Title: Effective Fly-Bys: Gravitational Waves from Dynamical Capture Binaries

Speakers: Nicholas Loutrel

Series: Strong Gravity

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Abstract: Compact binaries may be formed dynamically in globular clusters, with large (close to unity) orbital eccentricity and emitting gravitational waves within the detection band of ground based detectors. The gravitational waves from such sources resemble more a discrete set of bursts than the continuous signal of their quasi-circular counterparts. I here discuss the construction of new analytic waveforms for such systems, which rely on treating the problem as a perturbation of a parabolic fly-by. I will discuss the results of parameter estimation with these waveforms for binary black holes, and the inclusion of tidal effects for binary neutron stars.

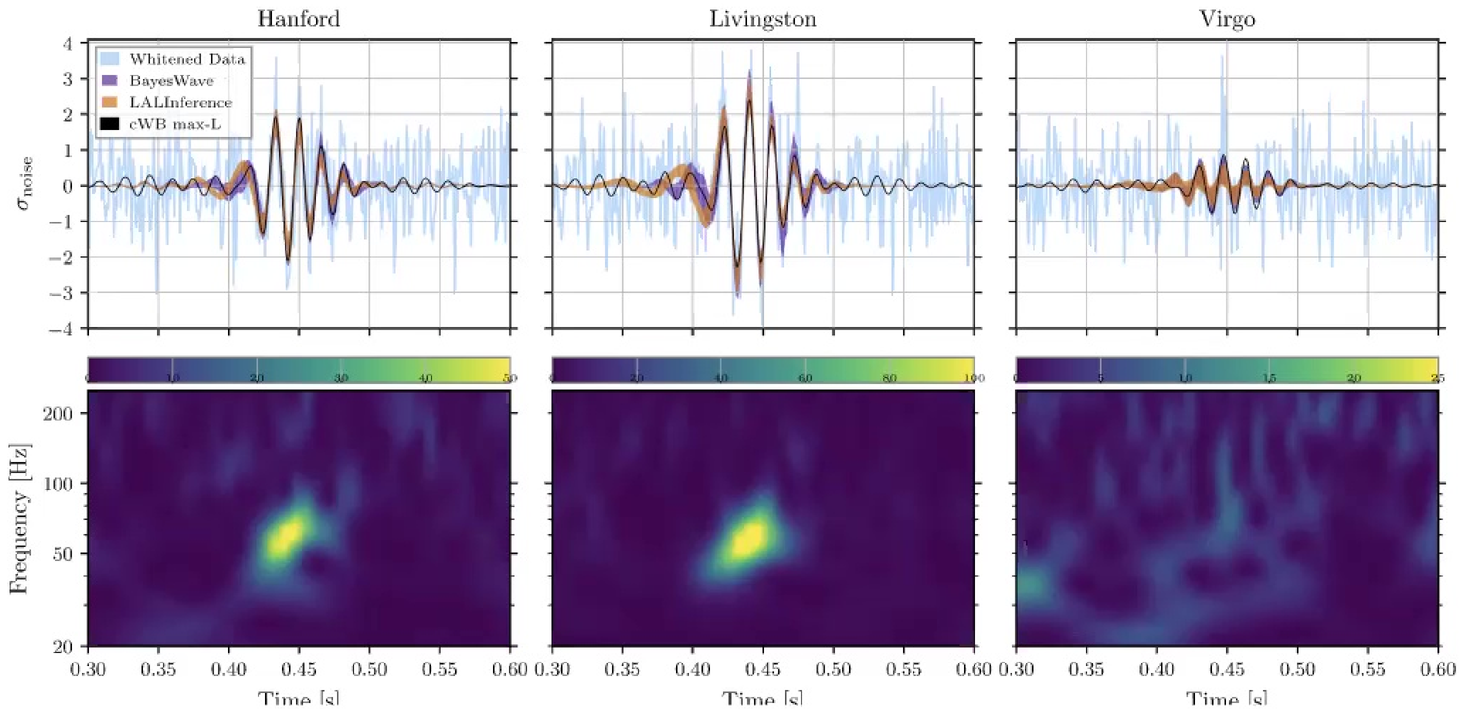
Effective Fly-Bys: Gravitational Waves from Dynamical Capture Binaries

