Title: Hamiltonian simulation meets holographic duality

Speakers: Toby Cubitt

Series: Perimeter Institute Quantum Discussions

Date: November 11, 2020 - 4:00 PM

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Abstract: "Analogue" Hamiltonian simulation involves engineering a Hamiltonian of interest in the laboratory and studying its properties experimentally. Large-scale Hamiltonian simulation experiments have been carried out in optical lattices, ion traps and other systems for two decades. Despite this, the theoretical basis for Hamiltonian simulation is surprisingly sparse. Even a precise definition of what it means to simulate a Hamiltonian was lacking.

AdS/CFT duality postulates that quantum gravity in a d-dimensional anti-de-Sitter bulk space is equivalent to a strongly interacting field theory on its d-1 dimensional boundary. Recently, connections between AdS/CFT duality and quantum error-correcting codes have led (amongst other things) to tensor network toy models that capture important aspects of this holographic duality. However, these toy models struggle to encompass dualities between bulk and boundary energy scales and dynamics.

On the face of it, these two topics seem to have nothing whatsoever to do with one another.

In my talk, I will explain how we put analogue Hamiltonian simulation on a rigorous theoretical footing, by drawing on techniques from Hamiltonian complexity theory and Jordan algebras. I will show how this proved far more fruitful than a mere mathematical tidying-up exercise, leading to the discovery of universal quantum Hamiltonians [Science, 351:6 278, p.1180, 2016], [Proc. Natl. Acad. Sci. 115:38 p.9497, 2018]. And I will explain how this new Hamiltonian simulation formalism, together with hyperbolic Coxeter groups, allowed us to extend the toy models of AdS/CFT to encompass energy scales, dynamics, and even (toy models of) black hole formation [arXiv:1810.08992].

Hamiltonian Simulation meets Holographic Duality

Toby Cubitt

TC, Ashley Montanaro, Stephen Piddock
PNAS 115:38, p.9497 (2018); arXiv: 1701.05182

Tamara Kohler & TC
J. High Energy Phys. (to appear); arXiv: 1810:08992

Perimeter, November 2020

UCL PHASECRAFT THE ROYAL SOCIETY

Talk Outline

- Hamiltonian simulation
  - What is H. simulation?
  - Jordan homomorphisms
  - Universal Hamiltonians

- Holographic duality
  - Holographic q. error-correcting codes
  - Hyperbolic Coxeter groups
  - Toy models of AdS/CFT

Lessons from computer science 1

Physical medium isn't important;
information it carries is what matters.

Virtualisation

Lessons from computer science 1

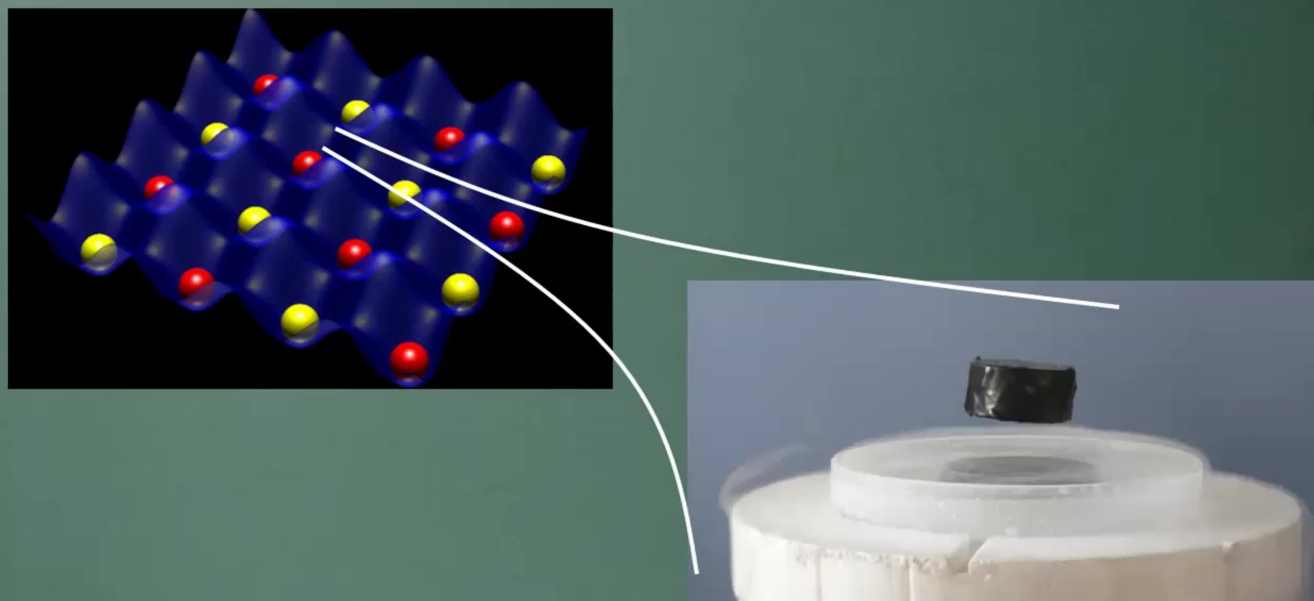
Physical medium isn't important;
information it carries is what matters.

Virtualisation

Virtualising Physics

Virtualising Physics

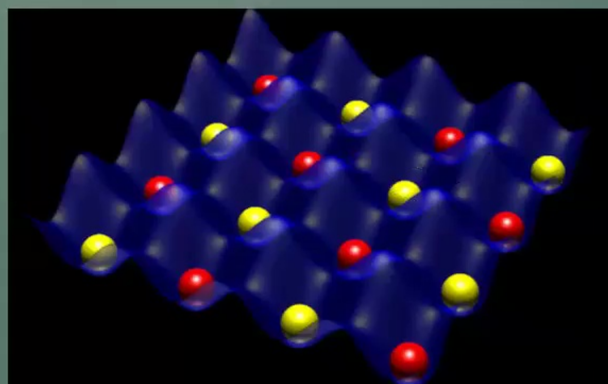
Virtualising Physics

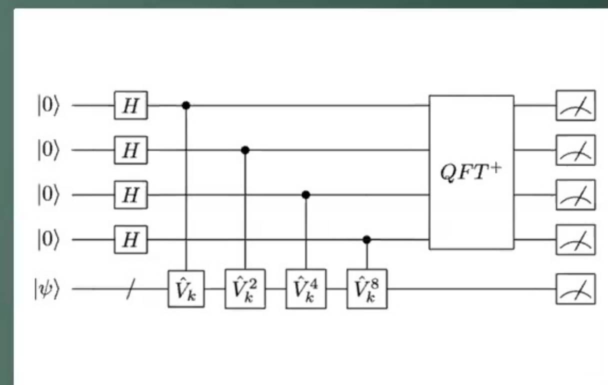


Virtualising Physics

Virtualising Physics

- Different from simulating Hamiltonian dynamics on a digital (quantum) computer.



