Title: Quantum Extremal Islands Made Easy: Complexity on the brane

Speakers: Shan-Ming Ruan

Collection: Tensor Networks: from Simulations to Holography III

Date: November 17, 2020 - 1:30 PM

URL: http://pirsa.org/20110047

Abstract: We examine holographic complexity in the doubly holographic model to study quantum extremal islands. We focus on the holographic complexity=volume (CV) proposal for boundary subregions in the island phase. Exploiting the Fefferman-Graham expansion of the metric and other geometric quantities near the brane, we derive the leading contributions to the complexity and interpret these in terms of the generalized volume of the island derived from the higher curvature action for the brane gravity. Motivated by these results, we propose a generalization of the CV proposal for higher curvature theories of gravity. Further, we provide two consistency checks of our proposal by studying Gauss-Bonnet gravity and f(R) gravity in the bulk.

Quantum Extremal Islands Made Easy: Complexity on the Brane

Shan-Ming Ruan
Perimeter Institute & University of Waterloo

Nov 2020 Tensor Networks: from Simulation to Holography

PERIMETER INSTITUTE

arXiv:2010.16398

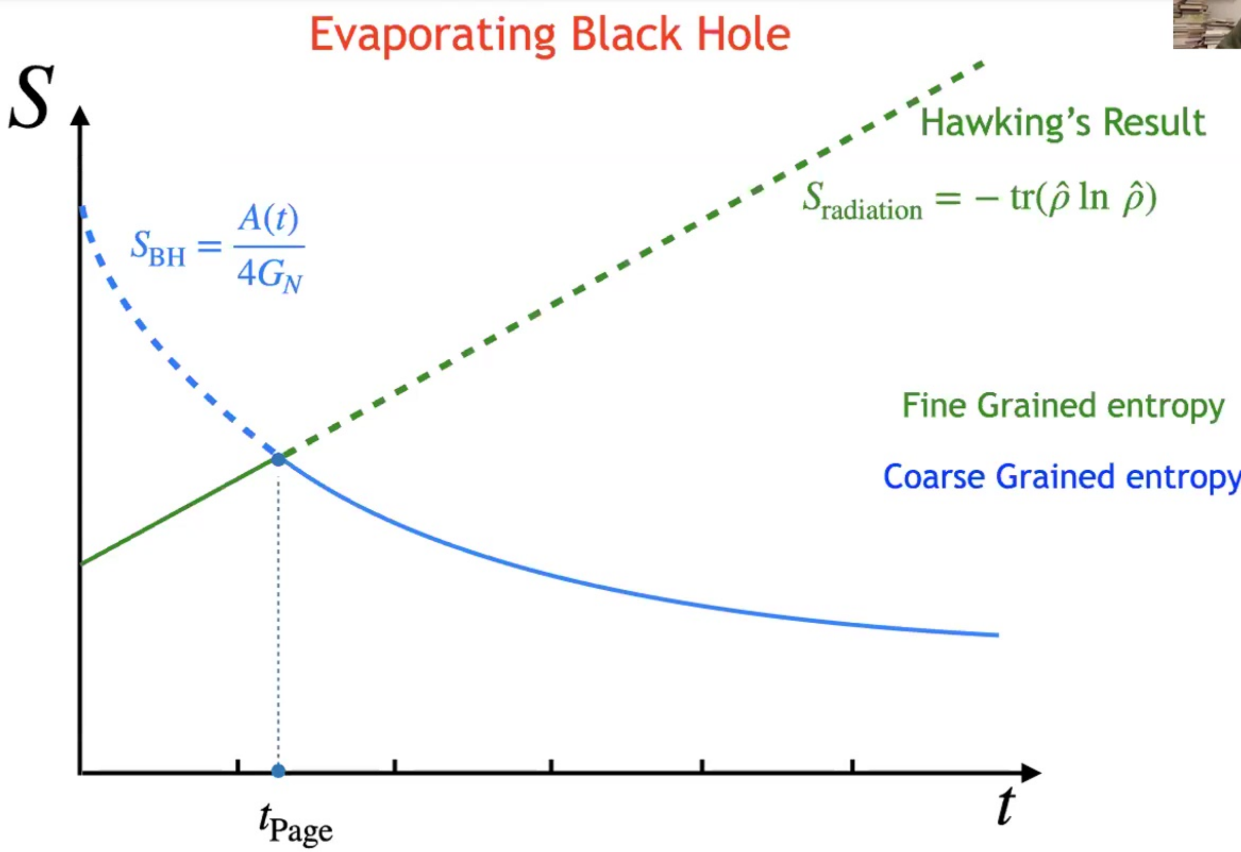
with Juan Hernandez and Robert C. Myers

1

Outline

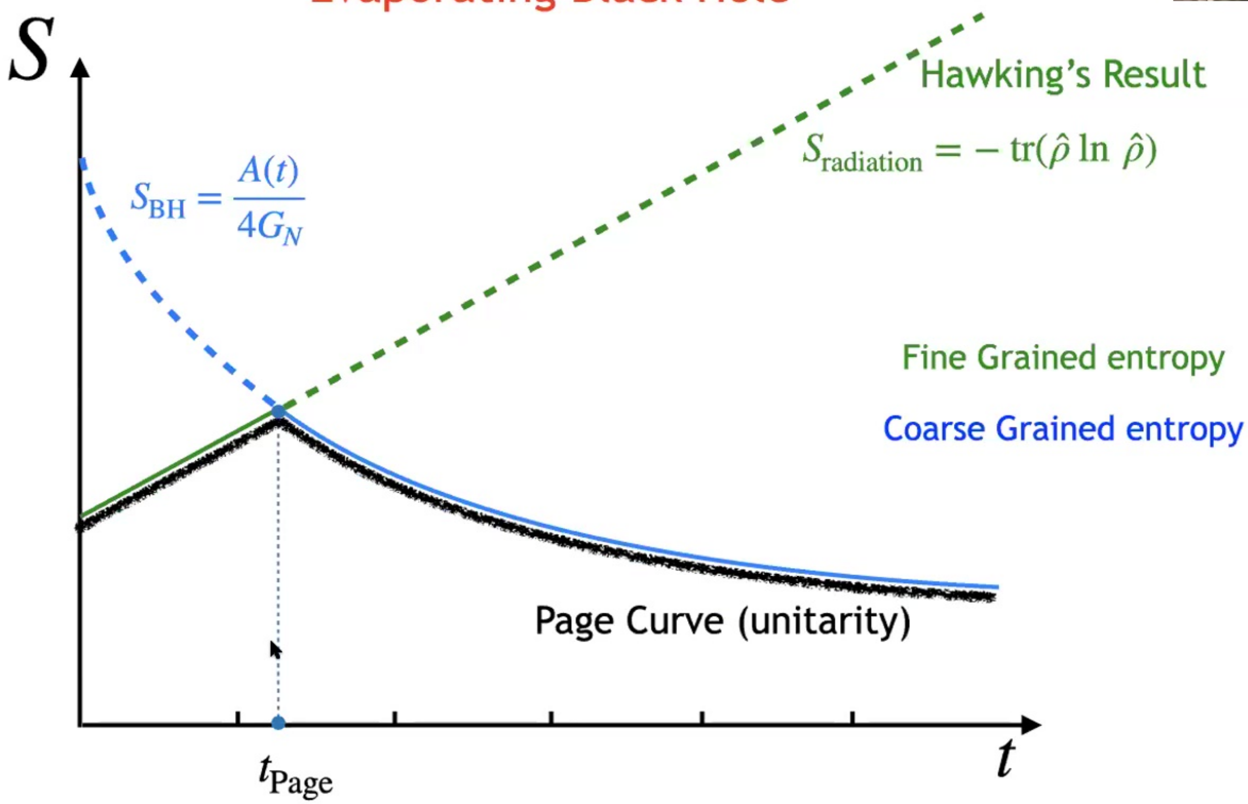
- Background
- Doubly Holographic Models
- Entanglement Entropy and Island Formula
- Complexity on the Brane

01.Background- Page Curve



01.Background- Page Curve

Evaporating Black Hole



Shan-Ming Ruan (PI) | BH coupled to a Thermal Bath | 3

01. Generalized Entropy and QES

Need a correct formula for entropy of BH/Radiations

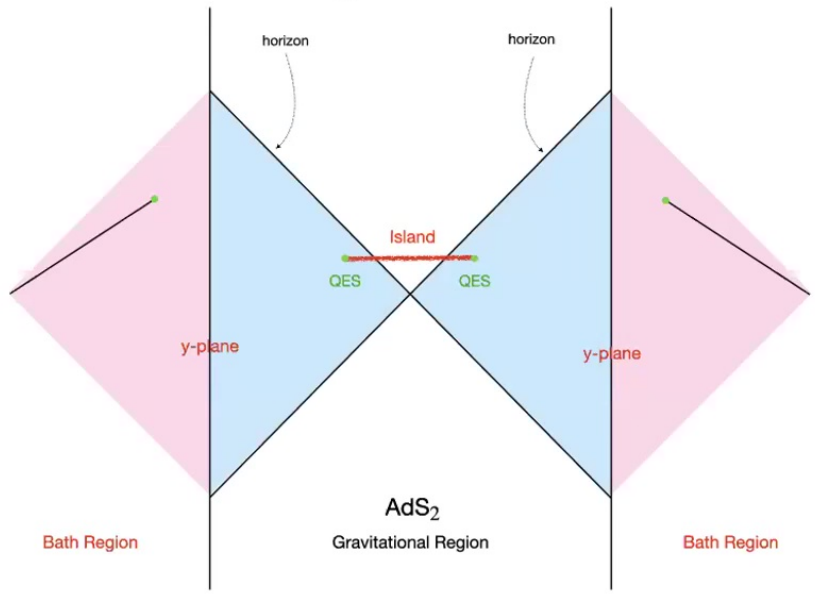
See 1905.08762, Almheiri, Engelhardt, Marolf, Maxfield
1905.08255, Penington

01. Island Formula

see: 1908.1
2006.06872

A correct formula for
the fine grained entropy of Hawking radiation

Island Formula： $S_{\text{radiation}} = \text{Min}_X \left[\text{Ext}_X \left(\frac{A[X]}{4G_N} + S_{\text{QFT}} \left(\Sigma_{\text{radiation}} \cup \Sigma_{\text{Island}} \right) \right) \right]$

