Title: Homological mirror symmetry for the universal centralizers

Speakers: Xin Jin

Series: Mathematical Physics

Date: November 05, 2020 - 1:30 PM

URL: http://pirsa.org/20110040

Abstract: I will present recent work (to appear soon) on the homological mirror symmetry about the universal centralizers J_G, for any complex semisimple Lie group G. The A-side is a partially wrapped Fukaya category of J_G, and the B-side is the category of coherent sheaves on the categorical quotient of a dual maximal torus by the Weyl group action (with some modification if G has a nontrivial center).

HMS for the universal centralizers

Xin Jin

11/5/2020

1

The goal of this talk is to present a HMS result that generalizes the following well known thm:

- **Thm:** For a complex torus $T \cong (\mathbb{C}^\times)^n$. Let $T^\vee$ be its dual torus. Then

$$\underbrace{\mathrm{WFuk}}_{\text{partially wrapped Fukaya category}}(T^*T) \simeq \mathrm{Coh}(T^\vee)$$

$$\simeq \mathrm{Loc}(T) \simeq \mathbb{C}[\pi_1(T)]\text{-Mod}$$

- We would like to generalize the above thm in the <u>nonabelian</u> setting. The natural replacement of T^*T, for a semisimple (reductive) complex Lie group G, is the universal centralizer J_G.

① Definition(s) of J_G.

②

There are several equivalent definitions of J_G and we will list two here, which show different features of it.

- **Def 1:** This explains the name "universal centralizer" and shows an integrable system structure on J_G. (SYZ mirror symmetry)

$$J_G = \underset{\subset\, G \times \mathfrak{g}^{*,reg}}{\overset{\subset\, T^{*,reg}G}{\{(g,\xi) : g\xi g^{-1} = \xi\}}} /\!/ \overset{Ad}{G}.$$

- ξ is regular $\Longleftrightarrow \dim C_G(\xi) = rk(G)$.

Example: $G = SL_n(\mathbb{C})$ $\quad \xi \overset{\text{similar}}{\sim} \left(\begin{array}{c|c|c} \begin{smallmatrix} * & 1 & \\ & * & 1 \\ & & * \end{smallmatrix} & & \\ \hline & \begin{smallmatrix} \circ & 1 \\ & \circ \end{smallmatrix} & \\ \hline & & \begin{smallmatrix} \star & 1 & \\ & \star & 1 \\ & & \star \end{smallmatrix} \end{array}\right)$ $\quad *, \circ, \star$ distinct.

Jordan normal form

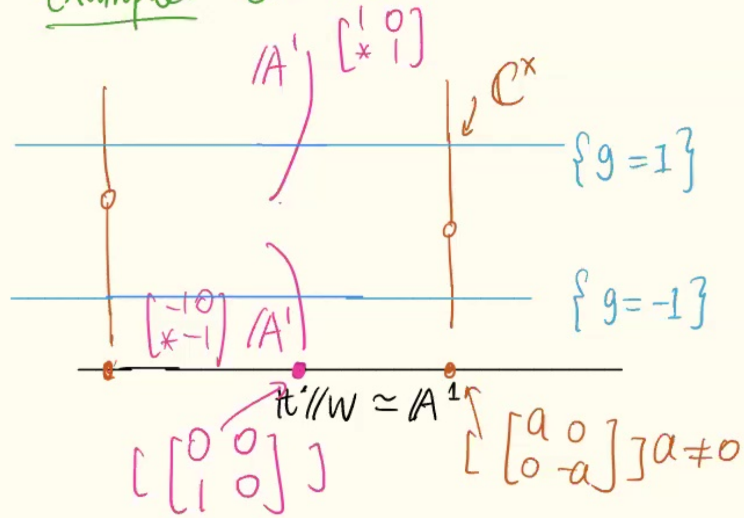
③

- **Def 1:** This explains the name "universal centralizer" and shows an integrable system structure on J_G. (SYZ mirror symmetry)

$$J_G = \underbrace{\{(g,\xi) : g\xi g^{-1} = \xi\}}_{\subset\, G \times \mathfrak{g}^{*,reg}} \,/\!/^{Ad}\, G \qquad \mathfrak{J}^{*,reg} G$$

- $$\begin{array}{ccc} J_G & \ni & (g,\xi) \\ \downarrow \chi & & \downarrow \\ \mathfrak{c} = \mathfrak{t}^*/\!/W & & [\xi] \end{array}$$

Example: $G = SL_2(\mathbb{C})$



③

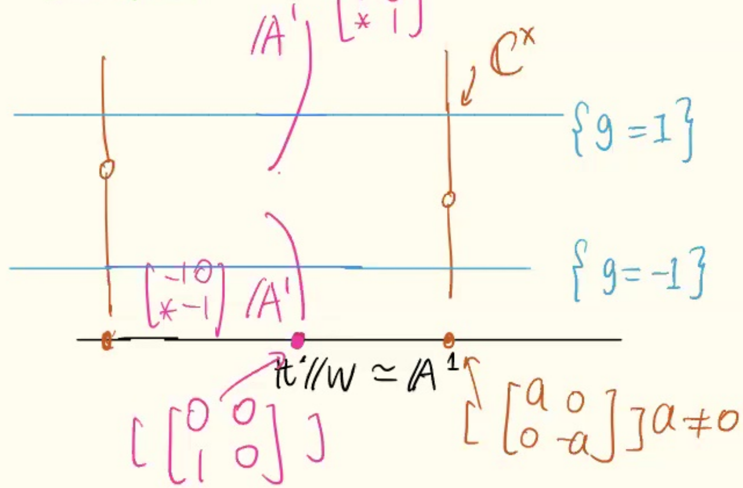
- **Def 1:** This explains the name "universal centralizer" and shows an integrable system structure on J_G. (SYZ mirror symmetry)

$$J_G = \{\underset{G\times\mathfrak{g}^{*,reg}}{(g,\xi)} : g\xi g^{-1} = \xi\} \overset{J^{*,reg}G}{} /\!/^{Ad} G.$$

- $$\begin{array}{ccc} J_G & \ni & (g,\xi) \\ \big\downarrow \chi & & \big\downarrow \\ \mathfrak{c} = \mathfrak{t}^*/\!/W & & [\xi] \end{array}$$

Kostant sections for each $g \in Z(G)$.