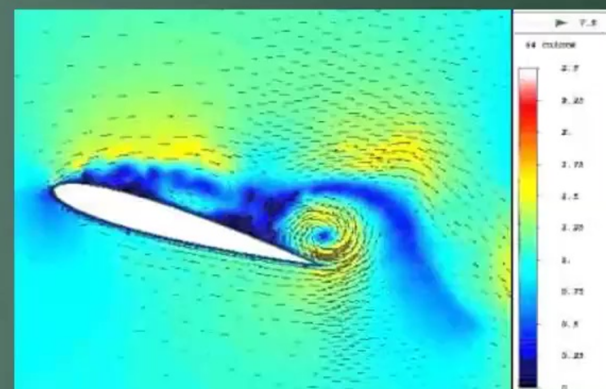


Physical simulation

Digital simulation

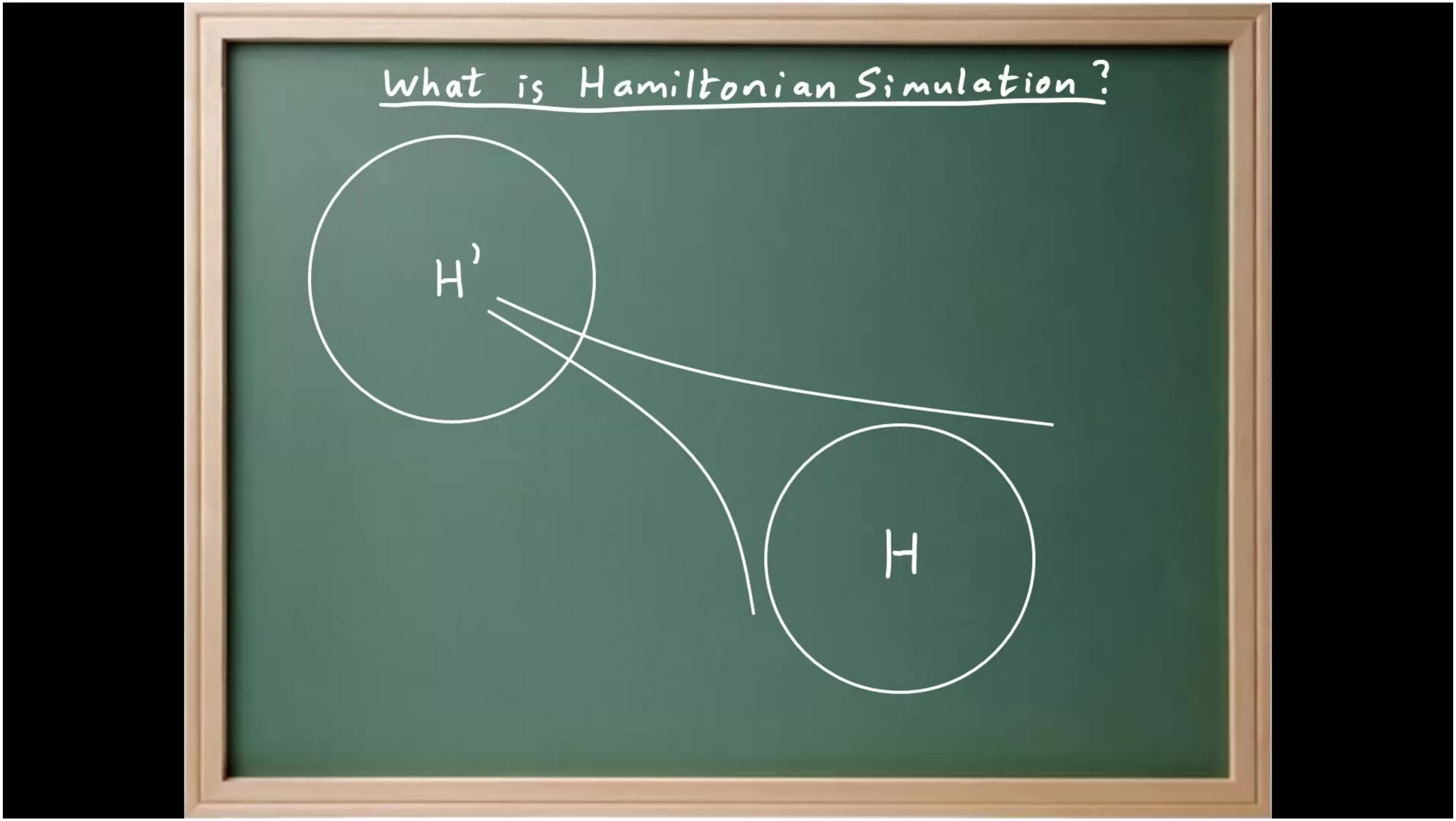
Virtualising Physics

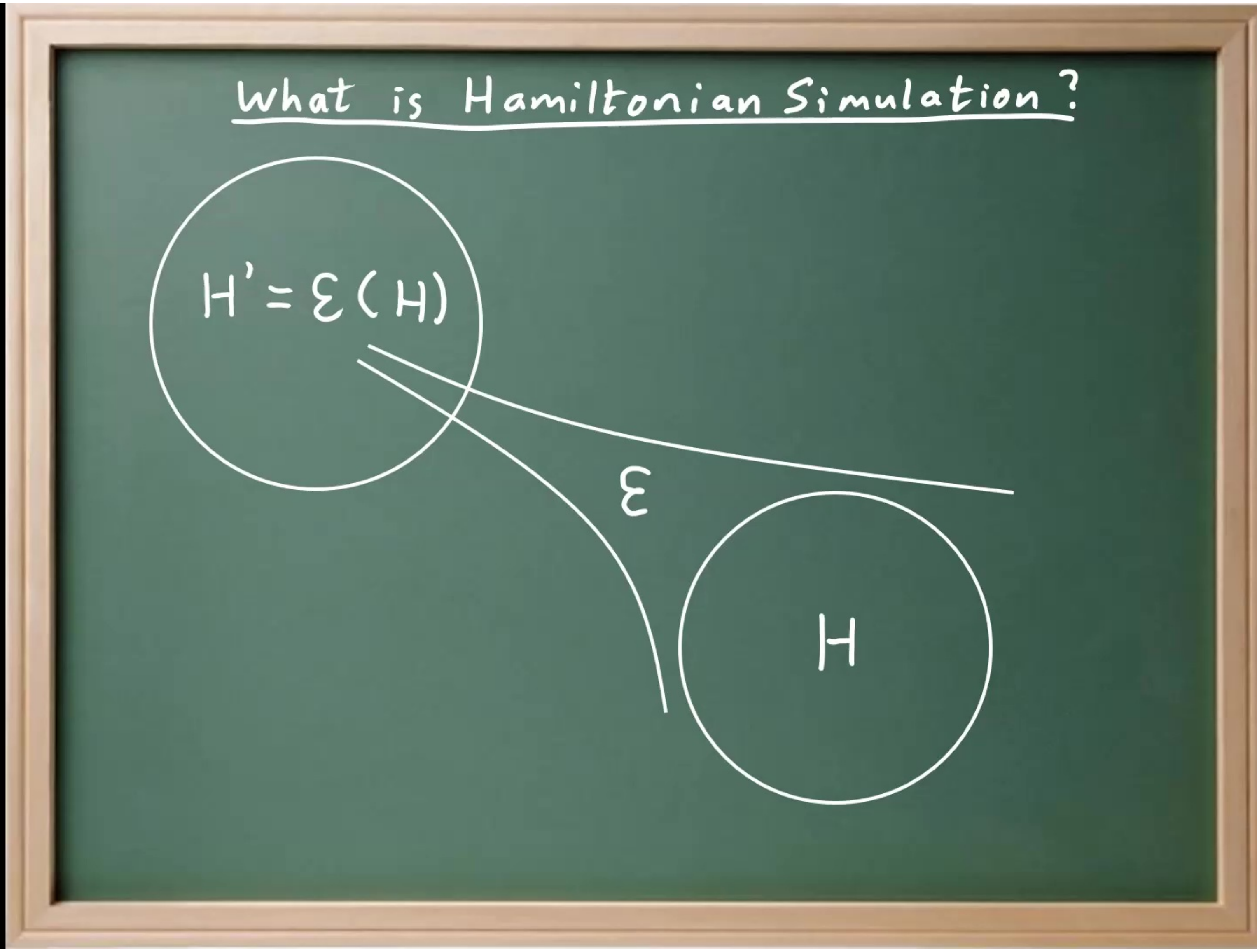
- Different from simulating Hamiltonian dynamics on a digital (quantum) computer.



