Title: Causal-Inferential theories: Realism revisited

Speakers: David Schmid

Series: Quantum Foundations

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Abstract: Using a process-theoretic formalism, we introduce the notion of a causal-inferential theory: a triple consisting of a theory of causal influences, a theory of inferences (of both the Boolean and Bayesian varieties), and a specification of how these interact. Recasting the notions of operational and realist theories in this mold clarifies what a realist account of an experiment offers beyond an operational account. It also yields a novel characterization of the assumptions and implications of standard no-go theorems for realist representations of operational quantum theory, namely, those based on Bell's notion of locality and those based on generalized noncontextuality. Moreover, our process-theoretic characterization of generalised noncontextuality is shown to be implied by an even more natural principle which we term Leibnizianity. Most strikingly, our framework offers a way forward in a research program that seeks to circumvent these no-go results. Specifically, we argue that if one can identify axioms for a realist causal-inferential theory such that the notions of causation and inference can differ from their conventional (classical) interpretations, then one has the means of defining an intrinsically quantum notion of realism, and thereby a realist representation of operational quantum theory that salvages the spirit of locality and of noncontextuality.

Causal-Inferential Theories:
Realism Revisited

David Schmid
John Selby
Rob Spekkens

arXiv:2009.03297

"[...] our present QM formalism is not purely epistemological; it is a peculiar mixture describing in part realities of Nature, in part incomplete human information about Nature all scrambled up by Heisenberg and Bohr into an omelette that nobody has seen how to unscramble.

— E.T. Jaynes, 1989

theory of inference

theory of causation

"[...] our present QM formalism is not purely epistemological; it is a peculiar mixture describing in part realities of Nature, in part incomplete human information about Nature all scrambled up by Heisenberg and Bohr into an omelette that nobody has seen how to unscramble.

Yet we think that the unscrambling is a prerequisite for any further advance in basic physical theory. For, if we cannot separate the subjective and objective aspects of the formalism, we cannot know what we are talking about; it is just that simple.

So we want to speculate on the proper tools to do this."

— E.T. Jaynes, 1989

causal-inferential framework

Simpson's paradox
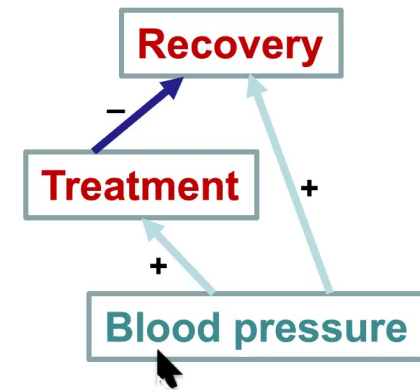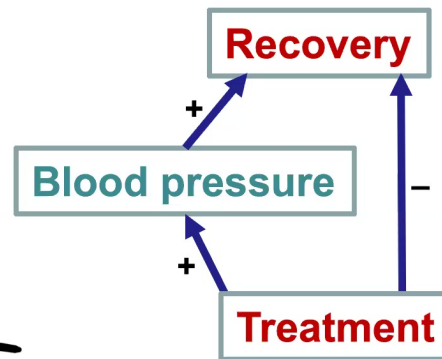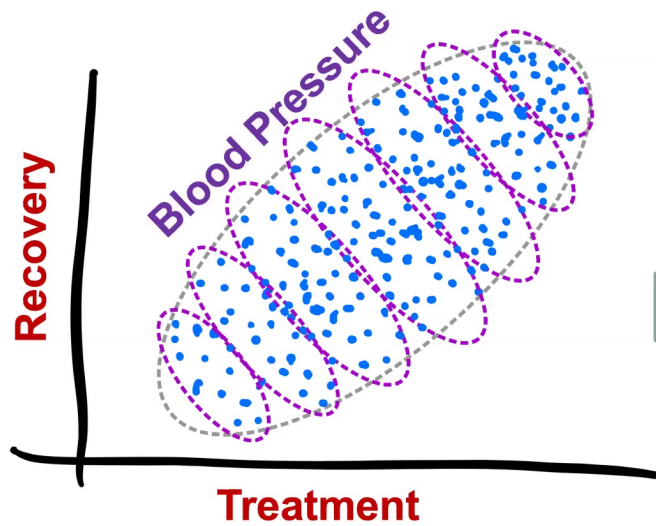
T is correlated with R
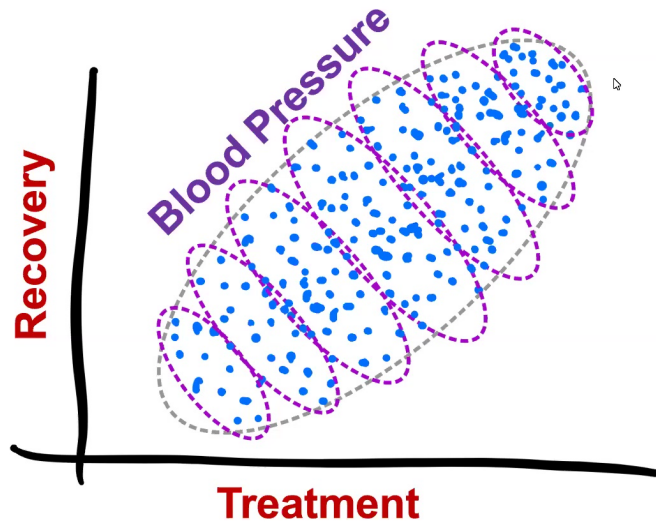
Simpson's paradox
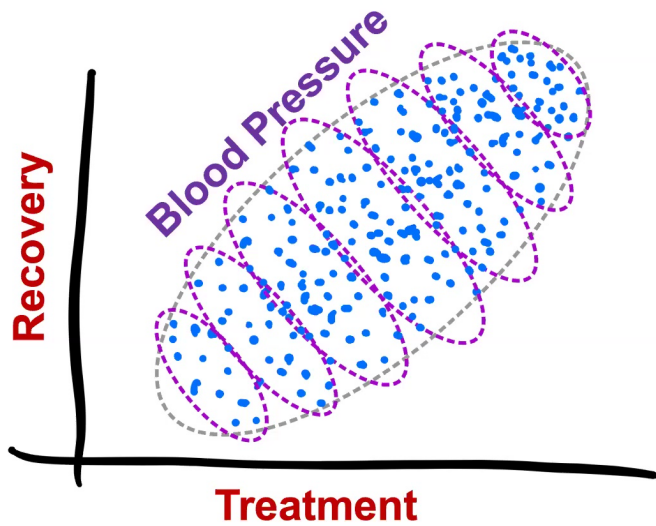
T is correlated with R

…but T is anticorrelated with R in **every** subpopulation!

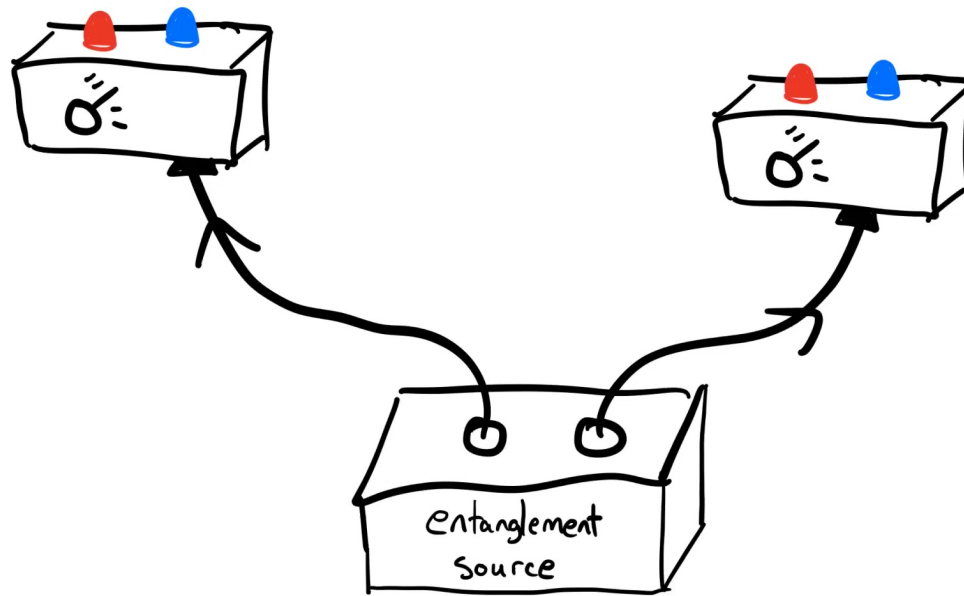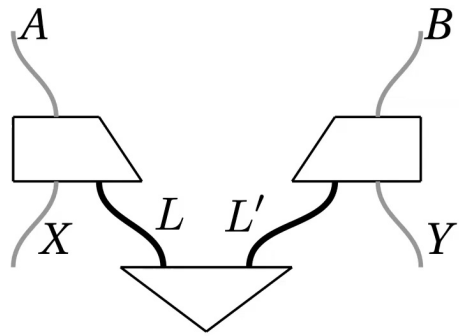If causation can be defined in terms of correlations… paradox!

Simpson's paradox

Framework of classical causal models
-causation is *primitive*, NOT defined by correlations
-causal conclusions require causal commitments
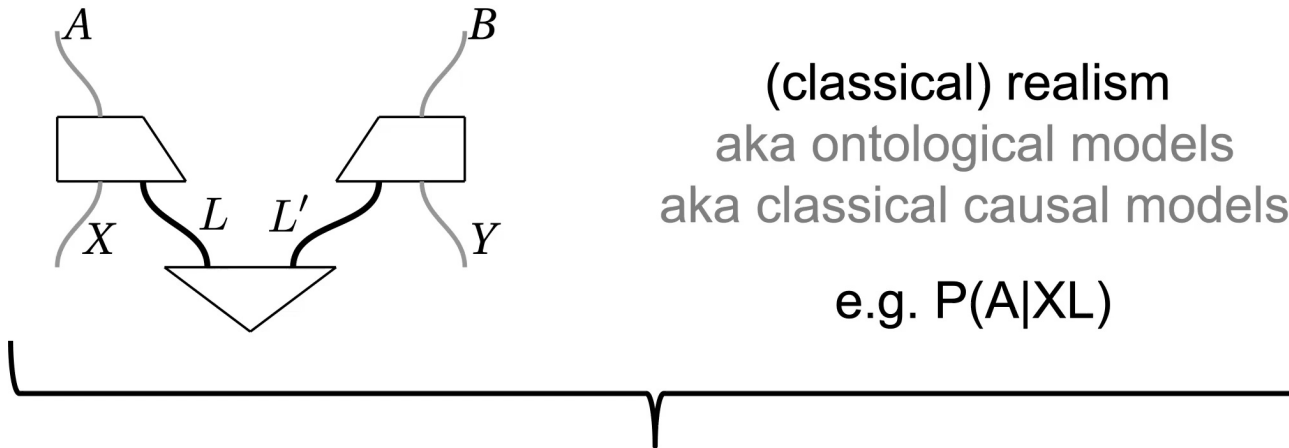
But this framework runs into trouble in a quantum world.

