Title: Symmetries at Null Boundaries

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Series: Quantum Gravity

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Abstract: I will present and motivate a program establishing, in full generality, the symmetries and charge analysis for gravitational theories near a generic null hypersurface without specifying any boundary condition. I will illustrate the first steps of this program on three dimensional Einstein gravity. In this case, there are three charges which are generic functions over the codimension one null surface. The integrability of the charges and the charge algebra depend on the state-dependence of symmetry generators which is a priori not specified. I will establish the existence of infinitely many choices that render the surface charges integrable. Then, I will argue that one expects this result to be valid for d > 3 when there is no Bondi news by developing some aspects of the four dimensional case. Finally, I will put our results in the context of earlier constructions of near horizon symmetries.

Based on: arxiv:2002.08346, arxiv:2005.06936 and arxiv:2007.12759

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Symmetries at Null Boundaries

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With H. Adami, D. Grumiller, M-M. Sheikh-Jabbari, S. Sadeghian, V. Taghiloo, H. Yavartanoo

Introduction

Goal: Determine the phase space of gravitational spacetimes with boundaries

Examples of boundaries

- Asymptotic boundary (e.g. $\mathscr{I}^+$)
- Causal boundary (e.g. horizon of black holes)

Why?

- Holographic dualities
- Entropy problem
- Describing matter falling from the point of view of the BH horizon
- Gravitational memory effect
- ...

Phase space = states such that

- Structure of boundary, i.e. behaviour of the fields near that boundary
- Dynamics

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Way to organise the states in the phase space is to consider the symmetry group preserving the boundary

The presence of a boundary breaks the full diffeomorphism invariance.

- Some diffeo are forbidden (those not preserving the boundary)
- Some diffeo are pure gauge (they have no effect on the states)
- Some diffeo preserve the boundary but change the state

Us: Study the phase space when the boundary is a codimension one null hypersurface in the bulk

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Table of contents

Phase space - Counting the degrees of freedom

Pure gravity in d dimensions with a boundary

$\frac{d(d+1)}{2}$ components

- d components are gauge fixed (consistently with boundary structure)
- $\frac{d(d-3)}{2}$ components correspond to the propagating gravitons - **bulk dof**
- d components correspond **boundary dof** - codimension 1 function, due to the residual gauge symmetries acting non trivially on the boundary

Depending on your gauge choice and boundary conditions you might kill some dof !

Program

Goal: Systematic treatment of the boundary degrees of freedom on a generic surface and their interaction with the bulk degrees of freedom

1. **How to realise the maximal number of boundary dof?**
2. What do they represent?
3. **Relation with bulk dof ?**

First step: use a model where there is no bulk dof.

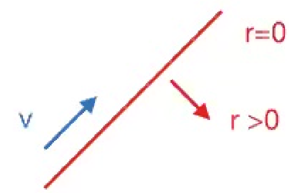
3d Einstein gravity because simple yet non trivial: solutions with Killing horizons

3d gravity

Maximal phase space content : boundary dof = 3 codimension 1 functions

Coordinates:

- Null hypersurface at $r = 0$
- « Advanced time » v
- Transverse direction 2π period: ϕ



Boundary structure given by metrics of the form :

$$ds^2 = -Fdv^2 + 2\eta dvdr + 2f dvd\phi + hd\phi^2$$

with $\eta(v,\phi)$ and

$$F(v,r,\phi) = F_0(v,\phi) + rF_1(v,\phi) + \mathcal{O}(r^2)$$
$$f(v,r,\phi) = f_0(v,\phi) + rf_1(v,\phi) + \mathcal{O}(r^2)$$
$$h(v,r,\phi) = \Omega(v,\phi)^2 + rh_1(v,\phi) + \mathcal{O}(r^2)$$

with $F_0 = -\left(\frac{f_0}{\Omega}\right)^2$ and $\Omega \neq 0$

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Symmetries

Looking at symmetries preserving the expansion -> ξ such that $g + \mathcal{L}_\xi g$ is still of the same form

