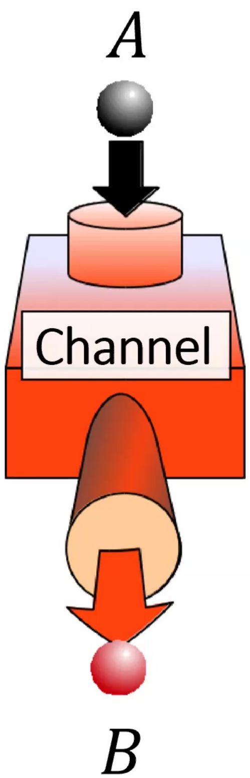
Interlude: Measure-and-prepare channels



Quantum channels $A \rightarrow B$ are maps from the space of density operators on system A to density operators on B, i.e.

$\rho_A \mapsto \rho_B$

"Measure-and-prepare" channel: Special type of channel that takes the form

$$\rho_A \mapsto \rho_B = \sum_\alpha Tr(M^\alpha \rho_A)\, \sigma_B^\alpha .$$

for some measurement operators $\{M^\alpha\}_\alpha$ and states $\{\sigma_B^\alpha\}_\alpha$ (e.g. orthogonal projectors $M^\alpha = |\alpha\rangle\langle\alpha|$)

Interlude: Measure-and-prepare channels

Evolutions of the form

$$\rho_A \mapsto \rho_B = \sum_\alpha Tr(M^\alpha \rho_A)\, \sigma_B^\alpha ,$$

represent measuring A in the basis associated to M^α and then preparing the state σ_B^α contingent on classical outcome α.

For Alice and Bob are at different labs A and B, they can implement such a map by sending only classical information: Alice measures A and then sends the outcome label α to Bob, who then prepares a state σ_B^α.

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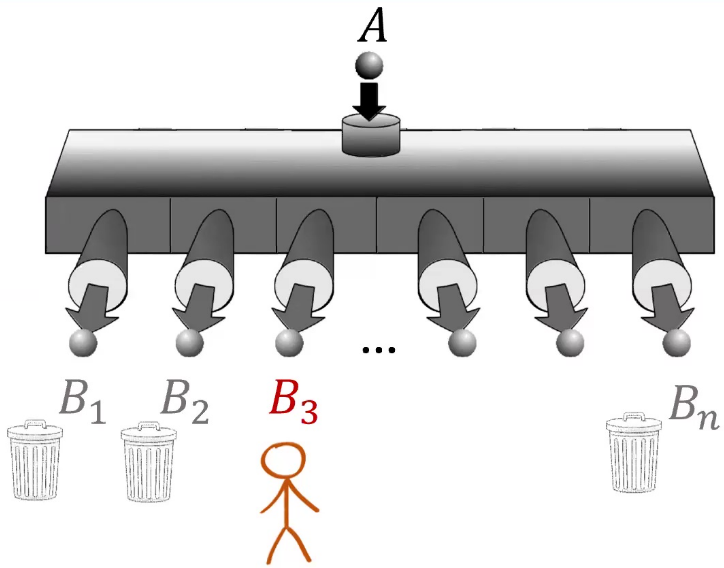
Our result: For any evolution $A \to B$ ...



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$$\approx \sum_{\alpha} Tr(M^{\alpha}\rho_A)\,\sigma_{B_i}^{\alpha}$$ "measure-and-prepare"

for almost all B_i
for some choice of measurement operators $\{M^{\alpha}\}$
(independent of B_i)

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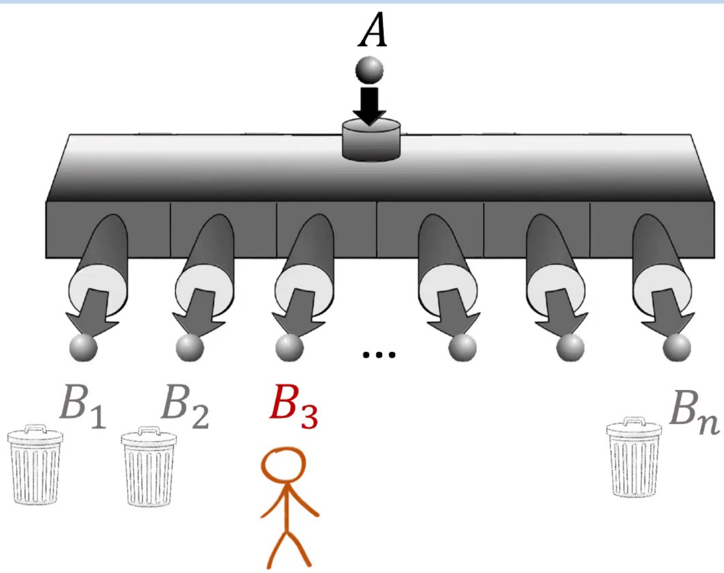

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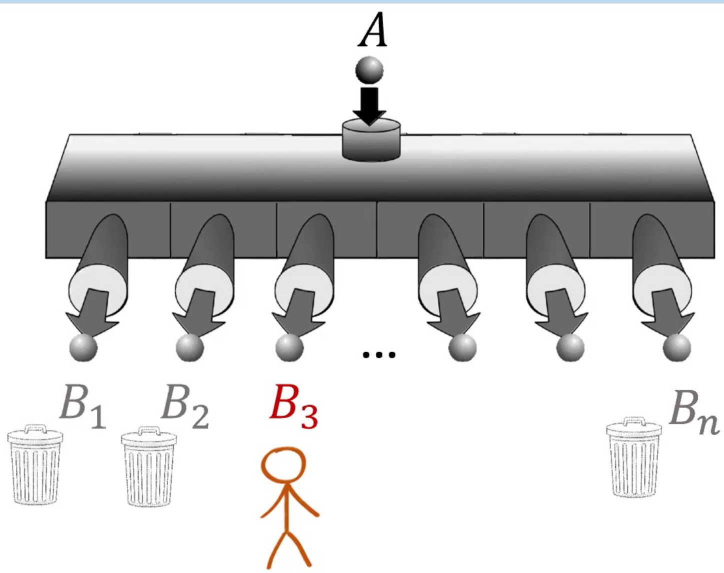

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Examples of applying theorem to evolutions

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$\downarrow$ Model evolution as coupling input A to environment B, evolving both, then tracing out all except B_i

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$\rho_A \to \rho_{B_1} = \rho_A$

$\rho_A \to \rho_{B_i} = Tr(\rho_A)|0\rangle\langle 0|_{B_i}$ (for $i > 1$)

Example:
Input system A sent faithfully to B_1
Other B_i sent to $|0\rangle$ (for $i > 1$)

Not measure-and-prepare ✗

Measure-and-prepare ✓ (Trivial prep.)