Bell's theorem

(classical) realism

Bell's theorem

(classical) realism
aka ontological models
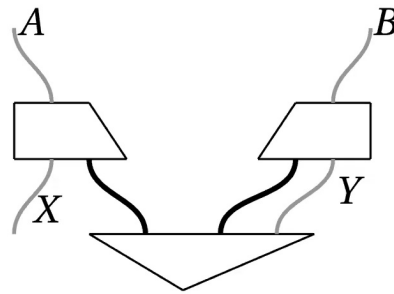aka classical causal models

e.g. P(A|XL)

Bell inequalities

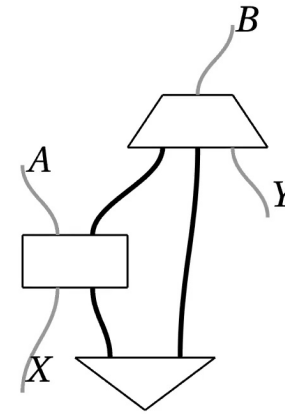What to do in light of quantum violations of these?

Responses to Bell's theorem

operationalism: only define directly measurable concepts

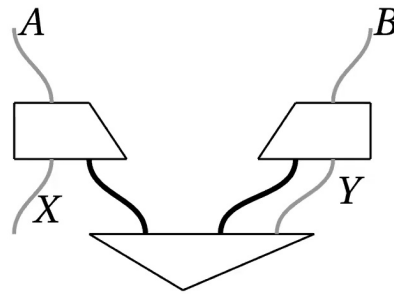classical realism, radical causal structure:

superdeterminism   superluminality
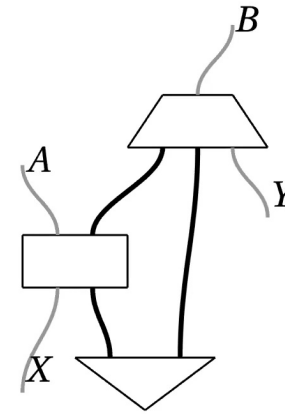
Responses to Bell's theorem

operationalism: only define directly measurable concepts

classical realism, radical causal structure:

superdeterminism   superluminality

radical realism, conservative causal structure:
- many worlds
- consistent histories
- causal-inferential framework

Constraints on nonclassical realist theories?

local
Leibnizian (noncontextual)
causation and inference unscrambled]
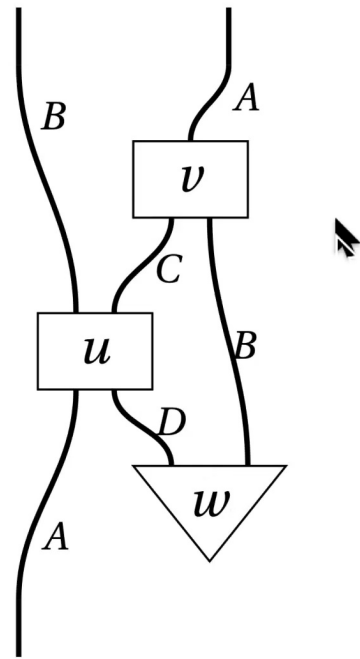
many worlds,
consistent histories
fail these

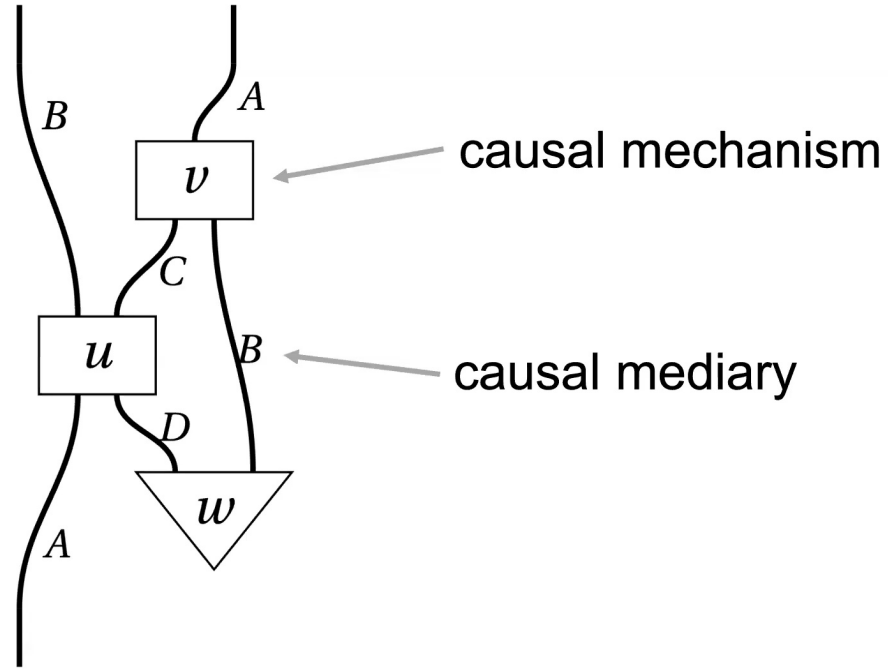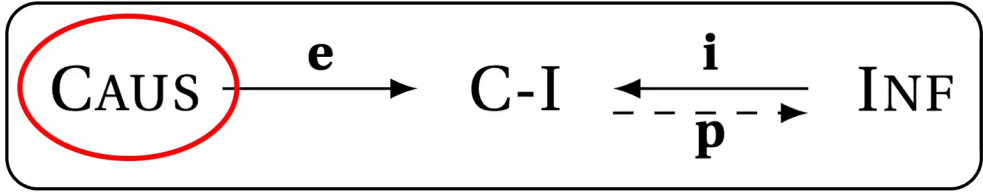How to even define these?

Need a formalism!

Process Theory G

-systems and processes
-closed under forming diagrams

The Framework

CAUS $\xrightarrow{\text{e}}$ C-I $\underset{\text{p}}{\overset{\text{i}}{\leftrightarrows}}$ INF
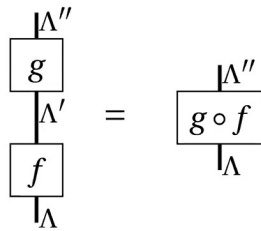
causal mechanism

causal mediary

Vertical diagrams

formal representations vary from one CI theory to another

FUNC

systems: sets (ontic state spaces)
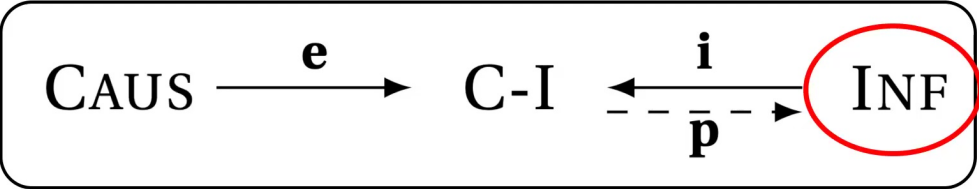processes: functional dynamics

Lots of structure:

$$\begin{array}{c} |\Lambda'' \\ \boxed{g} \\ |\Lambda' \\ \boxed{f} \\ |\Lambda \end{array} \quad = \quad \begin{array}{c} |\Lambda'' \\ \boxed{g \circ f} \\ |\Lambda \end{array}$$

-fully constrain causal relations

PROC

systems: abstract labels
processes: descriptions of lab procedures

No real structure:

 -no convex mixtures

$$\text{CAUS} \xrightarrow{\ \mathbf{e}\ } \text{C-I} \xleftarrow{\ \mathbf{i}\ } \underset{\mathbf{p}}{\dashrightarrow} \text{INF}$$

Horizontal
diagrams

$$\text{CAUS} \xrightarrow{\ \mathbf{e}\ } \text{C-I} \xleftarrow[\ \ \ \ \mathbf{p}\ \ \ \ ]{\ \mathbf{i}\ } \text{INF}$$