Perimeter Institute Strong Gravity Seminar

Nicholas Loutrel
Princeton University

Based on:
arXiv:1909.02143
arXiv:2003.13673
arXiv:2012.xxxxx

GW190521


Hanford
Livingston
Virgo
Whitened Data
BayesWave
LALInference
cWB max-L
σ_{noise}
Frequency [Hz]
Time [s]
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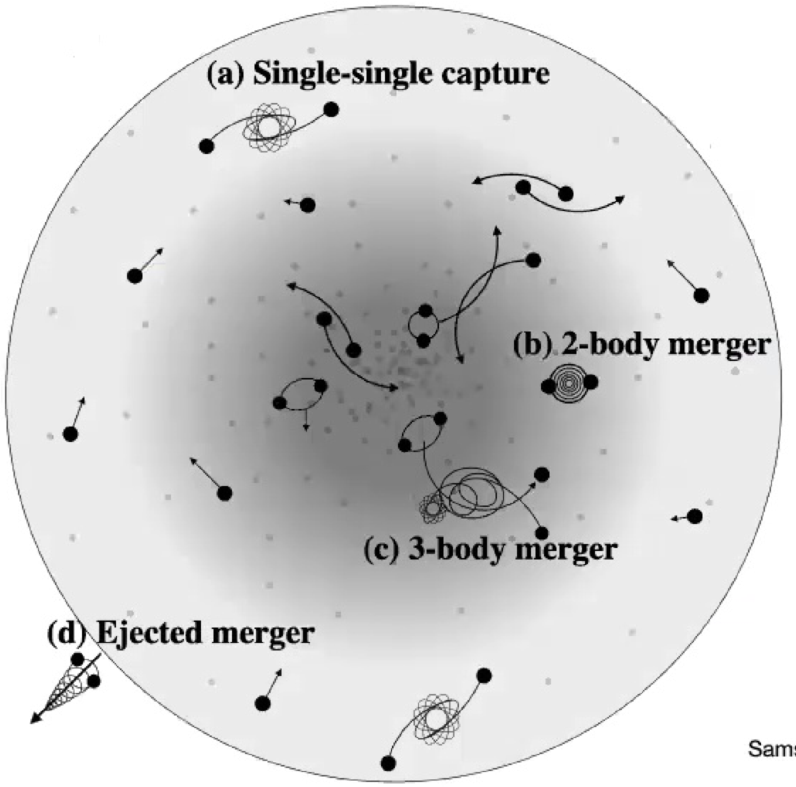
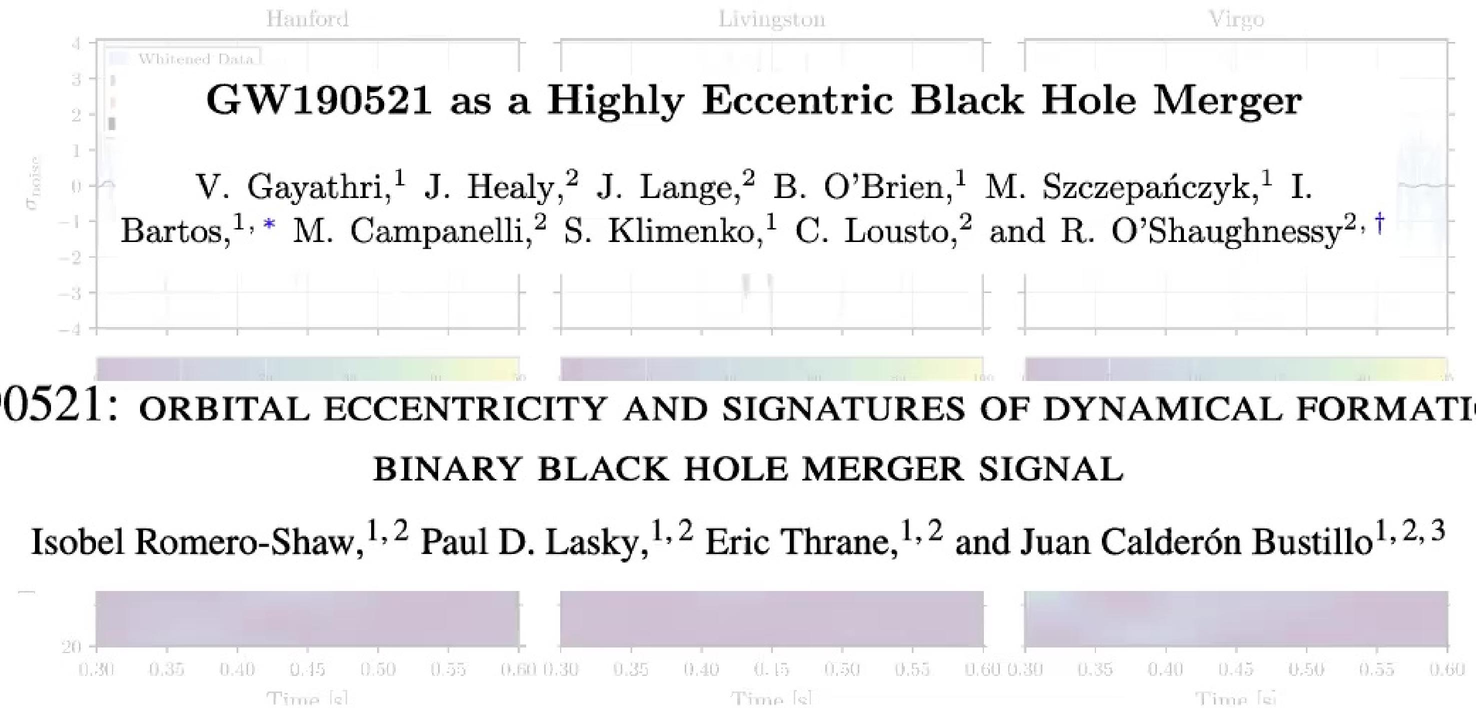
Time [s]

A number of studies over recent years have shown that **binaries can form in dense stellar environments with non-negligible eccentricity and within the LIGO band.**



Samsing et. al., 2019

GW190521



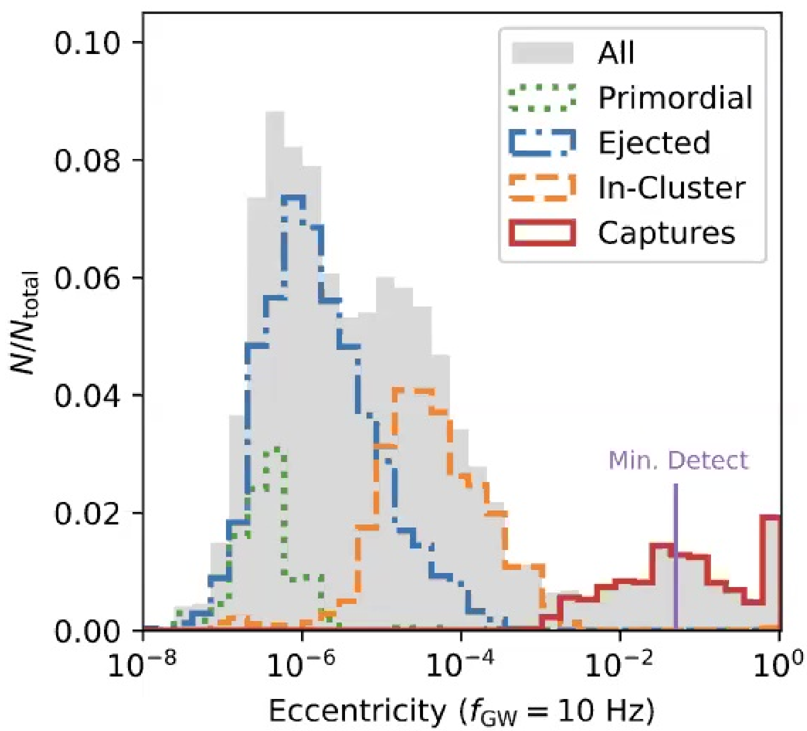
GW190521 as a Highly Eccentric Black Hole Merger

V. Gayathri,[1] J. Healy,[2] J. Lange,[2] B. O'Brien,[1] M. Szczepańczyk,[1] I. Bartos,[1,*] M. Campanelli,[2] S. Klimenko,[1] C. Lousto,[2] and R. O'Shaughnessy[2,†]

GW190521: ORBITAL ECCENTRICITY AND SIGNATURES OF DYNAMICAL FORMATION IN A BINARY BLACK HOLE MERGER SIGNAL

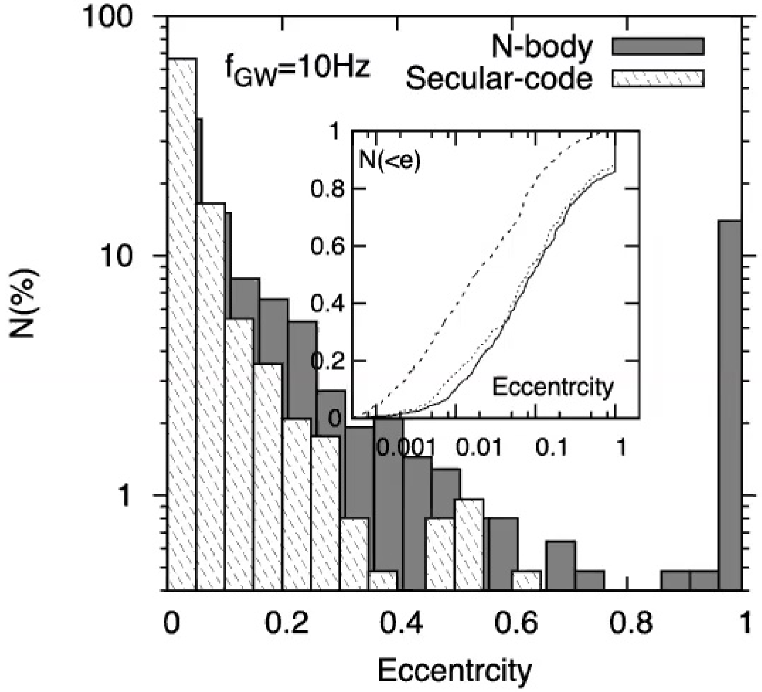
Isobel Romero-Shaw,[1,2] Paul D. Lasky,[1,2] Eric Thrane,[1,2] and Juan Calderón Bustillo[1,2,3]