Example: $G = SL_2(\mathbb{C})$



abelian group scheme.

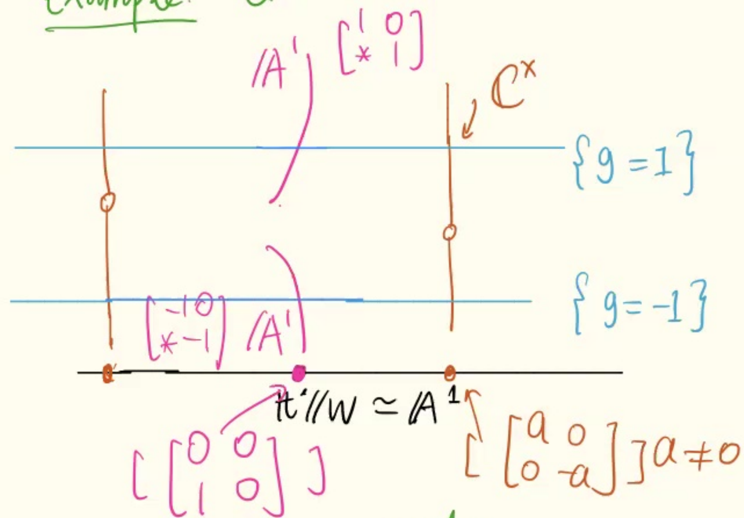
④

$$J_G \ni (g,\xi)$$

Kostant sections for each $g \in Z(G)$.

$$\begin{array}{ccc} J_G & \ni & (g,\xi) \\ \downarrow \chi & & \downarrow \\ \mathfrak{c} = \mathfrak{t}^*/\!/W & & [\xi] \end{array}$$

Example: $G = SL_2(\mathbb{C})$



abelian group scheme.

Thm of Kostant: fix a principal sl_2-triple $(\overset{\text{regular}}{e}, f, h)$

Kostant slice $\mathcal{S} \hookrightarrow \mathfrak{g}^{*,reg} \twoheadrightarrow \mathfrak{t}^*/\!/W$.

$\mathcal{S} = f + \ker \operatorname{ad} e$, $\quad \mathfrak{g}^{*,reg} \cong \mathfrak{g}^{reg}$ Killing form, $\quad \mathcal{S} \xrightarrow{\simeq} \mathfrak{t}^*/\!/W$

Then $J_G = \{(g,\xi) : g \in C_G(\xi), \ \xi \in \mathcal{S}\}$.

Def 2: J_G as a Hamiltonian reduction of T^*G, so it's holomorphic symplectic (actually it's hyperkahler).

Fix $T \subset B \subset G$ $\qquad$ $\mathfrak{t} \subset \mathfrak{b} \subset \mathfrak{g}$

$\quad\quad B \supset N$ $\qquad\qquad\qquad$ $\mathfrak{b} \supset \mathfrak{n}$

$$N \circlearrowright T^*G \circlearrowleft N \qquad (g, \xi)$$

$$f = \begin{bmatrix} 0 & & \\ 1 & 0 & \\ & \ddots & \ddots \\ 0 & & 1 \quad 0 \end{bmatrix}$$

$$\downarrow \mu \qquad\qquad \downarrow$$

$$\mathfrak{n}^* \oplus \mathfrak{n}^* \qquad (\underset{\text{mod } \mathfrak{b}}{g\xi g^{-1}}, \underset{\text{mod } \mathfrak{b}}{\xi})$$

$$\cong$$

$$(f, f) \in \mathfrak{n}^- \oplus \mathfrak{n}^-$$

$\{e, f, h\}$
principal
sl_2-triple

- Def 2: J_G as a Hamiltonian reduction of T^*G, so it's holomorphic symplectic (actually it's hyperkahler). ⑤

Fix $T \subset B \subset G$, $N \subset B$ $\qquad$ $\mathfrak{t} \subset \mathfrak{b} \subset \mathfrak{g}$, $\mathfrak{n} \subset \mathfrak{b}$

$$N \curvearrowright T^*G \curvearrowleft N \qquad (g, \xi)$$

$$\downarrow \mu \qquad\qquad \downarrow$$

$$\mathfrak{n}^* \oplus \mathfrak{n}^* \qquad (\underset{\mathrm{mod}\ \mathfrak{b}}{g\xi g^{-1}}, \underset{\mathrm{mod}\ \mathfrak{b}}{\xi})$$

$$\wr\!\mid$$

$$(f, f) \in \mathfrak{n}^- \oplus \mathfrak{n}^-$$

$$f = \begin{bmatrix} 0 & & & \\ 1 & 0 & & \\ & \ddots & \ddots & \\ 0 & & 1 & 0 \end{bmatrix}$$

$$J_G \cong \mu^{-1}(f, f) / (N \times N)$$

Page 17 of 64

⑥

$$N \subset T^*G \circlearrowleft N \qquad (g, \xi)$$

$$f = \begin{bmatrix} 0 & & & \\ 1 & 0 & & \\ & \ddots & \ddots & \\ 0 & & 1 & 0 \end{bmatrix} \qquad \downarrow \mu \qquad \mapsto$$

$$\mathfrak{n}^* \oplus \mathfrak{n}^* \qquad (\underset{\text{mod } b}{g \xi g^{-1}}, \underset{\text{mod } b}{\xi})$$

$$\text{\S}$$

$\{e, f, h\}$ principal sl_2-triple $\qquad (f, f) \in \mathfrak{n}^- \oplus \mathfrak{n}^-$

$$J_G \cong \mu^{-1}(f, f) / (N \times N)$$

- There's a natural $\mathbb{C}^\times$-action on J_G defined as follows:
  Use the above sl_2-triple with $h \in \mathfrak{t}$. ($h = 2\rho \in \Lambda^\vee_{\text{root}}$)

$$s \cdot \underbrace{(g, \xi)}_{\mu^{-1}(f,f)/N \times N} = (\mathrm{Ad}_{s^h} g, \underbrace{s^2}_{\mathrm{Ad}_{s^h}(f) = s^{-2} f.} \mathrm{Ad}_{s^h} \xi)$$

- The $\mathbb{C}^\times$-action scales the symplectic form ω by weight 2
  $\rightsquigarrow$ $\mathbb{R}^+$-action and takes square root gives a <u>Liouville vector field</u>: $\mathcal{L}_Z \omega = \omega$

• Main Theorem (J).