Physical simulation

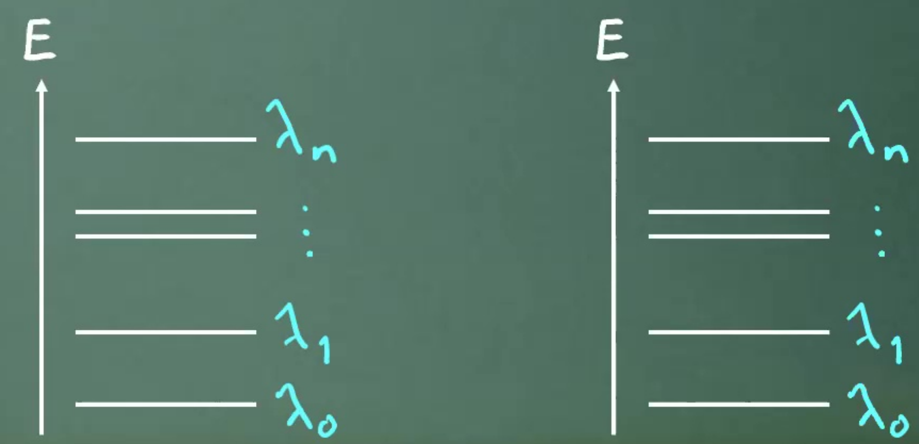
Digital simulation

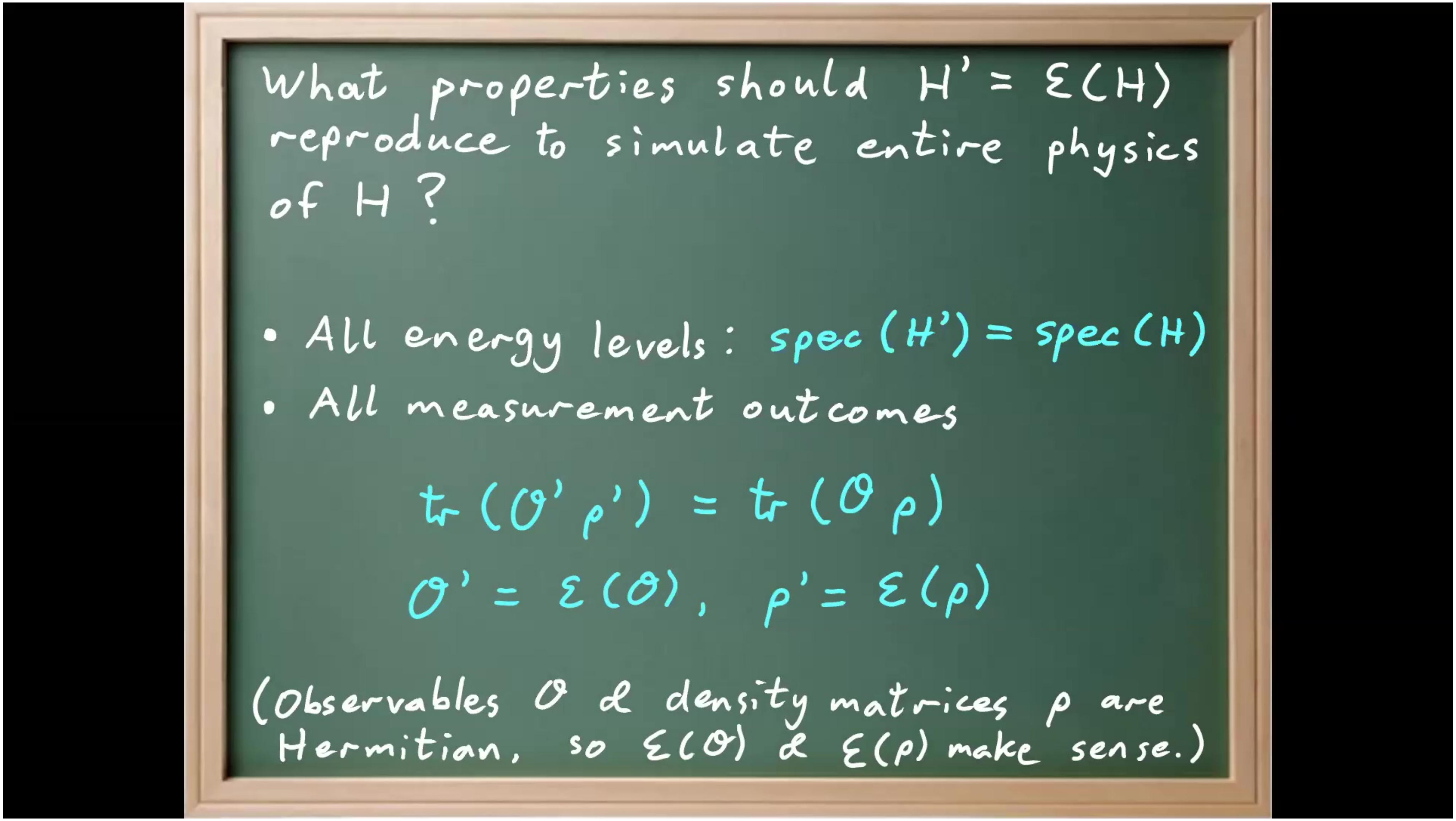

What is Hamiltonian Simulation?
H'
H


What is Hamiltonian Simulation?
H' = ε(H)
ε
H

What properties should $H' = \mathcal{E}(H)$ reproduce to simulate entire physics of H?

- All energy levels: $\text{spec}(H') = \text{spec}(H)$




What properties should $H' = \mathcal{E}(H)$ reproduce to simulate entire physics of H?
• All energy levels: $\mathrm{spec}(H') = \mathrm{spec}(H)$
• All measurement outcomes
$\mathrm{tr}(\mathcal{O}'\rho') = \mathrm{tr}(\mathcal{O}\rho)$
$\mathcal{O}' = \mathcal{E}(\mathcal{O})$, $\rho' = \mathcal{E}(\rho)$
(Observables $\mathcal{O}$ & density matrices ρ are Hermitian, so $\mathcal{E}(\mathcal{O})$ & $\mathcal{E}(\rho)$ make sense.)

What properties should $H' = \mathcal{E}(H)$ reproduce to simulate entire physics of H?

- All energy levels: $\mathrm{spec}(H') = \mathrm{spec}(H)$
- All measurement outcomes
- Time evolution
- Partition function

$$Z_{H'}(\beta) = \mathrm{tr}\left(e^{-\beta H'}\right)$$
$$= c\,\mathrm{tr}\left(e^{-\beta H}\right) = c\,Z_H(\beta)$$

(up to some constant c)

What properties should $H' = \mathcal{E}(H)$ reproduce to simulate entire physics of H?

- All energy levels: $\mathrm{spec}(H') = \mathrm{spec}(H)$
- All measurement outcomes
- Time evolution
- Partition function
- Local interactions:

$$H = \sum_i \alpha_i h_i \Rightarrow H' = \sum_i \alpha_i \mathcal{E}(h_i)$$

What properties should $H' = \mathcal{E}(H)$ reproduce to simulate entire physics of H?

- All energy levels: $\mathrm{spec}(H') = \mathrm{spec}(H)$
- All measurement outcomes
- Time evolution
- Partition function
- Local interactions: $H' = \sum_i \alpha_i \mathcal{E}(h_i)$
- Errors & noise

What properties should $H' = \mathcal{E}(H)$ reproduce to simulate entire physics of H?

- Must be a Hamiltonian! $H'^{\dagger} = H'$
- All energy levels: $\mathrm{spec}(H') = \mathrm{spec}(H)$
- All measurement outcomes
- Time evolution
- Partition function
- Local interactions: $H' = \sum_i \alpha_i \mathcal{E}(h_i)$
- Errors & noise
- ...

Physical Simulation

- Must be a Hamiltonian! $H'^{\dagger} = H'$
- All energy levels: $\mathrm{spec}(H') = \mathrm{spec}(H)$
- Local interactions: $H' = \sum_i \alpha_i \mathcal{E}(h_i)$

$$\Updownarrow$$

$$H' = \mathcal{E}(H) = U\left(H^{\oplus p} \oplus H^{* \oplus q}\right) U^{\dagger}$$

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$$H' = \mathcal{E}(H) = U\,(H^{\oplus p} \oplus H^{*\oplus q})\,U^{\dagger}$$

$$\Updownarrow$$

- Partition function
- All measurement outcomes
- Time evolution
- Errors & noise ...

Physical Simulation

$$H' = \mathcal{E}(H) = U\left(H^{\oplus p} \oplus H^{*\oplus q}\right)U^\dagger$$

$$\simeq U \begin{pmatrix} \overbrace{\boxed{H} \ddots \boxed{H}}^{p\text{ copies}} & 0 \\ 0 & \underbrace{\boxed{H^*} \ddots \boxed{H^*}}_{q\text{ copies}} \end{pmatrix} U^\dagger$$

Physical Simulation

- Must be a Hamiltonian! $H'^{\dagger} = H'$
- All energy levels: $\mathrm{spec}(H') = \mathrm{spec}(H)$
- Local interactions: $H' = \sum_i \alpha_i \mathcal{E}(h_i)$

$$\Updownarrow$$

$$H' = \mathcal{E}(H) = U\left(H^{\oplus p} \oplus H^{* \oplus q}\right) U^{\dagger}$$

$$\Updownarrow$$

- Partition function
- All measurement outcomes
- Time evolution • Errors & noise ...

Thm

$\mathcal{E}: \mathrm{Herm}_n \longrightarrow \mathrm{Herm}_m$. Following are equiv:

(i) $\mathcal{E}(H)^\dagger = \mathcal{E}(H)$

$\mathrm{spec}(\mathcal{E}(H)) = \mathrm{spec}(H)$

$\mathcal{E}(p H_1 + (1-p) H_2) = p\,\mathcal{E}(H_1) + (1-p)\,\mathcal{E}(H_2)$

(ii) $\exists$ unique extension $\mathcal{E}': \mathcal{M}_n \longrightarrow \mathcal{M}_m$ s.t.

$\mathcal{E}'(\mathbb{1}) = \mathbb{1}$

$\mathcal{E}'(A^\dagger) = \mathcal{E}'(A)^\dagger$

$\mathcal{E}'(A+B) = \mathcal{E}'(A) + \mathcal{E}(B)$

$\mathcal{E}(AB) = \mathcal{E}(A)\,\mathcal{E}(B)$

$\mathcal{E}'(xA) = x\,\mathcal{E}'(A)$, $x \in \mathbb{R}$

(iii) $\mathcal{E}'(A) = U\,(A^{\oplus p} \oplus A^{*\oplus q})\,U^\dagger$

Def. (special) Jordan algebra J_n

$\mathcal{M}_n$ with multiplication replaced by Jordan product: $A \circ B = AB + BA$

E.g. Herm_n, $A \circ B = AB + BA$

Def. Jordan homomorphism

$\phi : J_n \longrightarrow J'_m$

$\phi(A + B) = \phi(A) + \phi(B)$

$\phi(A \circ B) = \phi(A) \circ \phi(B)$

Thm 1 [Jacobson & Rikart '52 ; Martindale '67]

$\forall\, n \geq 2$, Jordan homomorphism $\phi : J_n \to J_m$

$\exists$ unique extension to homomorphism

$\phi' : \mathcal{M}_n \to \mathcal{M}_m$.

Thm 2

$\mathcal{E} : \mathrm{Herm}_n \to \mathrm{Herm}_m$

$\mathcal{E}(\mathbb{1}) = \mathbb{1}$

$\mathrm{spec}(\mathcal{E}(H)) = \mathrm{spec}(H)$

$\mathcal{E}(x H) = x\,(H)\,, \quad x \in \mathbb{R}$

$\Rightarrow \mathcal{E}$ is Jordan homomorphism

<u>Proof:</u> (i) $\Rightarrow$ (ii)

- spectrum-preserving $\Rightarrow \mathcal{E}(\mathbb{1}) = \mathbb{1}$, $\mathcal{E}(0) = 0$
  \+ convexity $\Rightarrow \mathcal{E}(x H) = x\,\mathcal{E}(H)$, $x \in \mathbb{R}$.
- Thm 2 $\Rightarrow$ $\mathcal{E}$ Jordon homomorphism.
- Thm 1 $\Rightarrow$ $\mathcal{E}'$ algebra homomorphism
  $\Rightarrow \mathcal{E}(AB) = \mathcal{E}(A)\,\mathcal{E}(B)$.
- Show remaing properties all lift from Herm_n to $\mathcal{M}_n$.
  (Only slightly tricky one is
  $\mathcal{E}(i\mathbb{1})^\dagger = -\mathcal{E}(i\mathbb{1})$)

Proof: (ii) $\Rightarrow$ (iii)

- Complex structure $J := \mathcal{E}(i\mathbb{1})$
  $J^2 = \mathcal{E}(i\mathbb{1})^2 = \mathcal{E}((i\mathbb{1})^2) = \mathcal{E}(-\mathbb{1}) = -\mathbb{1}$
  $\Rightarrow$ J has eigvals $\pm i$.

- $[J, \mathcal{E}(A)] = \mathcal{E}([i\mathbb{1}, A]) = \mathcal{E}(0) = 0$
  $\Rightarrow \mathcal{H} \cong \mathcal{H}_+ \oplus \mathcal{H}_-$, $\quad \mathcal{E}(J) = i\mathbb{1} \oplus (-i\mathbb{1})$.