Globular Clusters



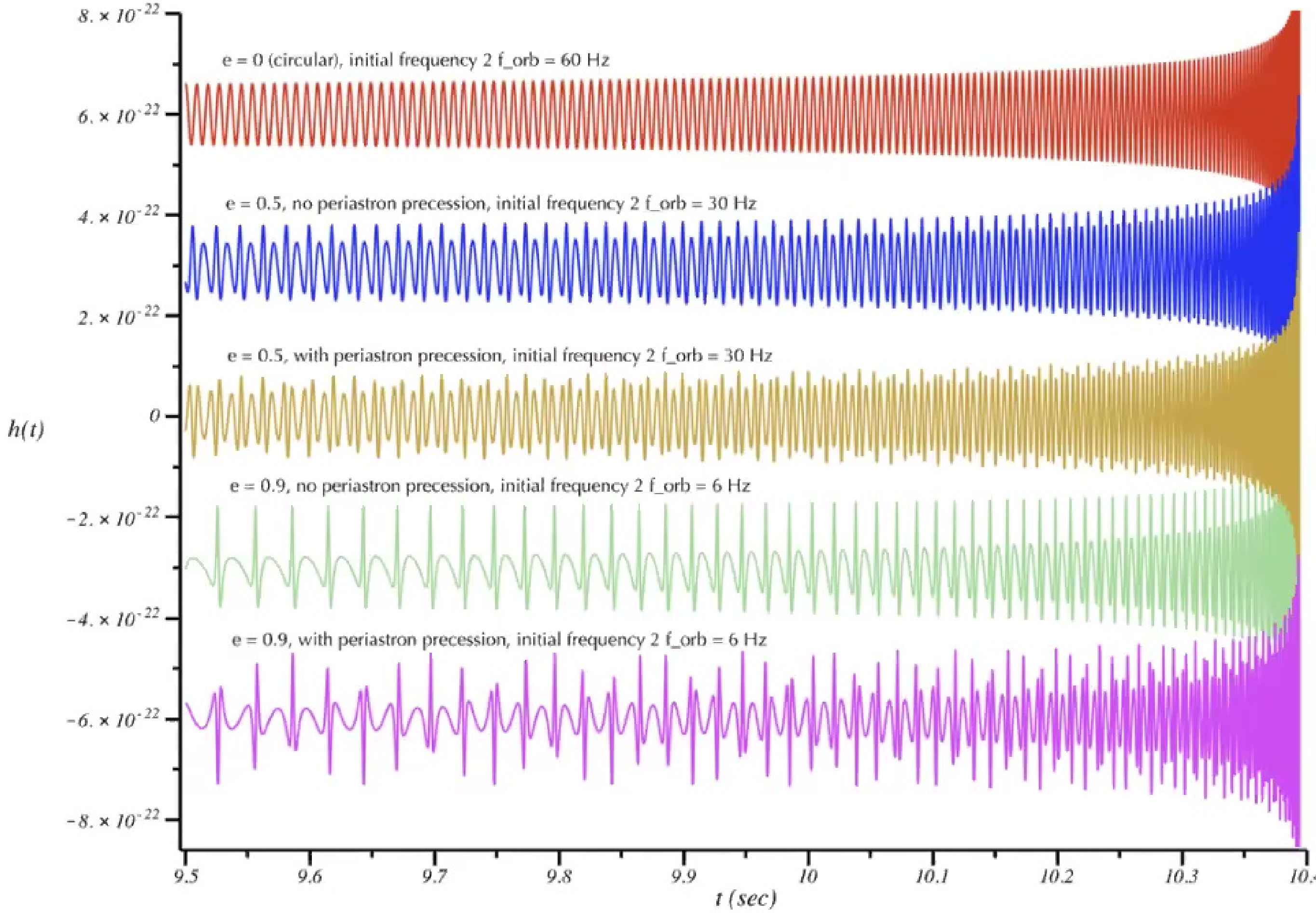
Rodriguez et. al., 2018

Hierarchical Triples



Antonini, Murray, & Mikkola, 2013

LIGO Event Rate @ Optimal Sensitivity: 1 - 2 per year



Sounds of Spacetime

Detection Strategy:

- Power stacking
  - Assume a set of *N* bursts are contained in a signal
  - Stack power in each burst to amplify signal-to-noise ratio (SNR)
  - Amplification scales as $N^{1/4}$
  - If the SNR per burst ~*1*, need 10,000 bursts to achieve detection
  - Sub-optimal

- Optimally want to use matched filtering
  - Currently few accurate numerical relativity waveforms

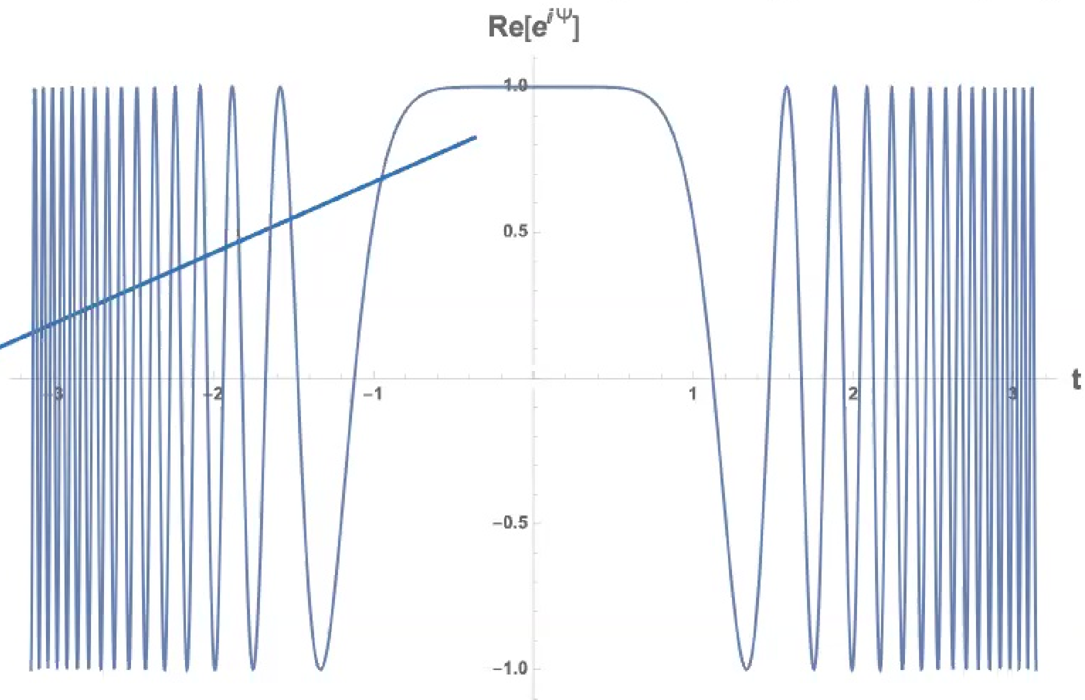
Want: Fast, analytic waveforms

Effective Fly-By Waveforms For Binary Black Holes

Quasi-circular TaylorF2:

Solve Radiation Reaction Eqns. ⟶ Time Domain Waveform ⟶ Stationary Phase Approximation (SPA)

$$\int A(t)e^{i\Psi(t)}dt$$



Stationary point

$$\Psi'(t_\star) = \Psi''(t_\star) = ... = \Psi^{(n-1)}(t_\star) = 0$$

$$\Psi(t) = \Psi(t_\star) + \frac{1}{n!}\Psi^{(n)}(t_\star)(t-t_\star)^n$$

Quasi-circular TaylorF2:

Solve Radiation Reaction Eqns. → Time Domain Waveform → Stationary Phase Approximation (SPA)

$$\tilde{h}(f) = \frac{\mathcal{A}}{f^{7/6}} e^{i\Psi(f)}$$

$$\Psi(f) = \boxed{2\pi f t_c - \phi_c} - \frac{\pi}{4} + \frac{3}{128(\pi\mathcal{M}f)^{5/3}}\left[1 + \sum_i \phi_i (\pi\mathcal{M}f)^{i/3}\right]$$

Time & Phase Of Coalescence

Chirp Mass

Post-Newtonian Coefficients

$$v \sim (\pi\mathcal{M}f)^{1/3}$$

Effective Fly-By TaylorF2 (EFB-F2):

Balance Laws

$$p(t) = p_0 + \left(\frac{dp}{d\ell}\right)_{\ell=0} \ell(t) + \mathcal{O}(\ell^2)$$

$$e(t) = e_0 + \left(\frac{de}{d\ell}\right)_{\ell=0} \ell(t) + \mathcal{O}(\ell^2)$$

$$\ell'(t) = n_0 + 2\pi F_{\rm rr} \ell(t) + \mathcal{O}(\ell^2)$$

$$n_0 = M^{1/2} \left(\frac{1-e_0^2}{p_0}\right)^{3/2} \sim \frac{1}{T_{\rm orb}}$$

$$F_{\rm rr} \sim \frac{M^3}{p_0^4}(1-e_0^2)^{1/2} \sim \frac{1}{T_{\rm coal}}$$

$$n_0/F_{\rm rr} \sim v^{-5}$$

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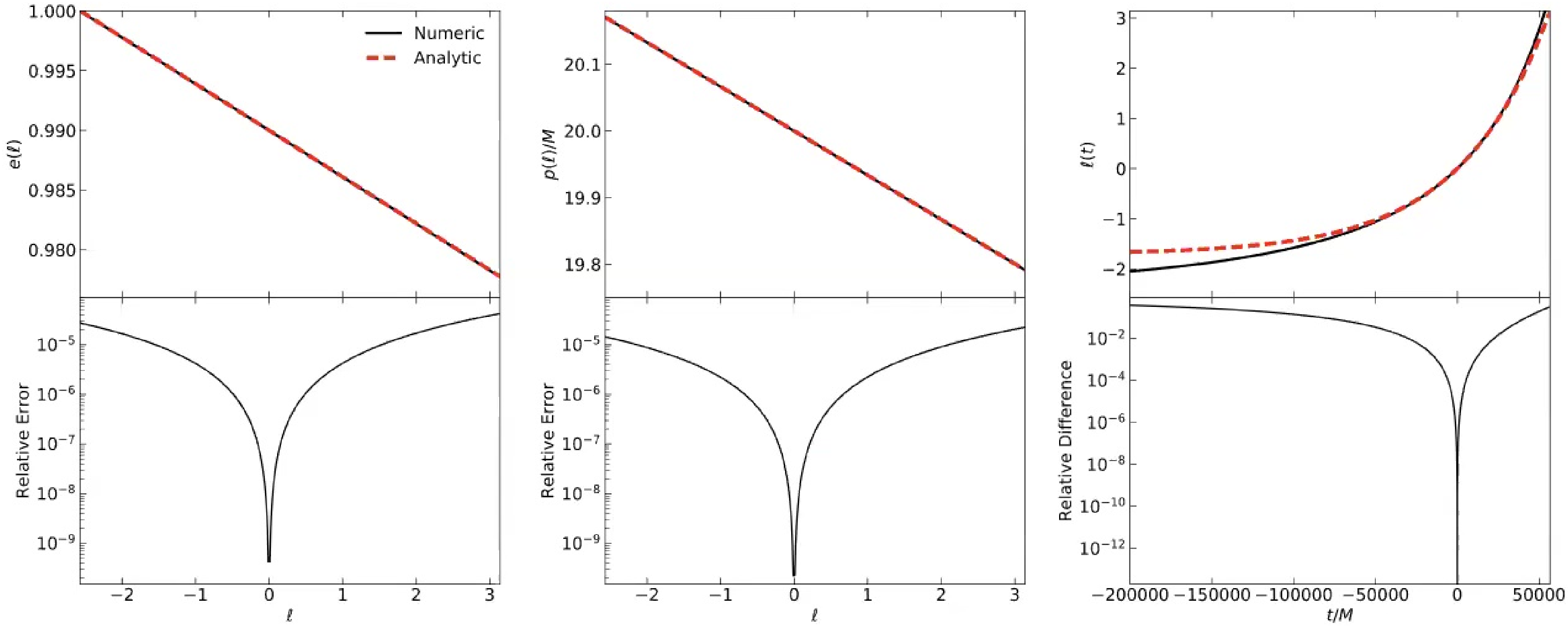
$$n_0/F_{\rm rr} \sim v^{-5}$$

$$\ell(t) = \frac{n_0}{2\pi F_{\rm rr}}\left(e^{t/F_{\rm rr}} - 1\right)$$

Effective Fly-By TaylorF2 (EFB-F2):



10

Effective Fly-By TaylorF2 (EFB-F2):

Time Domain:

$$h_{+,\times}(t) \sim \frac{\mathcal{M}}{D_L} \sum_{k=0}^{\infty} \left\{ E^k_{+,\times}[e(t)] e^{ik\ell(t)} + \text{c.c.} \right\}$$

$$J_k(ke), J'_k(ke)$$

Stationary Point:

$$t_{k,\star} = t_p + \frac{1}{2\pi F_{\text{rr}}} \log\left(\frac{2\pi f}{k n_0}\right)$$