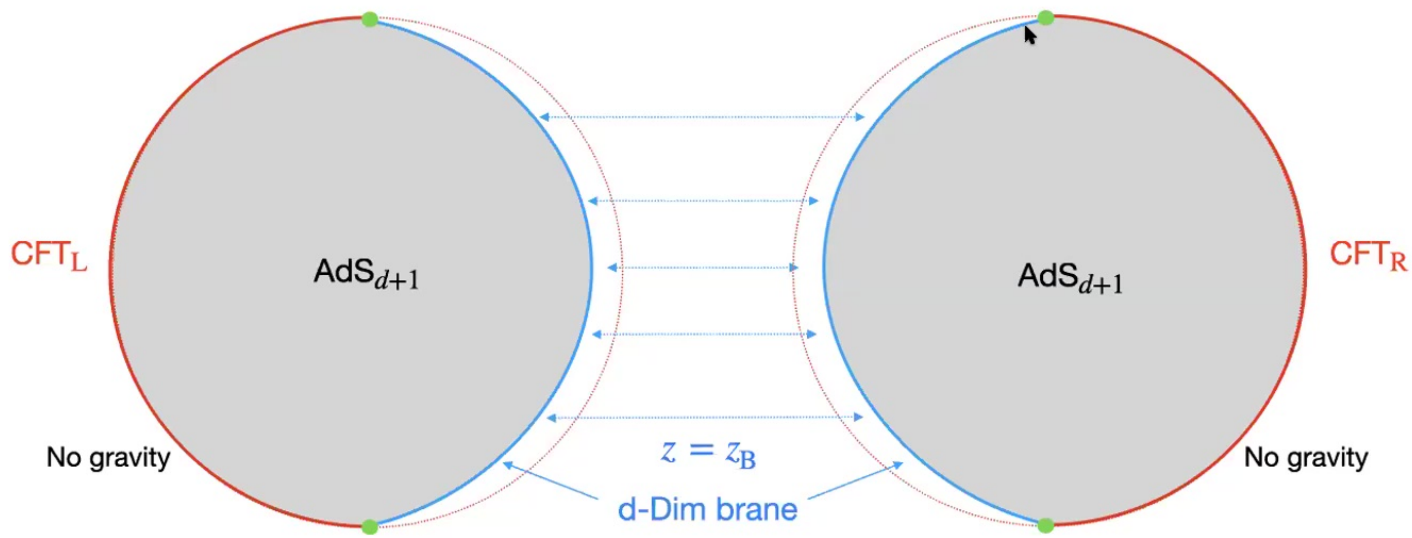


02. Doubly Holographic Model

arXiv:2006.04852,
2010.00018

Higher-Dimensional Construction

Randall-Sundrum scenario



$$ds^2 = \frac{L^2}{z^2}\left[dz^2 + \left(1 + \frac{z^2}{4L^2}\right)^2 g_{ij}^{\mathrm{AdS}_d} dx^i dx^j\right]$$

Shan-Ming Ruan (PI) Complexity on the brane 6

02. Doubly Holographic Model

(d+1)-Dim bulk gravity perspective: AdS$_{d+1}$ gravity coupled to brane

$$I_{\text{bulk}} = \frac{1}{16\pi G_{\text{bulk}}} \int_{\text{bulk}} d^{d+1}y\sqrt{-g}\left(\frac{d(d-1)}{L^2} + \mathscr{R}\left[g_{\mu\nu}\right]\right)$$

$$I_{\text{brane}} = -T_o \int d^d x\sqrt{-\tilde{g}}$$

Boundary perspective: Holographic CFT$_d$ coupled to defect (CFT$_{d-1}$)

arXiv:2006.04852, 2010.00018

02. Doubly Holographic Model

(d+1)-Dim bulk gravity perspective: AdS$_{d+1}$ gravity coupled to brane

$$I_{\text{bulk}} = \frac{1}{16\pi G_{\text{bulk}}} \int_{\text{bulk}} d^{d+1}y\sqrt{-g} \left(\frac{d(d-1)}{L^2} + \mathcal{R}\left[g_{\mu\nu}\right] \right)$$

$$I_{\text{brane}} = -T_o \int d^d x \sqrt{-\tilde{g}}$$

d-Dim brane perspective: CFT$_d$ living on asymptotic boundary and d-dim brane

$$I_{\text{ind}} = 2\left(I_{\text{bulk}} + I_{\text{bdy}}\right) + I_{\text{brane}}$$

contains induced gravity

Boundary perspective: Holographic CFT$_d$ coupled to defect (CFT$_{d-1}$)

arXiv:2006.04852, 2010.00018

Shan-Ming Ruan (PI) | Complexity on the brane | 7

02. Doubly Holographic Model

Adding gravity on the brane

Dvali, Gabadadze and Porrati
hep-th/0005016

$$I_{\text{brane}} = -\left(T_o - \Delta T\right)\int d^d x\sqrt{-\tilde{g}} + \frac{1}{16\pi G_{\text{brane}}}\int d^d x\sqrt{-\tilde{g}}\tilde{R}$$

Effective d-dim holographic gravity

$z = z_{\text{B}} \ll L$

$$I_{\text{eff}} = \frac{1}{16\pi G_{\text{eff}}}\int d^d x\sqrt{-\tilde{g}}\left[\frac{(d-1)(d-2)}{\ell_{\text{eff}}^2} + \tilde{R}(\tilde{g})\right] + \frac{1}{16\pi G_{\text{RS}}}\int d^d x\sqrt{-\tilde{g}}\left[\frac{L^2}{(d-4)(d-2)}\left(\tilde{R}^{ij}\tilde{R}_{ij} - \frac{d}{4(d-1)}\tilde{R}^2\right) + \cdots\right]$$

$$\frac{1}{G_{\text{eff}}} = \frac{2L}{(d-2)G_{\text{bulk}}} + \frac{1}{G_{\text{brane}}}, \qquad \frac{1}{G_{\text{RS}}} = \frac{2L}{(d-2)G_{\text{bulk}}}$$