(7)

(1) J_G can be partially compactified to be a Liouville (Weinstein) sector (canonical up to isotopy).

(defined by Sylvan, Ganatra-Pardon-Shende)

(2) $\mathcal{W}\mathrm{Fuk}(J_{G_{ad}}) \simeq \mathrm{Coh}(\check{T}_{sc}/\!/W)$.

partially wrapped Fukaya cat.

$\mathcal{W}\mathrm{Fuk}(J_G) \simeq \mathrm{Coh}((\check{T}_{sc}/\!/W)/\underbrace{Z(G)^\vee}_{=\pi_1(\check{G})})$

following Abouzaid-Seidel, Sylvan, GPS

- ## Main Theorem (J). ⑦

(1) J_G can be partially compactified to be a Liouville (Weinstein) Sector (canonical up to isotopy).

(2) $\mathcal{W}Fuk(J_{G_{ad}}) \simeq Coh(\check{T}_{sc} /\!/ W)$.

$$\mathcal{W}Fuk(J_G) \simeq Coh\big((\check{T}_{sc} /\!/ W) / \pi_1(\check{G}) \big)$$

Remark: This can be seen as an analytic version of a thm by Lonergan, Ginzburg (independently)

$$D\text{-mod}(\overset{\psi}{N}\backslash G / \overset{\psi}{N}) \simeq QCoh(``\check{T}^{\circ} /\!/ W_{aff}")$$

• Main Theorem (J).

(1) J_G can be partially compactified to be a Liouville (Weinstein) sector (canonical up to isotopy).

(2) $\mathcal{W}Fuk(J_{G_{ad}}) \simeq Coh(\check{T}_{sc}/\!/W)$.

$\mathcal{W}Fuk(J_G) \simeq Coh\left((\check{T}_{sc}/\!/W)/\pi_1(\check{G})\right)$

Remark: This can be seen as an analytic version of a thm by Lonergan, Ginzburg (independently)

$$\underbrace{D\text{-mod}}_{D_{hol}}(N^{\overset{f}{\psi}}\backslash G/_{\overset{f}{\psi}}N) \simeq \underbrace{QCoh}_{Coh.}(\text{``}\underbrace{\mathfrak{t}^*/\!/W_{aff}}_{\text{some coarse quotient}}\text{''})$$

"$(\mathfrak{t}^*/\Lambda)/\!/W$"

But algebraic $\not\simeq$ analytic

- **Main Theorem (J).**

(1) J_G can be partially compactified to be a Liouville (Weinstein) sector (canonical up to isotopy).

(2) $\mathrm{WFuk}(J_{G_{ad}}) \simeq \mathrm{Coh}(\check{T}_{sc} /\!/ W)$.

$\mathrm{WFuk}(J_G) \simeq \mathrm{Coh}((\check{T}_{sc} /\!/ W)/\pi_1(\check{G}))$

Remark: This can be seen as an analytic version of a thm by Lonergan, Ginzburg (independently)

$$\text{D-mod}(\overset{\psi}{N}\backslash G/\overset{\psi}{N}) \simeq \mathrm{QCoh}(``\mathfrak{t}^{\circ} /\!/ W_{aff}")$$

③

name "universal centralizer" and shows

cture on J_G. (SYZ mirror symmet

$J_G \ni (g, \xi)$

Kostant sections for each $g \in Z(G)$.

$$\mathfrak{c} = \mathfrak{t}^\vee /\!/ W \xleftarrow{\ \chi\ } J_G, \qquad [\xi] \mapsfrom (g,\xi)$$

$^{-1} = \xi \} /\!/^{Ad} G.$ $:1\}$

abelian group scheme.

$[\ast - 1]$ $/\mathbb{A}^1$ $-1\}$

$$\mathfrak{t}^\vee /\!/ W \simeq \mathbb{A}^1$$

$\left[\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\right]$ $\left\{\left[\begin{bmatrix} a & 0 \\ 0 & -a \end{bmatrix}\right]\right\}$

Thm of Kostant: fix a principal $\mathfrak{sl}_2$-triple $(\overset{\text{regular}}{e}, f, h)$

Kostant slice $S \hookrightarrow \mathfrak{g}^{\vee, reg} \twoheadrightarrow \mathfrak{t}^\vee /\!/ W$ Then $J_G = \{(g,$ $1\}$

$S = f + \ker \operatorname{ad} e$ $\mathfrak{g}^{reg}$ (Killing form) $\simeq$

$\neq 0$

Page 13 of 64

③

- **Def 1:** This explains the name "universal centralizer" and shows an integrable system structure on J_G. (SYZ mirror symmetry)

$$T^{*,\mathrm{reg}}G \hookleftarrow J_G = \{(g,\xi) : g\xi g^{-1} = \xi\} /\!/^{Ad}\, G.$$