Frequency Domain:

$$\tilde{h}_{+,\times} \sim \frac{\mathcal{M}}{D_L} \sum_{k=0}^{\infty} \frac{(E^{(k)}_{+,\times})^\dagger(t_{k,\star})}{F_{\text{rr}}\sqrt{2\pi\chi}} e^{i\Psi_k(\chi)}$$

$$\chi = f/F_{\text{rr}} \qquad \Psi_k(\chi) = 2\pi f t_p + k\chi_{\text{orb}} - \chi\left[1 - \ln(k\chi_{\text{orb}}/\chi)\right] - \pi/4$$

Effective Fly-By TaylorF2 (EFB-F2):

Re-summation: $J_k(ke) \sim K_{1/3}[(2/3)\zeta^{3/2}k] \longrightarrow \sum_{k=0}^{\infty} \sim \int_0^{\infty} dk$

$$\tilde{h}_{+,\times}^{\mathrm{EFB-F}}(f) \sim \mathcal{A}(f)\, {}_2F_1\left[\frac{l_1}{6} - i\frac{\chi}{2}, \frac{l_2}{6} - i\frac{\chi}{2}, s, X\right] \qquad X \sim v_p^{-5}$$

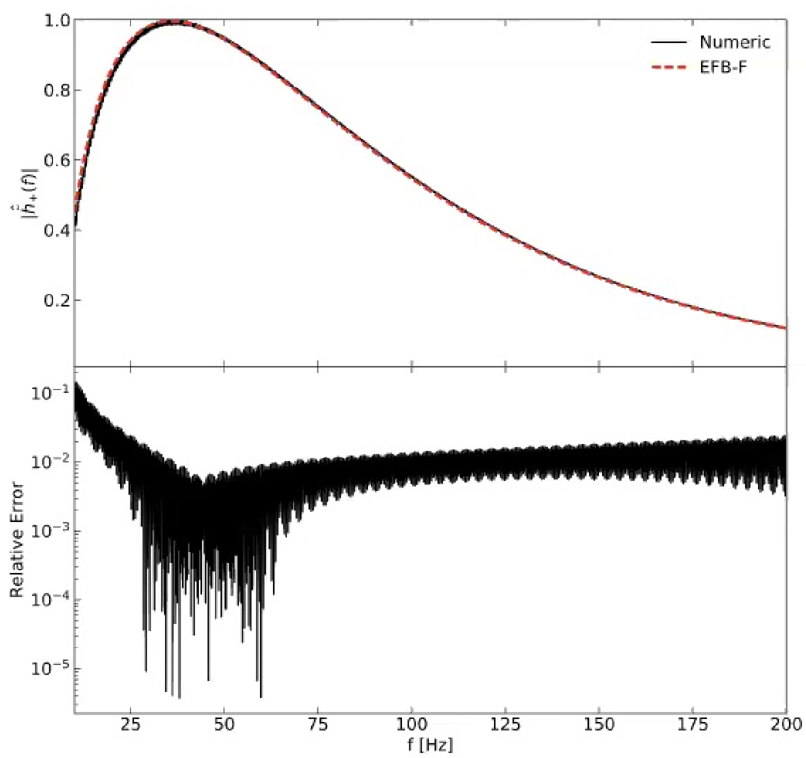
Problem: ~10 hours to evaluate

Asymptotic expansion $\chi \gg 1$

$$\tilde{h}_{+,\times}^{\mathrm{EFB-F2}}(f) \sim \sum_{n=0}^{2} \bar{A}(f) J_{\frac{1}{6}(l_1 - l_2)+n}\left(\frac{2e^{\Phi(\chi)/2}}{\sqrt{X-1}}\right)$$

$$\Phi(\chi) = 2\ln\left(\frac{\chi}{2}\right) + \frac{2i}{\chi}\left[\frac{1}{6}(l_1 + l_2) - s\right] + \mathcal{O}(\chi^{-2})$$

$$p_0 = 20M \qquad e_0 = 0.99$$

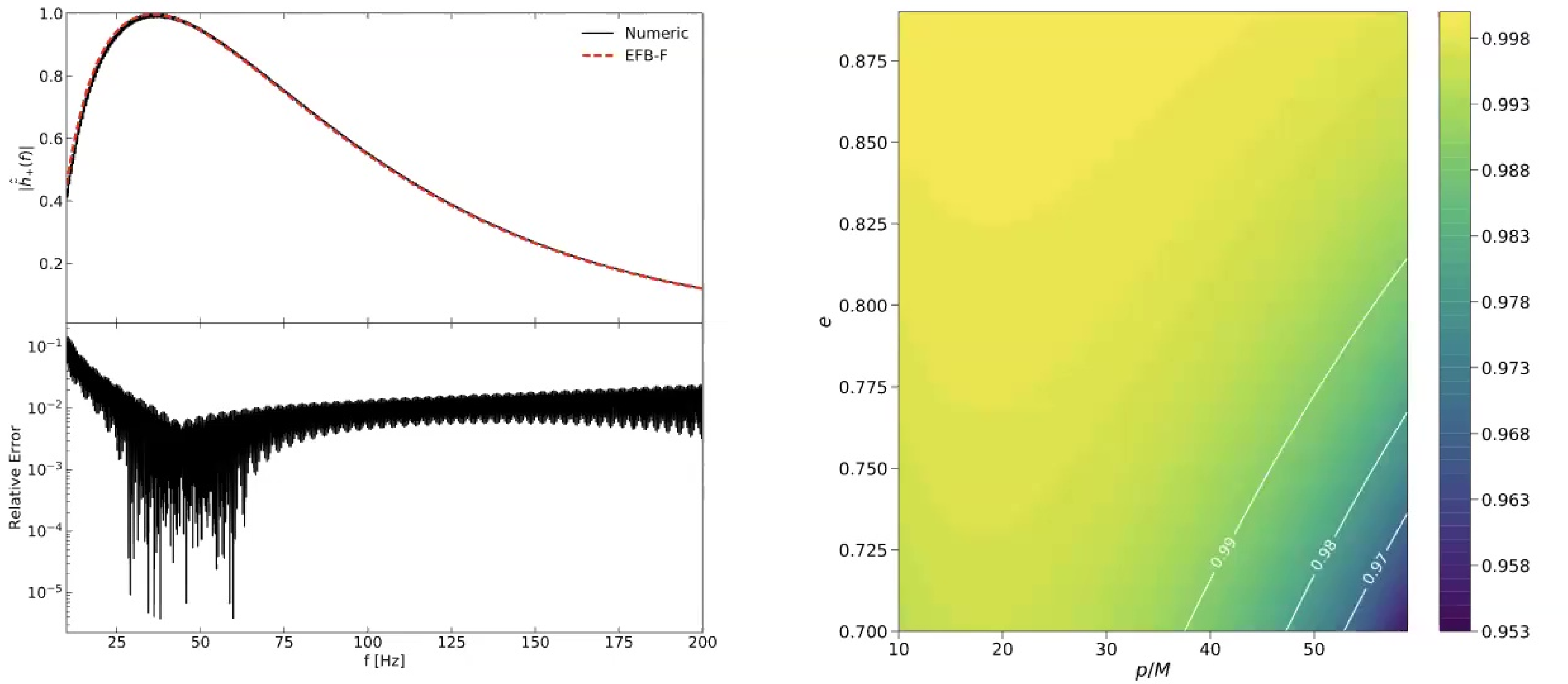


Match:

$$\mathcal{M} = \max_{t_p} (h_A | h_B)$$

$$(h_A | h_B) = 4\mathrm{Re} \int df \frac{\tilde{h}_A(f) \tilde{h}^\dagger_B(f)}{S_n(f)}$$