03. Entanglement Entropy&Island Rule

S. Ryu and T. Takaya
hep-th/0603001

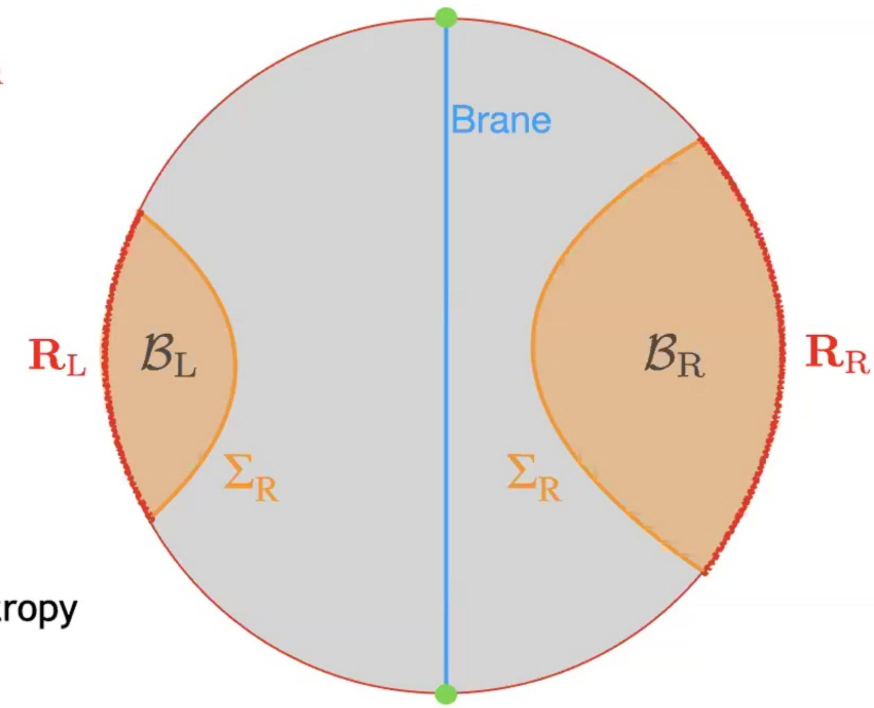
Holographic Entanglement Entropy in AAdS_{d+1}

boundary subregion: $\mathbf{R} = \mathbf{R}_\mathrm{L} \cup \mathbf{R}_\mathrm{R}$

RT surface: Σ_R

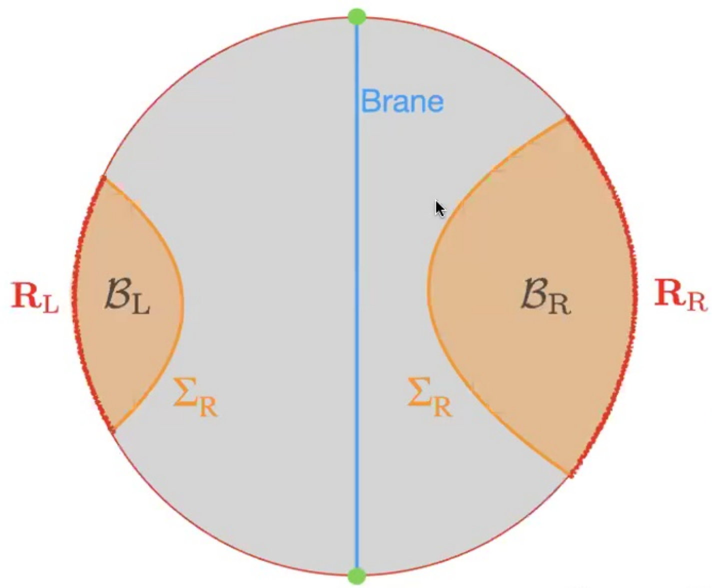
$$S_{\mathrm{EE}}(\mathbf{R}) = \min \left\{ \underset{\Sigma_\mathbf{R}}{\mathrm{ext}} \left(\frac{A\left(\Sigma_\mathbf{R}\right)}{4G_{\mathrm{bulk}}} \right) \right\}$$