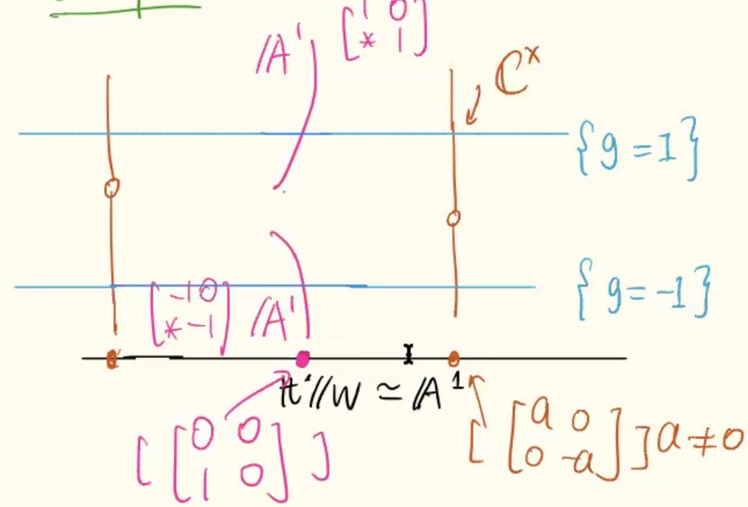
$$(g,\xi) \in G \times \mathfrak{g}^{*,\mathrm{reg}}$$

- $J_G \ni (g,\xi)$

$$\begin{array}{ccc} J_G & \ni & (g,\xi) \\ \downarrow \chi & & \downarrow \\ \mathfrak{c} =: \mathfrak{t}^*/\!/W & & [\xi] \end{array}$$

Kostant sections for each $g \in Z(G)$.

Example: $G = SL_2(\mathbb{C})$



abelian group scheme

Def 2: J_G as a Hamiltonian reduction of T^*G, so it's holomorphic symplectic (actually it's hyperkahler). ⑤

Fix $T \subset B \subset G$ $\qquad \mathfrak{t} \subset \mathfrak{b} \subset \mathfrak{g}$

$\qquad B \supset N \qquad \mathfrak{b} \supset \mathfrak{n}$

$$N \circlearrowright T^*G \circlearrowleft N \qquad (g, \xi)$$

$$T^*G \xrightarrow{\mu} \mathfrak{n}^* \oplus \mathfrak{n}^* \cong \mathfrak{n}^- \oplus \mathfrak{n}^- \qquad (g,\xi) \mapsto (\underbrace{g\xi g^{-1}}_{\mathrm{mod}\ \mathfrak{b}}, \underbrace{\xi}_{\mathrm{mod}\ \mathfrak{b}})$$

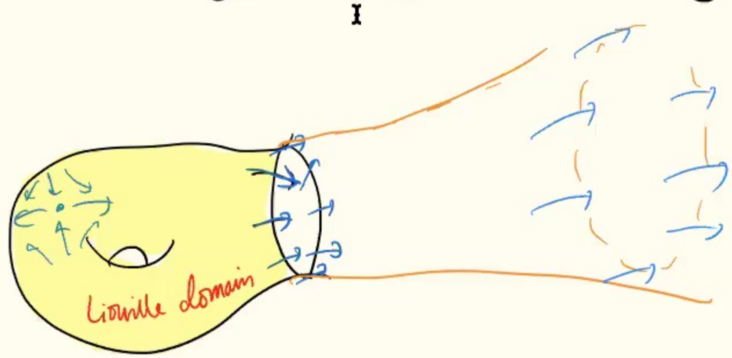
$$f = \begin{bmatrix} 0 & & & \\ 1 & 0 & & \\ & \ddots & \ddots & \\ 0 & & 1 & 0 \end{bmatrix} \qquad (f, f) \in \mathfrak{n}^- \oplus \mathfrak{n}^-$$

$\{e, f, h\}$ principal sl_2-triple

Page 16 of 64

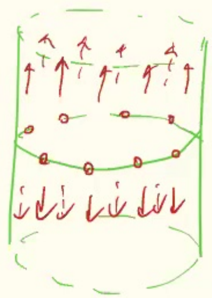
(8)

- (II) Background on partially wrapped Fukaya categories on Liouville/Weinstein sectors.
- Liouville mflds and Weinstein mflds.
- Def: $(X^{2n}, \omega = d\alpha)$ is a Liouville mfld if the Liouville v.f. Z defined by $\alpha = \iota_Z \omega$ satisfies
  - the flow of Z is complete;
  - Z is pointing outward everywhere along the ∞-boundary of X.



- Examples:

  - cotangent bundle of any closed mfld

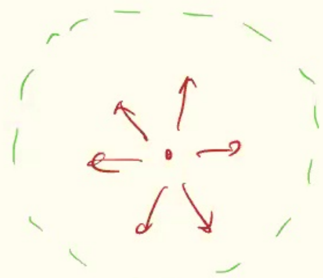


$\alpha = p\,dq$.

$Z = p\,\partial_p$

- $\mathbb{R}^{2n}$, $\alpha = \frac{1}{2}(p\,dq - q\,dp)$

  $Z = \frac{1}{2}(p\,\partial_p + q\,\partial_q)$



• Weinstein mflds. (Cieliebak – Eliashberg)

10

There is a nice class of Liouville mflds that can be built using handle attachments, called Weinstein mflds.

from Morse theory (-Bott)

symplectic analogue of Stein mflds.

index 1 handle.

Isotropic sphere

index 0 handle

- Weinstein mflds. (Cieliebeck – Eliashberg) (10)

There is a nice class of Liouville mflds that can be built using handle attachments, called Weinstein mflds.

from Morse theory (-Bott)

symplectic analogue of Stein mflds.

index 1 handle.

Isotropic sphere

index 0 handle

- index = dim of ascending mfld $\leq n$ if $\dim X = 2n$.
- terminology: index $k<n$ handle is called subcritical

  $k=n$ - - - - - critical ← these give rise to interesting symplectic invariants.

- Weinstein mflds. (Cieliebak – Eliashberg) (10)

There is a nice class of Liouville mflds that can be built using handle attachments, called Weinstein mflds.

from Morse theory (-Bott)

symplectic analogue of Stein mflds.