- $\mathcal{E}(AB)|_\pm = \mathcal{E}(A)|_\pm \, \mathcal{E}(B)|_+$
  $\mathcal{E}(A^\dagger)|_\pm = \mathcal{E}(A)^\dagger|_\pm$
  $\mathcal{E}(iA)|_\pm = (J \cdot \mathcal{E}(A))|_\pm = \pm i A'_\pm$
  $\Rightarrow \mathcal{E}(\cdot)|_\pm$ is an (anti)-$*$-homomorphism.

- $\mathcal{E}(A) = U(A^{\oplus p} \oplus A^{*\oplus q})U^\dagger$ by standard C^*-algebra representation theory.

Physical Simulation

Real Hamiltonian simulations:

- will never be perfect
  → account for approximation errors
- need to be efficient
  → account for simulation overhead
- should preserve locality
- might simulate only within a subspace
  (e.g. low-energy subspace)

Approximate Simulation

<u>Thm/Def</u>: Approximate simulation

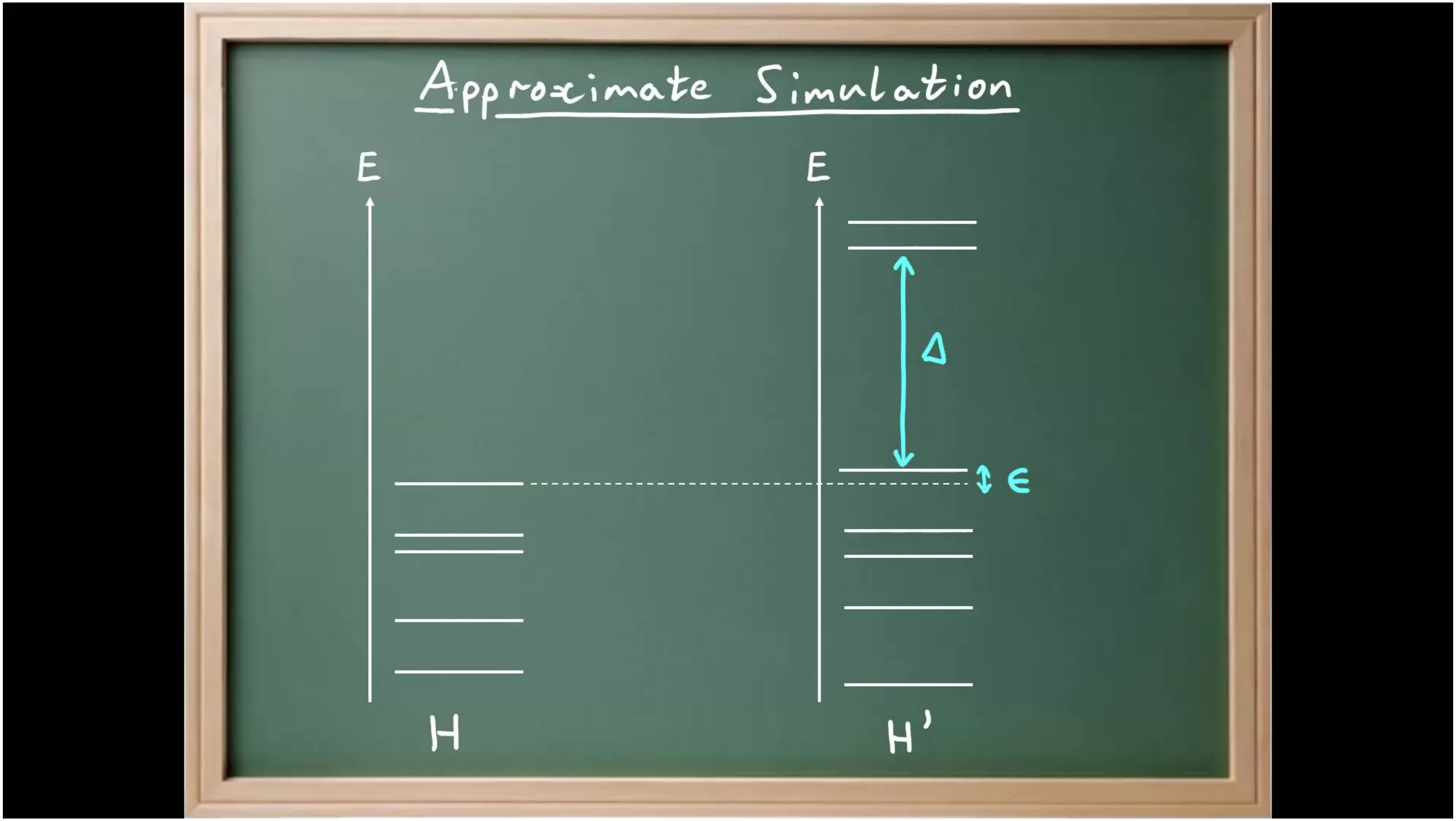
H' simulates H to precision ϵ, η up to cut-off Δ if $\exists$ subspace encodings $\mathcal{E}, \tilde{\mathcal{E}}$ s.t.

$$\| H'|_{\leq \Delta} - \tilde{\mathcal{E}}(H) \| \leq \epsilon$$

$$\| \tilde{\mathcal{E}} - \mathcal{E} \| \leq \eta$$

$$\mathcal{E}(H) = V^{\otimes n}(H \otimes P + H^* \otimes Q) V^{\otimes n \dagger}$$

P, Q locally orthogonal



Approximate Simulation
E
E
Δ
ϵ
H
H'

Approximate Simulation

Thm/Def: Approximate simulation

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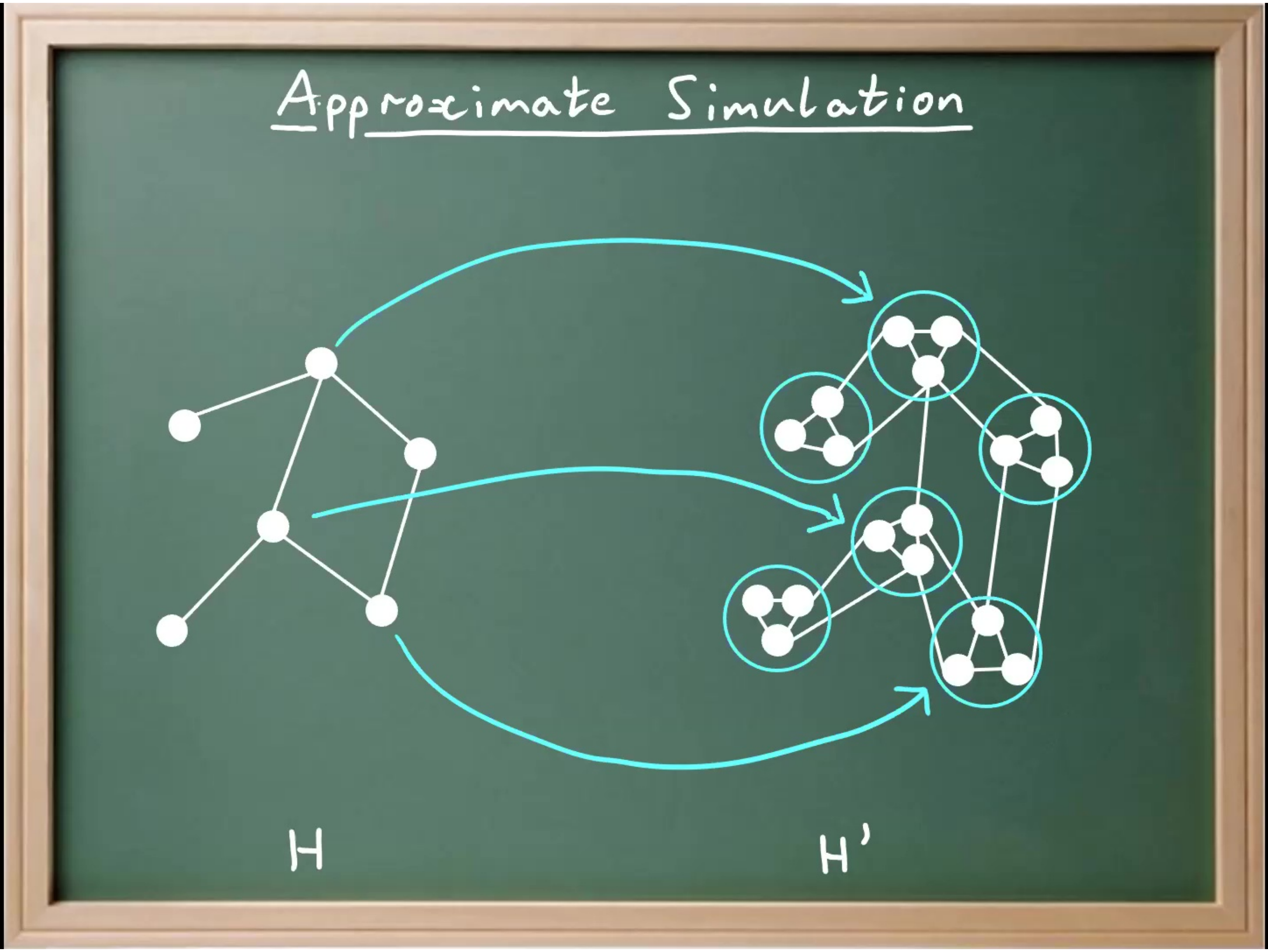
Approximate Simulation
H
H'

What physics are we replicating?

Everything!

