Title: Custom Fermionic Codes for Quantum Simulation

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Collection: Tensor Networks: from Simulations to Holography III

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Custom fermionic codes for quantum simulation

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[0]Based on RWC, JD Whitfield. arXiv:2009.11860

Quantum simulation of fermions

Typically when you think of quantum simulation, you think of the following procedure:

1. Prepare a state
2. Evolve the state
3. Measure the state

But first: **We have to decide how to embed the fermionic system into a qubit system.**

Fermion - qubit mappings

Overview of common transformations:

- Jordan-Wigner transformation
  - Maps N fermions to N qubits
  - 1 dimensional
- Higher dimensional generalizations
  - Maps N fermions to $M > N$ qubits
  - Restrict to a subspace of the M qubit system
  - Local fermion operators map to local qubit operators
  - Only encode even parity operators

Even parity operators

Majorana operators:

$$a_j = \frac{1}{2}(\gamma_{2j-1} + i\gamma_{2j}) \qquad \gamma_{2j-1} = a_j + a_j^\dagger$$

$$a_j^\dagger = \frac{1}{2}(\gamma_{2j-1} - i\gamma_{2j}) \qquad \gamma_{2j} = -i(a_j - a_j^\dagger)$$

$\{\gamma_j, \gamma_k\} = \gamma_j\gamma_k + \gamma_k\gamma_j = 2\delta_{jk}$

We only encode even parity fermion operators. Generators for parity preserving operators:

$$V_j = -i\gamma_{2j-1}\gamma_{2j} \quad \text{(vertex)}$$

$$= \mathbb{1} - 2a_j^\dagger a_j$$

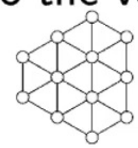
$$E_{jk} = -i\gamma_{2j-1}\gamma_{2k-1} \quad \text{(edge)}$$

$$= -i(a_j^\dagger a_k - a_k^\dagger a_j + a_j^\dagger a_k^\dagger + a_j a_k)$$

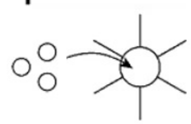
$E_{jk}^\dagger = E_{jk}$, $V_j^\dagger = V_j$, $E_{jk}^\dagger E_{jk} = \mathbb{1}$, $V_j^\dagger V_j = \mathbb{1}$

Code construction

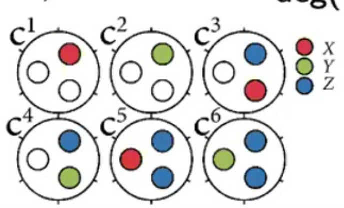
1. We have a model of fermions on a graph with modes associated to the vertices.



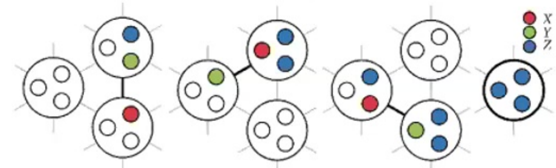
2. For each vertex, v, we assign $\deg(v)/2$ qubits to vertex, v.



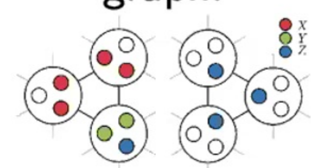
3. Define $\deg(v)$ Pauli operators acting on the $\deg(v)/2$ qubits $c^1, \ldots, c^{\deg(v)} \in \mathcal{P}_{\deg(v)/2}$.



4. Define encoded edge operators, $E_{jk} = \epsilon_{jk} c_j^p c_k^q$, and vertex operators, $V_j = i^{\deg(v)/2} c_j^1 \ldots c_j^{\deg(v)}$.



5. Restrict to the subspace in which $i^{|\partial p|} \prod_{(j,k) \in \partial p} E_{jk} = \mathbb{1}$ for all plaquettes, p, on the graph.



Custom codes

- The presence of edge operators in the Hamiltonian determine the **interaction graph**
- We're free to encode the system on some other graph, the **system graph**, as long as:
  1. The system graph has at least as many vertices as the interaction graph
  2. Any two modes that share an edge on the interaction graph are connected by a path on the system graph
- Use the presented mapping to encode the system graph
- If necessary, edge operators are realized as string operators traversing the system graph
- Simple example: The Jordan-Wigner transformation is an encoding of a system into a linear system graph. String operators traverse the chain

Outline

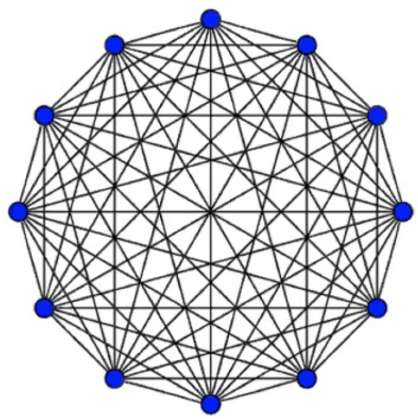
1. Quantum simulation of fermions
2. Systems with high connectivity
3. Lattice models

Highly connected systems

Example Hamiltonian - All quadratic interactions on N fermions:

$$H = \sum_{j}^{N} J_{j,j'} V_j + \sum_{jk}^{N} J_{j,k} E_{jk} + \sum_{jk}^{N} J_{j',k} E_{jk} V_j - \sum_{jk}^{N} J_{j',k'} E_{jk} V_j V_k$$