- index = dim of ascending mfld $\leq n$ if $\dim X = 2n$.
- terminology: index $k<n$ handle is called subcritical

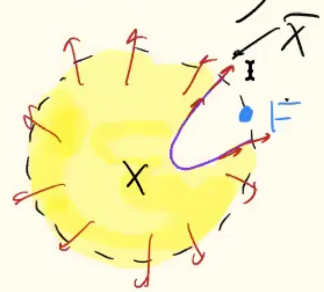
  $k=n$ - - - - - - critical

  these give rise to interesting symplectic invariants.
- Core

- Liouville / Weinstein sectors.

Liouville sectors are generalizations of Liouville mflds to mflds-with-boundary case.

- Def: A Liouville sector is a Liouville mfld-with-boundary X
(short version) which can be obtained from the following data:
  - $\bar{X}$ = a Liouville mfld, $F \subset \partial_\infty \bar{X}$ a Liouville hypersurface (i.e. $\alpha|_F$ is a Liouville domain structure)
  - Delete a standard nbhd of F. (can make Z tangent to ∂X)



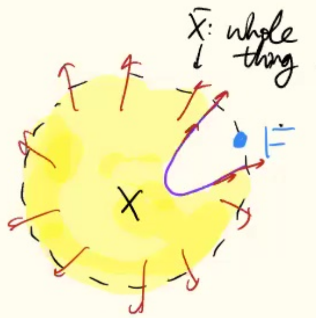
⑪

- Liouville / Weinstein Sectors.

Liouville sectors are generalizations of Liouville mflds to mflds-with-boundary case.

- Def (short version): A Liouville (Weinstein) sector is a Liouville (Weinstein) mfld-with-boundary X which can be obtained from the following data:
  - $\bar{X}$ = a Liouville (Weinstein) mfld, $F \subset \partial_\infty \bar{X}$ a Liouville (Weinstein) hypersurface
  - Delete a standard nbhd of F. (can make Z tangent to ∂X)

⑪

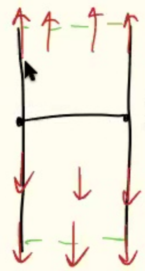


Xin Jin

• Examples:

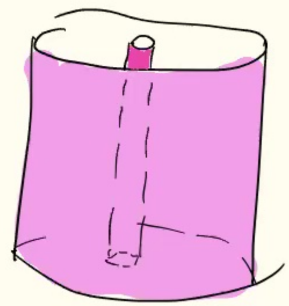
① Cotangent bundle of a compact mfld with boundary.

• $T^*[0,1]$



$\partial T^*[0,1]$

• $T^*T \cong \underbrace{T^*(S^1)^{\times n} \times (T^*[0,1])^{\times n}}_{\text{product of Weinstein sectors.}}$

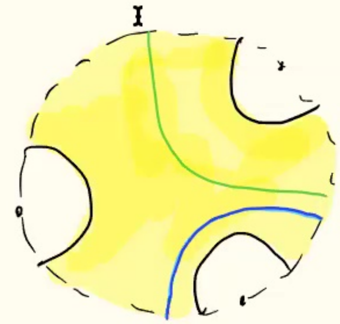


$\partial(T^*T)$

(13)

- Partially wrapped Fukaya category on Liouville / Weinstein Sectors.
  (Sylvan, GPS — generalizing $\left\{\begin{array}{l}\text{(fully) wrapped Fukaya cat.} \\ \text{Fukaya-Seidel cat.}\end{array}\right.$)

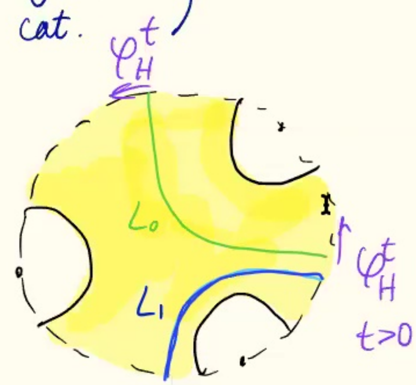
- Objects: $\left\{\begin{array}{l}\text{exact} \\ \text{Closed Lagrangians with conical}\end{array}\right.$
  ends (if not compact) + extra
  data : $\partial_\infty L \cap \partial X = \phi$.



• Partially wrapped Fukaya category on Liouville / Weinstein Sectors. ⑬

(Sylvan, GPS — generalizing $\left\{ \begin{array}{l} \text{(fully) wrapped Fukaya cat.} \\ \text{Fukaya-Seidel cat.} \end{array} \right.$)

• **Objects**: $\left\{ \begin{array}{l} \text{exact} \\ \text{Closed} \end{array} \right.$ Lagrangians with conical ends (if not compact) + extra data : $\partial_\infty L \cap \partial X = \phi$.

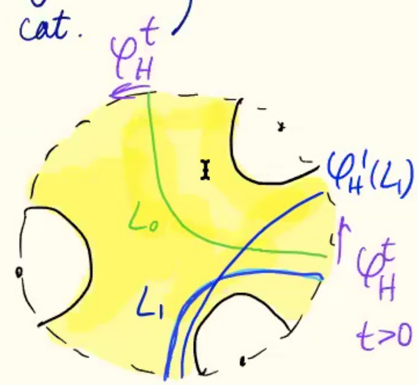


• **Morphisms**: Wrapped Floer complex.

$$W(L_0, L_1) = \varinjlim_{\text{"H: positive Hamiltonians"}} CF(\underbrace{\varphi_H^1(L_0)}_{\text{never touches } \partial X}, L_1)$$

- Partially wrapped Fukaya category on Liouville / Weinstein Sectors. (13)
  (Sylvan, GPS — generalizing { (fully) wrapped Fukaya cat. / Fukaya-Seidel cat. })

- Objects: { exact / Closed Lagrangians with conical ends (if not compact) + extra data: $\partial_\infty L \cap \partial X = \phi$.