<u>Thm</u>

If H' $(\Delta, \varepsilon, \eta)$-simulates H',

- Energy spectrum: $|\lambda'_i - \lambda_i| \leq \epsilon$
- Partition function: $Z_{H'} = cZ_H + O(e^{-\Delta} + \epsilon)$
- Time-evolution:

$$\| e^{-iH't} \rho' e^{iH't} - e^{-i\mathcal{E}(H)t} \rho' e^{i\mathcal{E}(H)t} \| \leq 2\epsilon t + 4\eta$$

- Observables: (note: $\mathcal{E}(\mathcal{O})$ local if $\mathcal{O}$ is)

$$|\mathrm{tr}(\mathcal{E}(\mathcal{O})\rho') - \mathrm{tr}(\mathcal{O}\rho)| \leq \epsilon$$

What physics are we replicating?

Everything!

<u>Thm</u>

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- Observables: (note: $\mathcal{E}(\mathcal{O})$ local if $\mathcal{O}$ is)

$$| \mathrm{tr}(\mathcal{E}(\mathcal{O})\rho') - \mathrm{tr}(\mathcal{O}\rho) | \le \epsilon$$

Simulation $\leftrightarrow$ QECC

- Perfect $(\Delta, 0, 0)$-simulation:

$$\mathcal{E}(A) = V\left(A^{\oplus p} \oplus A^{\oplus q}\right)V^{\dagger}$$

- QECC: $\mathcal{E}(A) = V A V^{\dagger}$

Simulation $\leftrightarrow$ QECC

- Perfect $(\Delta,0,0)$-simulation:

$$\mathcal{E}(A) = V(A^{\oplus p} \oplus A^{\oplus q})V^{\dagger}$$

- QECC: $\mathcal{E}(A) = VAV^{\dagger}$

$\rightarrow$ QECCs are special cases of perfect simulations with $p=1$, $q=0$.

- Approximate $(\Delta,\varepsilon,\eta)$-simulation for $p=1$, $q=0$

$\rightarrow$ approximate QECC

Simulation $\leftrightarrow$ QECC

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$\longrightarrow$ QECCs are special cases of perfect simulations with $p=1$, $q=0$.

- Approximate $(\Delta,\varepsilon,\eta)$-simulation for $p=1$, $q=0$

$\longrightarrow$ approximate QECC

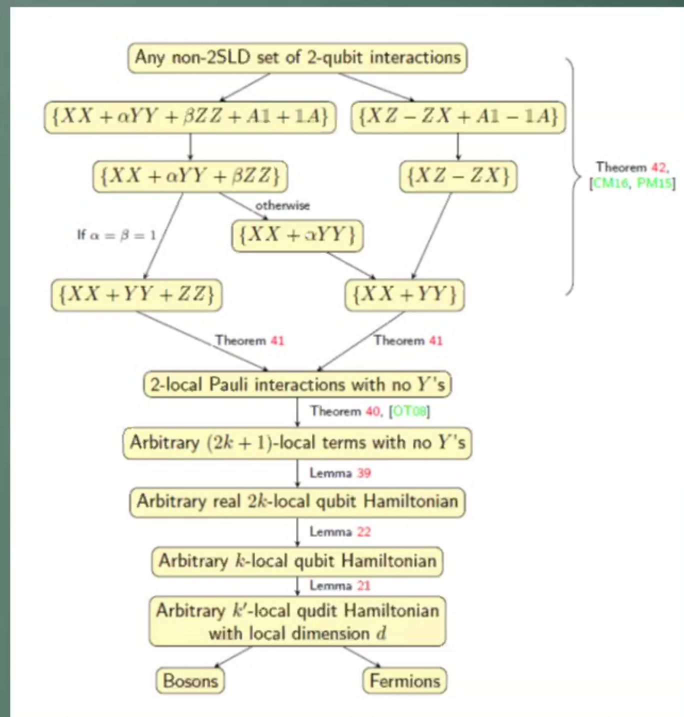
Do there exist "universal" quantum Hamiltonians?

Able to simulate:

- all physical properties
- of any other many-body model
- to any desired accuracy

($\neq$ universality classes in cond-mat.)

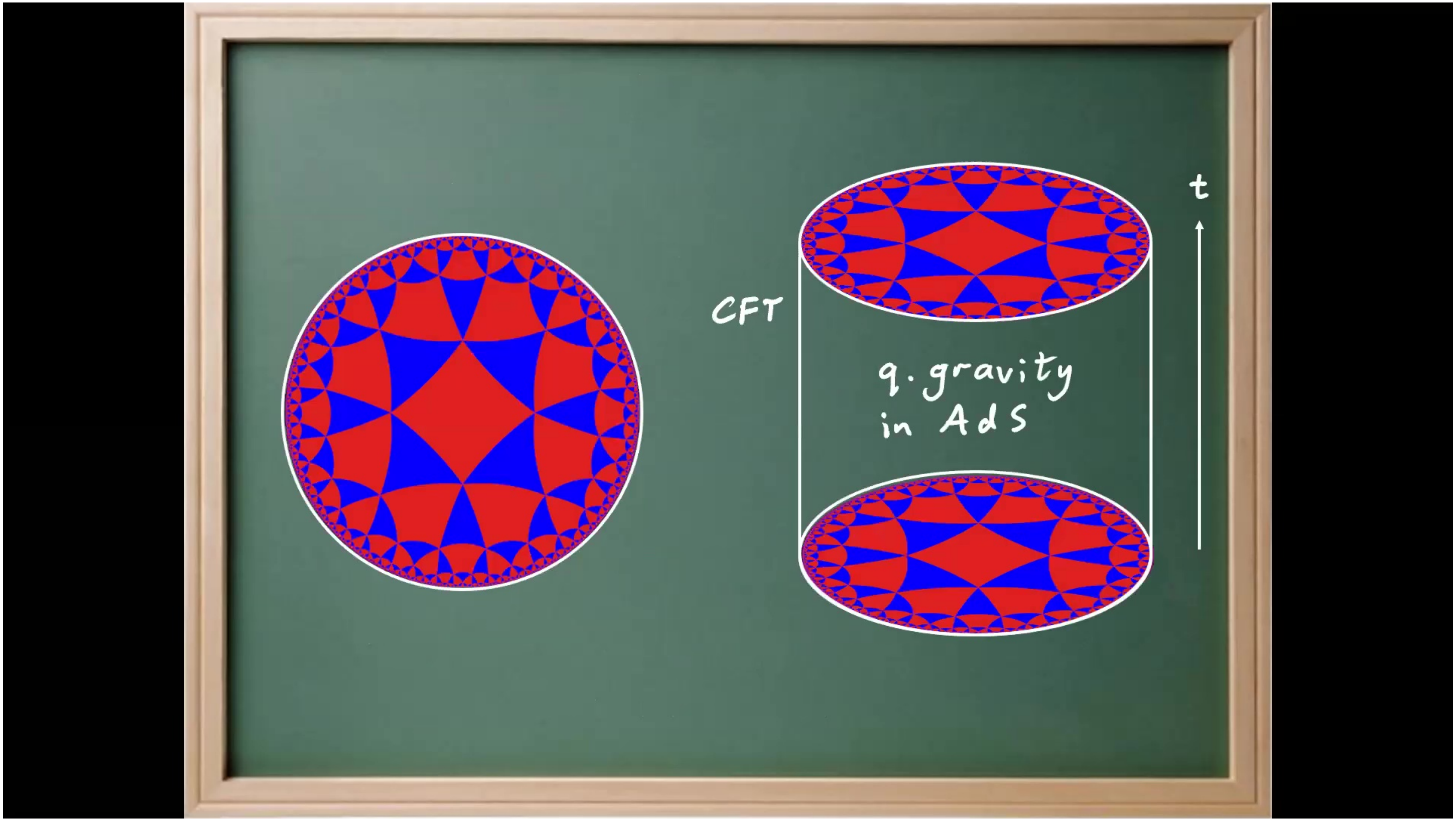
Proof idea: "Perturbative Gadgets"