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Example: Spin chain evolution

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τ_B = Groundstate of spin chain B

U_{AB} = Evolution of extended chain AB

Example:
Couple qubit A onto end of spin chain B, then evolve extended chain

$\rho_A \to \rho_{B_i} =$????

Measure-and-prepare ✓

Numerical examples work!

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e^{-iHt}

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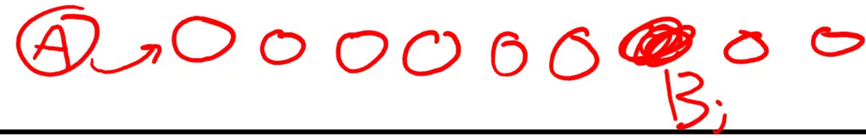

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Input system A is measured/decohered in $|0\rangle$, $|1\rangle$ basis
Outcome recorded on each B_i

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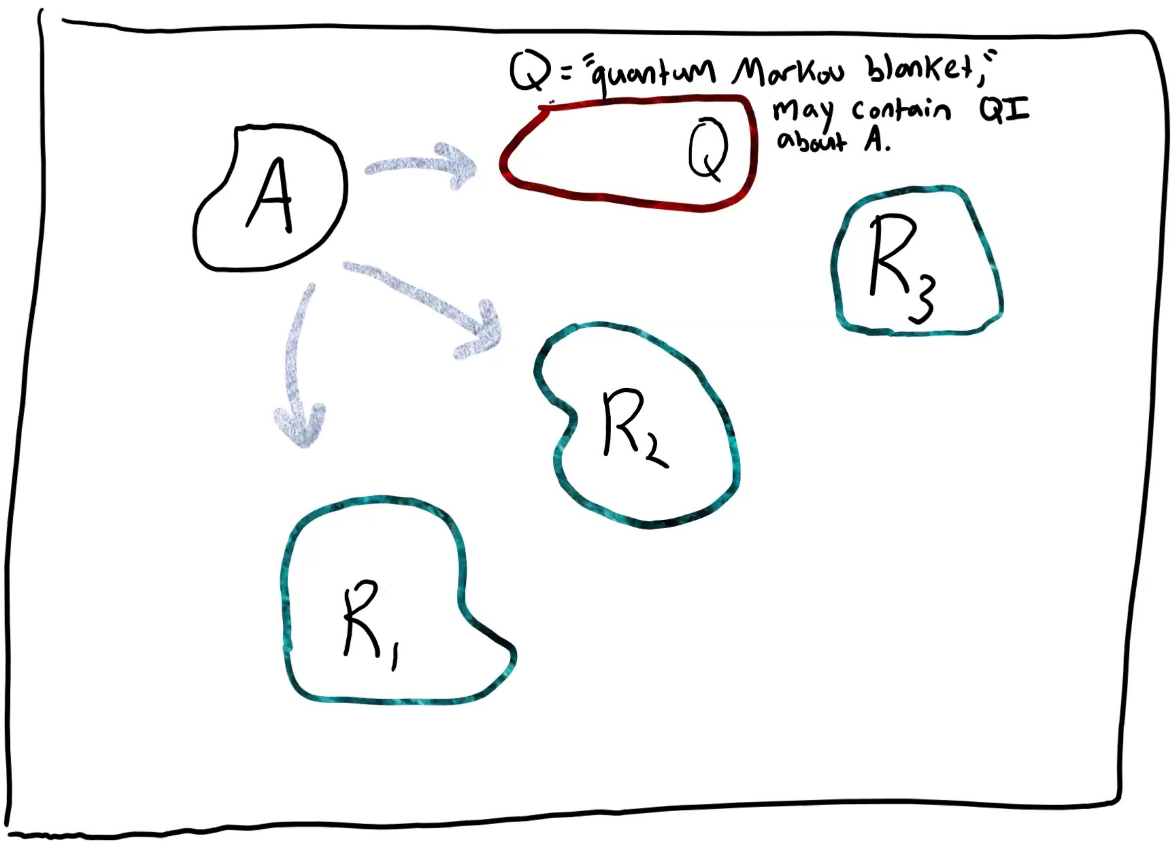
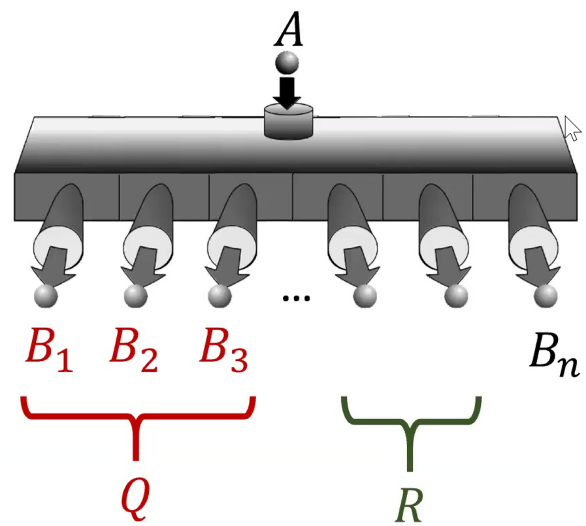
Measure-and-prepare ✓

Theorem statement (almost)