$W(L_0, L_1) \simeq 0$

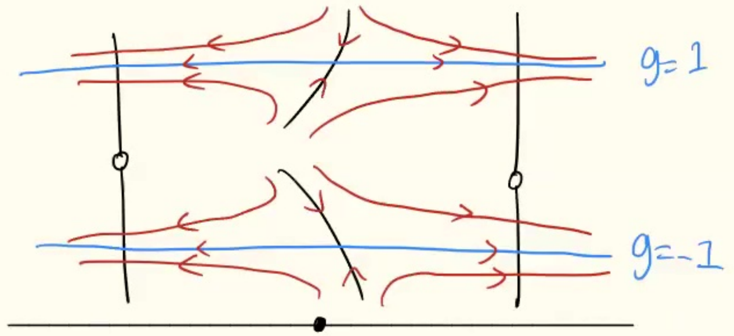
$W(L_1, L_0) \simeq \mathbb{K}$

- Morphisms: Wrapped Floer complex.

$$W(L_0, L_1) = \varinjlim_{\text{"H: positive Hamiltonians"}} CF(\varphi_H^1(L_0), L_1)$$

Ⅲ HMS for J_G.

- Example: $G = SL_2(\mathbb{C})$



$\mathfrak{c} = \mathfrak{t}/\!/W \cong \mathbb{A}^1$

14

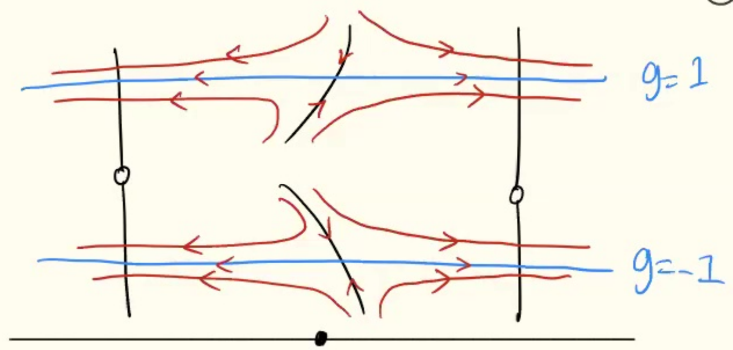
Page 45 of 64

Xin Jin

14

(III) HMS for J_G.

- Example: $G = SL_2(\mathbb{C})$



$\mathfrak{c} = \mathfrak{t}/\!/W \cong \mathbb{A}^1$

- Weinstein handle decomposition:
  - 0-handle (Morse-Bott)

$$J_G - \{g = \pm 1\} \cong T^*T$$

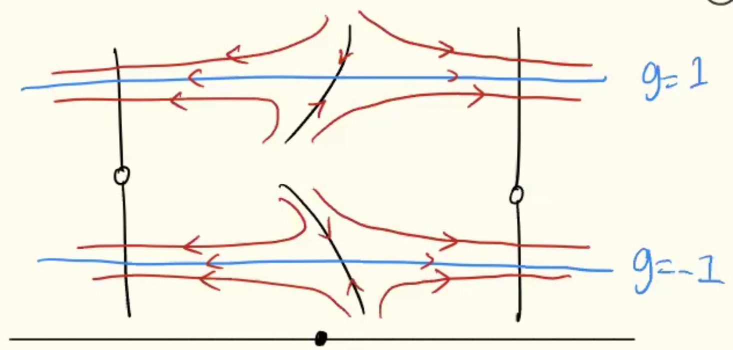
$$(b, x) \longmapsto \left(\overset{\substack{g \\ \shortparallel}}{\begin{bmatrix} 0 & -b^{-1} \\ b & 0 \end{bmatrix}}, \overset{\substack{\xi \\ \shortparallel}}{\begin{bmatrix} x & -b^{-2} \\ 1 & -x \end{bmatrix}} \right)$$

(In particular, zero-section in T^*T is a 2:1 cover of $\mathfrak{c} \setminus \{0\}$)

(14)

(III) HMS for J_G.

- Example: $G = SL_2(\mathbb{C})$



$\mathfrak{t}/\!/W \cong \mathbb{A}^1$

- Weinstein handle decomposition:
  - 0-handle (Morse-Bott)

$$J_G - \{g = \pm 1\} \cong T^*T$$

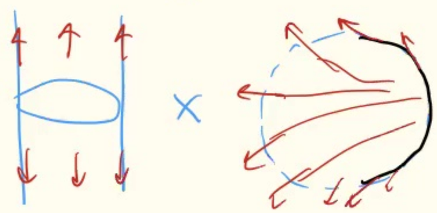
$$(b, x) \longmapsto \left(\underset{g}{\begin{bmatrix} 0 & -b^{-1} \\ b & 0 \end{bmatrix}}, \underset{\xi}{\begin{bmatrix} x & -b^{-2} \\ 1 & -x \end{bmatrix}} \right)$$

(In particular, zero-section in T^*T is a 2:1 cover of $\mathfrak{t}^{*,reg}/\!/W$.)

$T^*T \cong T^*S^1 \times \mathbb{R}^2$

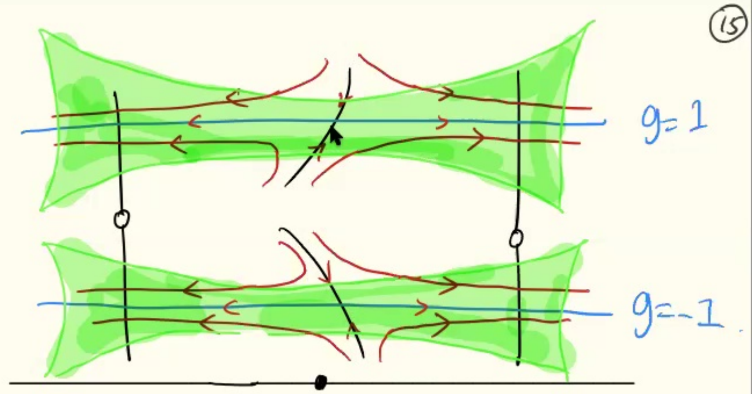
Core $= S^1 \times pt$

as a Weinstein sector



III HMS for J_G.