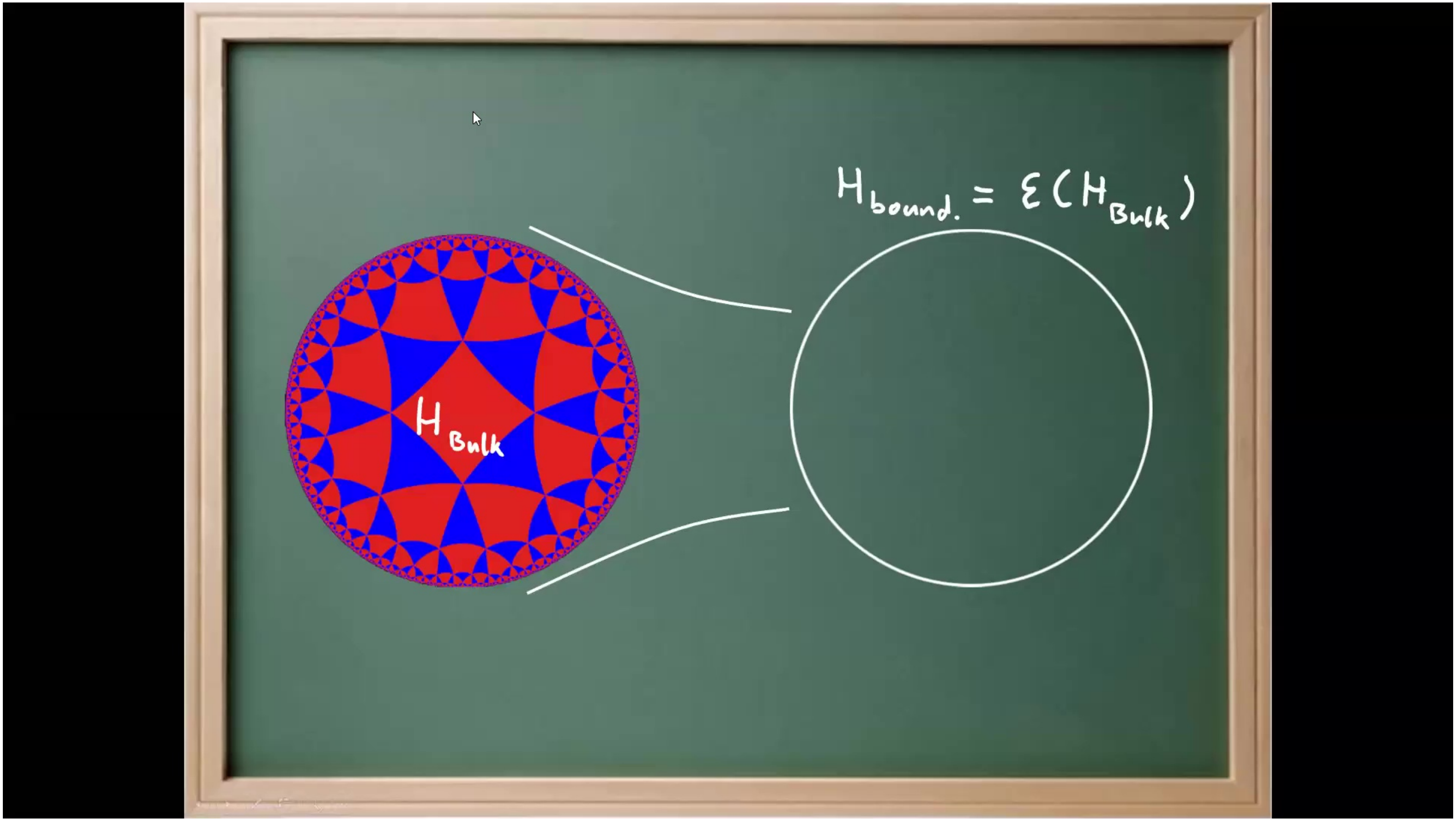


Cor: Half of H. complexity theory
+ many new results

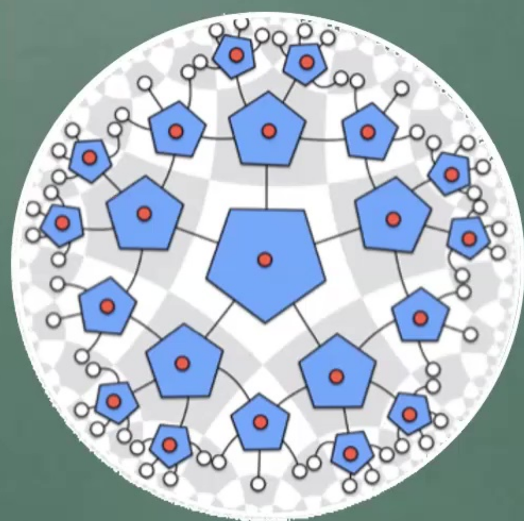
Talk Outline

- Hamiltonian simulation
  - What is H. simulation?
  - Jordan homomorphisms
  - Universal Hamiltonians
- Holographic duality
  - Holographic q. error-correcting codes
  - Hyperbolic Coxeter groups
  - Toy models of AdS/CFT

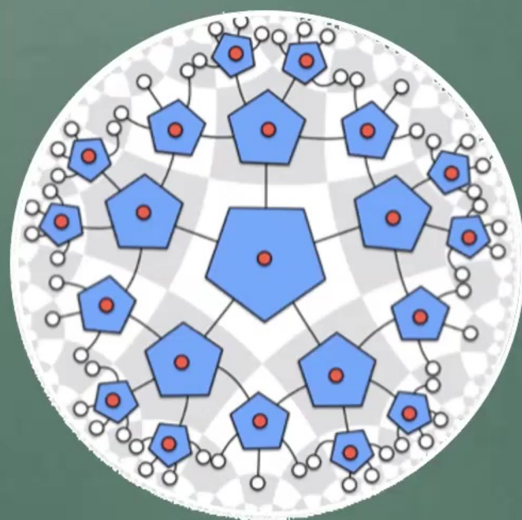

CFT
q. gravity
in AdS
t


$H_{bound.} = \mathcal{E}(H_{Bulk})$
H_{Bulk}

[HaPPY, 2015]:



- Tensor network construction of holographic q. error correcting code.
- Toy model exhibiting many non-trivial qualitative features of AdS/CFT duality.



- Tensor network
  $\longrightarrow$ encoding isometry V
  for QECC:

  $A_{\text{bound.}} = V A_{\text{Bulk}} V^\dagger$

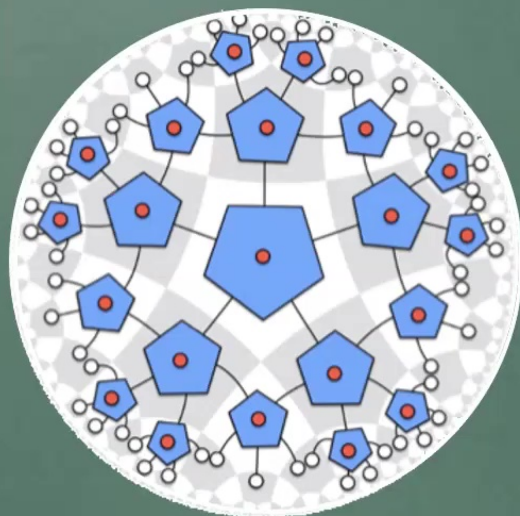
- Cf. Def. encoding in a subspace:

  $A' = V(A^{\oplus p} \oplus A^{* \oplus q}) V^\dagger$

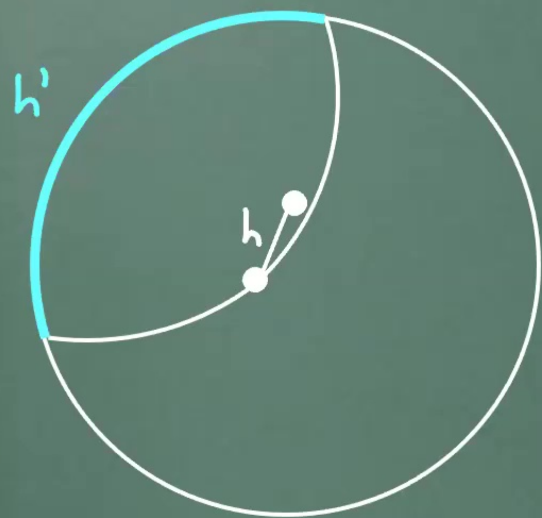
  $\longrightarrow$ QECCs are encodings with $p=1, q=0$.

- $H_{\text{bound}} = V H_{\text{Bulk}} V^\dagger + \Delta \sum_S h_S$

  $\longrightarrow$ $(\Delta, 0, 0)$-simulation.



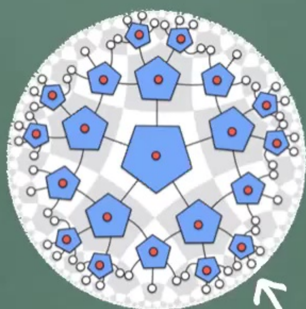
- Holographic error correcting codes give toy model of duality between bulk & boundary states & observables.
- But AdS/CFT is also a duality between models (= local Hamiltonians for toy model).



- What if we map local bulk Hamiltonians to boundary?
  → non-local Hamiltonian

- Holographic codes give good toy AdS/CFT for states & observables, but not for models (Hamiltonians, energy scales, dynamics).

- Universal Hamiltonian proof shows how anything can be simulated by e.g. 2D Heisenberg model.
- Combine tensor networks + H. simulation → toy models including H. duality?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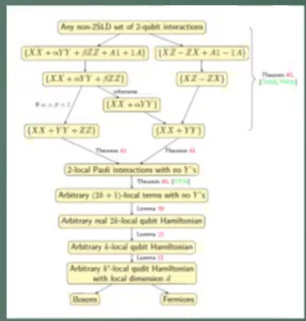

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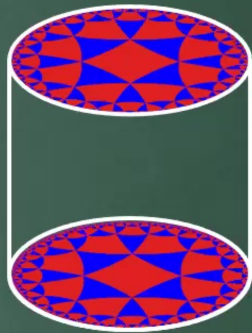
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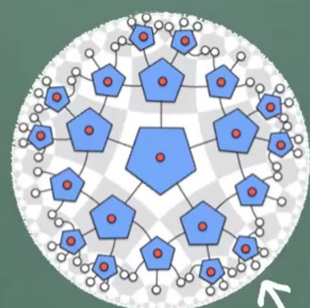
- Problem:

1D boundary   Works in ≥2D

- Universal Hamiltonian proof shows how anything can be simulated by e.g. 2D Heisenberg model.
- Combine tensor networks + H. simulation
→ toy models including H. duality?