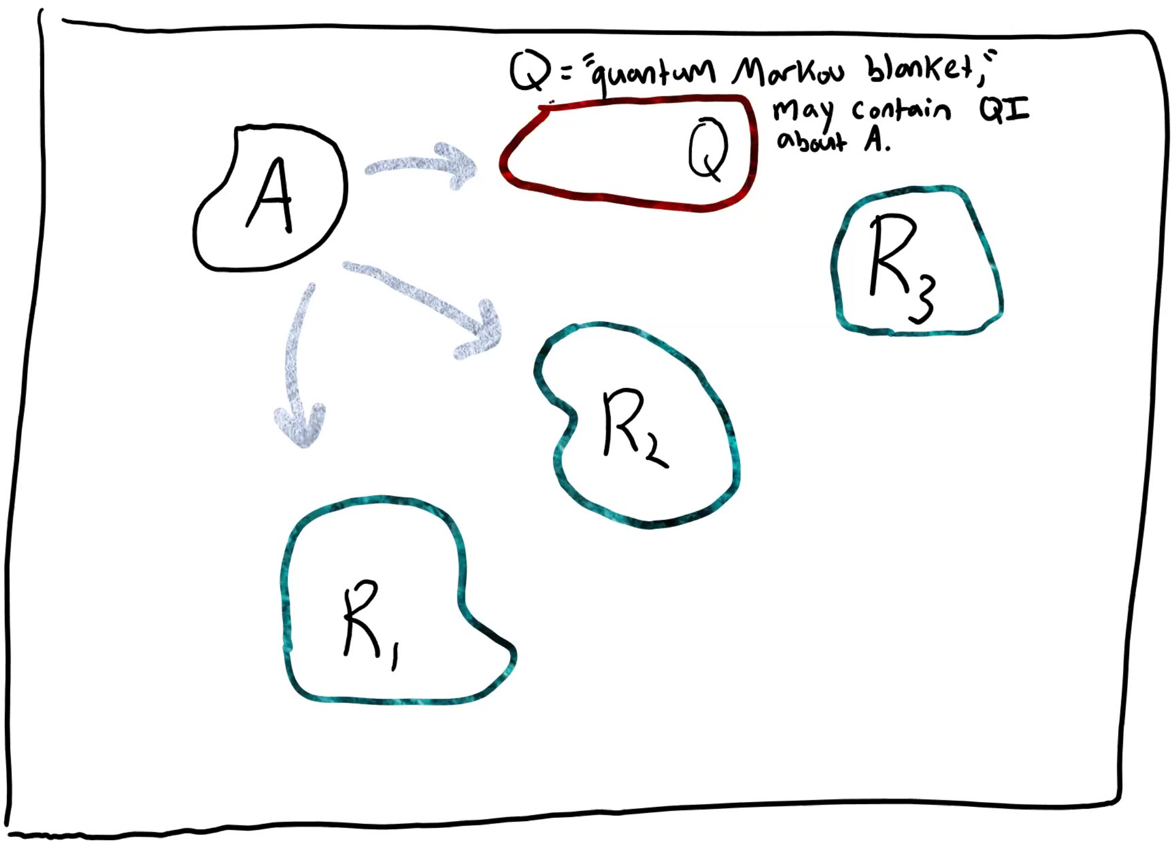


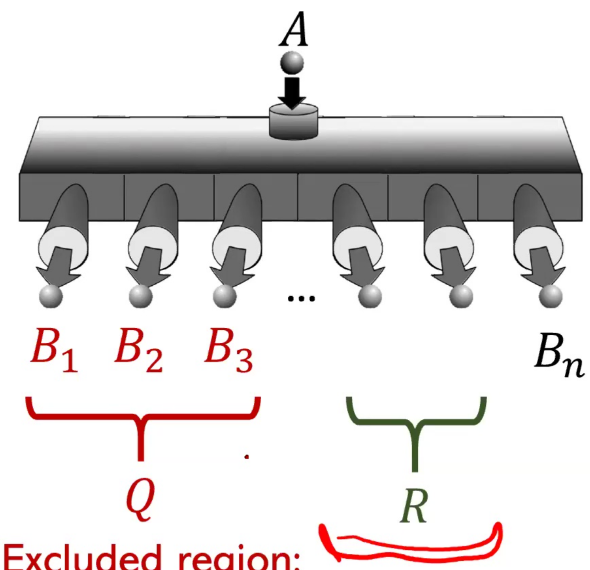
Excluded region:
"Quantum Markov blanket"

On all regions R away from Q, evolution $A \rightarrow R$ is approx. measure-and-prepare.

Allow larger $|Q|$ → smaller error.
Error indep. of total system size!
So we can take $|Q|$ = O(1).

Theorem statement (almost)





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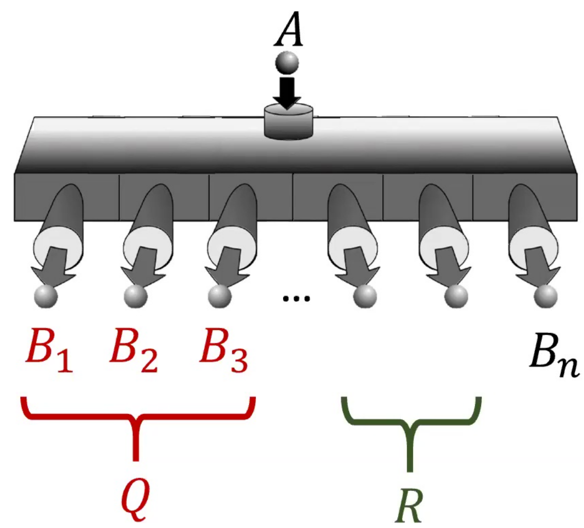
Quantum Markov blanket, Q

Q roughly includes outputs B_i with most information about A

Q "blankets" A:
Any information about A that's accessible on small regions R outside Q can be obtained from a classical measurement on just Q.

Q includes, at least, any region with locally accessible *quantum* information about A.