$$\xi^v = T$$

$$\xi^r = r(\partial_v T - W) + \frac{r^2 \partial_\phi T}{2\Omega^2}\left(f_1 + \partial_\phi \eta - \frac{f_0 h_1}{\Omega^2}\right) + \mathcal{O}(r^3)$$

$$\xi^\phi = Y - \frac{r\eta\partial_\phi T}{\Omega^2} + \frac{r^2 \eta h_1 \partial_\phi T}{2\Omega^4} + \mathcal{O}(r^3)$$

With $T(v,\phi), W(v,\phi), Y(v,\phi)$

The functions in the metric expansion are changing under the action of ξ

Ex:

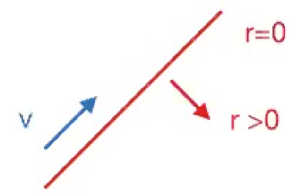
$$\delta_\xi \eta = T\partial_v \eta + 2\eta\partial_v T - \eta W + Y\partial_\phi \eta - \frac{f_0 \eta}{\Omega^2}\partial_\phi T$$

$$\delta f_0 = \partial_v(T f_0) + \partial_\phi(Y f_0) + \Omega^2 \partial_v Y + \left(\frac{f_0}{\Omega}\right)^2 \partial_\phi T$$

Note: if we go to the corotating frame -> $f_0 = 0 \Rightarrow Y = Y(\phi)$

3d gravity

Maximal phase space content : boundary dof = 3 codimension 1 functions



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Rechercher

Symmetry Algebra of ξ

$$[\xi_1, \xi_2]_{\text{adj.bracket}} = [\xi_1, \xi_2]_{\text{Dirac}} - \delta_2\xi_1 + \delta_1\xi_2$$

$$[\xi(W_1, T_1, Y_1), \xi(W_2, T_2, Y_2)]_{\text{adj.}} = \xi(W_{12}, T_{12}, Y_{12})$$

$$T_{12} = T_1\partial_v T_2 - T_2\partial_v T_1 + Y_1\partial_\phi T_2 - Y_2\partial_\phi T_1$$
$$W_{12} = T_1\partial_v W_2 - T_2\partial_v W_1 + Y_1\partial_\phi W_2 - Y_2\partial_\phi W_1 + \partial_v Y_1\partial_\phi T_2 - \partial_v Y_2\partial_\phi T_1$$
$$Y_{12} = Y_1\partial_\phi Y_2 - Y_2\partial_\phi Y_1 + T_1\partial_v Y_2 - T_2\partial_v Y_1\,,$$

Diff(C_2) $\oplus$ Weyl(C_2), where C_2 = null cylinder parameterised by v, ϕ

Are those symmetries pure gauge or not?

Symmetries

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Note: if we go to the corotating frame -> $f_0 = 0 \Rightarrow Y = Y(\phi)$

Charges

Generalised Noether theorem: gauge symmetry <-> (codimension 2) boundary charge Q

If $Q = 0$, the symmetry is pure gauge; If $Q \neq 0$, the symmetry is physical (or large);

However, no absolute charge formula in gravity, we rather compute a variation around a point in the phase space

$$\delta Q_\xi := \int_{\text{codim 2 boundary}} \mathcal{Q}_\xi^{\mu\nu}[g;h]dx_{\mu\nu} \text{ with}$$

$$\mathcal{Q}_\xi^{\mu\nu} = \frac{\sqrt{-g}}{8\pi G}\left(h^{\lambda[\mu}\nabla_\lambda \xi^{\nu]} - \xi^\lambda \nabla^{[\mu} h_\lambda^{\nu]} - \frac{1}{2} h \nabla^{[\mu}\xi^{\nu]} + \xi^{[\mu}\nabla_\lambda h^{\nu]\lambda} - \xi^{[\mu}\nabla^{\nu]} h\right),$$