state of knowledge

v

B

B

C

w

B

C

C

x

u

A

A

proposition

inferential system

inferential process
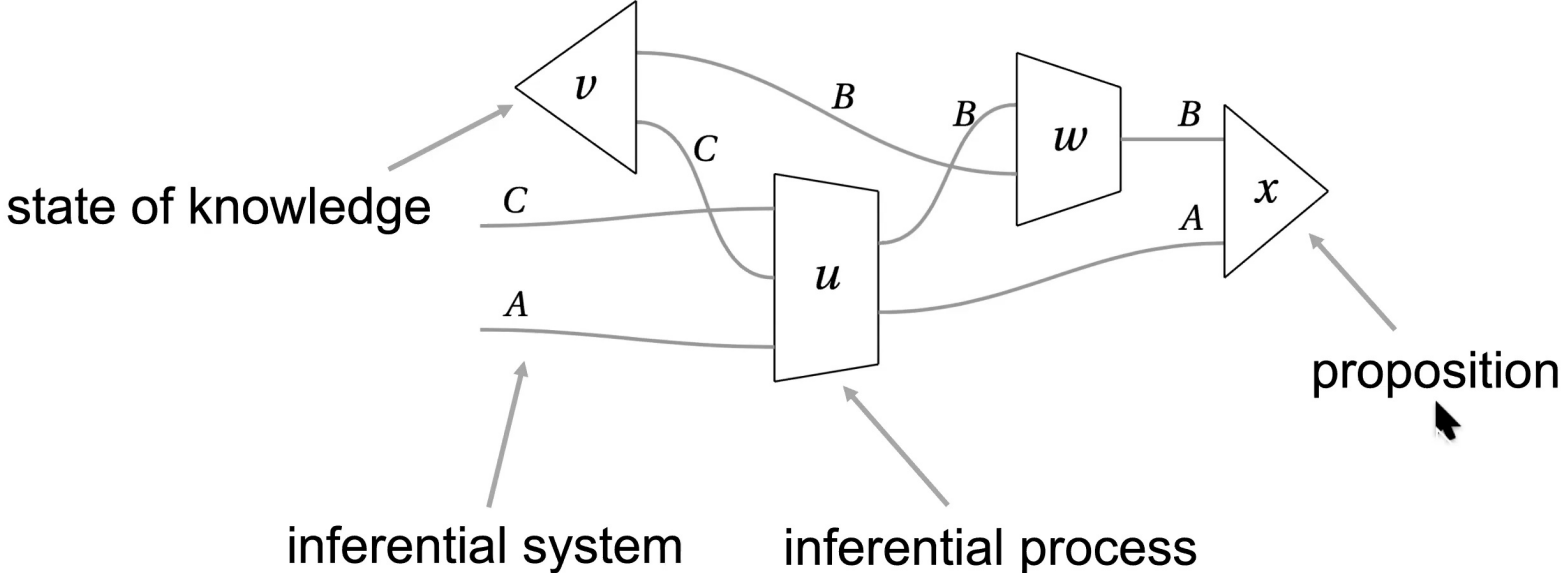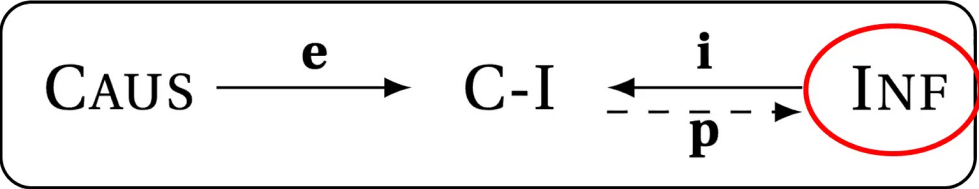
Horizontal diagrams

Classical theory of inference

     i) Bayesian probability theory   (stochastic maps)
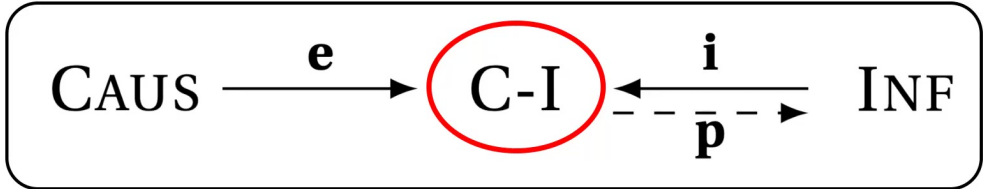   ii) Boolean propositional logic   (partial functions)

$$\subset \text{SUBSTOCH}$$

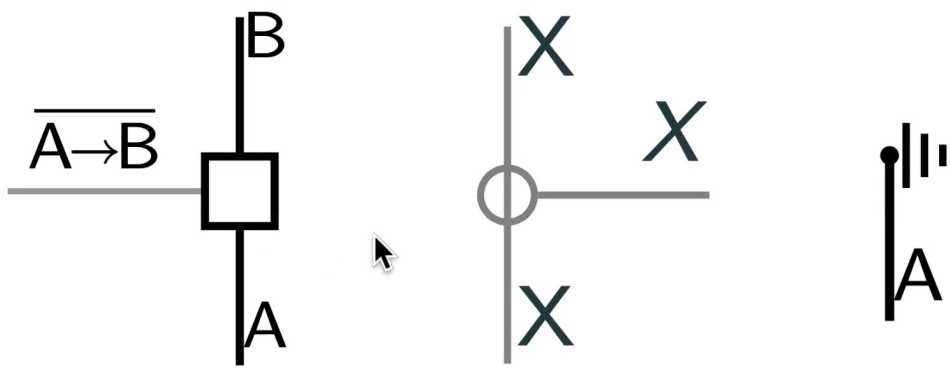Any substochastic map can
be constructed as
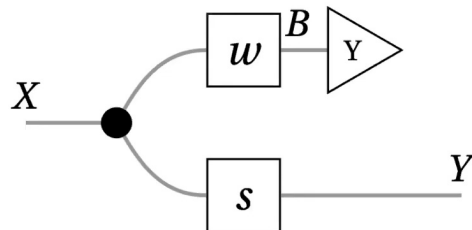
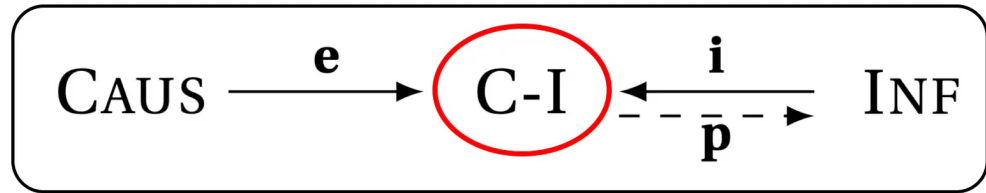So classical theory of inference = SUBSTOCH

C-I is constructed from the <u>interactions</u> between CAUS and INF
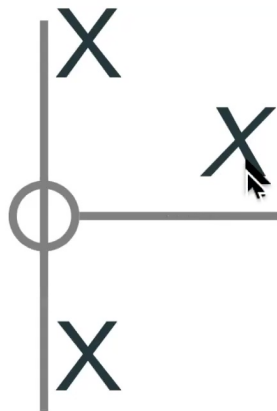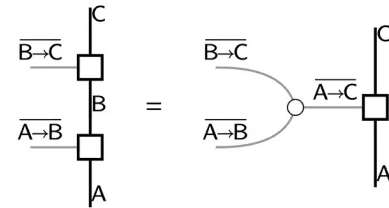
Three generators:

What we **learn/ask**

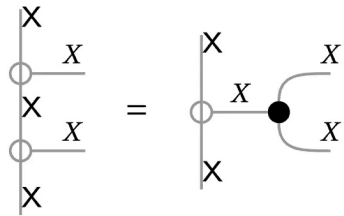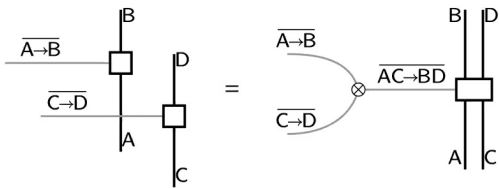$$\text{CAUS} \xrightarrow{\ e\ } \boxed{\text{C-I}} \xleftarrow[\ \bar{p}\ ]{\ i\ } \text{INF}$$

for classical systems only

Generator 2:

-generate states of knowledge
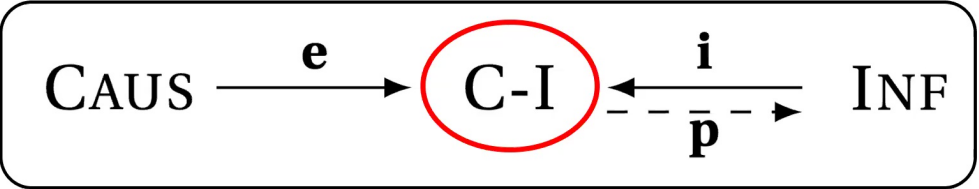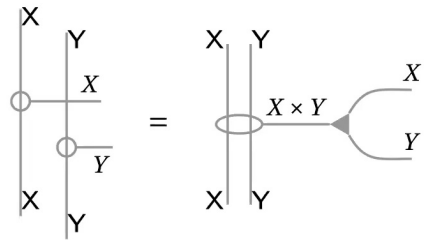
Consistency conditions!
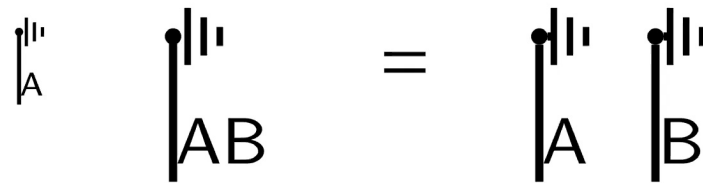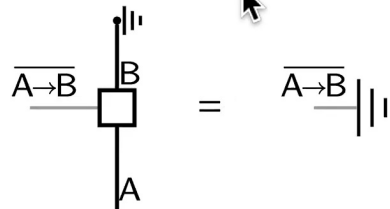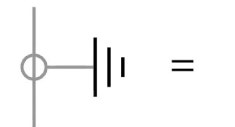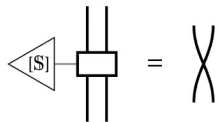
Embedding map

$$\text{CAUS} \xrightarrow{\; e \;} \text{C-I} \underset{p}{\overset{i}{\leftarrows}} \text{INF}$$
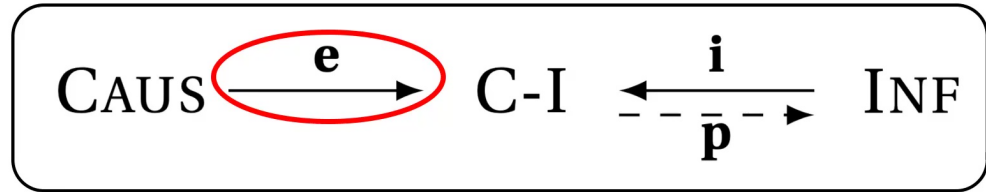
causal mechanism $\longrightarrow$ state of knowledge about causal mechanism
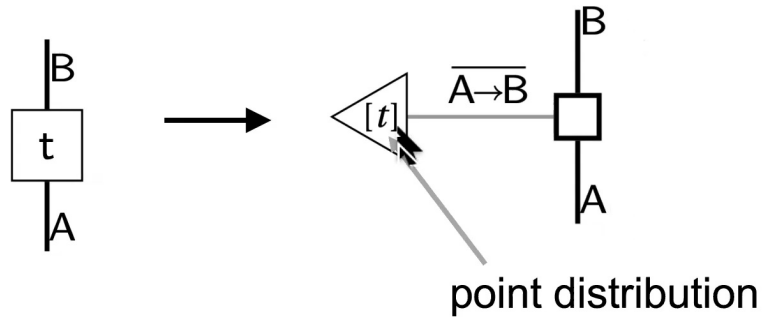
point distribution
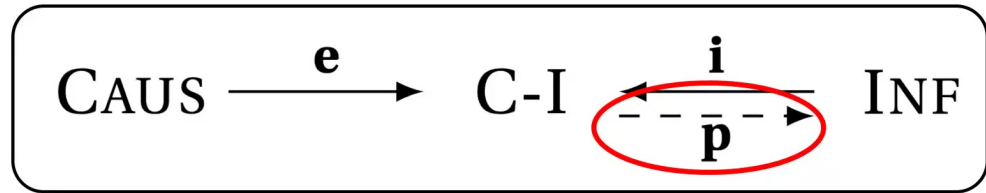
Prediction map

$$\text{CAUS} \xrightarrow{\ \mathbf{e}\ } \text{C-I} \underset{\mathbf{p}}{\overset{\mathbf{i}}{\rightleftarrows}} \text{INF}$$