For arbitrarily large environments, you can still "cover" A with an O(1)-sized blanket Q !



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Theorem statement

Consider a quantum channel

$$N : \mathcal{D}(A) \to \mathcal{D}(B_1 \otimes \cdots \otimes B_n)$$

$\mathcal{D}(X)$ = *density operators on* X

For general output subset R, let N

$$N_R \equiv Tr_{\bar{R}} \circ N \ : \ \mathcal{D}(A) \to \mathcal{D}(R)$$

denote the *reduced channel* obtained by tracing out the complement $\bar{R}$.

Theorem: For any $|Q|$, $|R| \in \{1, \ldots, n\}$, there exists a POVM $\{M_\alpha\}$ and an "excluded" output subset Q of size $|Q|$, such that for all output subsets R of size $|R|$ disjoint from Q,

$$||N_R - \mathcal{E}_R||_\diamond \leq d_A^3 \sqrt{2 \ln d_A \frac{|R|}{|Q|}}$$

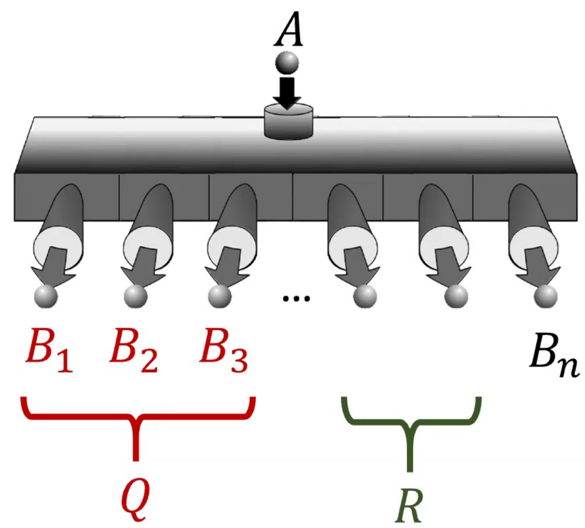
Take $|R| < |Q|$

$d_A = \dim(A)$

where $\mathcal{E}_R$ is *measure-and-prepare*,

$$\mathcal{E}_R(\rho) \equiv \sum_\alpha Tr(M_\alpha \rho)\sigma_R^\alpha$$

for some choice of states σ_R^α on R. Note $\{M_\alpha\}$ chosen independent of R.



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$R = \{B_1, B_2, B_5\}$

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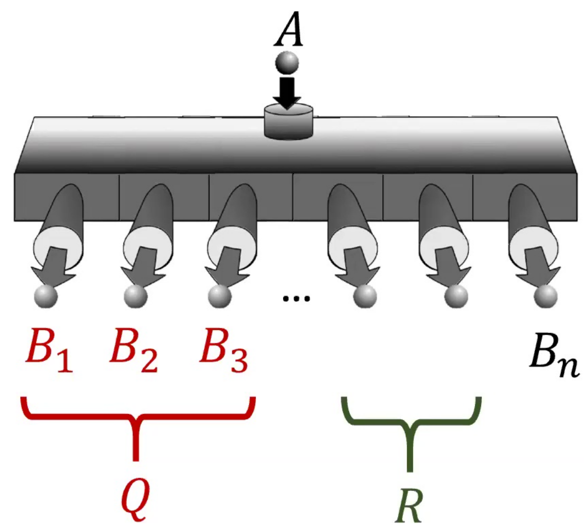

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On all regions R away from Q, evolution $A \rightarrow R$ is approx. measure-and-prepare.

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Error indep. of total system size!
So we can take $|Q|$ = O(1).

Theorem statement

Consider a quantum channel

$$N : \mathcal{D}(A) \rightarrow \mathcal{D}(B_1 \otimes \cdots \otimes B_n)$$

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For general output subset R, let N

$$N_R \equiv Tr_{\bar{R}} \circ N : \quad \mathcal{D}(A) \rightarrow \mathcal{D}(R)$$

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$R \cap Q = \phi$

$$||N_R - \mathcal{E}_R||_\diamond \leq d_A^3 \sqrt{2 \ln d_A \frac{|R|}{|Q|}}$$

Take $|R| < |Q|$

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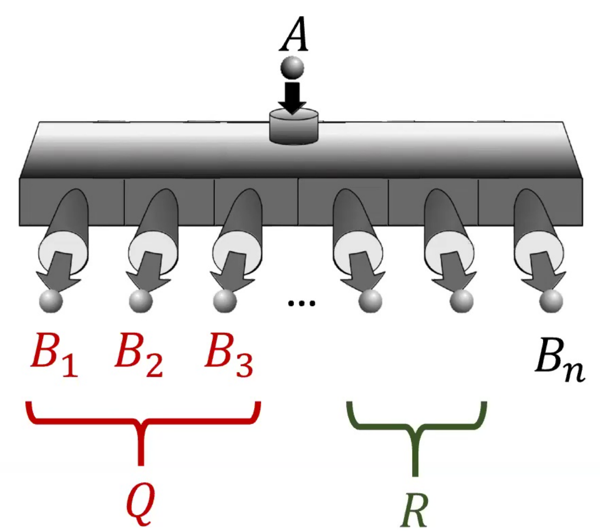

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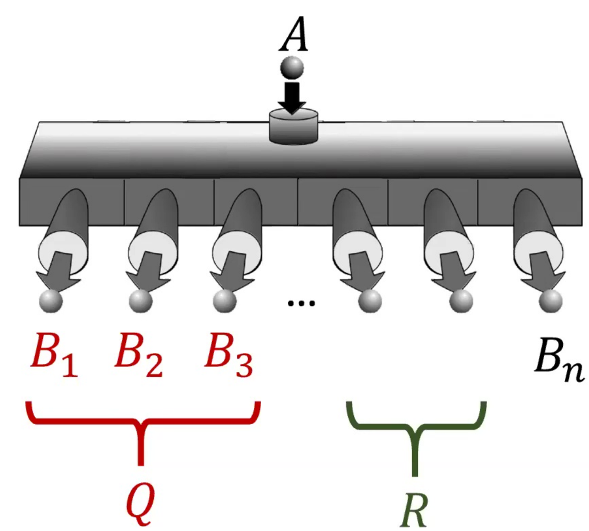