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Charges

$$\delta Q_\xi = \frac{1}{16\pi G}\int_0^{2\pi} d\phi \left[W\delta\Omega + Y\delta\Upsilon + T\left(-2\delta\chi - \Gamma\delta\Omega + \frac{f_0}{\Omega^2}\delta\Upsilon + \partial_\phi\left(\frac{f_0\delta\Omega}{\Omega^2}\right) + \frac{\chi\delta\eta}{\eta}\right)\right]$$

T, W, Y are large gauge symmetries

Γ, Υ, χ are given in terms of the metric components and are independent of each others

- For BTZ, $\Gamma = 2\kappa$
- Υ est proportionnel à $g_{v\phi}$
- $\chi \propto$ expansion of the null hypersurface

Integrability?

$$\delta Q_\xi = \frac{1}{16\pi G}\int_0^{2\pi} d\phi \left[W\delta\Omega + Y\delta\Upsilon + T\left(-2\delta\chi - \Gamma\delta\Omega + \frac{f_0}{\Omega^2}\delta\Upsilon + \partial_\phi\left(\frac{f_0\delta\Omega}{\Omega^2}\right) + \frac{\chi\delta\eta}{\eta}\right)\right]$$

- When the charges are integrable, i.e. Q_ξ exist, it is the Hamiltonian associated with the flow generated by ξ.
- **Non integrability** is related to the presence of a **flux** through the boundary

This flux is given by the non integrable part (which has some ambiguities in its definition) - see later

In 3d, what such a flux could it be, reminding that there is no propagating dof?

Actually, we realise that a state dependent redefinition of the symmetry generators would make the charges integrable.

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Making the charges integrable

$$\delta Q_\xi = \frac{1}{16\pi G}\int_0^{2\pi} d\phi \left[W\delta\Omega + Y\delta\Upsilon + T\left(-2\delta\chi - \Gamma\delta\Omega + \frac{f_0}{\Omega^2}\delta\Upsilon + \partial_\phi\left(\frac{f_0\delta\Omega}{\Omega^2}\right) + \frac{\chi\delta\eta}{\eta}\right)\right]$$

$$\hat{W} = W - T\Gamma - \frac{f_0}{\Omega^2}\,\partial_\phi T, \qquad \hat{Y} = Y + T\frac{f_0}{\Omega^2}, \qquad \hat{T} = \chi T,$$

$$\delta Q_\xi = \frac{1}{16\pi G}\int_0^{2\pi} d\phi \left(\hat{W}\delta\Omega + \hat{Y}\delta\Upsilon + \hat{T}\delta\mathscr{P}\right)$$

With $\mathscr{P} = \ln\frac{\eta}{\chi^2}$

Integrable if $\delta\hat{T} = \delta\hat{W} = \delta\hat{Y} = 0$, $Q_\xi = \frac{1}{16\pi G}\int_0^{2\pi} d\phi \left(\hat{W}\,\Omega + \hat{Y}\,\Upsilon + \hat{T}\mathscr{P}\right)$

The boundary dof are labelled by $(\Omega(v,\phi), \Upsilon(v,\phi), \mathscr{P}(v,\phi))$

What if $\chi = 0$? $\rightarrow \hat{T} = T$, $\delta Q = \int_{S^2} \hat{W}\delta\Omega + T\delta(-2\chi) + \hat{Y}\delta\Upsilon$

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Rechercher

Charge algebra

$$\delta_\chi Q_\xi := \{Q_\xi, Q_\chi\} = Q_{[\chi,\xi]_{\text{adj.}}} + \text{central extension}$$

$$\{\Omega(v,\phi), \Omega(v,\phi')\} = 0\,,$$
$$\{\mathscr{P}(v,\phi), \mathscr{P}(v,\phi')\} = 0\,,$$
$$\{\Omega(v,\phi), \mathscr{P}(v,\phi')\} = 16\pi G\, \delta(\phi-\phi')\,,$$