- Example: $G = SL_2(\mathbb{C})$

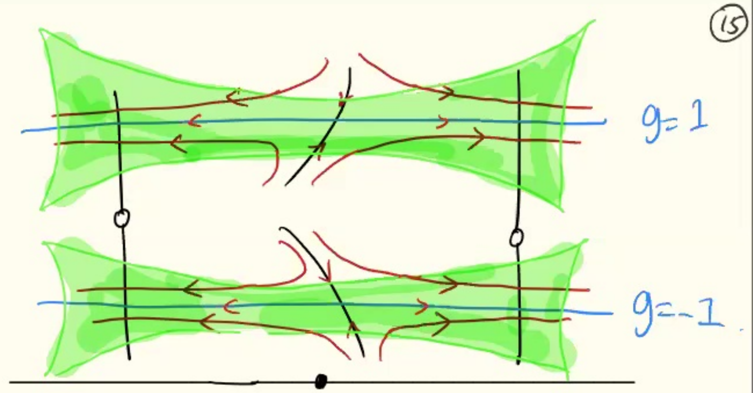


$\mathfrak{t}/\!/W \cong \mathbb{A}^1$

- Weinstein handle decomposition:
  - 2-handles: for each Kostant section (normal slice gives the core of the handle)

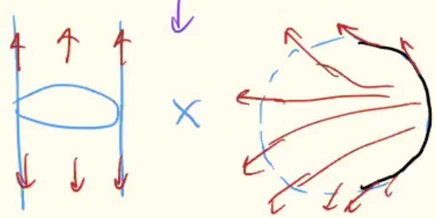
Ⅲ HMS for J_G.

- Example: $G = SL_2(\mathbb{C})$



- Weinstein handle decomposition: $\mathfrak{t}/\!/W \cong \mathbb{A}^1$
  - 2-handles: for each Kostant section
    (normal slice gives the core of the handle)
  - Attaching map: attached along 2 interesting Legendrian S^1

$T^*T \cong T^*S^1 \times \mathbb{R}^2$

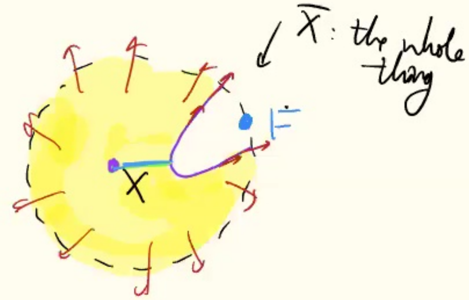


(16)

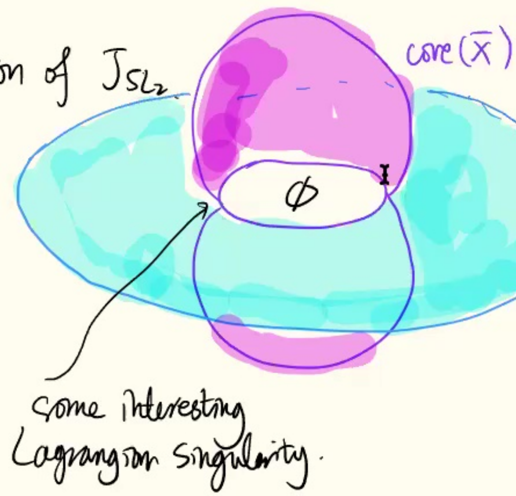
• Skeleton of J_{SL_2}

• Skeleton of a Weinstein sector.

$$= \mathrm{Core}(\bar{X}) \cup \mathrm{Cone}(\mathrm{Core}(F))$$



• Skeleton of J_{SL_2}

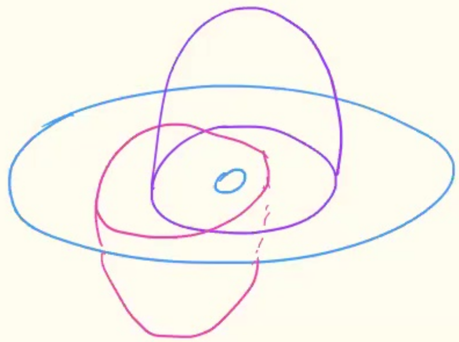


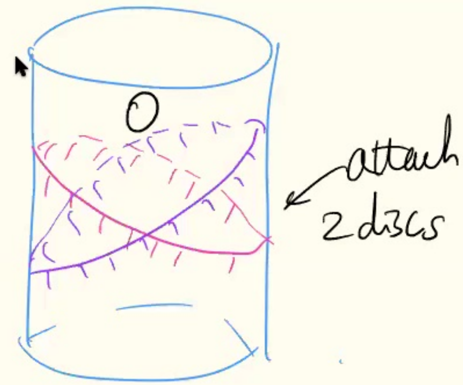
$F = T^*S^1$

• Optional: for people who are interested in Lagrangian singularities and sheaves.

(17)

Aborealize (Nadler) skeleton for J_{SL_2}

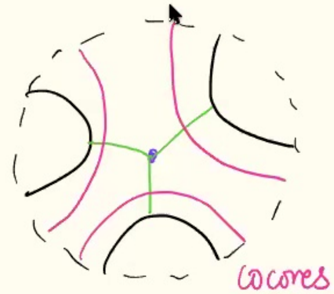




- WFuk (J_{SL_2}).

<u>Thm</u> (Chantraine-Rizell-Ghiggini-Golovko, GPS)

The cocores generate WFuk.



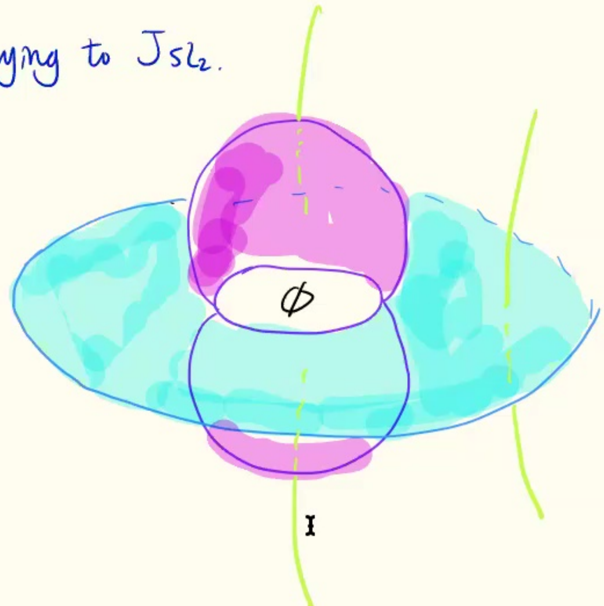
18

- WFuk (J_{sl_2}).