(partial map)

$$\mathcal{D}_{\mathbf{p}} \quad \in \quad \text{INF}$$

realities of
nature

$$\text{CAUS} \xrightarrow{\ \mathbf{e}\ } \text{C-I} \underset{\mathbf{p}}{\overset{\mathbf{i}}{\rightleftharpoons}} \text{INF}$$

Operational CI theories

$$\text{PROC} \xrightarrow{\ \ \mathbf{e}\ \ } \text{P-S} \xleftarrow[\ \ \mathbf{p}\ \ ]{\ \ \mathbf{i}\ \ } \text{SUBSTOCH}$$

Operational CI theories

$$\text{PROC} \xrightarrow{\ \ e\ \ } \text{P-S} \underset{p}{\overset{i}{\rightleftharpoons}} \text{SUBSTOCH}$$
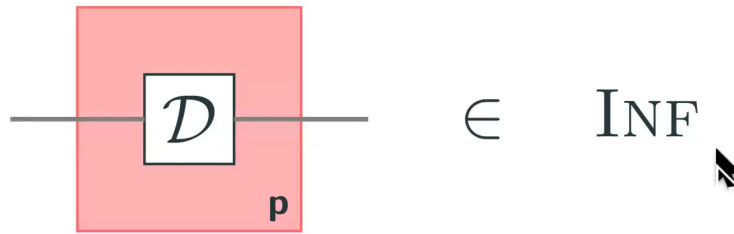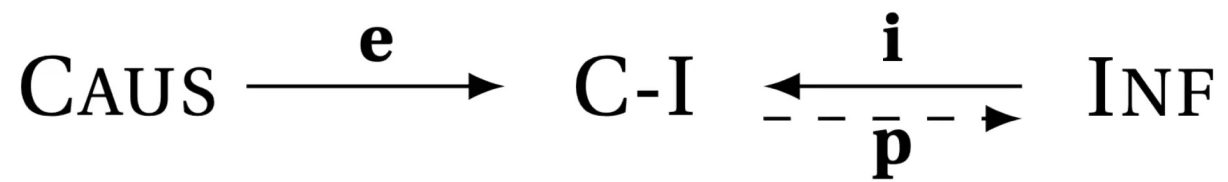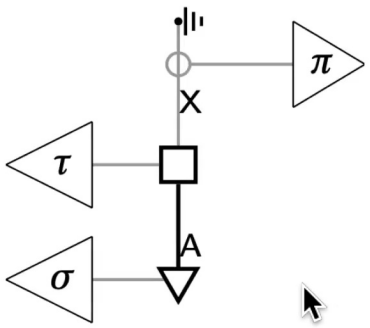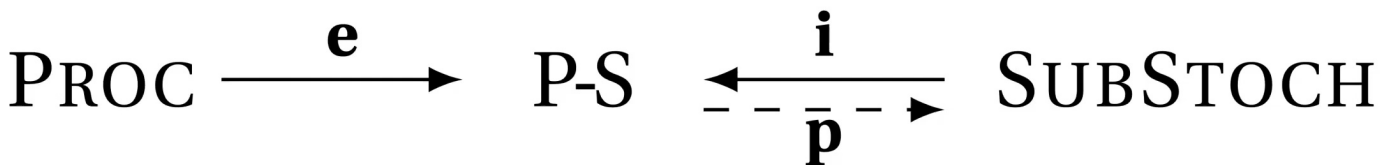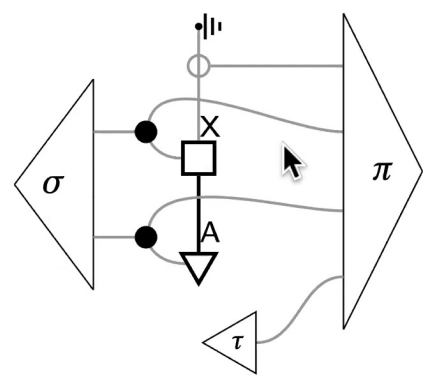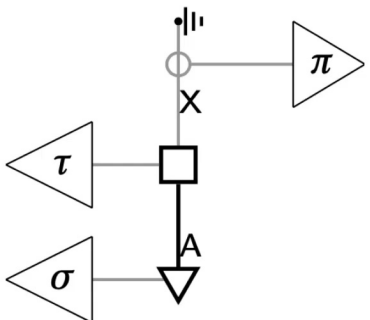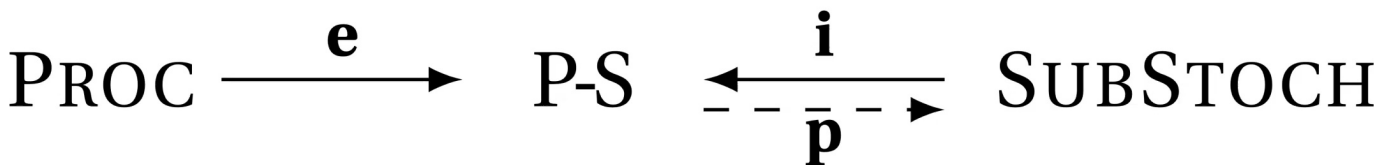
descriptive; not much structure

content of the theory is largely in the probability rule (which can vary)

Classical Realist CI theory "Ontological theories"

$$\text{FUNC} \xrightarrow{\ e'\ } \text{F-S} \xleftarrow[\ p^*\ ]{\ i'\ } \text{SUBSTOCH}$$

Classical Realist CI theory   "Ontological theories"

$$\textsc{Func} \xrightarrow{\ e'\ } \text{F-S} \xleftarrow{\ i'\ } \textsc{SubStoch}$$

$$\text{F-S} \xdashrightarrow{\ p^*\ } \textsc{SubStoch}$$
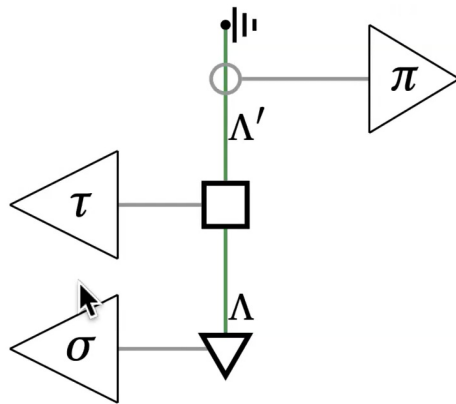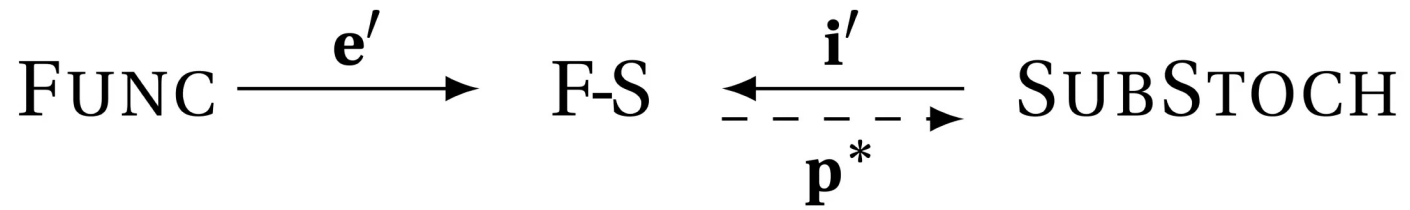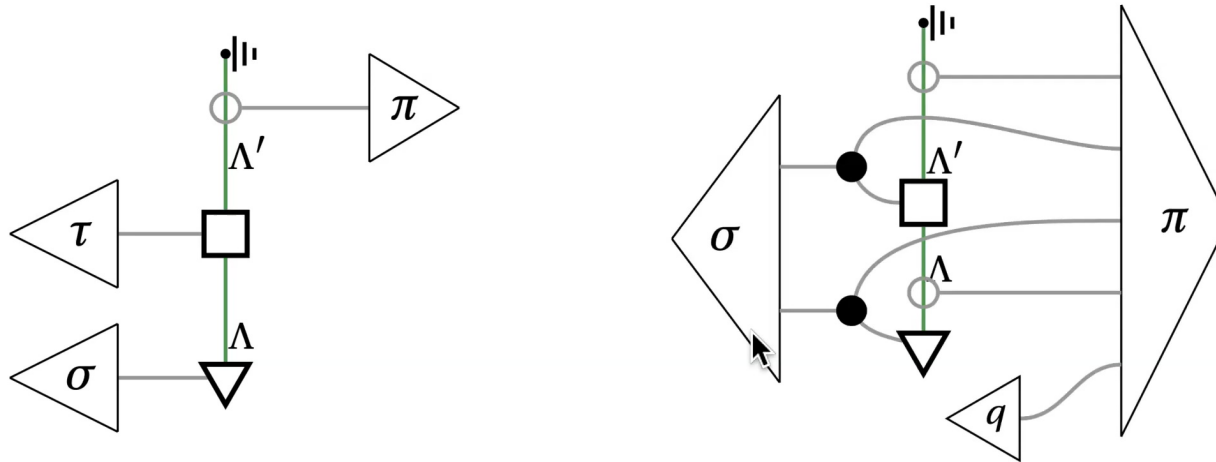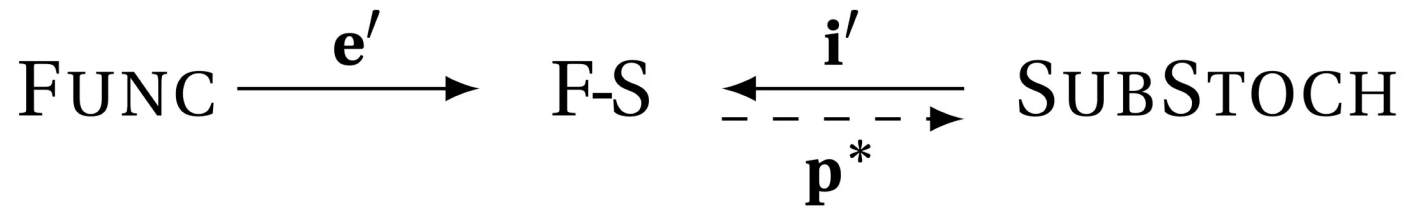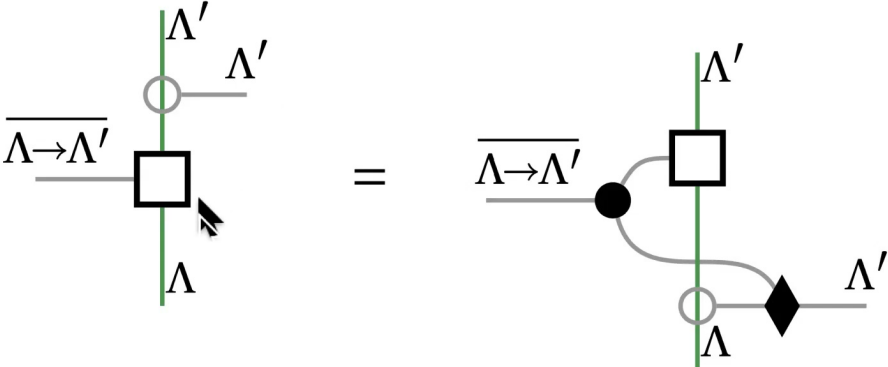
key equality in F-S:

key equality in F-S:
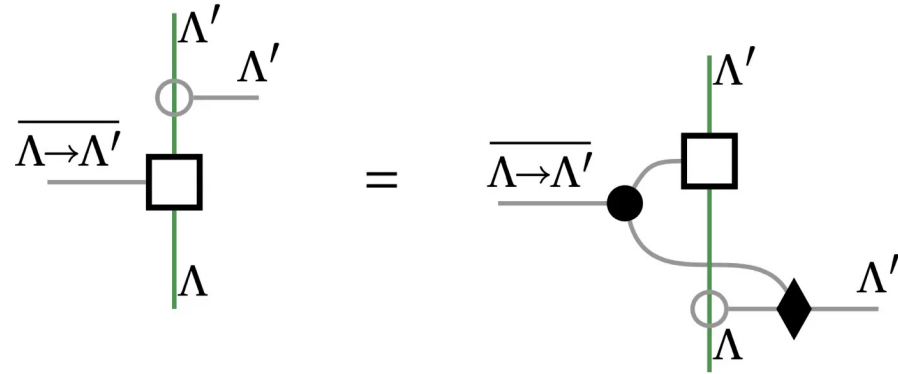
unique probability rule (**p***) for F-S

can consider propositions about every system

Example

$$\mathbb{1} := \Big|, \quad \neg := \boxed{\neg}, \quad \mathbf{0} := \begin{array}{c} \boxed{0} \\ \equiv \end{array}, \quad \mathbf{1} := \begin{array}{c} \boxed{1} \\ \equiv \end{array}$$

Example

$$\mathbb{1} := \Big|, \quad \neg := \boxed{\neg}, \quad \mathbf{0} := \underset{=}{\overset{0}{\nabla}}, \quad \mathbf{1} := \underset{=}{\overset{1}{\nabla}}$$
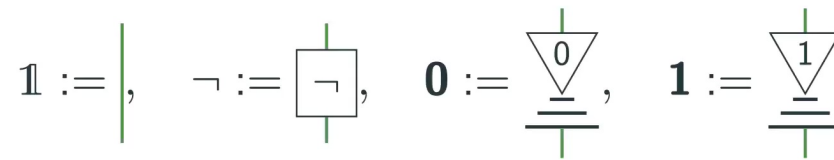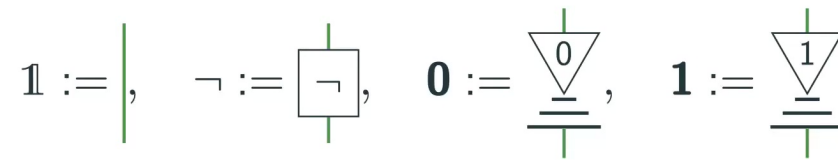
$$\sigma_{con} := 1/2[\mathbb{1}] + 1/2[\neg] \qquad \sigma_{dis} := 1/2[\mathbf{0}] + 1/2[\mathbf{1}]$$

same inferential equivalence class:
$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

Defining Quotiented Theories

$$\text{Caus} \xrightarrow{\ e\ } \text{C-I} \underset{p}{\overset{i}{\rightleftarrows}} \text{Inf}$$

$$\text{C-I} \xrightarrow{\sim p} \widetilde{\text{C-I}}$$

Defining Quotiented Theories

$$\widetilde{\text{C-I}}$$

$$\text{CAUS} \xrightarrow{\ e\ } \text{C-I} \xleftarrow[\ \bar{p}\ ]{\ i\ } \text{INF}$$

with $\sim p$ arrow from C-I to $\widetilde{\text{C-I}}$
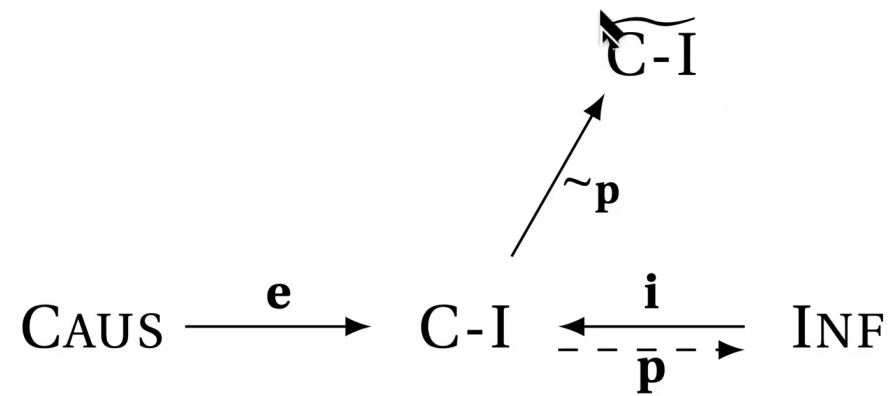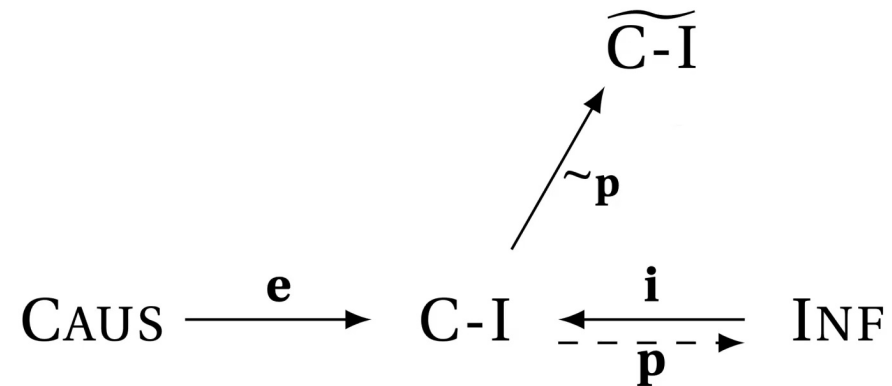
$\widetilde{\text{F-S}}$ subsumes traditional ontological models

$\widetilde{\text{P-S}}$ subsumes generalized probabilistic theories

Quotienting leads to scrambling

$$\mathbb{1} := \bigg|, \quad \neg := \boxed{\neg}, \quad \mathbf{0} := \underset{\equiv}{\nabla}_{\!0}, \quad \mathbf{1} := \underset{\equiv}{\nabla}_{\!1}$$

causal connection   causal disconnection
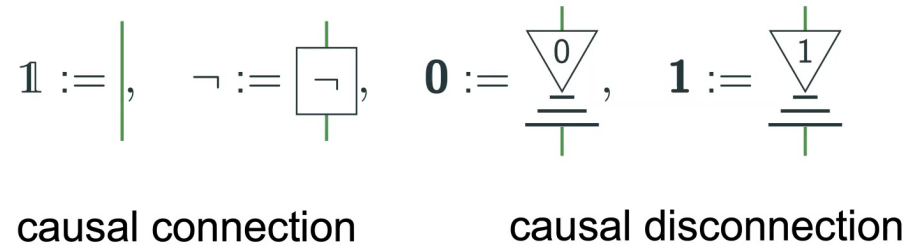
$$\sigma_{con} := 1/2[\mathbb{1}] + 1/2[\neg] \qquad \sigma_{dis} := 1/2[\mathbf{0}] + 1/2[\mathbf{1}]$$

inferentially equivalent!

Even the distinction between perfect causal connection and perfect causal disconnection is lost in quotiented theories!

GPTs are quotiented theories, hence scrambled.

Traditional ontological models are too!