Heisenberg algebra

$$\{\Upsilon(v,\phi), \Upsilon(v,\phi')\} = 16\pi G \Big(\Upsilon(v,\phi')\partial_\phi - \Upsilon(v,\phi)\partial_{\phi'}\Big)\, \delta(\phi-\phi')\,,$$

Diff(S^1)

$$\{\Upsilon(v,\phi), \Omega(v,\phi')\} = -\,16\pi G\, \Omega(v,\phi)\, \partial_{\phi'}\, \delta(\phi-\phi')\,,$$

$$\{\Upsilon(v,\phi), \mathscr{P}^{(s)}(v,\phi')\} = 16\pi G \Big(-\mathscr{P}(v,\phi)\partial_{\phi'} - \mathscr{P}(v,\phi')\,\partial_\phi + 2\partial_{\phi'}\Big)\, \delta(\phi-\phi')\,.$$

Does it exist other slicing? Yes, infinitely many

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Fundamental slicing

$\Upsilon = -2\partial_\phi\Omega - \Omega\partial_\phi\mathscr{P} + \mathscr{S}$ and

$$\hat{W} = \tilde{W} - 2\partial_\phi\tilde{Y} + \tilde{Y}\partial_\phi\mathscr{P}\,,$$
$$\hat{Y} = \tilde{Y}\,,$$
$$\hat{T} = \tilde{T} - \partial_\phi(\Omega\tilde{Y})\,.$$

$$\delta Q_\xi = \frac{1}{16\pi G}\int_0^{2\pi} d\phi\,\left(\tilde{W}\delta\Omega + \tilde{Y}\delta\mathscr{S} + \tilde{T}\delta\mathscr{P}\right)$$

If $\delta\tilde{T} = \delta\tilde{W} = \delta\tilde{Y} = 0$, the charges is integrable $Q_\xi = \frac{1}{16\pi G}\int_0^{2\pi} d\phi\,\left(\tilde{W}\,\Omega + \tilde{Y}\,\mathscr{S} + \tilde{T}\,\mathscr{P}\right)$

Algebra: Heisenberg direct sum with Diff(S^1)

$$\{\Omega(v,\phi),\Omega(v,\phi')\} = 0\,,\{\mathscr{P}(v,\phi),\mathscr{P}(v,\phi')\} = 0\,,\{\Omega(v,\phi),\mathscr{P}(v,\phi')\} = 16\pi G\,\delta(\phi-\phi')\,,$$
$$\{\mathscr{S}(v,\phi),\Omega(v,\phi')\} = \{\mathscr{S}(v,\phi),\mathscr{P}(v,\phi')\} = 0$$
$$\{\mathscr{S}(v,\phi),\mathscr{S}(v,\phi')\} = \Big(\mathscr{S}(v,\phi')\,\partial_\phi - \mathscr{S}(v,\phi)\,\partial_{\phi'}\Big)\,\delta(\phi-\phi')\,.$$

General method

$$\delta Q = \int \mu^i \delta Q_i \, d^{d-2}x \,, \qquad i = 1,2,\cdots,N \,.$$

In basis where $\delta\mu_i = 0$, one has $Q = \int \mu^i Q_i \, d^{d-2}x$

Consider $\tilde{Q}_i = \tilde{Q}_i[Q_j, \partial^n Q_k]$, one has $\delta Q = \int \tilde{\mu}^i \delta\tilde{Q}_i \, d^{d-2}x$, with $\tilde{\mu}^i = \dfrac{\delta Q_j}{\delta \tilde{Q}_i} \, \mu^j$

Note that the $\tilde{Q}$'s don't necessarily form an algebra.