$p_0 = 20M \qquad e_0 = 0.99$



- So far, have only focused on single bursts
- How do we go from single EFB's to full inspiral waveforms?
- **Solution:** Stitch EFB waveforms together

$$\tilde{h}(f) = \sum_I \tilde{h}_{\mathrm{EFB-F2}}(t_{p,I}, e_I, p_I; f)$$

$$t_{p,I} = t_{p,I-1} + \boxed{\frac{2\pi}{M^{1/2}}\left(\frac{p_I}{1-e_I^2}\right)^{3/2}}$$

$$p_I = p_{I-1}\left[1 - \frac{128\pi}{5}\eta \boxed{\left(\frac{M}{p_{I-1}}\right)^{5/2}} \left(1 + \frac{7}{8}e_I^2\right)\right]$$

$$e_I = e_{I-1} - \frac{608\pi}{15}\eta e_{I-1} \boxed{\left(\frac{M}{p_{I-1}}\right)^{5/2}} \left(1 + \frac{121}{304}e_I^2\right)$$

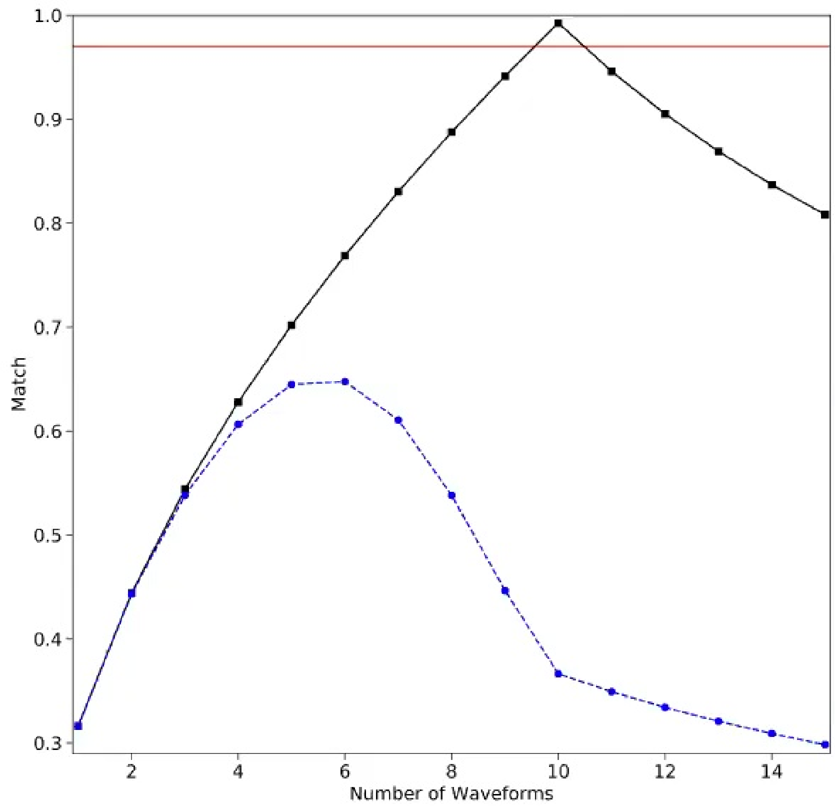
Orbital Period

Leading PN Order Balance Laws

- Compare to numerical PN waveform
- Generate 10 burst sequence
- Compute the match as a function of number of EFB waveforms

- **Result:** EFBs can recover a multi-burst sequence
- Predicts the correct number of bursts
- Achieves match > 0.97

- **Problem:** Requires a very accurate timing model
- Even with 1% error, lose detection
- PN timing model probably not sufficient



Parameter Estimation:

Fisher Analysis

$$\lambda^a = (\mathcal{M}, \eta, e_0, p_0, t_{p,0}, D_L, \iota, \beta)$$

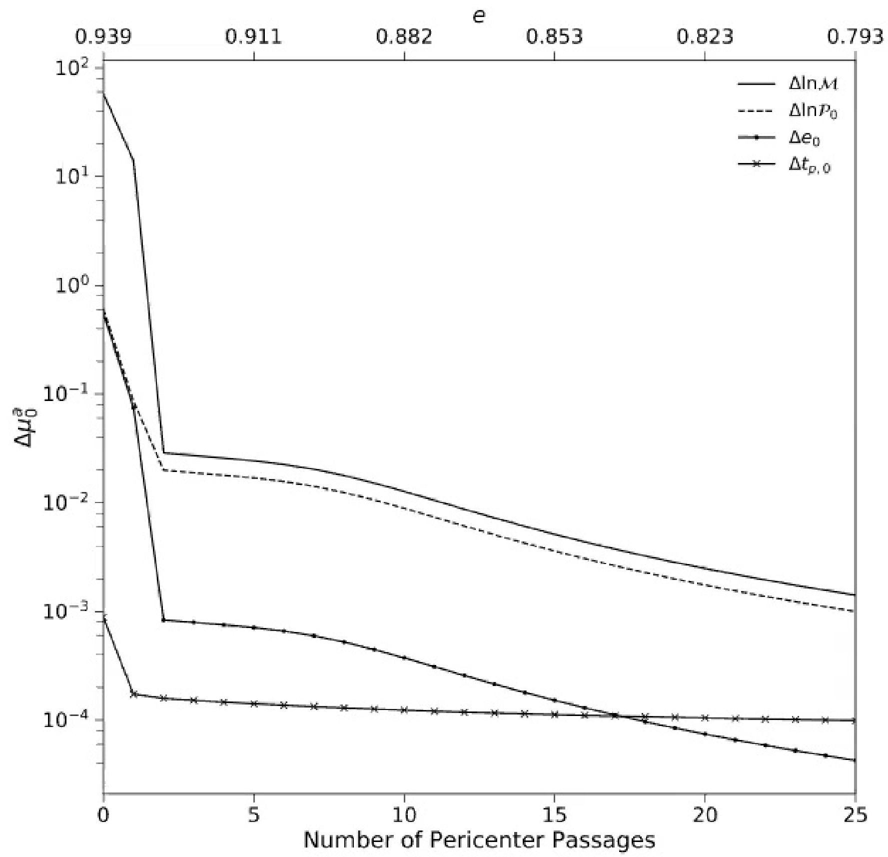
$$\Gamma_{ab} = \left(\frac{\partial h}{\partial \lambda^a} \middle| \frac{\partial h}{\partial \lambda^b} \right)$$

$$\Delta\lambda^a = \left(\Gamma^{-1}\right)_{aa}$$

Degeneracies:

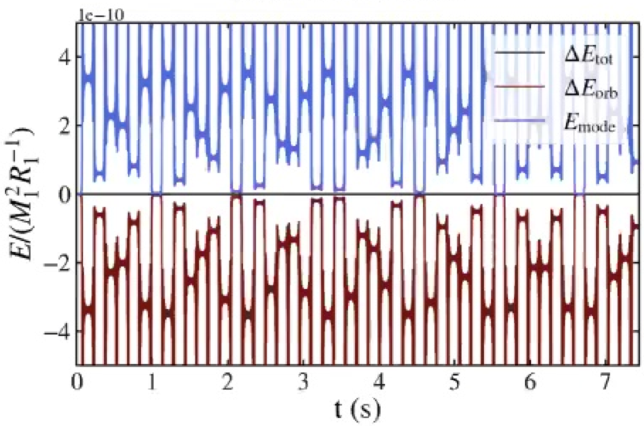
$$\mathcal{M} = M\eta^{3/5}$$

$$\mathcal{P} = p_0^{3/2}/M^{1/2}$$

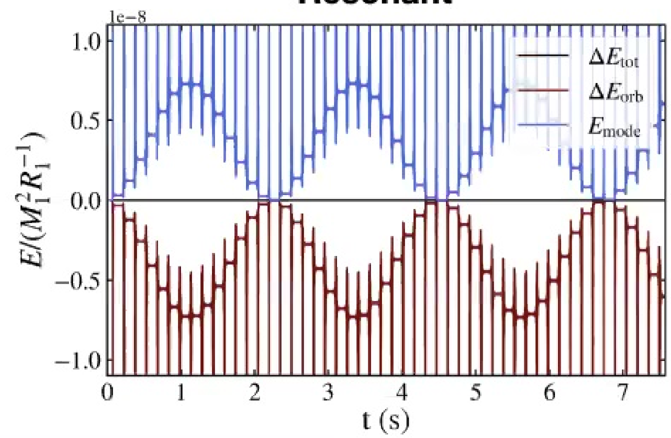


Effective Fly-By Waveforms
For Binary Neutron Stars

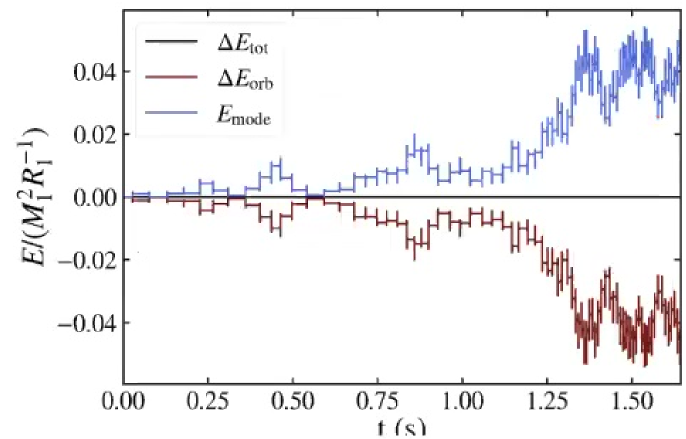
Non-resonant



Resonant

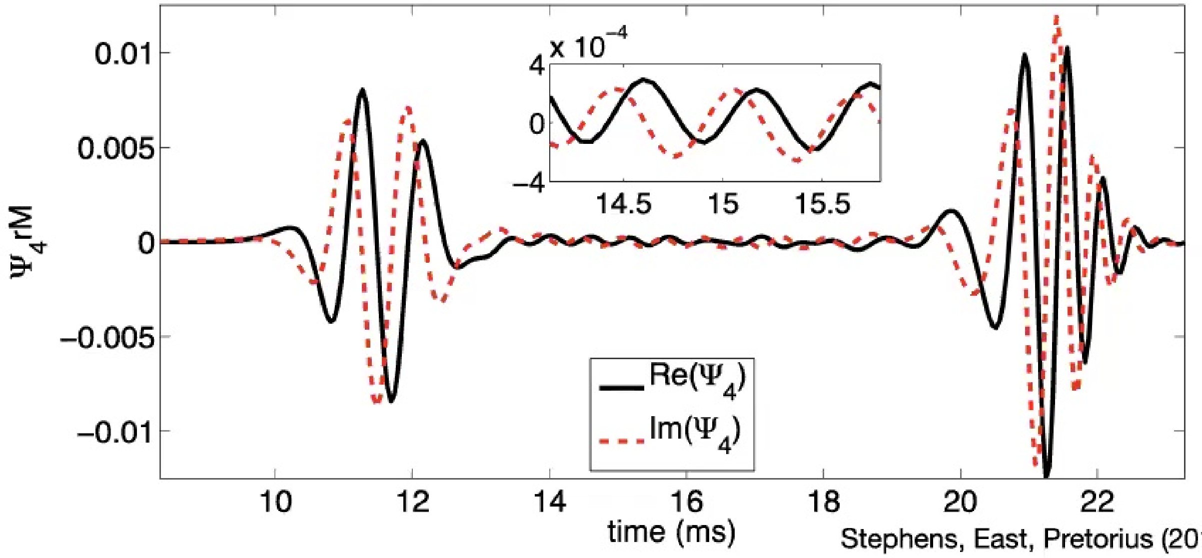


Chaotic

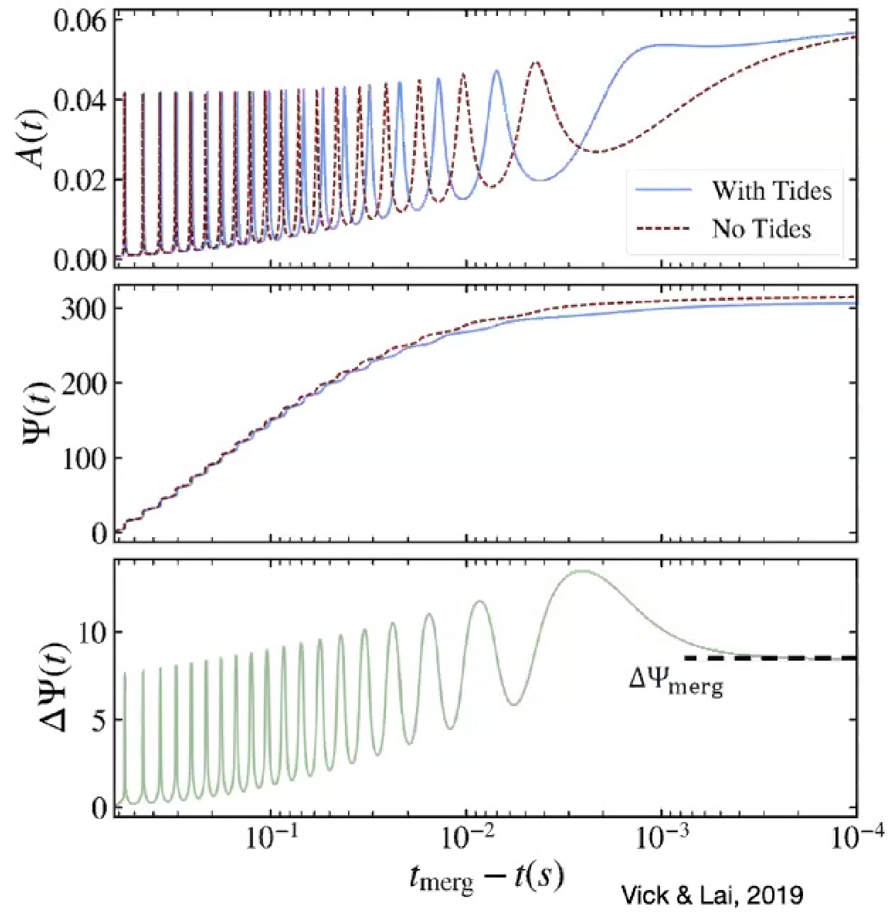


Vick & Lai, 2019

Black Hole-Neutron Star Binaries



Stephens, East, Pretorius (2011)



Vick & Lai, 2019

→ **Detectable by LIGO**

Neutron star f-modes: $\vec{x} \rightarrow \vec{x} + \vec{\xi}(t, \vec{x})$

$$\vec{\xi} = \sum_{\lambda} Q_{\lambda}(t)\, \vec{\xi}_{\lambda}(\vec{x})$$ → Eigenvalue problem

$$\ddot{Q}_m + \gamma \dot{Q}_m + \omega^2 Q_m \sim \frac{e^{im\phi}}{r^3}$$

Damping coefficient (γ)

Mode Frequency (ω^2)

Tidal Potential ($\frac{e^{im\phi}}{r^3}$)

Quadrupole moment: $I_{ij} \sim Q_m(t)$ → Dynamical tides

EFB Framework for f-modes:

$$\frac{p^3}{r^3}e^{im\phi} = \sum_{k=-\infty}^{\infty} X^k_{3,m}(e)\,e^{ik\ell} \qquad \ell = n(t - t_p)$$

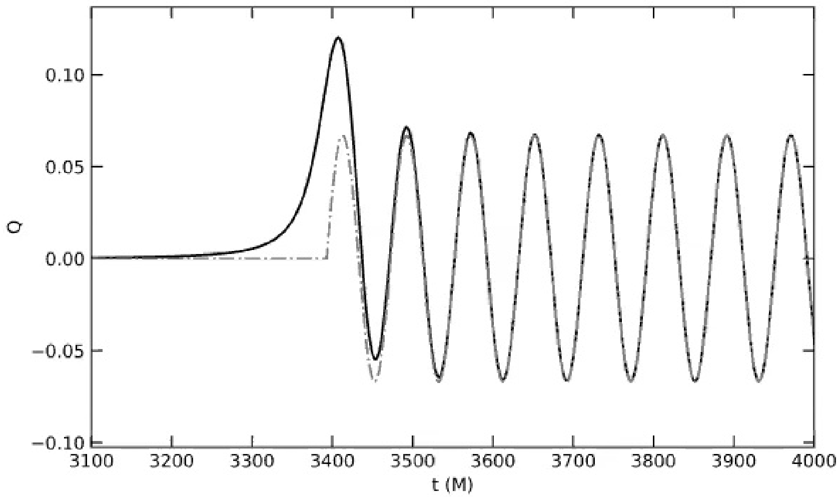
Hansen Coefficients

$$Q_m \sim \sum_{k=-\infty}^{\infty} \frac{X^k_{3,m}(e)}{\omega^2 - (kn)^2 + 2ikn\gamma}$$