Holography knows entanglement entropy



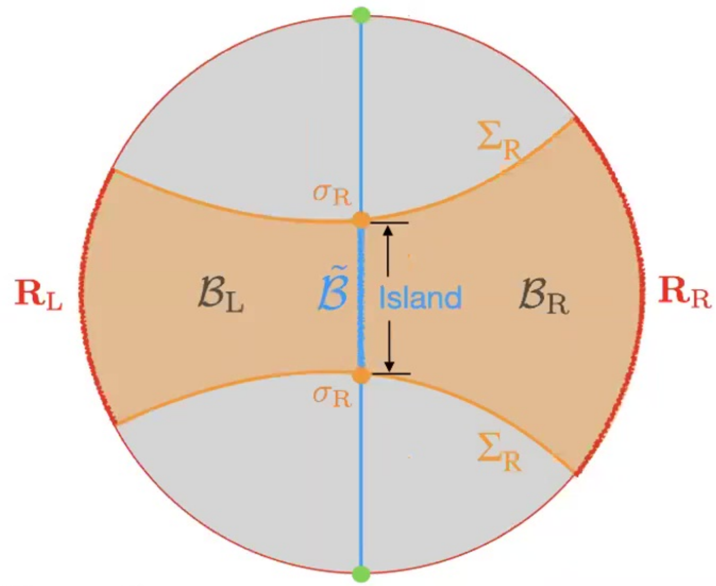
03. Entanglement Entropy&Island Rule

disconnected RT phase



non-island phase

connected RT phase



island phase

Competing Channels

$$S_{\mathrm{dis}}(\mathbf{R}) \geq S_{\mathrm{con}}(\mathbf{R})$$

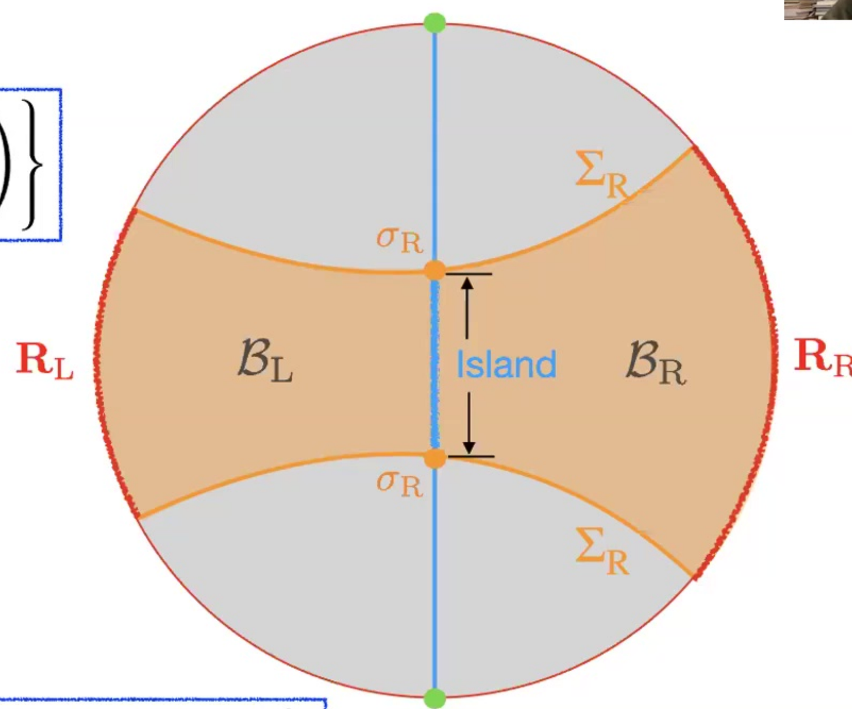
Shan-Ming Ruan (PI) | Complexity on the brane | 10

03. Entanglement Entropy&Island Rule

A generalization of RT formula

$$S_{\mathrm{EE}}(\mathbf{R}) = \min \left\{ \underset{\Sigma_{\mathbf{R}}}{\mathrm{ext}} \left(\frac{A\left(\Sigma_{\mathbf{R}}\right)}{4G_{\mathrm{bulk}}} + \frac{A\left(\sigma_{\mathbf{R}}\right)}{4G_{\mathrm{brane}}} \right) \right\}$$

A result from (d+1)-dim bulk



Island Formula

$$S_{\mathrm{EE}}(\mathbf{R}) = \min \left\{ \underset{\text{islands}}{\mathrm{ext}} \left(S_{\mathrm{QFT}}(R \cup \text{islands}) + \frac{A(\partial(\text{islands}))}{4G_{\mathrm{N}}} \right) \right\}$$

Quantum extremal surface
$\sigma_{\mathrm{R}} \equiv \partial(\text{Islands})$

Shan-Ming Ruan (PI) Complexity on the brane 11

03. Entanglement Entropy&Island Rule

Island Rule for Higher-Curvature Gravity

$$S_{\mathrm{EE}}(\mathbf{R}) = \min \left\{ \operatorname*{ext}_{\text{islands}} \left(S_{\mathrm{QFT}}(R \cup \text{islands}) + \frac{A(\partial(\text{islands}))}{4G_{\mathrm{N}}} \right) \right\}$$

Einstein gravity (area law)

$$\frac{A(\sigma_{\mathrm{R}})}{4G_{\mathrm{N}}} \longrightarrow S_{\mathrm{Wald-Dong}}$$

Higher-curvature gravity

$$S_{\mathrm{Wald-Dong}} = 2\pi \int d^{D-2}y\sqrt{g} \left\{ -\frac{\partial L}{\partial R_{\mu\rho\nu}} \varepsilon_{\mu\rho}\varepsilon_{\nu\sigma} + \sum_{\alpha} \left(\frac{\partial^2 L}{\partial R_{\mu_1\rho_1\nu_1\sigma_1} \partial R_{\mu_2\rho_2\nu_2\sigma_2}} \right)_{\alpha} \frac{2K_{\lambda_1\rho_1\sigma_1}K_{\lambda_2\rho_2\sigma_2}}{q_\alpha + 1} \times \right.$$
$$\left. \times \left[\left(n_{\mu_1\mu_2}n_{\nu_1\nu_2} - \varepsilon_{\mu_1\mu_2}\varepsilon_{\nu_1\nu_2} \right) n^{\lambda_1\lambda_2} + \left(n_{\mu_1\mu_2}\varepsilon_{\nu_1\nu_2} + \varepsilon_{\mu_1\mu_2}n_{\nu_1\nu_2} \right) \varepsilon^{\lambda_1\lambda_2} \right] \right\}$$