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Example of Lie algebra: Virasoro

Start: Fundamental slicing

Step 1: Fundamental slicing -> Two currents algebras and Diff(S^1)

$$J^{\pm} := \frac{1}{16\pi G}\left(\Omega \mp 2Gk\,\partial_\phi \mathscr{P}\right) \text{ and } \tilde{W} = \epsilon^+ + \epsilon^-, \quad \tilde{T} = 2Gk\,\partial_\phi\left(\epsilon^+ - \epsilon^-\right)$$

Charge: $\delta Q_\xi = \int_0^{2\pi} d\phi\left(\epsilon^+ \delta J^+ + \epsilon^- \delta J^- + \tilde{Y}\delta\mathcal{S}\right)$

Algebra:

$$\{J^\pm(v,\phi), J^\pm(v,\phi')\} = \pm\frac{k}{4\pi}\partial_\phi \delta(\phi-\phi'),$$

$$\{J^\pm(v,\phi), J^\mp(v,\phi')\} = 0$$

$$\{\mathcal{S}(v,\phi), \mathcal{S}(v,\phi')\} = \left(\mathcal{S}(v,\phi')\,\partial_\phi - \mathcal{S}(v,\phi)\,\partial_{\phi'}\right)\delta(\phi-\phi')$$

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Step 2: Current basis -> Virasoro

$$L^{\pm}(v,\phi) := \frac{2\pi}{k}\left[J^{\pm}(v,\pm\phi)\right]^2 + \beta_{\pm}\partial_{\phi}J^{\pm}(v,\pm\phi)$$

and $\epsilon^{\pm}(v,\phi) = \left(\frac{4\pi}{k}J^{\pm}(v,\phi) \mp \beta_{\pm}\partial_{\phi}\right)\chi^{\pm}(v,\pm\phi)$

Charge: $\delta Q_{\xi} = \int_0^{2\pi} d\phi\,(\chi^{+}\delta L^{+} + \chi^{-}\delta L^{-} + \tilde{Y}\delta\mathcal{S})\,.$

Algebra:

$$\{L^{\pm}(v,\phi), L^{\pm}(v,\phi')\} = \left(L^{\pm}(v,\phi')\,\partial_{\phi} - L^{\pm}(v,\phi)\,\partial_{\phi'} + \frac{k}{4\pi}\beta_{\pm}^{2}\partial_{\phi'}^{3}\right)\delta(\phi-\phi')$$

$$\{L^{\pm}(v,\phi), L^{\mp}(v,\phi')\} = 0, \qquad \{L^{\pm}(v,\phi), \mathcal{S}(v,\phi')\} = 0$$

$$\{\mathcal{S}(v,\phi), \mathcal{S}(v,\phi')\} = \left(\mathcal{S}(v,\phi')\,\partial_{\phi} - \mathcal{S}(v,\phi)\,\partial_{\phi'}\right)\delta(\phi-\phi')$$

-> Generating a central charge

Similar construction leads to $\text{BMS}_3 + \text{Diff}(S^1)$

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Rechercher

Recap' 3d case:

1. Boundary structure such that we have the maximal realisation of boundary dof
2. State-dependent redefinition of the symmetry generators, we have shown that the charges can be make integrable.
3. Various examples of choices leading to various symmetry algebra

In the paper, we also have treated the **2d Einstein-Dilaton gravity**

-> Also no local dof -> only boundary dof

-> switch on the maximal number 2 codimension 1 function

-> One slicing where the symmetry algebra is Heisenberg algebra

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4d gravity

Action: $S = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\,(R)$.

Bulk dof: 2 gravitons

Boundary dof: 4

Null hypersurface at $r = 0$ and coord: (v, r, x^A) where $A = 1,2$

Gauge $g_{rr} = 0\,, \quad g_{rA} = 0$

Expansion:

$$ds^2 = \eta\,\mathscr{K}\,dv^2 + 2\,\eta dvdr + 2\,\eta\,\mathscr{U}_A\,dvdx^A + q_{AB}dx^Adx^B$$