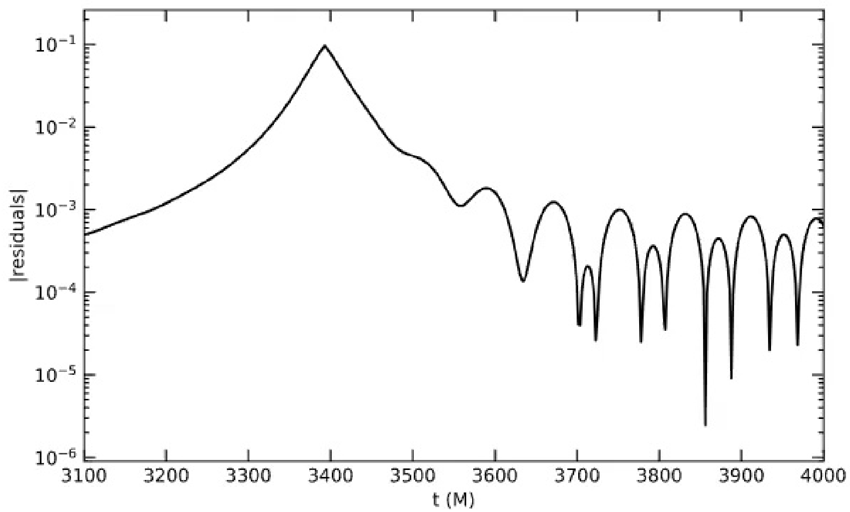
Re-summation (Contour Integration)

$$Q_m \sim e^{-\gamma(t-t_p)}\left[X^{\kappa_-}_{3,m}(e)e^{-i\omega'(t-t_p)} - X^{\kappa_+}_{3,m}(e)e^{i\omega'(t-t_p)}\right]$$

$$\kappa_\pm = \frac{1}{n}\left(i\gamma \pm \omega'\right) \qquad \omega' = \sqrt{\omega^2 - \gamma^2}$$



0.10
0.05
0.00
−0.05
−0.10
Q
3100
3200
3300
3400
3500
3600
3700
3800
3900
4000
t (M)



10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
|residuals|
3100
3200
3300
3400
3500
3600
3700
3800
3900
4000
t (M)

Future Prospects:

- Post-Newtonian EFB waveforms
  - Fourier coefficients become significantly more complicated
  - May be evaluated by method of stationary phase
- Tests of general relativity
  - Dispersion effects
  - Parameterized post-Einsteinian formalism
- Neutron stars
  - Backreaction of tides and precision timing model
  - Resonant tides
  - Constraints on equation of state
  - Source of damping (gravitational waves vs. viscosity)
- Chaotic f-modes
  - Mode energy can grow arbitrarily large at first order
  - Second order effects? Non-perturbative?
- Exotic compact objects
- Space-based observatories
  - Many more eccentric signals
  - ~ Two dozen expected within Milky Way

Thank you!