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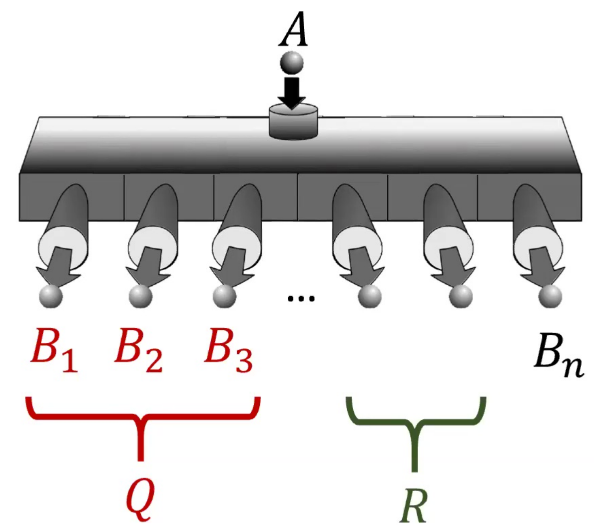

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$n = 10^{23}$
$|R| = 1$
$|Q| < 10, 100, 1000$

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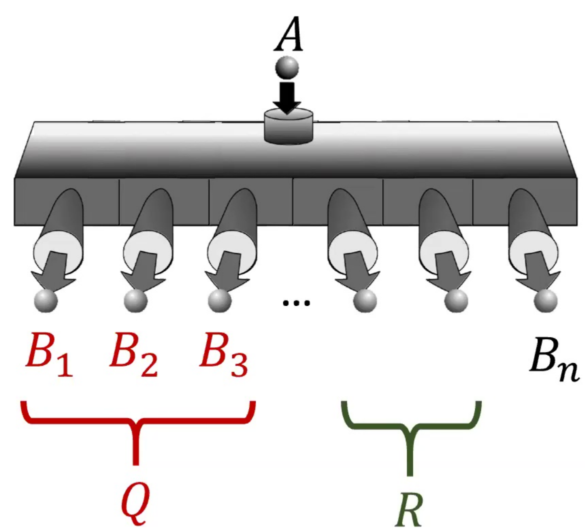

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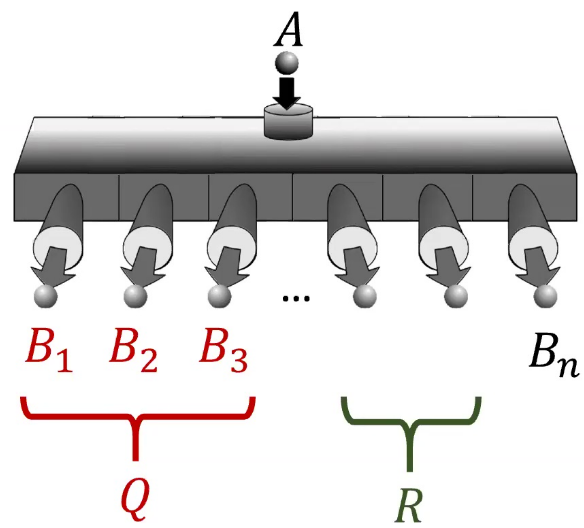

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Supplementary interlude: proof sketch

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Consider any quantum state $\rho_{AB_1 \ldots B_n}$ on $A \otimes B_1 \otimes \ldots \otimes B_n$.

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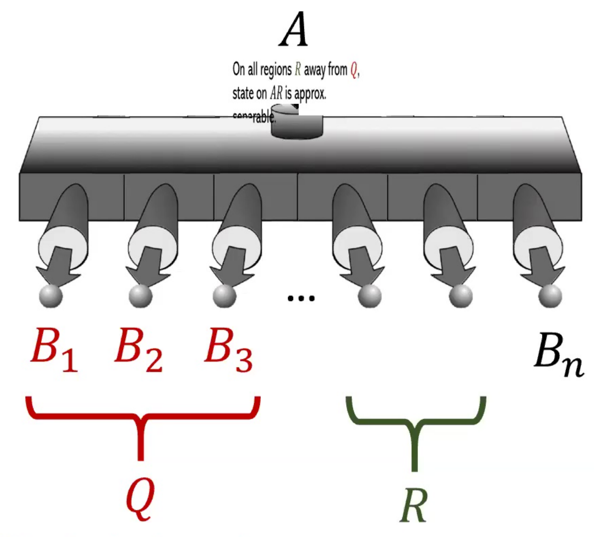

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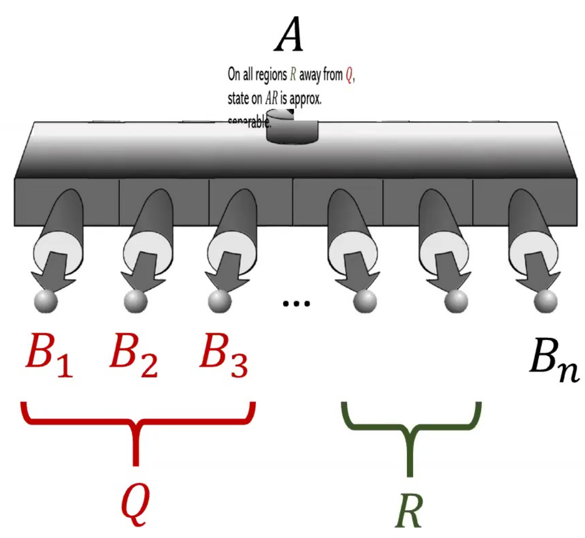

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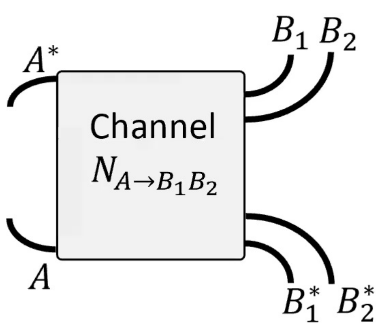
Channel-state duality

The main result may be formulated as a result about either (1) channels with multiple outputs, or (2) multipartite states.