A classical realist representation
of an operational CI theory is a
diagram preserving map

$$\xi : \text{P-S} \rightarrow \text{F-S}$$
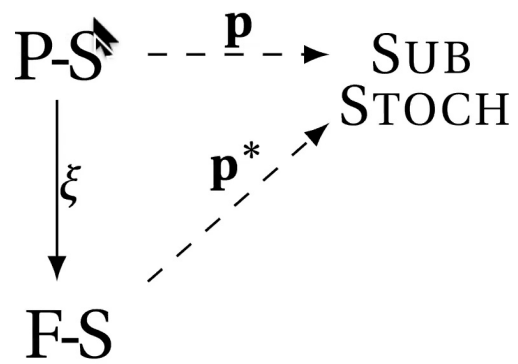
preserves predictions: preserves ignoring:

A classical realist representation of an operational CI theory is a diagram preserving map

$$\xi : \text{P-S} \to \text{F-S}$$

preserves predictions:

$$\text{P-S} \dashrightarrow^{\mathbf{p}} \text{SUB STOCH}$$

$$\downarrow \xi \qquad \nearrow \mathbf{p}^*$$

$$\text{F-S}$$

preserves ignoring:

Types of classical realist representations

The characterization of noncontextuality in the framework of generalized probabilistic theories arXiv:1911.10386

"ontological representation of a GPT"

the fundamental notion

Types of classical realist representations

$$\widetilde{\text{P-S}}$$

$\sim\!\mathbf{p}$

$\widetilde{\xi}$

P-S

ζ

ξ

$$\widetilde{\text{F-S}}$$

$\sim\!\mathbf{p}^{*}$

F-S

The characterization of noncontextuality in the framework of generalized probabilistic theories arXiv:1911.10386

"ontological representation of a GPT"

"traditional ontological model"

the fundamental notion

FUNC

F-S
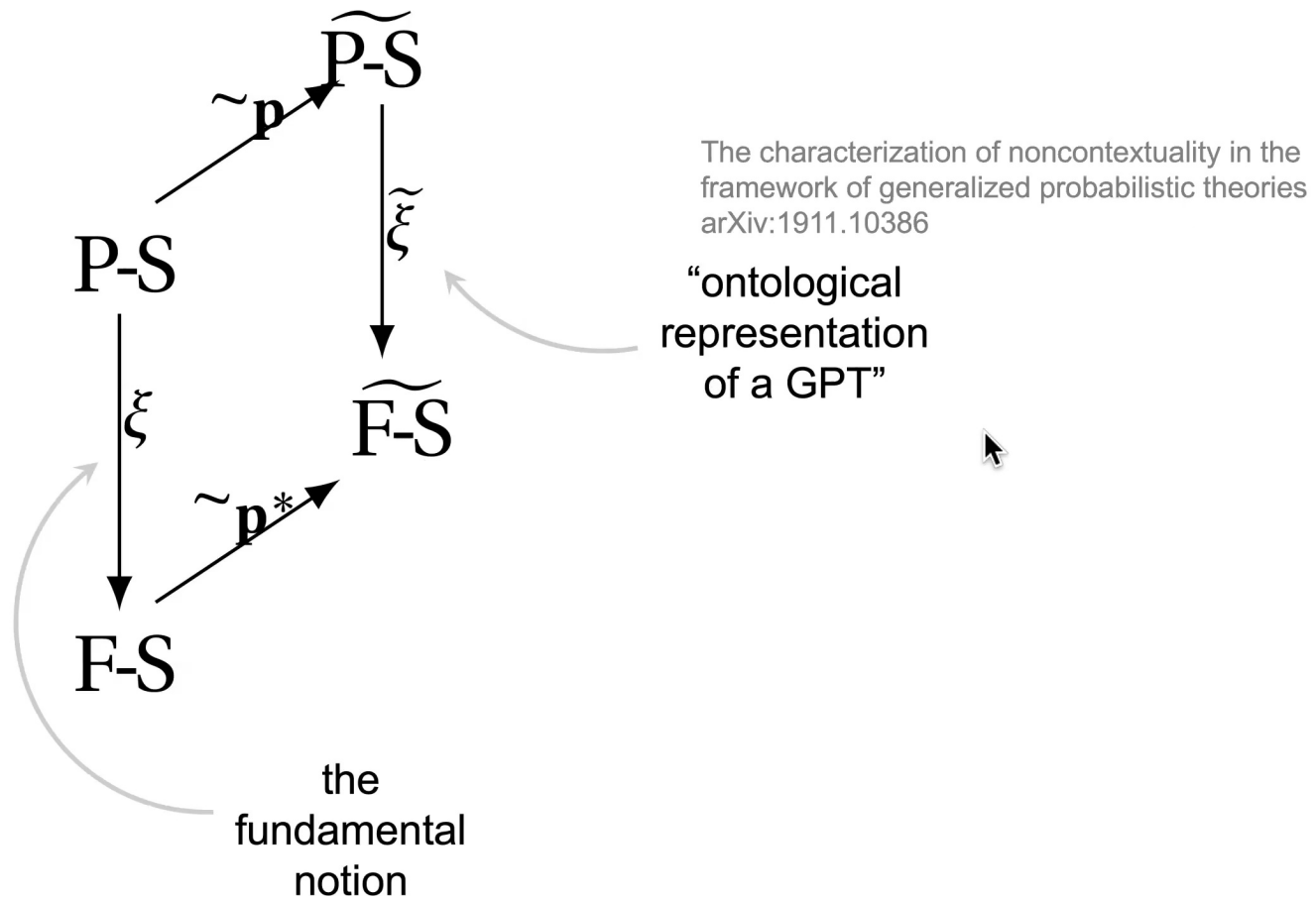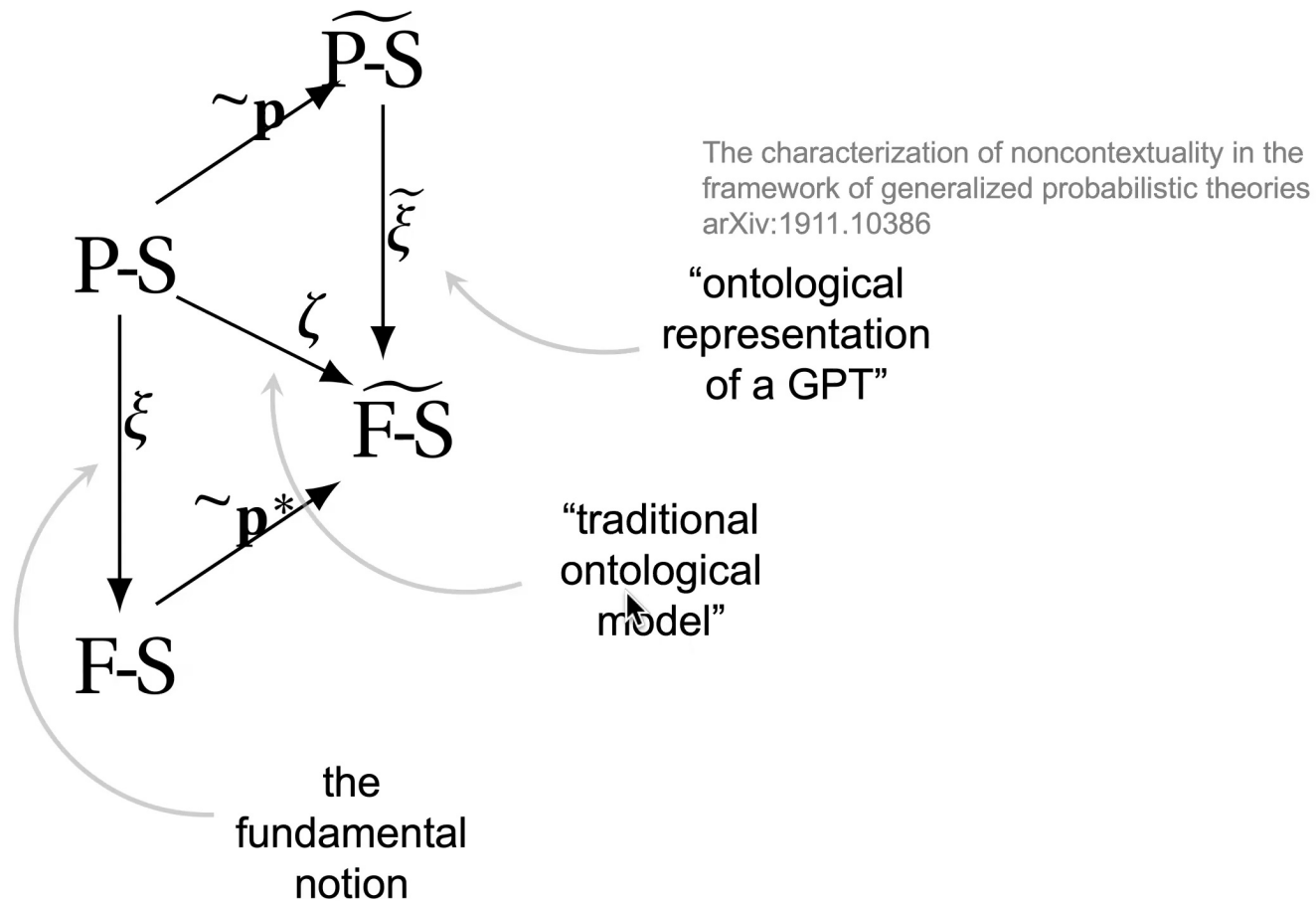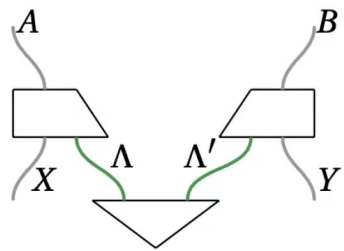
(no superluminality)
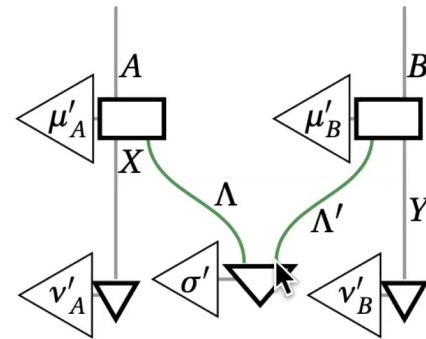
A correlation P(ABXY) that can be generated via
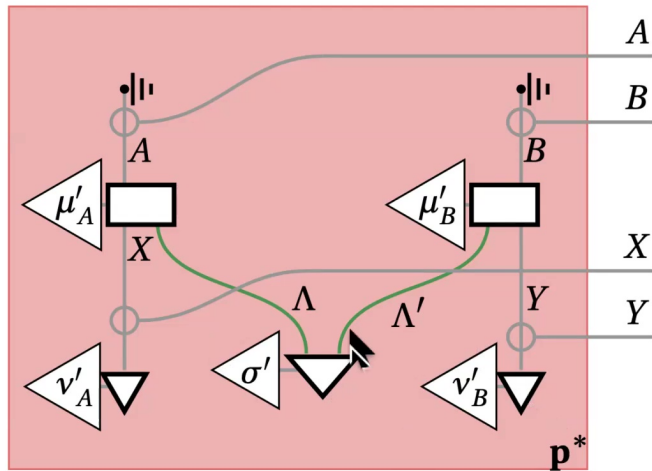
A correlation P(ABXY) that can be generated via
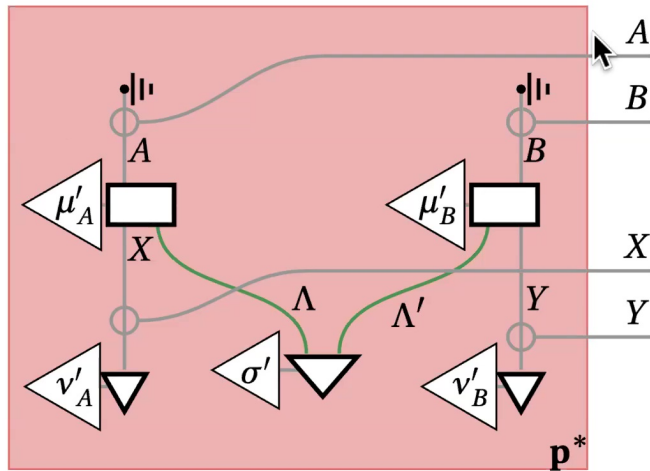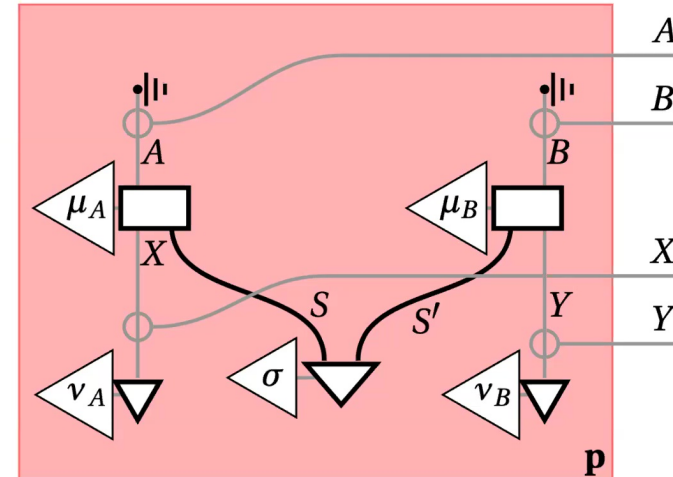
is "FS-realizable"   is "PS-realizable"