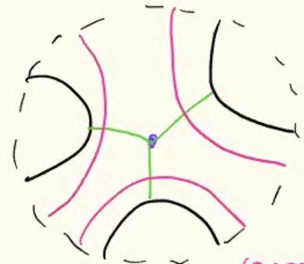
Thm (Chantraine-Rizell-Ghiggini-Golovko, GPS)

The cocores generate WFuk.

- Applying to J_{sl_2}.



cocores.



cocores "=" normal slices to Lag pieces.

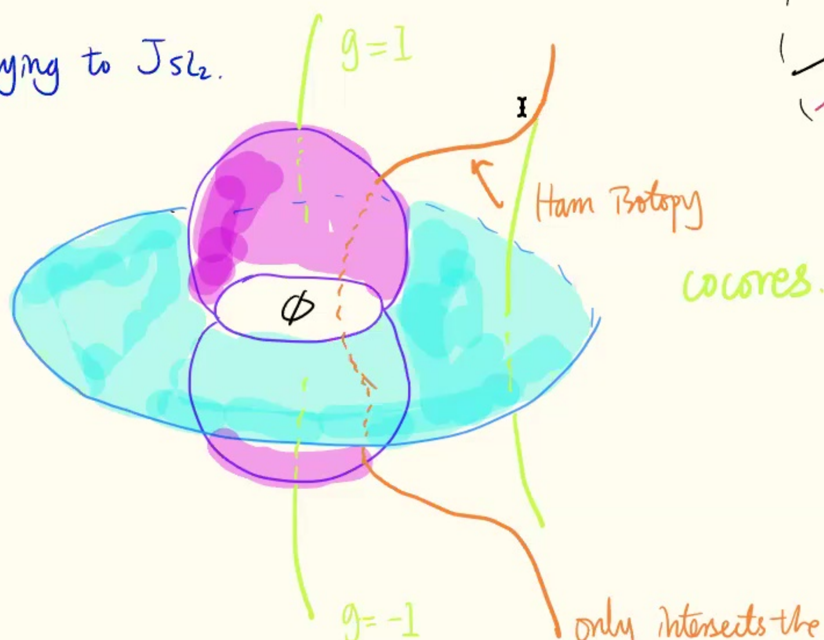
18

- $\mathrm{WFuk}(J_{sl_2})$.

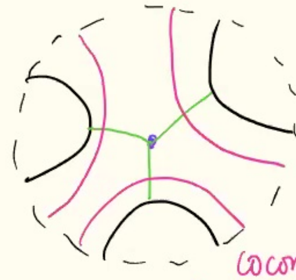
Thm (Chantraine-Rizell-Ghiggini-Golovko, GPS)

The cocores generate WFuk.

- Applying to J_{sl_2}.



only intersects the other 2 Lagrangian components $\Rightarrow$ it's generated by the other 2 cocores.



cocores "=" normal slices to Lag pieces.

19

- HMS for J_{SL_2}.

$$\mathcal{W}Fuk(J_{SL_2}) \simeq Coh\big((\check{T}_{sc}/\!/W)/(\mathbb{Z}/2\mathbb{Z})^*\big).$$

$$(\check{T}_{sc}/\!/W)/(\mathbb{Z}/2\mathbb{Z})^* = \mathbb{A}^1/(\mathbb{Z}/2\mathbb{Z})^*$$

$$\{g=1\} \longmapsto \Big(\mathcal{O} : \sigma^*\mathcal{O} \xrightarrow{\sim} \mathcal{O},\ f(-z) \mapsto f(z)\Big)$$

$$\{g=-1\} \longmapsto \Big(\mathcal{O} : \sigma^*\mathcal{O} \xrightarrow{\sim} \mathcal{O},\ f(-z) \mapsto -f(z)\Big)$$

• For general G.

① $-J_G$ admits a Bruhat decomposition that gives a Weinstein handle decomposition. (for Sl_2: $\underbrace{\{g = \pm 1\}}_{w=1}$, the complement $\underbrace{\left(\begin{bmatrix}0 & -b\\ b^{-1} & 0\end{bmatrix}\begin{bmatrix}x & -b^{2}\\ 1 & -x\end{bmatrix}\right)}_{w=w_0}$)

- For general G.

① – J_G admits a Bruhat decomposition that gives a Weinstein handle decomposition. (for SL_2: $\{\underset{w=1}{g=\pm 1}\}$, the complement $\underset{w=w_0}{\left(\begin{bmatrix}0 & -b\\ b^{-1} & 0\end{bmatrix}, \begin{bmatrix}x & -b^2\\ 1 & -x\end{bmatrix}\right)}$)

– J_G can be partially compactified to be a Weinstein sector (canonical up to isotopy).

– The handles are indexed by $\underset{w_0 w_S \in W}{S} \subseteq \overset{\text{simple roots}}{\Pi}$

② $\mathcal{W}Fuk(J_G)$ is generated by the Kostant sections

$$\{\text{Kostant sections}\} \longleftrightarrow Z(G) \longleftrightarrow \text{characters of } \pi_1(\check{G}) \longleftrightarrow \{\text{generators of } Coh((\check{T}_{sc}/\!/W)/\pi_1(\check{G}))\}$$

③ Calculation of wrapped Floer complexes

G_{ad}: one can choose nice Hamiltonians such that the intersection points are indexed by the dominant coweight lattice of T_{ad}.