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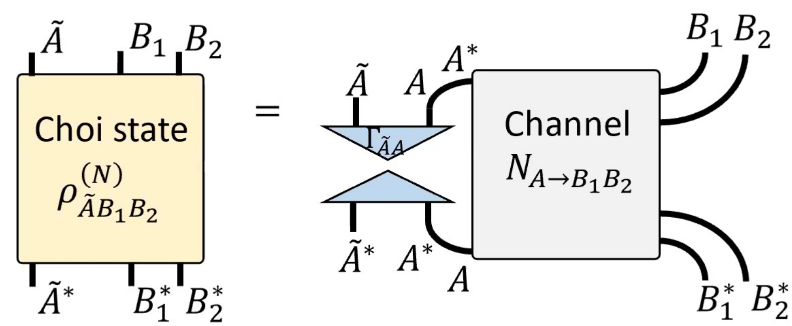

Constraints on **dynamical** properties of channels (e.g. no cloning)

imply

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Reduced channels are measure-and-prepare?

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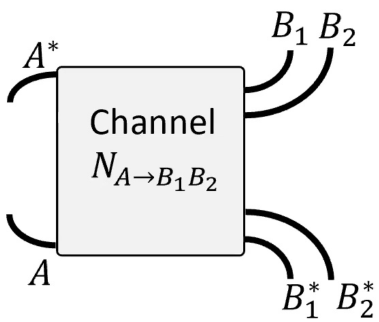
M.P. ⟷ sep. st.



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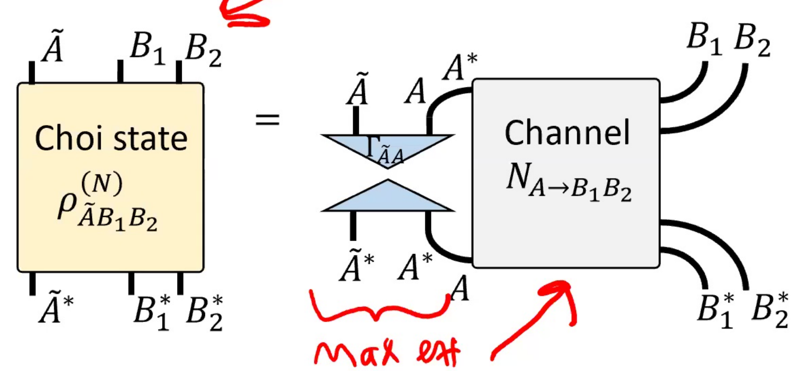

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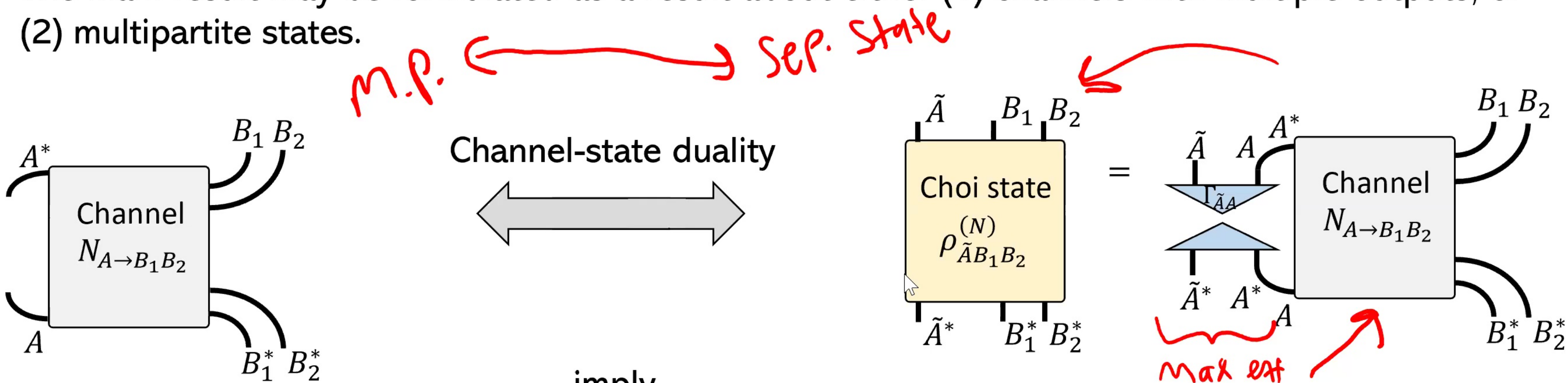

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Sketch of argument: Warm-up result

$S(X) = -Tr(\rho_X \log \rho_X)$ (entropy)

(how much there is to know about X)

$I(X,Y) = S(X) + S(Y) - S(XY)$ (mutual information)

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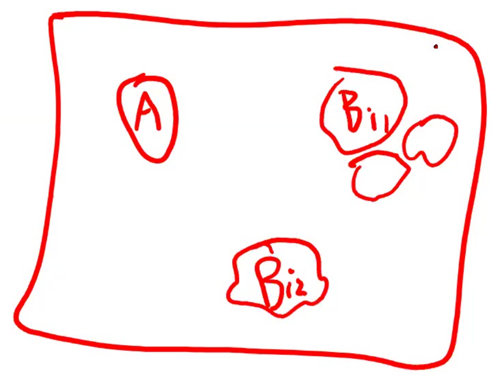