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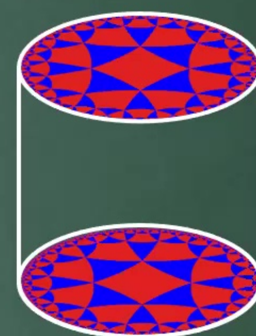


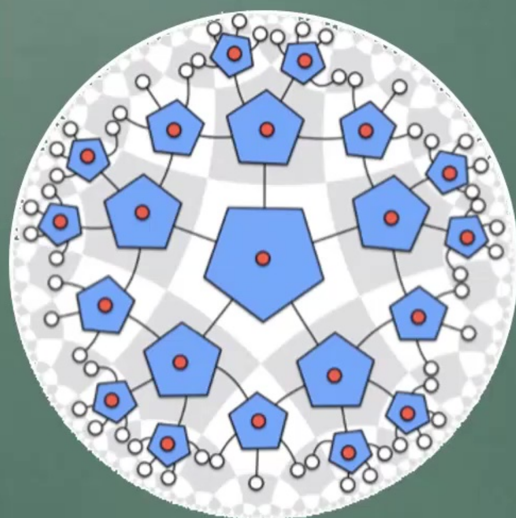
?

- Problem:

1D boundary

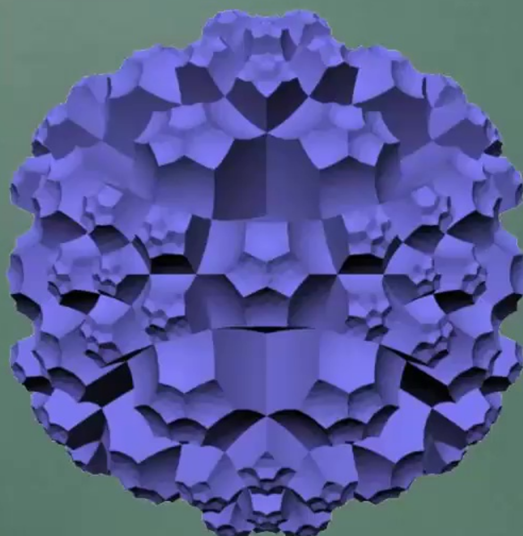
Works in ≥ 2D

→ Need ≥ 3D holographic codes first...



Tessellation of $\mathbb{H}_2$
by right-angled
pentagons

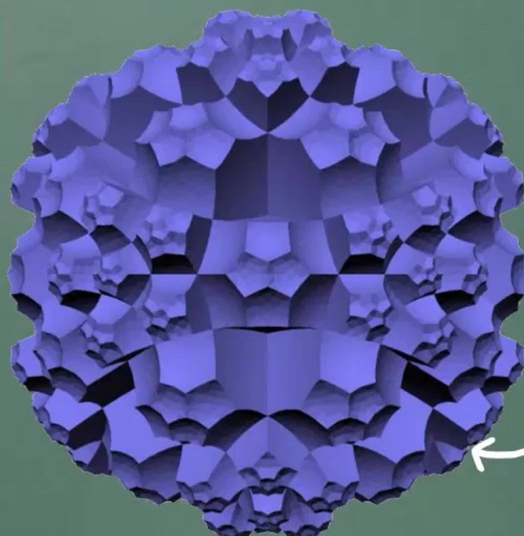
Coxeter
diagram: 



Tessellation of $\mathbb{H}_3$ by right-angled dodecahedra

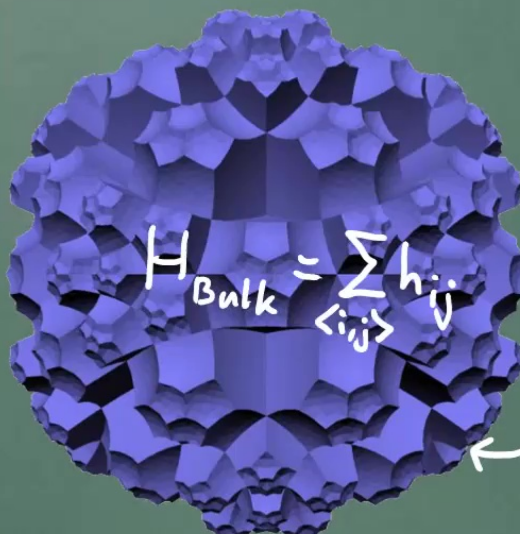
Coxeter diagram:

- V = encoding isometry for HQECC defined by tensor network



$$H' = \sum_{\langle i,j\rangle} V h_{Bulk} V^\dagger + \Delta_s \sum_{stabilizers} V h_s V^\dagger$$

- V = encoding isometry for HQECC defined by tensor network

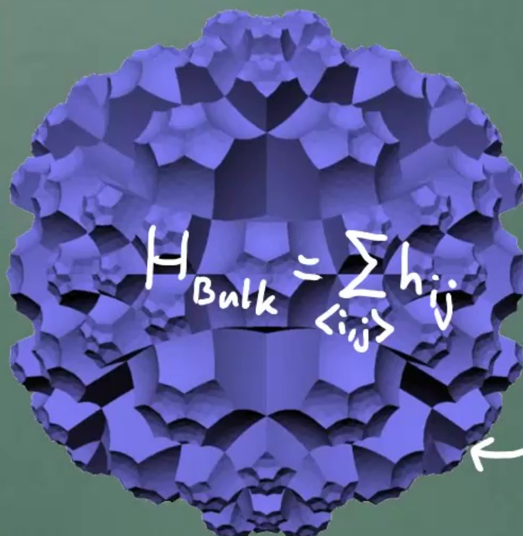


$$H_{Bulk} = \sum_{\langle i,j \rangle} h_{ij}$$

$$H' = \sum_{\langle i,j \rangle} V h_{Bulk} V^{\dagger} + \Delta_S \sum_{\text{stabilizers}} V h_S V^{\dagger}$$

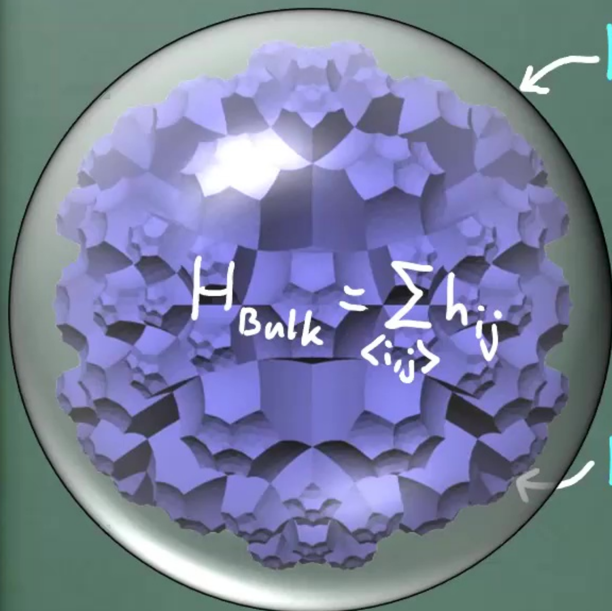
H' is $(\Delta_S, 0, 0)$-simulation of H_{Bulk}, but both $V h_{ij} V^{\dagger}$ & $V h_S V^{\dagger}$ non-local.

- Contruct $(\Delta_L, \varepsilon, \eta)$-simulation of H' using perturbation gadgets:



$$H' = \sum_{\langle i,j \rangle} V h_{Bulk} V^\dagger + \Delta_S \sum_{\text{stabilizers}} V h_S V^\dagger$$

- Contruct $(\Delta_L, \varepsilon, \eta)$-simulation of H' using perturbation gadgets:



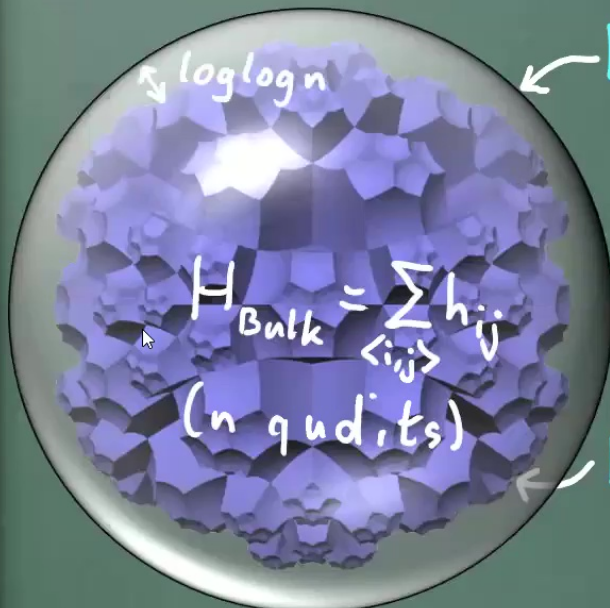
$$H_{bound} = \sum_{\langle i,j \rangle} \alpha_{ij} h_{Heis} + \Delta_L \sum_{\langle i',j' \rangle} h_{Heis}$$

$$H' = \sum_{\langle i,j \rangle} V h_{Bulk} V^{\dagger} + \Delta_S \sum_{stabilizers} V h_S V^{\dagger}$$

$\Rightarrow$ $H_{bound.}$ is $(\Delta_L, \varepsilon, \eta)$-simulation of H_{Bulk}

$\longrightarrow$ same physics

- Contruct $(\Delta_L, \varepsilon, \eta)$-simulation of H' using perturbation gadgets:



$H_{bound} = \sum_{\langle i,j \rangle} \alpha_{ij} h_{Heis} + \Delta_L \sum_{\langle i',j' \rangle} h_{Heis}$