03. Entanglement Entropy&Island Rule

(d+1)-dim
RT formula

$$S_{\mathrm{EE}}(\mathbf{R}) = \min \left\{ \operatorname*{ext}_{\Sigma_{\mathbf{R}}} \left(\frac{A\left(\Sigma_{\mathbf{R}}\right)}{4G_{\mathrm{bulk}}} + \frac{A\left(\sigma_{\mathbf{R}}\right)}{4G_{\mathrm{brane}}} \right) \right\}$$

Equivalence up to
quantum corrections

d-dim
Island Formula

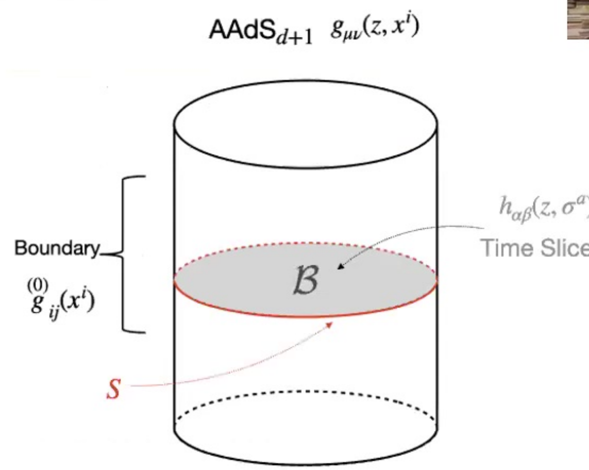
$$S_{\mathrm{EE}}(\mathbf{R}) = \min \left\{ \operatorname*{ext}_{\mathrm{islands}} \left(S_{\mathrm{QFT}}(R \cup \mathrm{islands}) + \underbrace{\frac{A(\partial(\mathrm{islands}))}{4G_{\mathrm{eff}}}}_{S_{\mathrm{Wald-Dong}}} \right) \right\}$$

04. Complexity on the Brane

Holographic Complexity

$$\mathbf{C}_{\mathrm{v}}(\mathbf{S}) = \max_{\partial \mathbf{B} = \mathbf{S}} \left[\frac{V(\mathbf{B})}{G_{\mathrm{N}} \ell} \right]$$

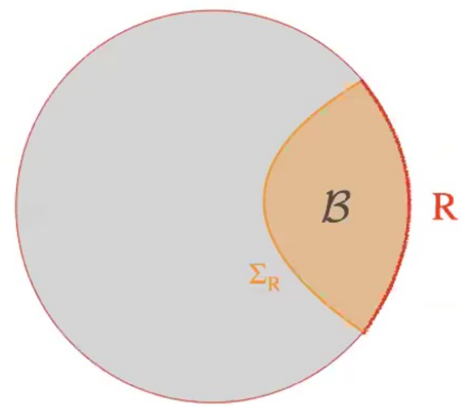
mysterious length scale



D. Stanford & L. Susskind : 1403.5698,1406.2678

Holographic Subregion-Complexity

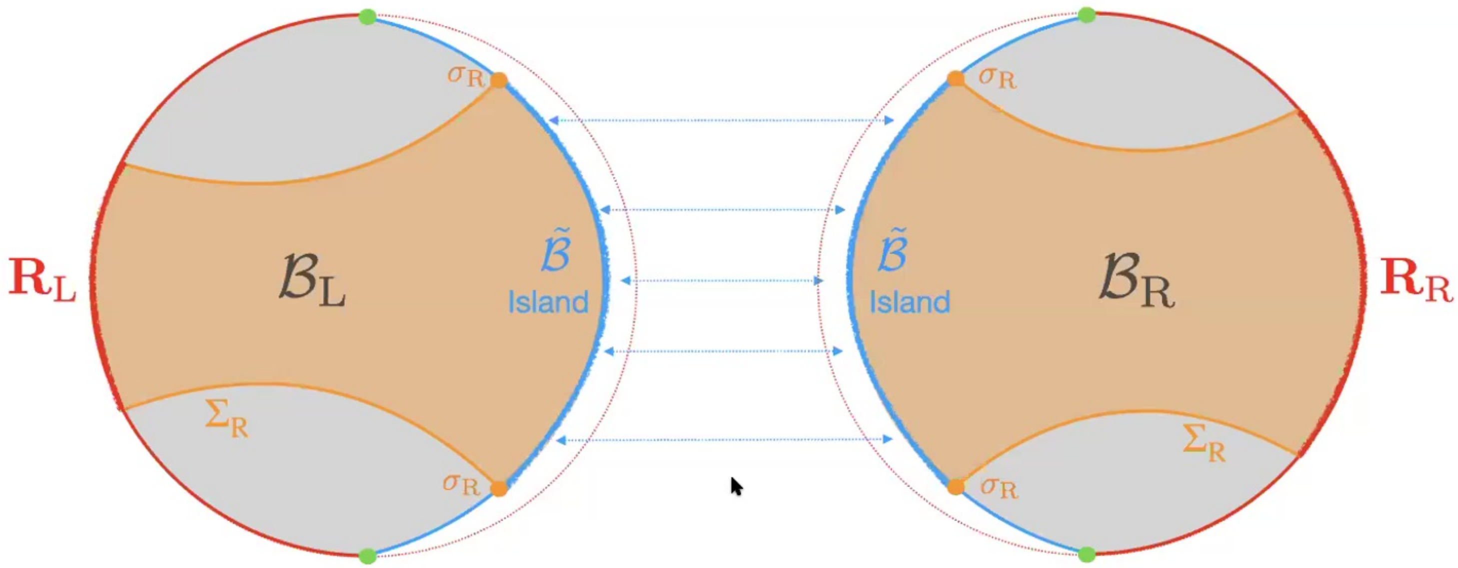
$$\mathbf{C}_{\mathrm{V}}^{\mathrm{sub}}(\mathbf{R}) = \max_{\partial \mathbf{B} = \mathbf{R} \cup \Sigma_{\mathbf{R}}} \left[\frac{V(\mathbf{B})}{G_{\mathrm{N}} \ell} \right]$$