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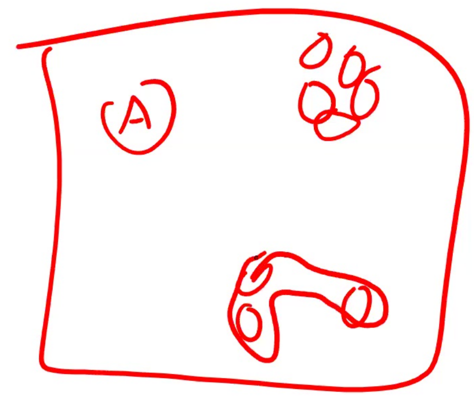

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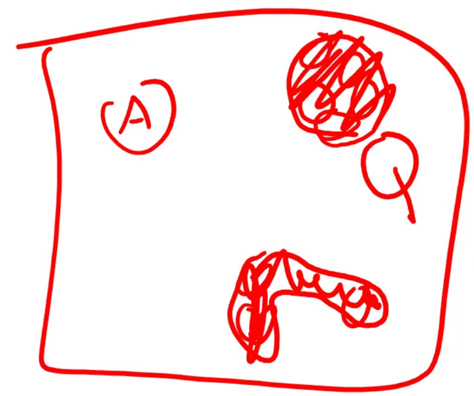

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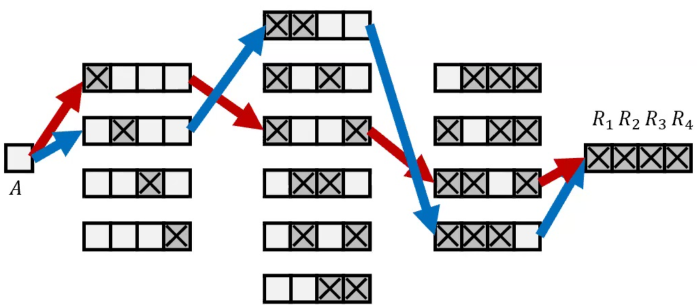
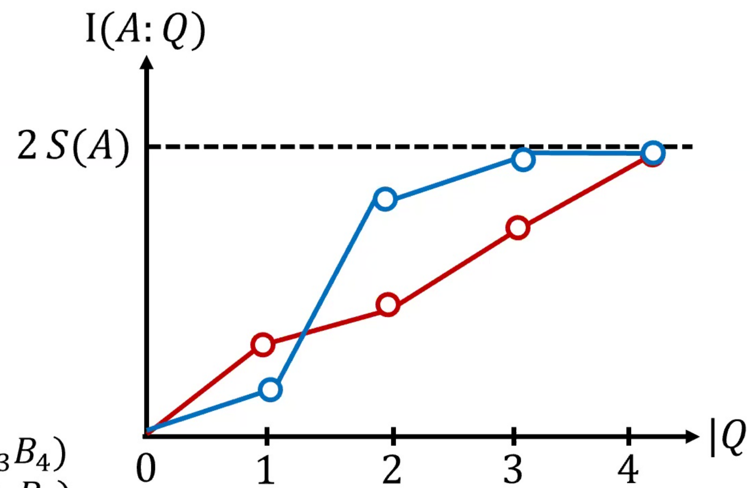
Visual constructive proof: To find the region Q, optimize over paths below.





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$= 2\,S(A)$ (if pure)

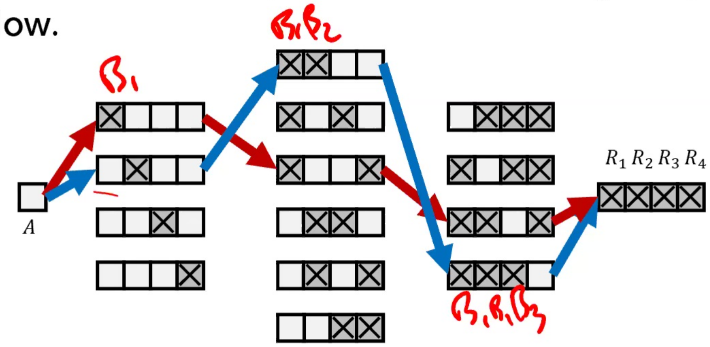
Each node is a candidate region Q. Arrows indicate inclusions. Each path is an expanding subset of outputs. Strategy: **Gradually expand Q to learn as much as possible about A, until further expansion yields no further knowledge.** Stop there to obtain final Q.

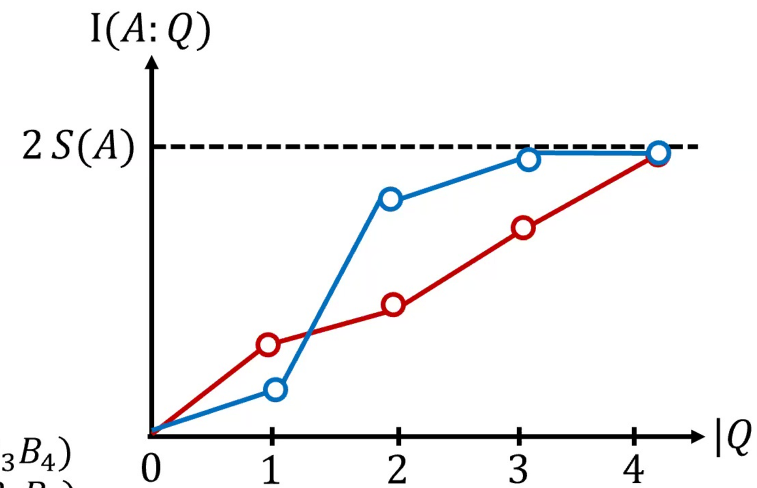
$I(X,Y|Z) = I(X,YZ) - I(X,Z)$ (conditional mutual information)

For any state $\rho_{AB_1 \ldots B_n}$, for any size q:
There exists region $Q \subset \{B_1, \ldots, B_n\}$ of size $|Q| \leq q$ such that for all $B_i \notin Q$,

$$I(A, B_i|Q) \leq \frac{1}{|Q|} 2 \log(d_A).$$

Visual constructive proof: To find the region Q, optimize over paths below.