With: $\lim_{r\to 0}\left(-\frac{\mathscr{K}}{\eta} + q^{AB}\mathscr{U}_A\mathscr{U}_B\right) = 0$

$$\mathscr{K} = \eta\,\Omega^{AB}f_Af_B + r\mathscr{G} + \mathscr{O}[r^2]\,,$$
$$\mathscr{U}_A = f_A + r\theta_A + \mathscr{O}[r^2]\,,$$
$$q_{AB} = \Omega_{AB} + r\lambda_{AB} + \mathscr{O}[r^2]\,.$$

and take corotating frame $g_{vA}\big|_{r=0} = 0 \Leftrightarrow f_A = 0$

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Rechercher

Symmetries

$$\xi^v = T(v, x^A)$$
$$\xi^r = -r\,W(v, x^A) + \mathcal{O}(r)$$
$$\xi^\phi = Y(x^A) + \mathcal{O}(r)$$

Algebra

$$\left[\xi(T_1, W_1, Y_1^A), \xi(T_2, W_2, Y_2^A)\right] = \xi(T_{12}, W_{12}, Y_{12}^A)$$

$$T_{12} = T_1\partial_v T_2 - T_2\partial_v T_1 + Y_1^A\partial_A T_2 - Y_2^A\partial_A T_1$$

With $W_{12} = T_1\partial_v W_2 - T_2\partial_v W_1 + Y_1^A\partial_A W_2 - Y_2^A\partial_A W_1$

$$Y_{12}^A = Y_1^B\partial_B Y_2^A - Y_2^B\partial_B Y_1^A\,.$$

Charges

Defining:

$$\det \Omega_{AB} = \Omega^2 , \quad \text{Shear/News: } N_{AB} := \partial_v \Omega_{AB} - \frac{\partial_v \Omega}{\Omega} \Omega_{AB} , \quad \Gamma = \mathscr{G} - \frac{\partial_v \eta}{\eta}$$

Charges:

$$\delta Q_\xi = \int_{S^2} \delta\Omega \left(\partial_v T + W - T\Gamma + T \frac{\partial_v \Omega}{\Omega} \right) + T \left(-2\partial_v \delta\Omega + \frac{\partial_v \Omega}{\eta} \delta\eta \right) + Y^A \delta \Upsilon_A$$
$$- \frac{1}{2} \Omega N^{AB} \delta\Omega_{AB} T$$

$T(v, x^A)$, $W(v, x^A)$, $Y^A(x^A)$ are large gauge symmetries -> 4 boundary dof

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Rechercher

News

$$\delta Q = \int_{S^2} \delta\Omega \left(\partial_v T + W - T\Gamma + T\frac{\partial_v\Omega}{\Omega}\right) + T\left(-2\partial_v\delta\Omega + \frac{\partial_v\Omega}{\eta}\delta\eta\right) + Y^A\delta\Upsilon_A - \frac{1}{2}\Omega N^{AB}\delta\Omega_{AB}T$$

Case where $N_{AB} = 0$ (Ex: $\Omega_{AB} = \omega\,\gamma^{S^2}_{AB}$)

$$\hat{W} = W - T\,\Gamma + \partial_v T + T\frac{\partial_v\Omega}{\Omega}, \quad \hat{T} = \partial_v\Omega\,T.$$

Charges

$$\delta Q = \int_{S^2} \hat{W}\delta\Omega + \hat{T}\delta\mathscr{P} + Y^A\delta\Upsilon_A$$

What if $\partial_v\Omega = 0$?

$$\hat{T} = T$$

$$\delta Q = \int_{S^2} \hat{W}\delta\Omega + T\delta(-2\partial_v\Omega) + Y^A\delta\Upsilon_A$$

Case where $N_{AB} \neq 0$

$$\delta Q = \int_{S^2} \hat{W}\delta\Omega + \hat{T}\delta\mathscr{P} + Y^A\delta\Upsilon_A - \frac{1}{2}\frac{\Omega}{\partial_v\Omega}N^{AB}\delta\Omega_{AB}\hat{T}$$