See: 1509.06614; 1612.00433

04. Complexity on the Brane

Island Phase



CV from (d+1)-dim bulk

$$\mathbf{C}_\mathrm{V}^{\mathrm{sub}}(\mathbf{R}) = \max_{\partial \mathbf{B} = \mathbf{R} \cup \Sigma_\mathbf{R}} \left[\frac{V(\mathbf{B}_\mathrm{L}) + V(\mathbf{B}_\mathrm{R})}{G_{\mathrm{bulk}} \ell} \right]$$

Shan-Ming Ruan (PI) | Complexity on the brane 15

04. Complexity on the Brane

Holographic Subregion-Complexity

CV from (d+1)-dim bulk

$$\mathbf{C}_{\mathrm{V}}^{\mathrm{sub}}(\mathbf{R}) = \max_{\partial \mathbf{B} = \mathbf{R} \cup \Sigma_{\mathbf{R}}} \left[\frac{V(\mathbf{B}_{\mathrm{L}}) + V(\mathbf{B}_{\mathrm{R}})}{G_{\mathrm{bulk}} \ell} + \frac{V(\widetilde{\mathbf{B}})}{G_{\mathrm{brane}} \ell'} \right]$$

DGP term

d-dim subregion $\mathbf{B} \equiv \mathbf{B}_{\mathrm{L}} \cup \mathbf{B}_{\mathrm{R}}$

(d-1)-dim Islands $\widetilde{\mathbf{B}} \equiv \mathbf{B}_{\mathrm{L}} \cap \mathbf{B}_{\mathrm{R}}$

Expectation from d-dim brane theory

$$\mathbf{C}_{\mathrm{V}}^{\mathrm{sub}}(\mathbf{R}) \simeq \max_{\partial \widetilde{\mathbf{B}} = \sigma_{\mathrm{R}}} \left[\frac{V(\widetilde{\mathbf{B}})}{G_{\mathrm{eff}} \ell'} + \cdots \right]$$

volume of the island

$$\ell' = \frac{d-1}{d-2} \ell$$

Shan-Ming Ruan (PI) | Complexity on the brane | 16

04. Complexity on the Brane

$$\mathbf{C}_{\mathrm{V}}^{\mathrm{sub}}(\mathbf{R}) \longrightarrow \frac{V(\mathbf{B})}{G_{\mathrm{bulk}}\ell} = \frac{1}{G_{\mathrm{bulk}}\ell}\int_{\mathbf{B}} d^{d-1}\sigma dz \sqrt{\det h_{\alpha\beta}}$$

$$\simeq \frac{2LV(\widetilde{\mathbf{B}})}{(d-1)G_{\mathrm{bulk}}\ell} + \frac{2L^3}{G_{\mathrm{bulk}}\ell}\int_{\tilde{\mathbf{B}}} d^{d-1}\sigma \sqrt{\det \tilde{h}_{ab}} \left(\frac{\tilde{K}^2}{2(d-1)^2(d-3)} - \frac{\tilde{R}_{ij}\tilde{n}^i\tilde{n}^j + \frac{1}{2}\tilde{R}}{(d-1)(d-2)(d-3)} \right) + \cdots$$

Extrinsic — Intrinsic

With the volume of islands

$$V(\widetilde{\mathbf{B}}) = \int_{\tilde{\mathbf{B}}} d^{d-1}\sigma \sqrt{\det \tilde{h}_{ab}}$$

$$= L^{d-1}\int_{\tilde{\mathbf{B}}} d^{d-1}\sigma \sqrt{\det{}^{(0)} h_{ab}} \left[\frac{1}{z_{\mathrm{B}}^{d-1}} + \frac{1}{z_{\mathrm{B}}^{d-3}} \left(\frac{K^2}{2(d-1)} - \frac{R_a^a - \frac{1}{2}R}{2(d-2)} \right) + \cdots \right]$$

04. Complexity on the Brane

Holographic Subregion-Complexity

CV from (d+1)-dim bulk

$$\mathbf{C}_{\mathrm{V}}^{\mathrm{sub}}(\mathbf{R}) = \max_{\partial \mathbf{B} = \mathbf{R} \cup \Sigma_{\mathbf{R}}} \left[\frac{V(\mathbf{B}_{\mathrm{L}}) + V(\mathbf{B}_{\mathrm{R}})}{G_{\mathrm{bulk}} \ell} + \frac{V(\widetilde{\mathbf{B}})}{G_{\mathrm{brane}} \ell'} \right]$$

=

Island Formula for Complexity ?