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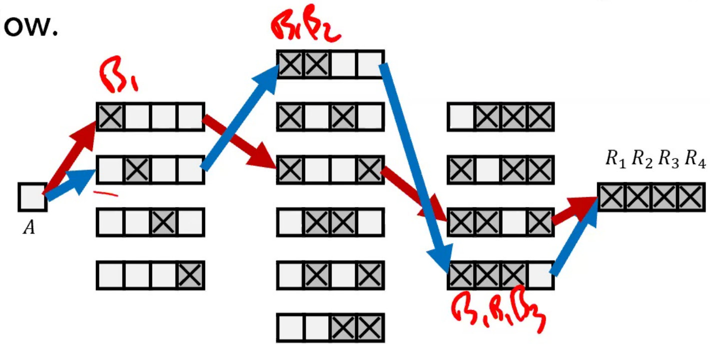

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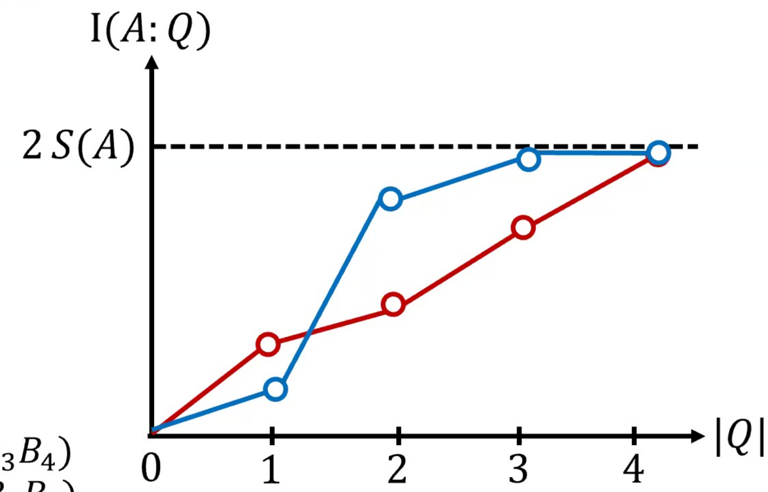

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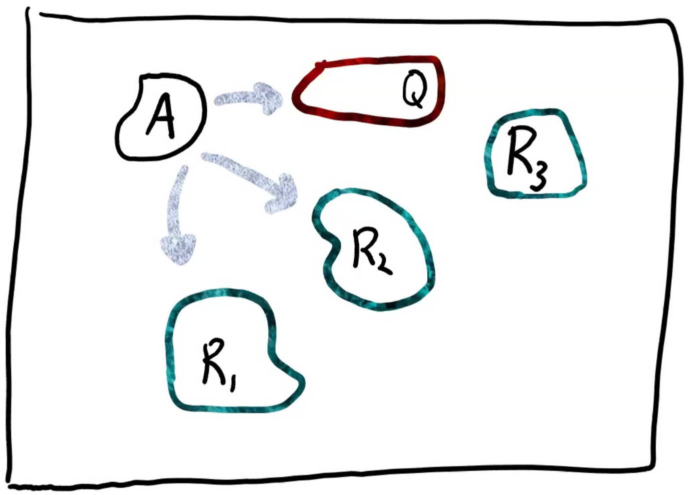
Implications for many-body dynamics?

Constructive method identifies “basis” $\{M^\alpha\}$ on A that is effectively measured/decohered by the rest of the system.

Helps identify emergent classical variables in many-body systems?



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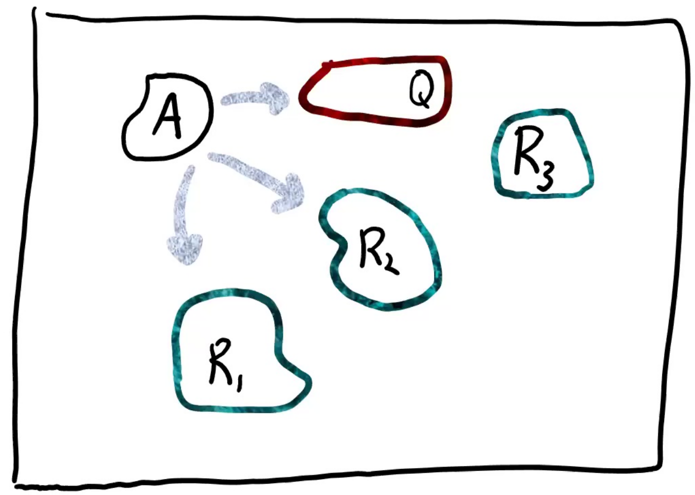
Example: Hydrodynamics

In charge-conserving random circuits, the observable "measured" on A roughly coincides with the charge (confirmed numerically).

Explore more examples? Apply analysis where we don't already understand what's going on?



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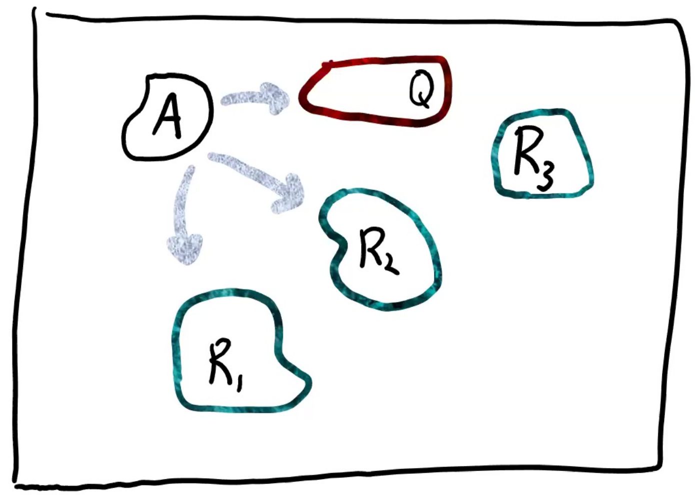

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- How tight is the bound? How can it be improved when assuming additional structure to the dynamics, like spatial geometry or local conserved quantities?

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Thank you!