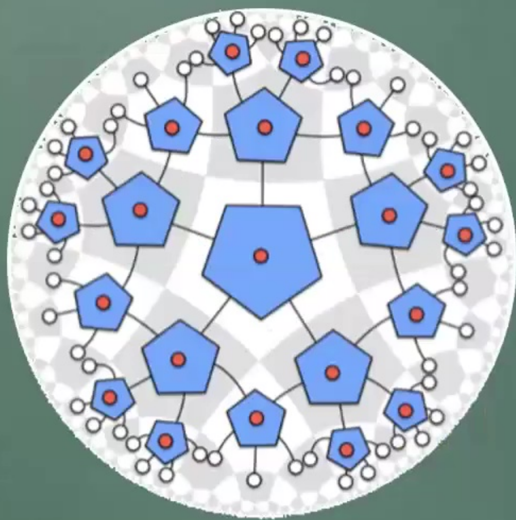
$O(n \log^4 n)$ qubits

$H' = \sum_{\langle i,j \rangle} V h_{Bulk} V^\dagger + \Delta_S \sum_{\text{stabilizers}} V h_S V^\dagger$

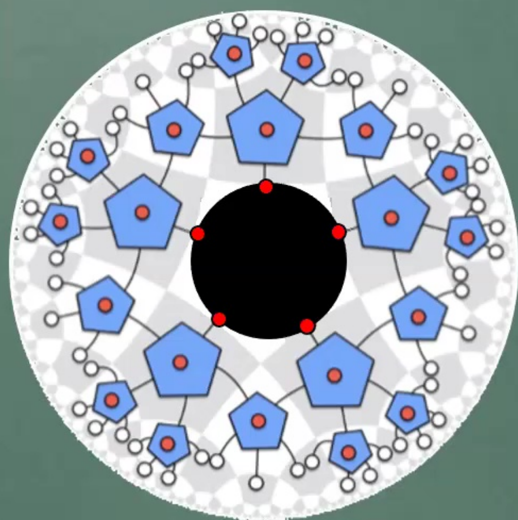
$\Rightarrow H_{bound.}$ is $(\Delta_L, \varepsilon, \eta)$-simulation of H_{Bulk}

$\longrightarrow$ same physics $(\varepsilon, \eta \searrow 0$ as $\Delta_L \nearrow \infty)$

$V_{n,c}$:= encoding isometry for tensor network corresponding to $\mathcal{H}_{n,c}$



$$U := \bigoplus_{n} \bigoplus_{c} V_{n,c}$$

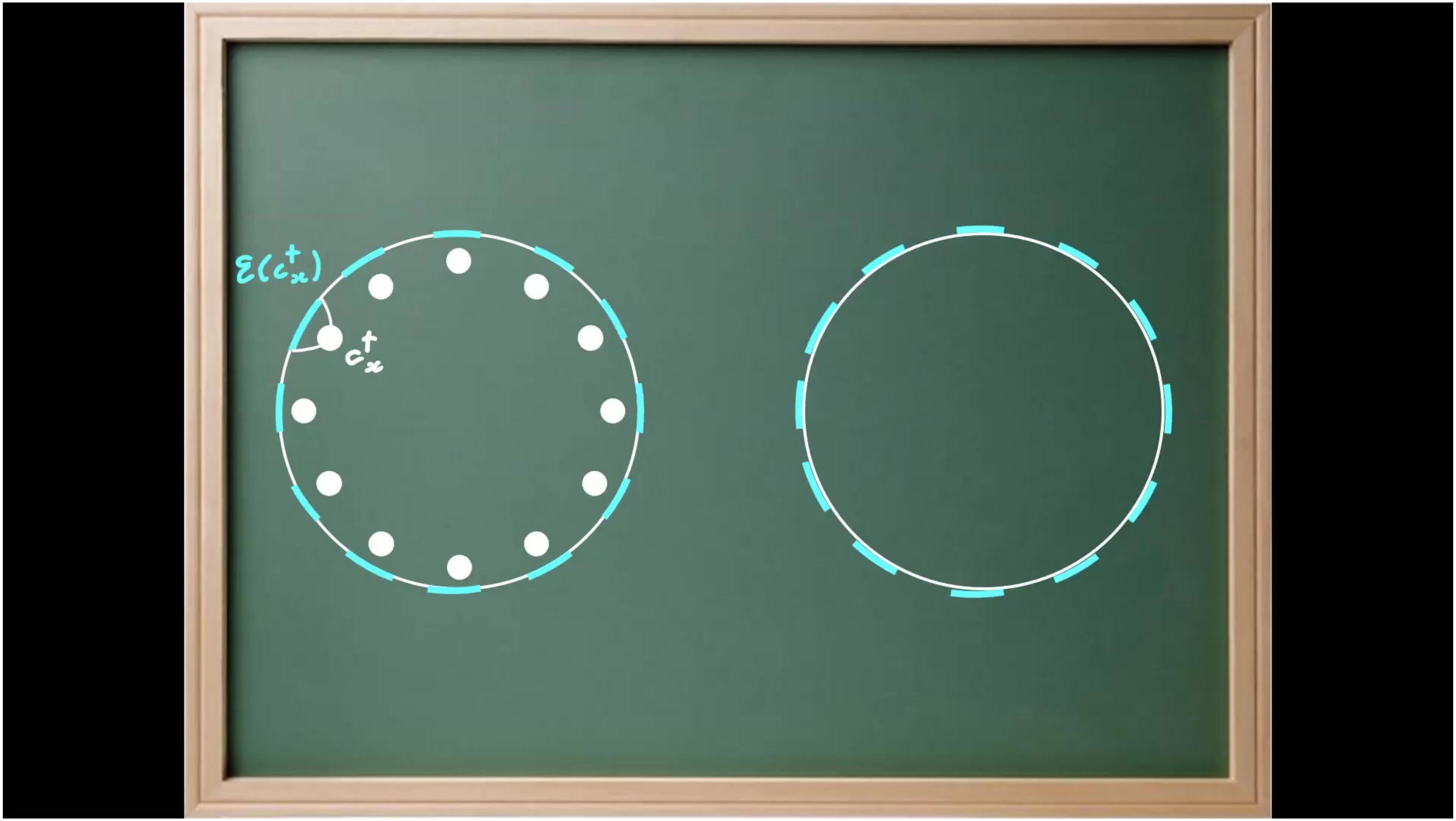


Toy model of black hole [HaPPY 2015]:

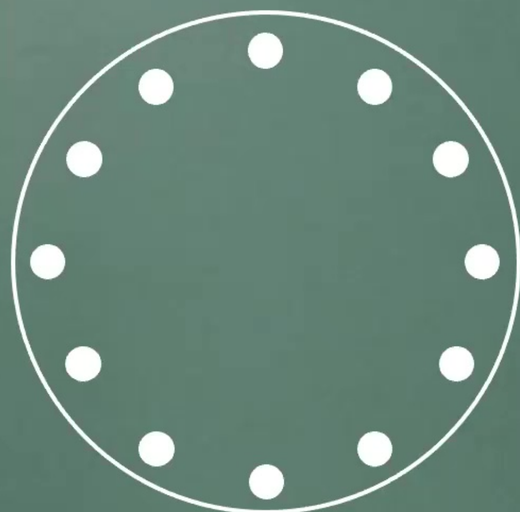
- New free tensor indices on boundary of hole = internal black hole d.o.f.
- # dof scale with surface area of hole $\approx$ Bekenstein-Hawking
- New: black hole states have high energy $\geqslant \Delta_s$ w.r.t. H_{bound}

- Have H_{bound} now
  $\rightarrow$ can study time dynamics ...
- Assume bulk dynamics is that of gravity.
- Start in state $\in \mathcal{H}_0$ corresponding to low-density shell of matter near boundary, $E > \Delta_s$

$\rightarrow$ collapses inwards to centre.

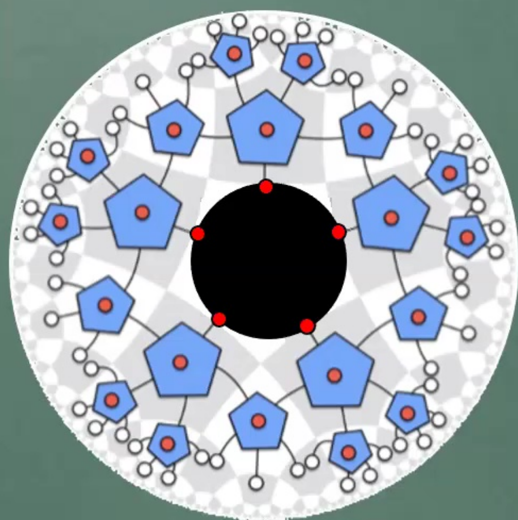


$\mathcal{E}(c_x^\dagger)$
$c_x^\dagger$



- Initial state $\in \mathcal{H}_0$ & energy $> \Delta_s$ (by assumption)
- Doesn't violate any stabilizers ($\mathcal{H}_0$)

$\longrightarrow E > \Delta_s$ from many contributions $\ll \Delta_s$ from local H terms



- Unitary dynamics
  → energy conserved
  ⇒ final state has energy $> \Delta_s$
- Can only pick up energy from few local H terms at centre
  ⇒ must violate ≥ 1 stabilizer term $\Delta_s h_s$
  → black hole state

Gravitational dynamics
→ formation of toy model black holes.

Conclusions

- Constructed rigorous holographic duality for any local bulk model.

  → Where is gravity? Is holography just a consequence of IH?

- Our constructions work in Euclidean & spherical space (worse scalings)

  → What qualitative features should we aim for?

Conclusions

- Have bulk interpretation of **any** boundary state (not just code subspace)
  → Extract more? E.g. non-flat S_α

- Construction gives full local SU(2) symmetry on boundary
  → More symmetries? Poincaré? Conformal?
  [Stiegemann, Osborne '17]

References

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TC, Ashley Montanaro, Stephen Piddock
Proc. Natl. Acad. Sci. 115:38, p.9497 (2018)
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Tamara Kohler & TC
J. High Energy Phys. 2019:17
arXiv: 1810:08992 [hep-th] (62 pages)