Generalized CV from d-dim brane theory

$$\mathbf{C}_{\mathrm{v}}^{\mathrm{sub}}(\mathbf{R}) = \max_{\partial \widetilde{\mathbf{B}} = \sigma_{\mathrm{R}}} \left[\frac{\widetilde{W}_{\mathrm{gen}}(\widetilde{\mathbf{B}}) + \widetilde{W}_{K}(\widetilde{\mathbf{B}})}{G_{\mathrm{eff}} \ell'} + C_{\mathrm{bulk}}(R \cup \widetilde{\mathbf{B}}) \right]$$

$$\equiv \mathbf{C}_{\mathrm{v}}^{\mathrm{Island}}(\widetilde{\mathbf{B}})$$

Shan-Ming Ruan (PI) | Complexity on the brane | 18

05. Complexity for Higher-Curvature Gravity

A proposal:

Generalized CV for any (d+1)-dim higher-curvature gravity

$$\mathcal{C}_{\rm v}(\mathbf{R}) = \max_{\partial\mathcal{B}=\mathbf{R}} \left[\frac{W_{\rm gen}(\mathcal{B}) + W_K(\mathcal{B})}{G_{\rm N}\ell} \right], \qquad (d>2)$$

$$W_{\rm gen}(\mathbf{B}) = \frac{2}{(d-1)(d-2)} \int_{\mathbf{B}} d^d\sigma \sqrt{\det h} \left(1 + (d-3) \frac{\partial \mathbf{L}_{\rm bulk}}{\partial \mathcal{R}_{\mu\nu\rho\sigma}} n_\mu h_{\nu\rho} n_\sigma \right)$$

$$W_K(\mathbf{B}) = \frac{4(d-3)}{(d-1)^2(d-2)} \int_{\mathbf{B}} d^d\sigma \sqrt{\det h} \frac{\partial^2 \mathbf{L}_{\rm bulk}}{\partial \mathcal{R}_{\mu_1\nu_1\rho_1\sigma_1} \partial \mathcal{R}_{\mu_2\nu_2\rho_2\sigma_2}}$$

$$\times \left[\mathbf{K}_{\nu_1\sigma_1} \left(h_{\mu_1\rho_1} + (d-2) n_{\mu_1} n_{\rho_1} \right) \mathbf{K}_{\nu_2\sigma_2} \left(h_{\mu_2\rho_2} + (d-2) n_{\mu_2} n_{\rho_2} \right) \right] + \cdots$$

05. Complexity for Higher-Curvature Gravity

A test from higher curvature gravity in the bulk

Gauss-Bonnet gravity in the bulk

$$I^{\mathrm{GB}}_{\mathrm{bulk}} = \frac{1}{16\pi G_{\mathrm{bulk}}} \int d^{d+1}y \sqrt{-g} \left[\frac{d(d-1)}{L^2} + \mathcal{R}\left[g_{\mu\nu}\right] + \lambda_{\mathrm{GB}} \mathcal{L}_{\mathrm{GB}} \right] + I^{\mathrm{GB}}_{\mathrm{surf}}$$

with $\lambda_{\mathrm{GB}} = \dfrac{L^2 \lambda}{(d-2)(d-3)}, \quad \mathcal{L}_{\mathrm{GB}} = \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma} - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}^2$

Generalized CV from (d+1)-dim GB gravity in the bulk

$$\mathbf{C}^{\mathrm{sub}}_{\mathrm{V}}(\mathbf{R}) = \max_{\partial \mathbf{B} = \mathbf{R} \cup \Sigma_{\mathbf{R}}} \left[\frac{V(\mathbf{B})}{G_{\mathrm{bulk}}\ell} + \frac{1}{G_{\mathrm{bulk}}\ell} \frac{2L^2\lambda}{(d-1)(d-2)} \left(\int_{\mathbf{B}} d^{d-1}\sigma dz \sqrt{\det h_{\alpha\beta}}\, R_{\mathbf{B}} + 2\int_{\tilde{\mathbf{B}}} d^{d-1}\sigma \sqrt{\det \tilde{h}}\, \left(K_{\mathrm{L}} + K_{\mathrm{R}}\right) \right) \right]$$

05. Complexity for Higher-Curvature Gravity

A test from GB gravity in the bulk

$$\mathbf{C}_{\mathrm{V}}^{\mathrm{sub}}(\mathbf{R}) = \max_{\partial\mathbf{B}=\mathbf{R}\cup\Sigma_{\mathbf{R}}} \left[\frac{W_{\mathrm{gen}}(\mathbf{B}) + W_{K}(\mathbf{B}) + W_{\mathrm{bdy}}\left(\partial\mathbf{B}_{\mathrm{L}} \cup \partial\mathbf{B}_{\mathrm{R}}\right)}{G_{\mathrm{bulk}}\ell} \right]$$

$$\simeq \mathbf{C}_{\mathrm{V}}^{\mathrm{Island}}(\widetilde{\mathbf{B}}) \equiv \max_{\partial\widetilde{\mathbf{B}}=\sigma_{\mathrm{R}}} \left[\frac{\widetilde{W}_{\mathrm{gen}}(\widetilde{\mathbf{B}}) + \widetilde{W}_{K}(\widetilde{\mathbf{B}})}{G_{\mathrm{eff}}\ell'} + \cdots \right]$$

We can also check this equivalence for f(R) gravity in the bulk

Shan-Ming Ruan (PI) Complexity on the brane

Remarks and Further Directions

- Generalized CV for higher-curvature gravity
- Generalized volume and K-term, boundary term
- Holography makes things easier
- Similar Story for CA?
- Island formula for complexity?
- Generalized complexity (what is quantum corrections C_{bulk}?)
- A derivation of holographic complexity?
- Relation between EE (Wald-Dong) and generalized CV?
- ....

Thanks for your attention!

Shan-Ming Ruan (PI) Complexity on the brane 23