One has that N_{AB} is traceless so there is no way to write some scalar involved in $\hat{T}$ which are gonna make the charge integrable

$\Rightarrow N_{AB}$ is responsible for the fact that the charges are intrinsically non integrable

This is related to the fact that there is some true flux through the surfaces

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Algebra of non integrable charges

We have intrinsically non integrable charge -> Use Modified Bracket (Barnich/Troessaert)

$$\delta_\chi Q_\xi := \{Q_\xi, Q_\chi\} = Q_{[\chi,\xi]_{\text{adj.}}} + K_{\xi,\chi} \rightarrow \delta_\chi Q^I_\xi + F_\xi(\delta_\chi g) = \{Q^I_\xi, Q^I_\chi\}^* = Q^I_{[\chi,\xi]_{\text{adj.}}} + K_{\xi,\chi}$$

Ambiguities:

$$Q^I \rightarrow Q^I - N$$
$$F \rightarrow Q^I + \delta N$$
$$K_{\chi,\xi} \rightarrow K_{\chi,\xi} + \delta_\xi N_\chi - \delta_\chi N_\xi + N_{\{\xi,\chi\}_{\text{adj.}}}$$

$$Q^I = (\partial_v T + W)\delta\Omega - T(\Omega\Gamma + 2\partial_v\Omega) + Y^A \delta\Upsilon_A$$
$$F_\xi = T\,\Omega(\delta\Gamma + \frac{\partial_v\Omega}{\Omega}\delta \ln(\eta\,\Omega) + \frac{1}{2}N_{AB}\delta\Omega^{AB})$$

Charge Algebra is the same as the KV without a central charge

Flux is the non integrable part

1. No central charge
2. When $\delta_{\partial_v}\phi = \partial_v\phi$

One obtained the **generalized conservation equation**

$$\frac{d}{dv}Q^I_\xi = - F_{\partial_v}(\delta_\xi g)$$

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Rechercher

Imposing further conditions

$$ds^2 = \mathscr{G}\,\eta\, r dv^2 + 2\,\eta dv dr + 2\,\eta\,\theta_A\, r\, dv dx^A + \Omega_{AB} dx^A dx^B + \mathcal{O}(r^2)$$

1. $\eta = 1 \Rightarrow \delta\eta = 0$

We have $\delta_\xi \eta = \partial_v(\eta T) - W\eta + Y^A \partial_A \eta \Rightarrow W = \partial_v T$ Instead of 4 symm. Generators, we have 3

2. **Constant surface gravity**

Generator of the null hypersurface: $\ell^a = \partial_v - r\,\frac{1}{2}\mathscr{G}\,\partial_v T + \mathcal{O}(r^2)$

We have $\kappa = -\frac{1}{2}\mathscr{G} + \partial_v \ln \eta$, $\delta\kappa = 0 \Rightarrow \partial_v^2 T + \kappa \partial_v T = 0 \Rightarrow T = T^0(x^A) + T^1(x^A) e^{-\kappa v}$

1 + 2 = Symmetries found by Donnay et al. (1607.05703) and Chandrasekaran/Prabhu (1908.00017)

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Conclusion

Recap'

- We have switched on the maximal number of boundary dof 3d gravity: d functions of codimension 1
- There exist always integrable slicing (more than one). One of them being this fundamental slicing: Heisenberg direct sum with Diff(S^{d-2})
- In 4d, we have seen that the presence of News make the charge intrinsically non integrable
- Choice of gauge or boundary condition can kill some of the boundary dof

Proposal

No propagating dof passing through the boundary <-> Existence of the fundamental slicing where the charges are integrable

Proof in progress

Further questions?

1. What does slicing represent physically?

How those are related to the existing boundary conditions for these cases?

2. Surface at infinity where the metric diverges?

3. What does this apparent flux represent (when you do not change the slicing)?

4. In the case of integrable charges, there are not conserved. What can be the dynamics of these boundary dof?

5. Generalisation to timelike surface

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Thanks!