(wrt the Bell scenario)

Bell: There are PS-realizable distributions that are not FS-realizable.
⇒ no diagram-preserving classical realist representation exists

Fundamental notion of classical-explainability: Leibnizianity

Fundamental notion of classical-explainability: Leibnizianity

A classical realist representation map
is **Leibnizian** if it preserves inferential
equivalence relations.

context: information that distinguishes
elements of the same equivalence class

e.g. $\sigma_{con} := 1/2[\mathbb{1}] + 1/2[\neg]$

vs

$\sigma_{dis} := 1/2[\mathbf{0}] + 1/2[\mathbf{1}]$

every (unquotiented) process
is defined by its equivalence
class and its context

For ζ , Leibnizianity is exactly generalized noncontextuality.

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$\widetilde{\text{P-S}}$

$\sim\mathbf{p}$

$\widetilde{\xi}$

no
contexts

P-S

ζ

have
contexts

ξ

$\widetilde{\text{F-S}}$

$\sim\mathbf{p}*$

F-S

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P-S

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ξ

$\sim\mathbf{p}^*$

F-S
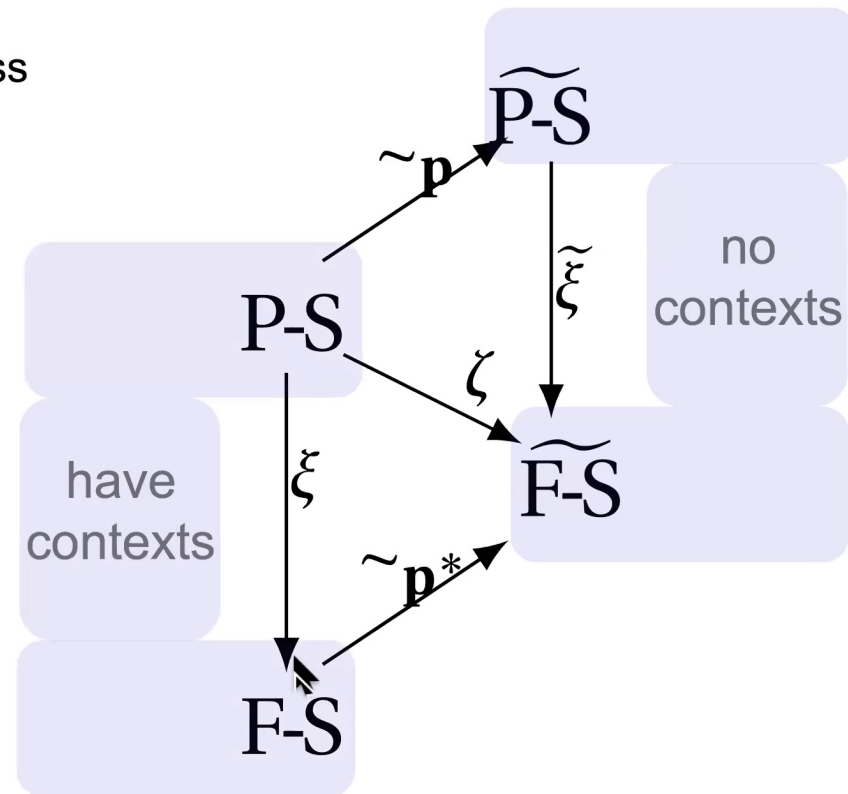
For ζ, Leibnizianity is exactly generalized noncontextuality.

"*equivalence class* of representation
cannot depend on operational contexts"

"representation cannot depend
on operational contexts"

No classical realist representation preserves…

inferential equivalences
and causal-inferential structure

Leibnizianity
(NC) no-go

Bell-like
no-go

An analogy with non-Euclidean geometry

Different geometries can be defined in terms of axioms.

The axioms give meaning to geometrical concepts.

Some geometries challenge our intuition.

But, they preserve *some* intuitions.

These are often the most physically useful!
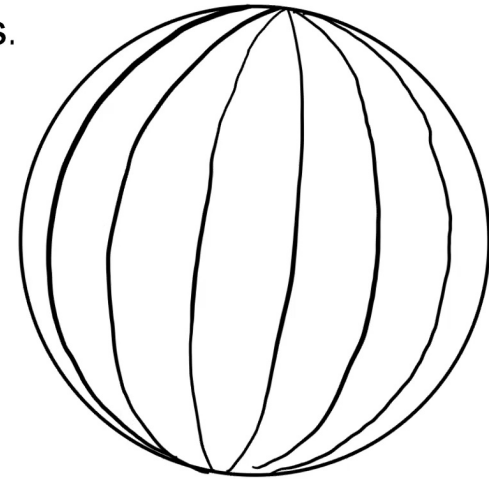
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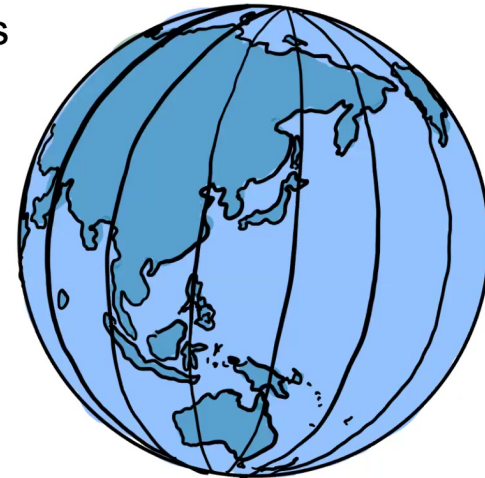
$$\frac{\text{nonclassical realist theories}}{\text{classical realist theory}} \sim \frac{\text{nonEuclidean geometries}}{\text{Euclidean geometry}}$$

Nonclassical realist theories:

$$\text{XFUNC} \xrightarrow{\ e'\ } \text{XF-XS} \underset{p^*}{\overset{i'}{\longleftrightarrow}} \text{XSUBSTOCH}$$

Features of F-S

Λ

Λ

Λ
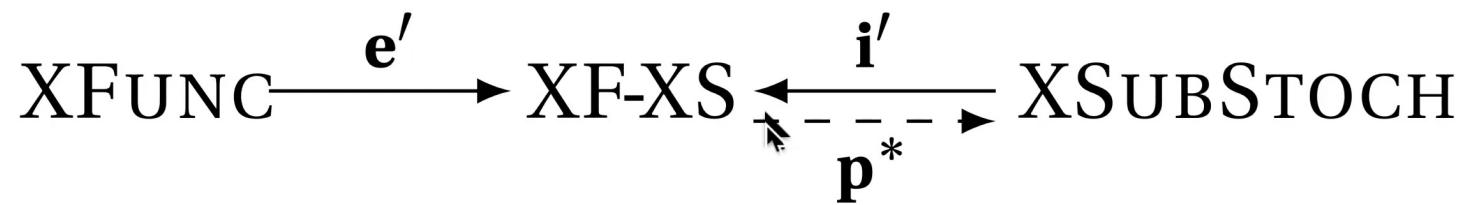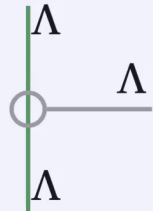
(Desiderata for XF-XS)

Can attach propositions
to all systems

Features of F-S   (Desiderata for XF-XS)
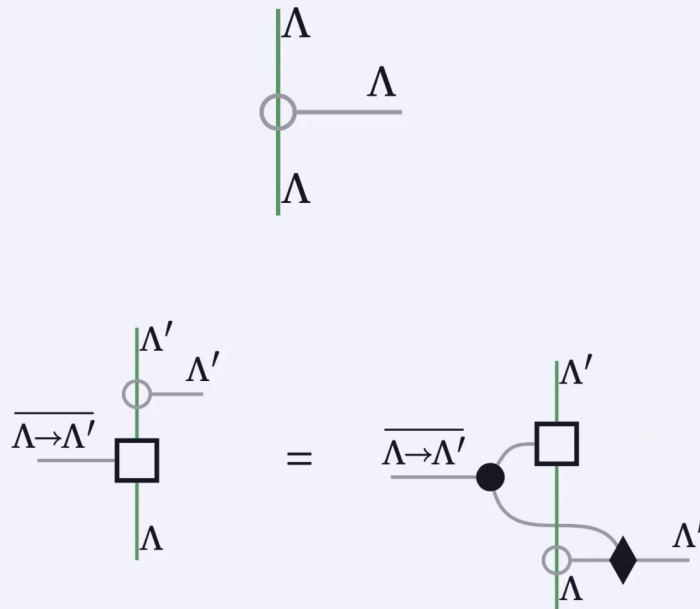
Unique prediction map

Can attach propositions
to all systems

Can propagate knowledge
through all mechanisms

Unique prediction map

Desiderata for XF~UNC~

Enough structure to capture
the notion of causal influence

Analogues of
d-separation,
common causes,
Reichenbach's principle,
counterfactuals,
etc

Desiderata for XS~UB~S~TOCH~

Enough structure to capture
probabilistic and logical inference

. . .