Complete interaction graphs for N modes - no notion of locality



Two naive geometries:

- Linear:
  N qubits
  $O(N^3)$ Pauli weight
- Complete graph:
  $(N^2 - N)/2$ qubits
  $O(N^2 \log N)$ Pauli weight

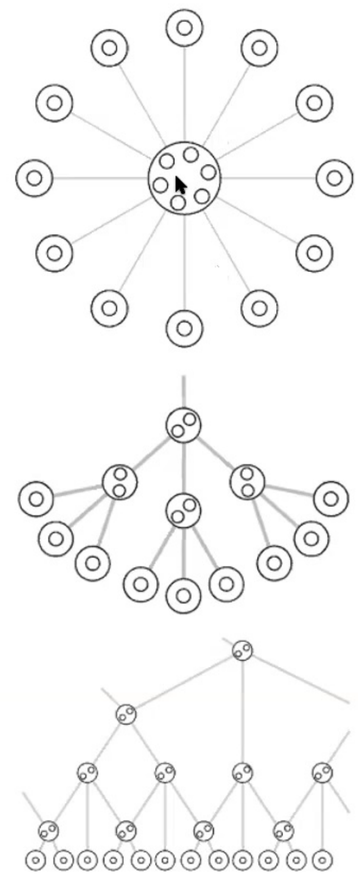
Highly connected systems

We considered a number of geometries:

- Linear
- Complete graph
- One central node with N branches
- Hierarchical geometries like trees and hyperbolic lattices

Can simultaneously achieve

- linear qubit scaling
- $O(N^2 \log N)$ Pauli weight for the qubit Hamiltonian



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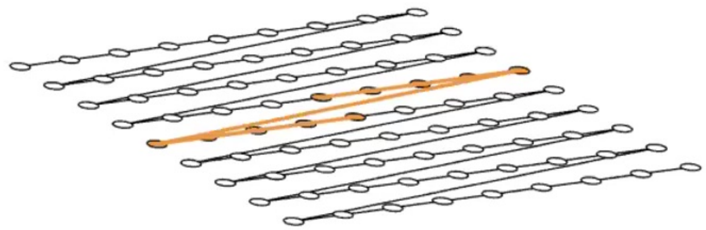
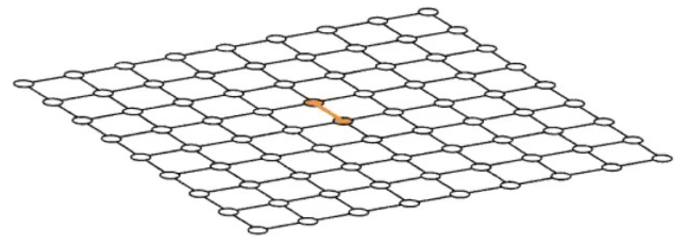
Quasilocal codes for lattices





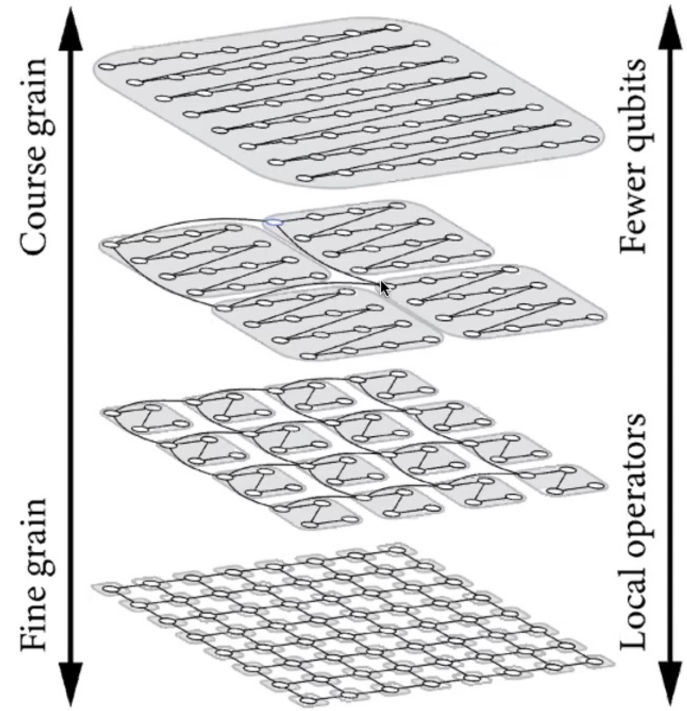
Non-local operators
No vacuum entanglement
$N_{qubits} = N_{fermions}$

Local operators
Entangled vacuum
$N_{qubits} = \deg(v) N_{fermions}/2$

Filling in this gap

Quasilocal codes for lattices

- Control non-locality by setting a lattice spacing for the system graph
- Below this lattice spacing operators will be non-local
- Above this scale we have local operators
- We generate a toric code state on a lattice with spacing set by the system graph.



Thank you for your time

Summary:

1. Quantum simulation of fermions
2. Systems with high connectivity
3. Lattice models

Questions?