Goal: find a representation of quantum theory

Goal: find a representation of quantum theory

$$\xi : \text{P-S} \to \text{XF-XS}$$

as a nonclassical realist theory

$$\text{XFUNC} \xrightarrow{\ e'\ } \text{XF-XS} \underset{p^*}{\overset{i'}{\leftarrow\;\text{-}\;\text{-}\;\text{-}\;\text{-}\rightarrow}} \text{XSUBSTOCH}$$

that preserves causal-inferential structure
and preserves inferential equivalences.

Locality

Leibnizianity
(noncontextuality)

(diagram preservation and inferential equivalence are defined for all CI theories)

Should quantum theory be understood as

…a nonclassical logic?   Quantum Logic

…a nonclassical probability theory?   Leifer-Spekkens (2013)

…a nonclassical causal model?   Barrett et al (2019)
                                       Allen et al (2017)
                                       Costa-Shrapnel (2016)

Our formalism:

    unscrambles operational theories

    unscrambles classical realist theories

    can describe nonclassical realist theories

        -we give concrete constraints on these

        -including definitions of locality and Leibnizianity (NC)

We have given a formalism for going beyond standard ontological models in order to salvage the spirit of locality and noncontextuality.

Our formalism:

    unscrambles operational theories

    unscrambles classical realist theories

    can describe nonclassical realist theories

       -we give concrete constraints on these

       -including definitions of locality and Leibnizianity (NC)

We have given a formalism for going beyond standard ontological models in order to salvage the spirit of locality and noncontextuality.

Thanks for your attention!

Other dividends of our framework:

-subsumes GPTs and ontological models as quotiented theories

-clarifies structure common to all points in the landscape of GPTs

    (forthcoming)

-clarifies distinction between Bell and noncontextuality no-gos

-formalizes "mixtures of procedures"

-fully compositional ⇒ new kinds of proofs

A structure theorem for generalized-noncontextual ontological models
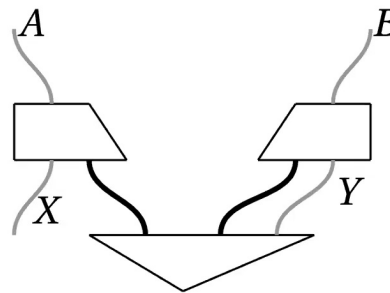arXiv:2005.07161

The only noncontextual model of the stabilizer subtheory is Gross's discrete Wigner function
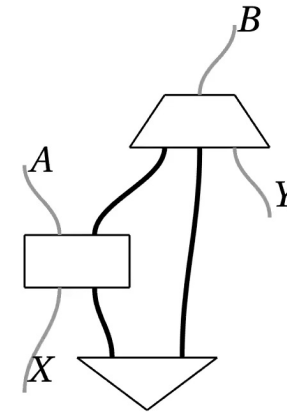forthcoming

Responses to Bell's theorem

operationalism: only define directly measurable concepts

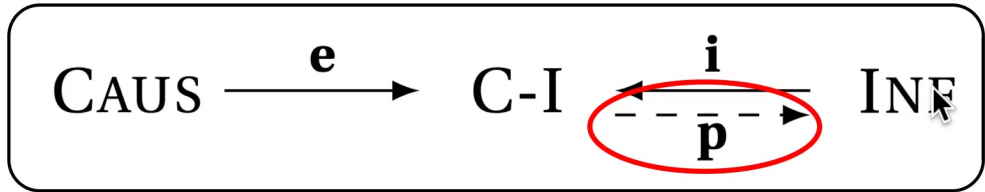classical realism, radical causal structure:
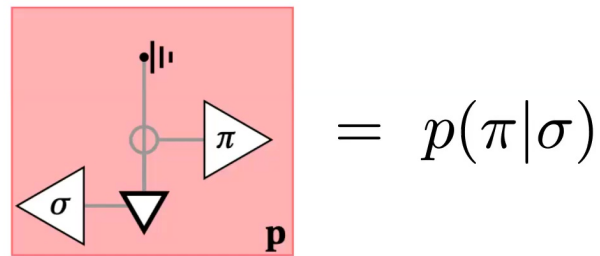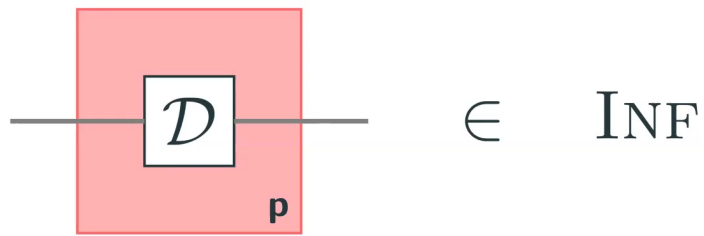

superdeterminism   superluminality

radical realism, conservative causal structure:
- many worlds
- consistent histories
- causal-inferential framework

Prediction map
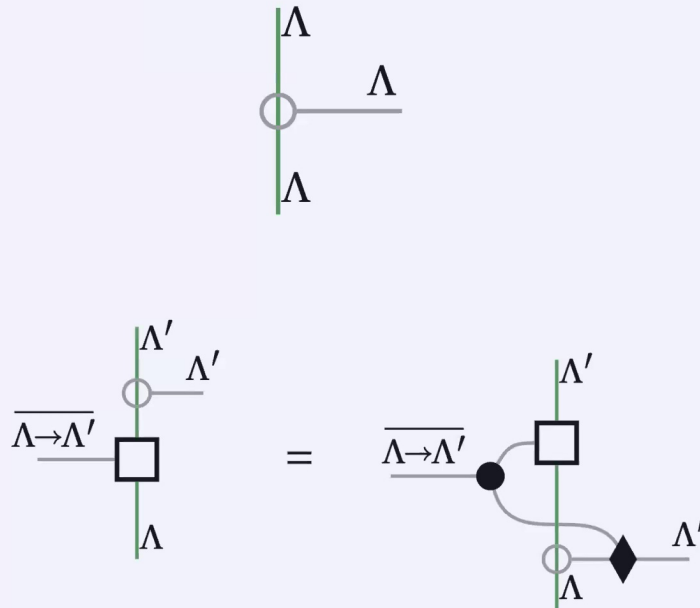
$$\textsc{Caus} \xrightarrow{\ \mathbf{e}\ } \text{C-I} \underset{\mathbf{p}}{\overset{\mathbf{i}}{\rightleftarrows}} \textsc{Inf}$$

(partial map)

$$\in \quad \textsc{Inf}$$

$$= \ p(\pi|\sigma)$$

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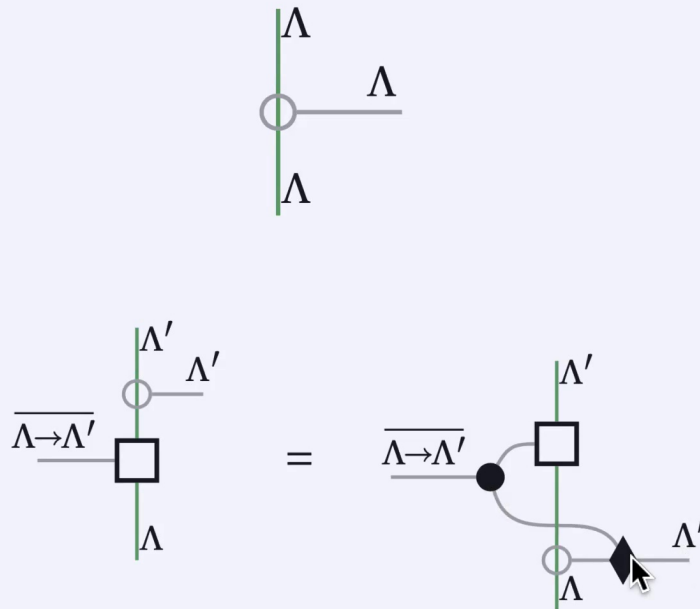common causes,
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. . .

Features of F-S

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Λ

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Λ'

Λ'

$\overline{\Lambda \to \Lambda'}$

Λ

$=$

$\overline{\Lambda \to \Lambda'}$

Λ'

Λ'

Λ

Can propagate knowledge
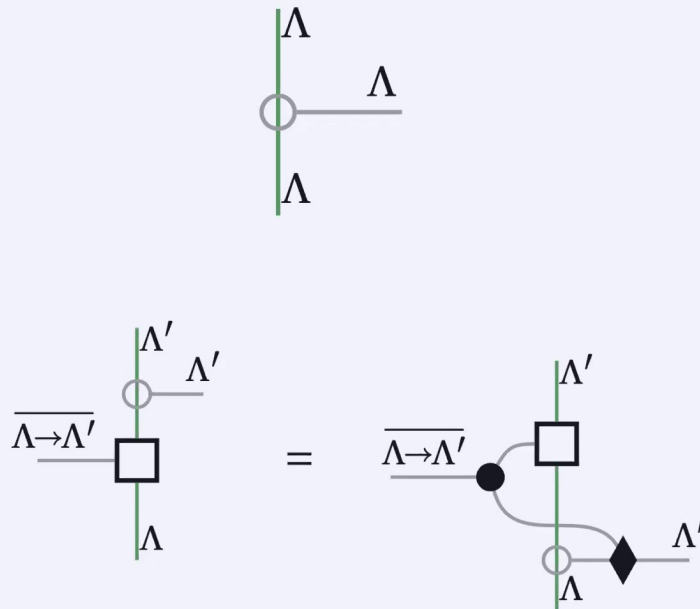through all mechanisms

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